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Effect of demand boosting policy on optimal inventory policy for imperfect lot size with backorder in fuzzy environment*

by

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Abstract: This paper investigates an Economic Order Quantity (EOQ) model with backorder by taking imprecise demand rate with dependence upon the frequency of advertisement. The formulated model also incorporates learning effects on percentage of defective items present in each lot. Due to imprecision in demand, the obtained profit function is fuzzy. To determine the optimal values, we determine the equivalent crisp profit function by applying the signed distance method. Optimal order quantity and backorder level are obtained by using algebraic method in place of differential calculus. A numerical example is used to study the behavior of the model with respect to different inventory parameters. All calculations are performed with MATLAB 7.4.

Keywords: learning curve, advertisement, signed distance, triangular fuzzy number, backorder, inventory model.

1. Introduction

In the classical economic production/order quantity models, the items produced/received are implicitly assumed to be of perfect quality. However, it may not always be the case. Due to imperfect production process, natural disasters, damage or breakage in transit, or for many other reasons, the lots considered

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may contain some defective items. To increase the applicability of the inventory models, several researchers studied the respective scenarios in formulating the production/inventory models and the effect of imperfect quality on lot sizing policy. Specifically, we note that in Rosenblatt and Lee's (1986) study, it was assumed that the defective items could be reworked instantaneously at a cost and found that the presence of defective products motivates smaller lot sizes. Salameh and Jaber (2000) assumed that the defective items could be sold as a single batch at a discounted price prior to receiving the next shipment, and found that the economic lot size tends to increase as the average percentage of imperfect quality items decreases. Goyal and Cádenas-Barrón (2002) reconsidered the task of Salameh and Jaber (2000) and presented a simple approach for determining the optimal lot size. Besides, a broader survey related to the studies of imperfect quality can be found in Salameh and Jaber (2000). Chang (2004) determined the optimal order lot size to maximize the total profit when the lot contains imperfect quality items. Papachristos and Konstantaras (2006) extended the work of Salameh and Jaber (2000), focusing on the timing of withdrawing the imperfect quality units from stock. Wee and Chung (2007) developed an optimal inventory model for items with imperfect quality and shortage backordering. Eroglu and Ozdemir (2007) studied the effect of share of defective items on optimal solution. Maddah and Jaber (2008) enhanced the results of Salameh and Jaber (2000) by applying the renewal theory to obtain the expected profit per unit time. The work of Salameh and Jaber (2000) was explored by Hsu and Yu (2009) in relation to quality issues.

The above literature survey reveals that most of the researchers assumed that the defective rate in lot sizes produced/received is a fixed constant, while others assumed it as a random variable with known probability distribution to predict the uncertainty of imperfect quantity. Everyone considers that characteristics of defective items remain the same in each production run/lot. This assumption does not fit the real life situations, and, in particular, the case of a new product, where the historical data are not available for establishing the probability density function. To make the study more realistic, we assume that the percentage of defective items per lot decreases with cumulative number of shipments conforming to a learning curve. This is not surprising; knowing that some studies reported that quality improves per lot because of learning, this is due to the acquaintance with the set-up, the tooling, instructions, blueprints, the workplace arrangement, and the condition of the process.

A look at the available literature on inventory reveals that several models have been formulated in a static environment, where the demand for the item under consideration was assumed to be constant, for the sake of simplicity. However, it is observed that in practical situations, constant demand can be justified only for the maturity phase of the product. There are many products, like clothes, fashion accessories, mobile phones etc., for which demand cannot be predicted accurately. In these types of inventory models, a major difficulty faced by a decision maker is to forecast the demand. It is not possible for decision maker to decide the exact demand in such complex and uncertain environment as highlighted by Guiffrida (1998). So, demand is imprecise by its very nature. To overcome this, we consider the demand rate as represented by the triangular fuzzy number.

Further, Salameh and Jaber (2000), Chang (2004), Huang (2004), Papachristos and Konstantaras (2006), Jaber et al. (2008) all discussed imperfect items present in a lot, but no one considered shortages. Shortages of an item may occur in stock. Over-production/holding is not a solution due to high inventory holding cost. As demand is known imprecisely, shortages cannot be ignored in inventory model. So, we consider backorder level as decision variable.

In the present competitive market situations, a product is promoted in the society through glamorous advertisements in the electronic media. Observing this phenomenon, many marketing researchers and practitioners are motivated to investigate its modeling aspects. Time-to-time advertisement (in such media as TV, radio, newspapers, magazines, etc., and through the sales representatives) of an item also changes the demand for that item. Therefore, we can conclude that there is a functional relationship between the demand for an item and the frequency/cost of the advertisement of that item. This type of demand was reported in the inventory models developed by Pal et al. (2006), Mondal et al. (2007). Few researchers studied the effects of the frequency of advertisement on the demand for an item. Among them, one may refer to the work of Urban (1992), Abad (2000), Bhunia and Maiti (1997), Goyal and Gunasekarn (1995), Luo (1998), Pal et al. (2004, 2006), Mondal et al. (2007) etc. By taking the motivation from the above work, we assume demand for an item to depend on the frequency of advertisement, meant to capture the untouched part of the demand.

The classical optimization technique, based on differential calculus undoubtedly is a powerful and useful method for solving inventory decision models. The methodology mentioned is used to find the optimal solutions or derive the conditions for optimality, but it is replaceable. Grubbström and Erdem (1999) derived the classical EOQ formula algebraically. Their result received considerable attention and encouraged many researchers to propose various algebraic methods to solve inventory related models with or without shortages (see, e.g., Cádenas-Barrón, 2000, 2001, 2008; Chung and Wee, 2007; Wee and Chung, 2007). Teng (2009) suggested the arithmetic-geometric mean (AM-GM) inequality theorem. Hsieh et al. (2008) provide Cauchy-Schwarz inequality and Wee et al. (2009) modified the cost-difference comparison method. In the recent studies Pentico et al. (2009) and Zhang (2009) also proposed new and/or alternative approaches to solve the EOQ model with partial backordering.

Based on the above arguments, this article incorporates the following points:

(1). Imprecision of demand is handled by triangular fuzzy numbers, with assumption that that it is proportional to the frequency of advertisement.

(2). The number of defective items in each lot follows the learning curve and the lots are continuously screened at the rate x and after the completion of the screening process, defective items are withdrawn from the lot prior to next shipment.

