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# Finding an optimum dynamic spatial structure of information systems

by

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In the present paper questions connected with the formulation of problems of finding optimum dynamic spatial structures of information systems are discussed in the form of integer programming problems. Possibilities of linearization of a nonlinear problem are indicated and the computational algorithm ARO for reduction of the original set of linear constraints is given.

## 1. Introduction

We shall call an information system (SI) [1], a complex of technical equipment together with connecting lines and operating personnel designed for the purposes of receiving, storing, processing, transmitting, diffusion and delivery of information.

Radio-television broadcasting systems, postal and telecommunicational systems, systems of elaboration and diffusion of scientific and technical information, systems of computer centers, etc. may be regarded as examples of the SI.

The spatial structure of systems of this type is, as a rule, clearly determined, that is the allocation of devices and links between them are given.

We shall introduce a certain classification of spatial structures of the SI.

We shall call a spatial structure static if the allocation of its stations and links between them do not change within the time interval under consideration.

If we consider a developing information system in which, for example, new stations and links are built, then we say that its spatial structure is dynamic.

Among all dynamic structures we distinguish those that are subject to rapid changes (reorganization, displacement) due to changes in the environment from which information is collected or to which information is delivered. These we shall call nonstationary spatial structures.

Information systems of nonstationary spatial structures will be the subject of the present paper. Radar detection system the spatial structure of which should be adjusted to rapid changes of a site and a shape of a sounded area will serve as an example.

In the recent ten years much attention has been given to problems connected with the synthesis of optimal static spatial structures of the SI. Different aspects of these problems with reference to communication systems and particularly to telephony systems have been considered in [2—6]. In [7] the problem of finding a cost-optimal number of links between the fixed nodes of a telephony network in presence of some reliability requirements has been formulated as an integer programming problems (with a linear goal function and linear constraints). Reference [8] is a trial of a comprehensive study of a question of complex computer systems synthesis taking into account their spatial structure. In [9] and [10] a problem of optimum locations of radar stations has been presented. In the former, the minimization of the highest probability of not detecting a space object has been taken as a criterion and an algorithm based on successive approximations has been applied to obtain a solution. In the latter, the problem has been reduced to a linear integer programming problem and one of the Gomory's algorithms has been used for its solution.

Much less attention has been given so far to problems concerned with dynamic structures. In this area reference [11] can be mentioned, in which dynamic programming methods are used in optimum planning of developing power supply networks. In [12] a problem of optimum planning of nonstationary spatial structure of a radar system has been formulated in outline. Generally speaking, the problem has been brought to a nonlinear integer programming problem.

This paper is thought to be a continuation and extention of [12]. In section 2, apart from repeating the basic hypotheses of the problem, two criterions of optimal selection of spatial structure are presented. Special cases that lead to irregularities appearing as terms of "if ... then..." type are discussed. In section 3, the ways of eliminating the irregularities in a cost function and the ways of linearization of both a cost function and constraints are given. An ARO algorithm that allows for the computer reduction of a large set of constraints obtained either directly for the problem or as a result of problems linearization is described in section 4. In the conclusion the possibilities of effective solution of the problem under consideration are briefly discussed.

## 2. Mathematical model of a detection system with nonstationary spatial structure

### 2.1. General remarks

Let us consider a sector $Q$ of a threedimensional Euclidean space $R^3$ ($Q \subset R^3$) shape and position (related to predetermined coordinates) of which change in time $t \in [0, T)$ in the following way:

$$Q_r = \text{const. for } t_{r-1} \leqslant t < t_r, \quad r = 1, 2, \ldots, s,$$

$$Q=\bigcup_{r=1}^{r=s} Q_r, \quad t_0=0, \quad t_s=T.$$

Assume that for each $r$ a certain partition of $Q_r$ is made, that is each $Q_r$ is devided into a finite number $m_r$ of exhaustive and not intersecting one another subregions. One of the possible ways of discretization of $Q_r$ has been given in [12]. We shall assume for convenience, that the subregions are numbered by a variable $i\in I=\bigcup_{r=1}^{s} I^r\{1, 2, ..., m\}$, where $I^r=\{m_{r-1}+1, m_{r-1}+2, ..., m_r\}$, $r=1, 2, ..., s$, $m_0=$ $=0$, $m_s=m$.

The detecting devices that are at our disposal, e.g. radars are numbered by variable $u\in U=\{1, 2, ..., w\}$. These devices may be devided into groups according to their basic technical parameters, for instance, according to the operating frequency of a transmitter-receiver system. In these cases we shall agree that the group $g$ is a subset of numbers $U^g=\{w_{g-1}+1, w_{g-1}+2, ..., w_g\}$; $g=1, 2, ..., h$, $w_0=0$, $w_h=w$.

It is assumed in the following, that each group of devices is characterized by two parameters: sounding zone, i.e. the set of points belonging to the region $Q$ in which an object is detected with the desired probability, and an average velocity $v_g$ of transporting the devices belonging to the group $g$.

The devices can be installed in places that are chosen from a certain predetermined, finite set $J=\{1, 2, ..., j, ..., n\}$.