(3). Backorder level is considered as one of the decision variables.

Profit expression in fuzzy sense is defuzzified by using the signed distance method. We derive the optimal lot size and backorder level without using differential calculus.

This paper is organized as follows. In Section 2, some definitions and properties of fuzzy sets related to this study are introduced. We also introduce the learning curve and the arithmetic-geometric inequality in this section. In Section 3 notations and assumptions are given, used to develop the proposed model. Section 4 presents fuzzy model. Section 5 contains analysis conducted directly through the obtained expression. Section 6 provides the numerical illustration of the proposed model. Section 7 summarizes the work done in this paper and formulates future extension of this work.

2. Preliminaries

2.1. Some basic definitions and properties of fuzzy sets

DEFINITION 1 *For $0 \leqslant \alpha \leqslant 1$, a fuzzy set $\tilde{a}_\alpha$ defined on $R = (-\infty, \infty)$ is called an α-level fuzzy point if the membership function of $\tilde{a}_\alpha$ is given by*

$$\mu_{\tilde{a}_\alpha}(x) = \begin{cases} \alpha, & x = a \\ 0 & x \neq a \end{cases} . \tag{1}$$

DEFINITION 2 *The fuzzy set $\tilde{A} = (a, b, c)$, where $a<b<c$, defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by*

$$\mu_{\tilde{a}_\alpha}(x) = \begin{cases} (x-a)/(b-a), & a \leqslant x \leqslant b, \\ (c-x)/(c-b) & b \leqslant x \leqslant c, \\ 0, & otherwise. \end{cases} \tag{2}$$

DEFINITION 3 *For $0 \leqslant \alpha \leqslant 1$, the fuzzy set $[a_\alpha, b_\alpha]$, defined on R is called an α-level fuzzy interval if the membership function of $[a_\alpha, b_\alpha]$ is given by*

$$\mu_{[a_\alpha, b_\alpha]}(x) = \begin{cases} \alpha, & a \leqslant x \leqslant b \\ 0 & otherwise. \end{cases} \tag{3}$$

DEFINITION 4 *Let $\tilde{A}$be a fuzzy set on R, and $0 \leqslant \alpha \leqslant 1$. The α-cut $A(\alpha)$ of $\tilde{A}$consists of points x such that$\mu_{\tilde{A}}(x) \geqslant \alpha$, that is,*

$$A(\alpha) = \{x : \mu_{\tilde{A}}(x) \geqslant \alpha\}. \tag{4}$$

Decomposition Principle: Let $\tilde{A}$ be a fuzzy set on R and $0 \leqslant \alpha \leqslant 1$. Suppose the$\alpha$-cut of $\tilde{A}$ to be a closed interval $[A_L(\alpha), A_U(\alpha)]$, that is, $A(\alpha) =$

$[A_L(\alpha), A_U(\alpha)]$. Then, we have (see, e.g., Kaufman and Gupta, 1991)

$$\tilde{A} = \bigcup_{0\leqslant\alpha\leqslant 0} \alpha A(\alpha) \tag{5}$$

or $\mu_{\tilde{A}} = \bigcup_{0\leqslant\alpha\leqslant 0} \alpha C_{A(\alpha)}(x)$

where

(1) $\alpha A(\alpha)$ isafuzzy set with membership function $\mu_{\alpha A(\alpha)}(x)$ $= \begin{cases} \alpha, & x \in A(\alpha) \\ 0 & otherwise. \end{cases}$

(2) $C_{A(\alpha)}(x)$ is a characteristic function of $A(\alpha)$, that is, $C_{A(\alpha)}(x)$ $= \begin{cases} 1, & x \in A(\alpha) \\ 0 & otherwise. \end{cases}$

Remark 1 *From the decomposition principle and the relation* $\mu_{[a_\alpha, b_\alpha]}(x) = \begin{cases} \alpha, & a \leqslant x \leqslant b \\ 0 & otherwise. \end{cases}$ *, we obtain*

$$\tilde{A} = \bigcup_{0\leqslant\alpha\leqslant 0} \alpha A(\alpha) = \bigcup_{0\leqslant\alpha\leqslant 0} [A_L(\alpha), A_U(\alpha)] \tag{6}$$

or $\mu_{\tilde{A}}(x) = \bigcup_{0\leqslant\alpha\leqslant 0} \alpha C_{A(\alpha)}(x) = \bigcup_{0\leqslant\alpha\leqslant 1} \mu_{[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]}(x).$

For any a, b, c, d, k $\in$ R, $a<b$, and $c<d$, the interval operations are as follows:

(1). $[a, b] \oplus [c, d] = [a + c, b + d]$

(2). $[a, b]\, \Theta\, [c, d] = [a - d, b - c]$

(3).$k.\, [a, b] = \begin{cases} [ka, kb] & k > 0 \\ [kb, ka] & k < 0. \end{cases}$

(4). For $a > 0$ and $c > 0$, $[a, b] \otimes [c, d] = [ac, bd]$.

(5). For $a > 0$ and $c > 0$, $[a, b] \emptyset [c, d] = \left[\frac{a}{d}, \frac{b}{c}\right]$. (7)

Next, as in Yao and Wu (2000), we introduce the concept of signed distance of fuzzy set. We first consider the signed distance in R.

Definition 5 *For any a and 0 ∈ R, define the signed distance from a to 0 as* $d_0(a,0)= a$. *If* $a>0$, *the distance from a to 0 is* $a = d_0(a,0)$; *if* $a<0$, *the distance from a to 0 is* $-a = -d_0(a,0)$. *Hence,* $d_0(a,0)= a$ *is called the signed distance from a to 0.*

Let Ω *be the family of all fuzzy sets* $\tilde{A}$ *defined on R, for which the* α*-cut* $A(\alpha) = [A_L(\alpha), A_U(\alpha)]$ *exists for every* $\alpha \in [0,1]$ *and both* $A_L(\alpha)$ *and* $A_U(\alpha)$

are continuous functions on $\alpha \in$[0,1]. Then, for any $\tilde{A}\in \Omega$ from equation (6) we have

$$\tilde{A} = \bigcup_{0\leqslant\alpha\leqslant1} [A_L(\alpha), A_U(\alpha)]. \tag{8}$$

From Definition 5, the signed distance of two end points $A_L(\alpha)$ and $A_U(\alpha)$, of the α-cut $A(\alpha)$ =[$A_L(\alpha)$,$A_U(\alpha)$] of $\tilde{A}$ to the origin 0 is $d_0(A_L(\alpha),0)= A_L(\alpha)$ and $d_0(A_U(\alpha),0)= A_U(\alpha)$, respectively. Their average, [$A_L(\alpha)$+ $A_U(\alpha)$] /2, is taken as the signed distance of α-cut [$A_L(\alpha)$,$A_U(\alpha)$] to 0. That is, the signed distance of the interval [$A_L(\alpha)$,$A_U(\alpha)$] to 0 is defined as $d_0([A_L(\alpha),A_U(\alpha)],0)$ = [$d_0(A_L(\alpha),0)$+ $d_0(A_U(\alpha),0)$] /2 = [$A_L(\alpha)$+ $A_U(\alpha)$] /2.

In addition, for every $\alpha \in$[0,1], there is a one-to one mapping between the α-level fuzzy interval [$A_L(\alpha)_\alpha$,$A_U(\alpha)_\alpha$] and the real interval [$A_L(\alpha)$, $A_U(\alpha)$], that is, the following correspondence is one-to-one mapping:

$$[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha] \leftrightarrow [A_L(\alpha), A_U(\alpha)]. \tag{9}$$

Also, the 1-level fuzzy point $\tilde{0}_1$is mapping to the real number 0. Hence, the signed distance of [$A_L(\alpha)_\alpha$,$A_U(\alpha)_\alpha$] to $\tilde{0}_1$can be defined as d([$A_L(\alpha)_\alpha$,$A_U(\alpha)_\alpha$], $\tilde{0}_1$) = d_0([$A_L(\alpha)$, $A_U(\alpha)$],0) = ($A_L(\alpha)$+$A_U(\alpha)$)/2. Moreover, for $\tilde{A}\in \Omega$, since the above function is continuous on$0 \leqslant \alpha \leqslant 1$, we can use the integration to obtain the mean value of the signed distance as follows:

$$\int_0^1 \mathrm{d}([A_L(\alpha)_\alpha,A_U(\alpha)_\alpha],\tilde{0}_1)d\alpha = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_U(\alpha)]d\alpha. \tag{10}$$

Then, from equations (5) and (6), we have the following definition:

Definition 6 *For $\tilde{A}\in \Omega$, define the signed distance of $\tilde{A}$ to $\tilde{0}_1$(i.e., y-axis) as*

$$d(\tilde{A}, \tilde{0}_0) = \int_0^0 d([A_L(\alpha)_\alpha,A_U(\alpha)_\alpha], \tilde{0}_0)d\alpha = \frac{0}{2}\int_0^0 [A_L(\alpha) + A_U(\alpha)]d\alpha. \tag{11}$$

According to Definition 6, we obtain the following property.

Property 1 *For the triangular fuzzy number $\tilde{A}$= (a,b,c), the α-cut of $\tilde{A}$ is $A(\alpha)$ = [$A_L(\alpha)$, $A_U(\alpha)$], $\alpha \in$[0,1], where $A_L(\alpha)$ = a + (b-a)α and $A_U(\alpha)$ = c-(c-b)α. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is*

$$d(\tilde{A}, \tilde{0}_1) = (a + 2b + c)/4. \tag{12}$$

Furthermore, for two fuzzy sets $\tilde{A}, \tilde{G} \in \Omega$, where $\tilde{A} = \bigcup_{0\leqslant\alpha\leqslant1} [A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]$ and $\tilde{G} = \bigcup_{0\leqslant\alpha\leqslant1} [G_L(\alpha)_\alpha, G_U(\alpha)_\alpha]$ and $k \in R$, using equations (7) and (9), we have

$$(1).\tilde{A}\oplus\tilde{G}=\bigcup_{0\leqslant\alpha\leqslant1}[(A_L(\alpha)+G_L(\alpha))_\alpha,(A_U(\alpha)+G_U(\alpha))_\alpha].$$

$$(2).\tilde{A}\Theta\tilde{G}=\tilde{A}\bigcup_{0\leqslant\alpha\leqslant1}[(A_L(\alpha)-G_U(\alpha))_\alpha,(A_U(\alpha)-G_L(\alpha))_\alpha].$$

$$(3).\tilde{k}\odot\tilde{A}=\begin{cases}\bigcup_{0\leqslant\alpha\leqslant1}[(kA_U(\alpha))_\alpha,(kA_L(\alpha))_\epsilon] & k<0\\ \bigcup_{0\leqslant\alpha\leqslant1}[(kA_L(\alpha))_\alpha,(kA_U(\alpha))_\epsilon]. & k>0\\ & \tilde{0}_1k=0\end{cases} \tag{13}$$

From the above and Definition 6, we obtain the following property:

Property 2 *For two fuzzy sets $\tilde{A}$, $\tilde{G}\in\Omega$ and $k\in R$,*

$$(1).\ d(\tilde{A}\oplus\tilde{G},\tilde{0}_1)=d(\tilde{A},\tilde{0}_1)+d(\tilde{G},\tilde{0}_1).$$

$$(2).\ d(\tilde{A}\Theta\tilde{G},\tilde{0}_1)=d(\tilde{A},\tilde{0}_1)-d(\tilde{G},\tilde{0}_1).$$

$$(3).\ d(\tilde{k}\odot\tilde{A},\tilde{0}_1)=kd(\tilde{A},\tilde{0}_1). \tag{14}$$

2.2. Learning curve

The earliest learning curve representation is a geometric progression that expresses the decreasing cost required to accomplish any repetitive operation. This theory in its most popular form states that as the total quantity of units produced doubles, the cost per unit declines by some constant percentage (e.g., Yelle, 1979; Jaber, 2006).

The learning curve (e.g., power versus exponential) has been debated by several authors; see Jaber (2006) for a discussion. There is almost unanimous agreement among practitioners and academicians that the learning curve is best described by a power as suggested by Wright (1936). It is worth noting that the learning curve in practice is an 'S'-shaped curve (Jordan, 1958; Carlson, 1973), as shown in Fig.1. The first phase (incipient) is the phase during which the worker is getting acquainted with the set-up, the tooling, instruction, blueprints, the workplace arrangement, and the conditions of the process. In this phase improvement is slow. The second phase (learning) is where most of the improvement, e.g., reduction in errors, changes in the distance moved takes place. The third and last phase (maturity) represents the learning of the curve. The S-shaped logistic learning curve is of the form

$$p(n)=\frac{a}{f+e^{gn}}, \tag{15}$$

where a, g, and $f>0$ are the model parameters, n is the cumulative number of shipments, and $p(n)$ is the percentage defective item per shipment.

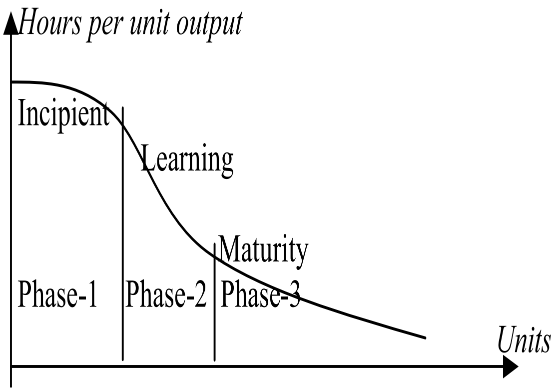

Figure 1. Three phases of the learning curve

2.3. Arithmetic-geometric inequality

Let a_1, a_2, ...,a_n be n positive real numbers, then

$$\frac{\sum_{k=1}^{n} a_k}{n} \geqslant \sqrt[n]{\prod_{k=1}^{n} a_k} \text{ with equality iff } a_1 = a_2 = \ldots\ldots = a_n.$$

3. Notations and assumptions

The following notations and assumptions are used to develop the model.

3.1. Notations

Notations as given in the subsequent table are used in the formulation of the model.

3.2. Assumptions

To develop the mathematical model presented in this study, the following assumptions are made:

1. The replenishment is instantaneous.
2. The screening process and demand proceed simultaneously, but the screening rate is greater than demand rate, i.e., $x>D$.
3. The defective items exit and percentage of defective items present in each lot follows a learning curve.
4. 100% screening of items is done in each shipment.
5. Defective items are sold at a discounted price at the end.
6. Shortages are completely backordered.

$\tilde{D}$	N^{γ} d where N is frequency of advertisement (γ>0) and d is scale parameter which is triangular fuzzy number
y_n	Order quantity of units for the n^{th} shipment, where n⩾1
c	Unit purchasing cost
K	Fixed ordering cost per order
h	Holding cost per unit per unit of time
s	Unit selling price per good quality unit
v	Unit discounted price per defective unit, $v < c$
T_n	Cycle time for shipment per order
x	Screening rate measured in units per unit of time, where $x > D$
S_c	Unit screening cost
t_n	Time to screen y_n unit, where $t_n = y_n/x < T_n$
B	The maximum backordering quantity in units
b	Backordering cost per unit
G	Cost of advertisement
N	Frequency of advertisement
t_{1n}	Time where inventory level reaches to zero
t_{2n}	Duration of backorder

7. Items of poor quality are kept in stock and sold prior to receiving the next shipment as a single batch.
8. A single product is considered.
9. The backordered products will be delivered without any defect.

4. Mathematical model

The behavior of the inventory level is illustrated in Fig. 2. It is assumed that a lot of size y_n is replenished instantaneously at the beginning of each cycle, and contains good as well as poor quality of items. From the beginning of the cycle, inspection process starts and it is assumed that inspection rate is higher than the demand rate; t_n is time when screening process is completed and at that moment, items of poor quality are sorted, kept in stock and sold at a salvage value prior to receiving the next shipment. At t_{1_n}, inventory level reaches zero and after that shortages occur up to t_{2_n}. T_n is total replenishment cycle duration. The optimum operating inventory strategy is obtained by trading off the total revenues per unit of time so as to derive an optimal solution.

Now, $TR(y_n, B)$ is the total revenue, which is the sum of total sale volume of good quality and imperfect quality items and is given by

$$TR(y_n, B) = (1 - p(n))y_n s + p(n)y_n v. \tag{16}$$

$TC(y_n, B)$ is the sum of ordering cost, purchasing cost, screening cost, holding cost, backordering cost and advertisement cost:

$$TC(y_n, B) = K + cy_n + S_c y_n + h\{(y_n - p(n)y_n - B)^2/2dN^{\gamma} \\ +p(n)y_n^2/x\} + bB^2/2dN^{\gamma} + NG. \tag{17}$$

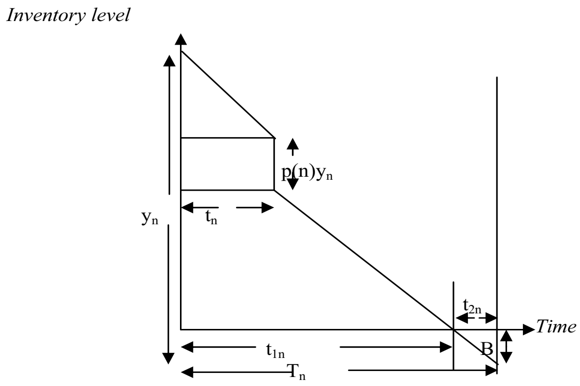

Figure 2. Inventory system with backordering

The net profit per unit time, $TP(y_n, B)$, is determined by the revenue per unit time, $TR(y_n, B)/T_n$, less the average cost per unit time $TC(y_n, B)/\mathrm{T}_n$. The equation can be formulated as

$$\text{Net Profit} = \frac{TR(y_n, B) - TC(y_n, B)}{T_n}. \tag{18}$$

Since the replenishment cycle length T_n=$(1 - p(n))y_n/dN^\gamma$, so

$$TP(y_n, B) = \frac{dN^\gamma}{(1-p(n))y_n}\Bigg[(1-p(n))y_n s + p(n)y_n v - K - cy_n - S_c y_n$$
$$-h\left\{\frac{(y_n - p(n)y_n - B)^2}{2dN^\gamma} + \frac{p(n)y_n^2}{x}\right\} - \frac{bB^2}{2dN^\gamma - NG}\Bigg].$$

By rearranging the terms, we get

$$TP(y_n, B) = dN^\gamma\left[s + \frac{p(n)v}{1-p(n)} - \frac{K}{(1-p(n))y_n} - \frac{c}{1-p(n)} - \frac{S_c}{1-p(n)}\right.$$
$$\left.-\frac{hp(n)y_n}{(1-p(n))x} - \frac{NG}{(1-p(n))y_n}\right] - \frac{h}{2}\frac{(y_n - p(n)y_n - B)^2}{(1-p(n))y_n} - \frac{bB^2}{2(1-p(n))y_n}. \tag{19}$$

As mentioned earlier, although the statistical method can be used to deal with the uncertainty, however, in some cases, it may lack historical data to estimate the probability distribution function. Since the demand rate may change due to high complexities of the market from one lot to another, it can be described in fuzzy terms as "$\tilde{D}$ =demand rate is about D". In this study it is assumed that demand rate is a fuzzy variable.

4.1. Mathematical model with fuzzy annual demand

In this subsection, we modify the crisp demand rate model shown in equation (9) by incorporating fuzziness. The crisp annual demand D in equation (9) is fuzzified as the triangular fuzzy number, $\tilde{D} = N^{\gamma}(d-\Delta_1, d, d-\Delta_2)$, where Δ_1 and Δ_2 are determined by the decision-makers and should satisfy the conditions $0<\Delta_1<$d, $0<\Delta_2$. For this, we express fuzzy total profit per unit time as

$$T\tilde{P}(y_n, B) = \tilde{D}\left[s + \frac{p(n)v}{1-p(n)} - \frac{K}{(1-p(n))y}_n - \frac{c}{1-p(n)} - \frac{S_c}{1-p(n)} - \frac{hp(n)y}{}_n(1-p(n))x - \frac{NG}{(1-p(n))y}_n\right] - \frac{h}{2}\frac{(y_n - p(n)y_n - B)^2}{(1-p(n))y}_n - \frac{bB^2}{2(1-p(n))y_n}. \quad (20)$$

Now, we defuzzify $T\tilde{P}(y_n, B)$ by using the signed distance method. From Property 2, the signed distance of $T\tilde{P}(y_n, B)$ to $\tilde{0}_1$ is given by

$$d\left(T\tilde{P}(y_n, B), \tilde{0}_1\right) = d\left(\tilde{D}, \tilde{0}_1\right)\left[s + \frac{p(n)v}{1-p(n)} - \frac{K}{(1-p(n))y_n} - \frac{c}{1-p(n)} - \frac{S_c}{1-p(n)} - \frac{hp(n)y_n}{(1-p(n))x} - \frac{NG}{(1-p(n))y_n}\right] - \frac{h}{2}\frac{(y_n - p(n)y_n - B)^2}{(1-p(n))y_n} - \frac{bB^2}{2(1-p(n))y_n} \quad (21)$$

where $d\left(\tilde{D}, \tilde{0}_1\right)$ are measured as follows. From Property 1, the signed distance of fuzzy number $\tilde{D}$ to $\tilde{0}_1$ is

$$d\left(\tilde{D}, \tilde{0}_1\right) = \frac{N^{\gamma}}{4}\left[(d-\Delta_1) + 2d + (d-\Delta_2)\right] = dN^{\gamma} + \frac{N^{\gamma}}{4}(\Delta_2 - \Delta_1). \quad (22)$$

On substituting result of equation (22) in equation (21), we have

$$TP^*(y_n, B) = \left(dN^{\gamma} + \frac{N^{\gamma}}{4}(\Delta_2 - \Delta_1)\right)\left[s + \frac{p(n)v}{1-p(n)} - \frac{K}{(1-p(n))y_n} - \frac{c}{1-p(n)} - \frac{S_c}{1-p(n)} - \frac{hp(n)y_n}{(1-p(n))x} - \frac{NG}{(1-p(n))y_n}\right] - \frac{h}{2}\frac{(y_n - p(n)y_n - B)^2}{(1-p(n))y_n} - \frac{bB^2}{2(1-p(n))y_n}.$$

$TP^*(y_n, B)$is regarded as the estimate of total profit per unit time in the crisp sense. Rearranging the terms of above equation yields

$$TP^*(y_n, B) = dN^\gamma s + \frac{p(n)vdN^\gamma}{1-p(n)}$$
$$+\frac{(\Delta_2-\Delta_1)sN^\gamma}{4} + \frac{(\Delta_2-\Delta_1)p(n)vN^\gamma}{4(1-p(n))}$$
$$-\frac{cdN^\gamma}{1-p(n)} - \frac{S_c dN^\gamma}{1-p(n)} - \frac{c(\Delta_2-\Delta_1)N^\gamma}{4(1-p(n))} - \frac{S_c(\Delta_2-\Delta_1)dN^\gamma}{4(1-p(n))}$$
$$-\left\{\frac{KdN^\gamma}{(1-p(n))y_n} + \frac{hp(n)y_n dN^\gamma}{(1-p(n))x} + \frac{N^{\gamma+1}Gd}{(1-p(n))y_n}\right.$$
$$+\frac{KN^\gamma(\Delta_2-\Delta_1)}{4(1-p(n))y_n} + \frac{(\Delta_2-\Delta_1)hp(n)y_nN^\gamma}{4x(1-p(n))} +$$
$$\left.\frac{(\Delta_2-\Delta_1)GN^{\gamma+1}}{4(1-p(n))y_n} + \frac{h}{2}\frac{(y_n - p(n)y_n - B)^2}{(1-p(n))y_n} + \frac{bB^2}{2(1-p(n))y_n}\right\}. \tag{23}$$

Since the first eight terms in equation (23) are free of the decision variable (y_n, B), they can be dropped in determining (y_n^*, B^*). Consequently, maximizing $TP^*(y_n, B)$ is equivalent to minimizing the cost terms in brace (we denote it using $f(y_n, B)$). Thus

$$f(y_n, B) = y_n\left[\frac{hp(n)dN^\gamma}{(1-p(n))x} + \frac{hp(n)(\Delta_2-\Delta_1)N^\gamma}{4x(1-p(n))} + \frac{h}{2}(1-p(n))\right]$$
$$+\frac{1}{y_n}\left[\frac{KdN^\gamma}{(1-p(n))} + \frac{GdN^{\gamma+1}}{(1-p(n))} + \frac{K(\Delta_2-\Delta_1)N^\gamma}{4(1-p(n))} + \frac{G(\Delta_2-\Delta_1)N^{\gamma+1}}{4(1-p(n))}\right.$$
$$\left.+\frac{h}{2}\frac{B^2}{(1-p(n))} + \frac{bB^2}{2(1-p(n))}\right] - hB.$$

Now, we apply the method given by Chang et al. (1998) to obtain the exact closed form solution for optimal (y_n^*, B^*) and minimum cost $f(y_n^*, B^*)$ without derivatives. The above expression can be rearranged as follows

$$f(y_n, B) = \frac{(h+b)}{2(1-p(n))y_n}\left[B - \frac{h(1-p(n))y_n}{(h+b)}\right]^2 + y_n\left[\frac{hp(n)dN^\gamma}{(1-p(n))x}\right.$$
$$\left.+\frac{hp(n)(\Delta_2-\Delta_1)N^\gamma}{4x(1-p(n))} + \frac{h}{2}(1-p(n)) - \frac{h^2}{2}\frac{(1-p(n))}{(h+b)}\right]$$
$$+\frac{1}{y_n}\left[\frac{KdN^\gamma}{(1-p(n))} + \frac{GdN^{\gamma+1}}{(1-p(n))} + \frac{K(\Delta_2-\Delta_1)N^\gamma}{4(1-p(n))} + \frac{N^{\gamma+1}G(\Delta_2-\Delta_1)}{4(1-p(n))}\right]. \tag{24}$$

Thus, for given y_n, setting the square term in equation (24) to zero results in

$$B^* = \frac{h(1-p(n))y_n}{(h+b)} \tag{25}$$

and equation (24) reduces to

$$f(y_n) = \frac{h}{2}\left[\frac{2p(n)dN^{\gamma}}{(1-p(n))x} + \frac{p(n)(\Delta_2-\Delta_1)N^{\gamma}}{2x(1-p(n))} + (1-p(n)) - \frac{h(1-p(n))}{(h+b)}\right] y_n + \frac{1}{y_n}\left[\frac{KdN^{\gamma}}{(1-p(n))} + \frac{dGN^{\gamma+1}}{(1-p(n))} + \frac{K(\Delta_2-\Delta_1)N^{\gamma}}{4(1-p(n))} + \frac{N^{\gamma+1}G(\Delta_2-\Delta_1)}{4(1-p(n))}\right]. \tag{26}$$

Next, from Teng (2009), using the arithmetic-geometric mean inequality (AM-GM) theorem,

$$f(y_n) \geqslant \sqrt{2h\left[\frac{KdN^{\gamma}}{(1-p(n))} + \frac{GdN^{\gamma+1}}{(1-p(n))} + \frac{K(\Delta_2-\Delta_1)N^{\gamma}}{4(1-p(n))} + \frac{N^{\gamma+1}G(\Delta_2-\Delta_1)}{4(1-p(n))}\right]\left[\frac{2p(n)dN^{\gamma}}{(1-p(n))x} + \frac{p(n)(\Delta_2-\Delta_1)N^{\gamma}}{2x(1-p(n))} + (1-p(n)) - \frac{h(1-p(n))}{(h+b)}\right]}. \tag{27}$$

When two terms related to y_n in equation (27) are equal, then

$$y_n^* = \sqrt{\frac{2\left[\frac{KdN^{\gamma}}{(1-p(n))} + \frac{GdN^{\gamma+1}}{(1-p(n))} + \frac{k(\Delta_2-\Delta_1)N^{\gamma}}{4(1-p(n))} + \frac{N^{\gamma+1}G(\Delta_2-\Delta_1)}{4(1-p(n))}\right]}{h\left[\frac{2p(n)dN^{\gamma}}{(1-p(n))x} + \frac{p(n)(\Delta_2-\Delta_1)N^{\gamma}}{2x(1-p(n))} + (1-p(n)) - \frac{h(1-p(n))}{(h+b)}\right]}}. \tag{28}$$

Once y_n^* is obtained, B^* follows from equation (25), and equation (26) reduces to equality, i.e., the minimum cost is

$$f(y_n) = \sqrt{2h\left[\frac{KdN^{\gamma}}{(1-p(n))} + \frac{GdN^{\gamma+1}}{(1-p(n))} + \frac{K(\Delta_2-\Delta_1)N^{\gamma}}{4(1-p(n))} + \frac{N^{\gamma+1}G(\Delta_2-\Delta_1)}{4(1-p(n))}\right]\left[\frac{2p(n)dN^{\gamma}}{(1-p(n))x} + \frac{p(n)(\Delta_2-\Delta_1)N^{\gamma}}{2x(1-p(n))} + (1-p(n)) - \frac{h(1-p(n))}{(h+b)}\right]}. \tag{29}$$

Using equations (25) and (28), we can obtain the net estimate of total profit, $NETP^*$:

$$NETP^* = N^{\gamma}\left(d + \frac{(\Delta_2-\Delta_1)}{4}\right)\left(s + \frac{p(n)}{1-p(n)}v - \frac{c}{1-p(n)} - \frac{S_c}{1-p(n)}\right) - \sqrt{2h\left(\frac{(KN^{\gamma}+GN^{\gamma+1})(4d+\Delta_2-\Delta_1)}{4(1-p(n))}\right)\left[\frac{p(n)N^{\gamma}(4d+\Delta_2-\Delta_1)}{2x(1-p(n))} + \frac{b(1-p(n))}{(h+b)}\right]}. \tag{30}$$

5. Analysis

Once we obtained the exact closed-form solutions, the effects of problem parameters on the optimal solutions can be easily analyzed.

1. From equation (30), if there is no salvage value for defective items, i.e., v=0, to make sure that the policy will generate a positive net profit i.e., $NETP^* \geqslant 0$, the selling price per unit should be set higher than the threshold

$$s > \left[\frac{(c+S_c)}{1-p(n)} - \frac{1}{N^\gamma \left(d + \frac{\Delta_2 - \Delta_1}{4}\right)} \cdot \sqrt{2h \left(\frac{(KN^\gamma + GN^{\gamma+1})(4d+\Delta_2-\Delta_1)}{4(1-p(n))} \right) \left(\frac{p(n)N^\gamma(4d+\Delta_2-\Delta_1)}{2x(1-p(n))} + \frac{b(1-p(n))}{(h+b)} \right)} \right. . \tag{31}$$

2. As screening rate x increases, y^* increases (so does B^*), f^* decreases and hence $NETP^*$ increases.

6. Numerical example

To illustrate the results of the proposed models, we consider an inventory system with the following data: screening cost Sc=\$0.5/unit, purchase cost c=\$25/unit, selling price of good-quality items s=\$50/unit, selling price of imperfect-quality items v=\$20/unit, G=10,000. We assume p is deterministic and follows the learning curve of the form described in equation (15). The values of the parameters in equation (15) are: a=40, f=999, d=50,000 units/year, ordering cost K=\$100/cycle, holding cost h=\$5/unit/year, screening rate x= 175200 units/year.

Example 1 *For the model proposed in Section 4, with D fuzzified as the triangular fuzzy number $\tilde{D} = N^\gamma(d-\Delta_1,\, d,\, d-\Delta_2)$, we solve to obtain the optimal order lot size, backorder level and the maximum total profit per year in the fuzzy sense for various given sets of g and (Δ_1, Δ_2) that satisfy the conditions $0<\Delta_1<D$ and $0<\Delta_2$. We compare the solutions of fuzzy case with those of crisp case by calculating the relative variation between them. The results are summarized in Table 3. We also compare profits when demand is crisp and fuzzy.*

Observations

From Table1, which shows the impact of the boosting factor and the number of advertisement on optimal solution, we observe that

(1) as the boosting factor i.e., γ increases, the saturation level of frequency of advertisement is reached faster; if γ =0.02 then profit increases up to the 3^{rd} frequency of advertisement and after that it decreases; similarly for γ =0.04, profit increases up to the 6^{th} frequency of advertisement and after that it decreases;

(2) as the boosting factor increases, the respective change in profit is very high.

So, we can say that if the manager properly selects the frequency and mode of advertisement then company shall grow fast. It all depends on the efficiency

Table 1. Profit w.r.t different values of γ

N	NETP*		
	γ=0.02	γ=0.03	γ=0.04
1	1184000	1184000	1184000
2	1188500	1196910	1205370
3	**1188910**	1202320	1215880
4	1187930	1204920	1222150
5	1186330	1206100	1226200
6	1184390	**1206450**	1228910
7	1182280	1206260	1230730
8	1180060	1205720	1231920
9	1177780	1204910	1232660
10	1175470	1203930	1233050
11	1173150	1202800	1233180
12	1170830	1201560	**1233080**
13	1168520	1200250	1232820
14	1166220	1198870	1232410
15	1163940	1197440	1231890

Table 2. Optimal values of order and backorder quantities w.r.t different

b	y^*	B^*	NETP*
0	13243.7	12714.5	1225630
2	3743.14	2566.85	1223630
4	3044.94	1624.04	1222990
6	2762.13	1205.35	1222630
8	2606.91	962.59	1222410
10	2511.39	803.684	1235100
12	2443.08	689.842	1234990
14	2392.91	604.554	1234900
16	2354.49	538.195	1234830
18	2324.11	485.054	1234770
20	2299.48	441.521	1234720
∞	2060.44	000.000	1221340

of manager's selection of mode of advertisement and on the analysis as to which method is more effective in capturing the untouched demand of the market.

Table 2 shows the effect of backorder cost. We observe that as the backordering cost increases, the optimal order size, backorder level and net total profit gradually decrease when we move across the table from top to bottom.

Rel $NETP$'=[$NETP^*$-$NETP_c$]$NETP_c$ $\times$ 100%.

In Table 3 we analyze the effects of the learning factor and fuzzy demand on optimal solution. We get the following observations from that table:

(1) for fixed value of learning rate i.e., g, as the tolerance level of annual demand increases, the optimal lot size, backorder level as well as profit increase;

(2) Table 3 shows the variation in lot size, backorder level and profit with the variation in demand; there is very negligible change in lot size, backordering level and profit due to high change in Δ_1 and Δ_2; this shows that our model

Table 3. Effect of learning and imprecision in demand of optimal values y and B

g	Δ_1	Δ_2	y^*	B^*	TP^*	Rel(*NETP`*)
g =0.0	500	350	2504.52	801.44	1205530	-0.318
		500	2505.43	801.73	1209380	0.000
		650	2506.33	802.02	1213240	0.319
	1000	750	2503.92	801.25	1202960	-0.530
		1000	2505.43	801.73	1209380	0.000
		1250	2506.93	802.21	1215810	0.531
	1500	1000	2502.40	800.76	1196530	-1.062
		1500	2505.43	801.73	1209380	0.000
		2000	2508.44	802.70	1222240	1.063
g =0.25	500	350	2504.38	801.48	1205560	-0.318
		500	2505.29	801.77	1209410	0.000
		650	2506.19	802.06	1213270	0.319
	1000	750	2503.78	801.29	1202990	-0.530
		1000	2505.29	801.77	1209410	0.000
		1250	2506.80	802.25	1215840	0.531
	1500	1000	2502.27	800.80	1196560	-1.062
		1500	2505.29	801.77	1209410	0.000
		2000	2508.30	802.74	1222270	1.063
g =0.50	500	350	2503.90	801.61	1205660	-0.319
		500	2504.81	801.90	1209520	0.000
		650	2505.71	802.19	1213370	0.318
	1000	750	2503.30	801.42	1203090	-0.531
		1000	2504.81	801.90	1209520	0.000
		1250	2506.32	802.39	1215940	0.530
	1500	1000	2501.79	800.94	1196670	-1.062
		1500	2504.81	801.90	1209520	0.000
		2000	2507.83	802.87	1222370	1.062
g =0.75	500	350	2502.30	802.06	1206010	-0.318
		500	2503.20	802.35	1209860	0.000
		650	2504.11	802.64	1213710	0.318
	1000	750	2501.69	801.87	1203440	-0.530
		1000	2503.20	802.35	1209860	0.000
		1250	2504.71	802.83	1216280	0.530
	1500	1000	2500.18	801.38	1197020	-1.061
		1500	2503.20	802.35	1209860	0.000
		2000	2506.22	803.32	1222700	1.061

Rel *NETP`*= $[NETP^*\text{-}NETP_c]/NETP_c \times 100\%$

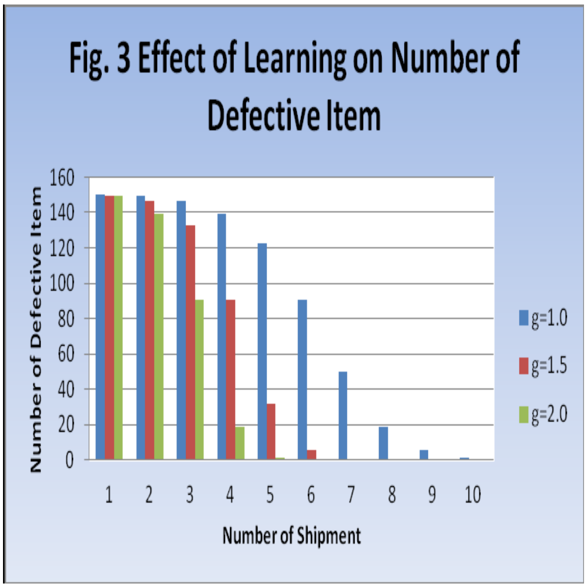

Figure 3. Effect of learning on number of defective item

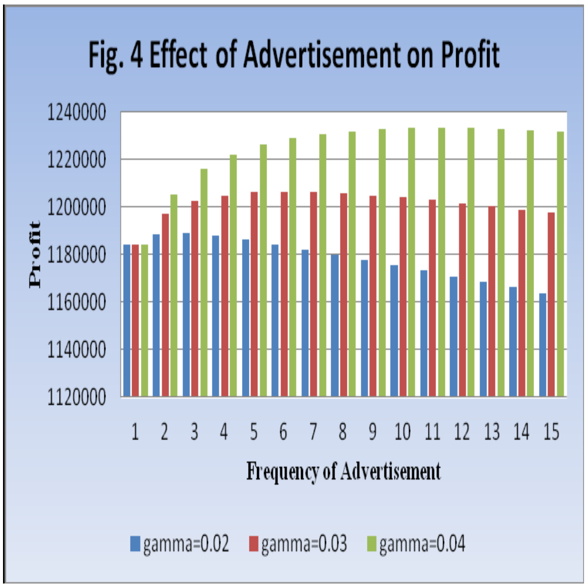

Figure 4. Effect of advertisement on profit

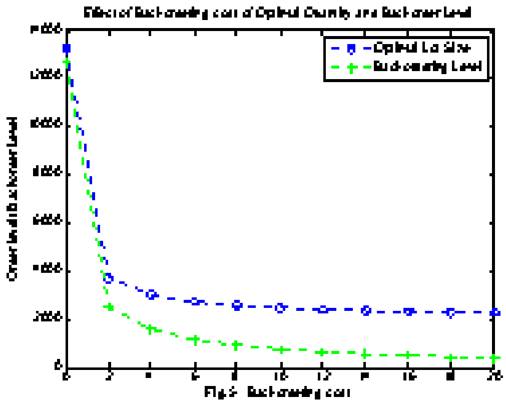

Figure 5. Backordering cost

is stable, the optimal solution not being significantly affected by the change of demand.

Table 4. Optimal values of the order and backorder quantities w.r.t. the value of n

	g =0.2			g =0.50			g =0.75		
n	y^*	B^*	NETP*	y^*	B^*	NETP*	y^*	B^*	NETP*
1	2505.41	801.740	1209390	2505.36	801.746	1209390	2505.36	801.360	1209400
2	2505.40	801.744	1209390	2505.33	801.763	1209400	2505.23	801.790	1209430
3	2505.38	801.749	1209390	2505.23	801.790	1209430	2504.96	801.960	1209490
4	2505.36	801.755	1209400	2505.07	801.835	1209460	2504.38	802.027	1209610
5	2505.33	801.763	1209400	2504.81	801.908	1209520	2503.20	802.355	1209860
6	2505.30	801.772	1209410	2504.38	802.027	1209610	2500.87	803.003	1210360
7	2505.26	801.784	1209420	2503.69	802.220	1209760	2496.53	84.195	1211290
8	2505.21	801.797	1209430	2502.59	802.526	1209990	2489.40	806.134	1212790
9	2505.15	801.814	1209440	2500.87	803.003	1210360	2479.71	808.727	1214810
10	2505.07	801.865	1209460	2498.26	803.722	1210920	2469.45	811.419	1216920

From Table 4 it can be observed that as ‘g’ increases, the profit of the organization increases; saturation level of profit is reached faster as ‘g’ increases; so, it clear that if a manager learns effectively from the past, company will grow faster; if ‘g’ is considered as a proxy for attitude and managerial capacity of a manager, then we observe that a higher level of ‘g’ helps in a faster growth of the company.

From Fig. 3 it can be observed that the number of defective item present in each lot gradually decreases as learning effect increases.

7. Conclusions

This article addressed fuzzy model for an inventory problem for single item with shortages backordered. Each lot contains imperfect-quality items. We investigated the effect of the learning factor and a boosting factor, such as advertisement, meant to promote demand, on the lot size per shipment, backordering level and net profit. Annual demand is considered as triangular fuzzy number. For the fuzzy model, a method of defuzzification, namely signed distance, is employed to find the estimate of net profit per unit time. Without using the method of differential calculus, we obtained globally optimal lot size and backorder levels by applying arithmetic-geometric mean inequality theorem, which is easy to apply and simple to understand.

We examined the effects of problem parameters on the optimal policies analytically and numerically. The result shows that the number of defective units and the shipment size decrease, whereas backordering level and net profit increase as learning increases following the form of the logistic curve. As learning becomes faster it is recommended to order in smaller lots less frequently. As we increase frequency of advertisement, before saturation level net total profit increases, after that it decreases gradually. By increasing the backordering cost, the shipment size and backorder level decrease whereas profit increases.

Finally, we would like to point out that most of researchers studying the fuzzy production/ inventory problems (e.g. Chang et al., 1998; Lee and Yao, 1998, 2000; Yao et al., 2000) often employed the centroid method to obtain the estimate of total cost in the fuzzy sense. To achieve this, the membership function of fuzzy total cost has to be found first using the Extension Principle, while this derivation is very complex. To avoid this problem we use signed distance method for defuzzification in this study.

A possible future research issue is to study the impact of stochastic learning curve. Another interesting factor would be to consider a multi-item EOQ model.

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