Numbers of stations attainable at stage $r$ will be the elements of the set $J^r=$ $=\{n_{r-1}+1, n_{r-1}+2, ..., n_r\}$, $r=1, 2, ..., s$, $n_0=0$, $n_s=n$. Detection networks designer now faces a problem of locating devices at stations (taking account of a limited number of devices that cause a necessity for reiterated use of some of them) so as to optimize a predetermined criterion provided that at each stage $r$ certain requirements are satisfied by the network designed. Such a criterion may be, for instance, maximum probability of detecting on object within a region $Q$, minimum number of devices necessary for task accomplishment or minimum cost of an enterprise provided that appropriate cost coefficients are introduced.

On the basis of the above assumptions potential possibilities of a network can be described by means of two matrices called detection matrix and transportation matrix respectively, and defined as follows:

$$A^g=\|a^g_{ij}\|,\ i\in I,\ j\in J, \quad g=1, 2, ..., h,$$

$$T^g=\|t^g_{j_1 j_2}\|,\ j_1 j_2\in J,\ j_1\neq j_2, \quad g=1, 2, ..., h,$$

where

$$a^g_{ij}=\begin{cases}1, & \text{when subregion } i \text{ is covered by a detection zone of a device belonging} \\ & \text{to group } g \text{ and placed at station } j,\\ 0, & \text{otherwise}\end{cases}$$

and $t^g_{j_1 j_2}$ is a transportation time necessary for a device belonging to group $g$ to be transported from station $j_1$ to station $j_2$ (hypotheses made further on this work allow for consideration of cases in which $j_1\in J^{r_1}$, $j_2\in J^{r_2}$, $r_2>r_1$ only), between the stations considered. It may also comprise times of assembly and disassembly of a device at a station.

We shall introduce a decision variable $x_{ju}$ defined as follows:

$$x_{ju} \overset{\text{def}}{=} \begin{cases} 1, & \text{when a device } u \text{ is placed at station } j \\ 0, & \text{otherwise.} \end{cases}$$

The performance criterion and constraints representing demands for the network are in general composite function of the decision variable.

### 2.2. Minimum number of devices criterion

It should be taken into account in formulation of this criterion, that the same device may be used several times in the planned structure. Having this in mind we can write the minimum number of devices criterion in the form:

$$\sum_{u=1}^{u=w} f^u(z_u) \to \min,$$

where

$$f^u(z_u) = \begin{cases} 1, & \text{when } z_u = \sum\limits_{j \in J} x_{ju} \geqslant 1 \\ 0, & \text{when } z_u = \sum\limits_{j \in J} x_{ju} = 0 \end{cases}$$

and $u = 1, 2, \ldots, w$.

### 2.3. Minimum cost criterion

To formulate this criterion we shall introduce two kinds of cost. The first one is connected with the preparation of a station for installing a device (e.g. earthworks, building operations, etc.) and we shall assume that it takes the following form:

$$c_j(\eta_j) = \begin{cases} c_j \eta_j + d_j, & \text{when } \eta_j = \sum\limits_{u \in U} x_{ju} \geqslant 1 \\ 0, & \text{when } \eta_j = 0 \end{cases}$$

where $c_j$, $d_j$ – constants, $j = 1, 2, \ldots, n$.

A case when some stations are attainable at more than one stage should be considered separately. In the system of numbering used here such a station will have different numbers at different stages. In this case the cost of preparation of a station should be calculated as follows:

$$c_e(\eta_e) = \begin{cases} c_e \eta_e + d_e, & \text{when } \eta_e = \sum\limits_{j \in J^e} \sum\limits_{u \in U} x_{ju} \geqslant 1 \\ 0, & \text{when } \eta_e = 0, \end{cases}$$

where $J^e$, $e = 1, 2, \ldots, v$ is a set that consists of numbers of the station at all stages at which it is attainable and $c_e$, $d_e$ and constants.

The second kind of cost is connected with a change of stations by devices and it can include cost of transportations well as costs of assembly and disassembly. This cost is expressed by the coefficient $c_{j_1 j_2 u}$, $u = 1, 2, \ldots, w$; $(j_1, j_2) \in J$, $(c_{jju} = 0)$. As follows from the definition of the decision variable, the cost $c_{j_1 j_2 u}$ will be calculated whenever $x_{j_1 u} x_{j_2 u} = 1$. It can be seen in Fig. 1 that the above equality does not imply an actual change of stations by the device $u$. To avoid the incorrect cost calculation we shall introduce a function

$$c_{j_1 j_2 u}(x_{j_1 j_2 u}) = \begin{cases} c_{j_1 j_2 u}, \text{ when } x_{j_1 j_2 u} = \sum_{r=r_1+1}^{r=r_2-1} \sum_{j \in J^r} x_{ju} = 0 \\ 0, \text{ when } x_{j_1 j_2 u} \geqslant 1 \end{cases}$$

where $r_1, r_2 = 1, 2, \ldots, s,\ r_2 > r_1,\ j_1 \in J^{r_1},\ j_2 \in J^{r_2},\ u = 1, 2, \ldots, w.$

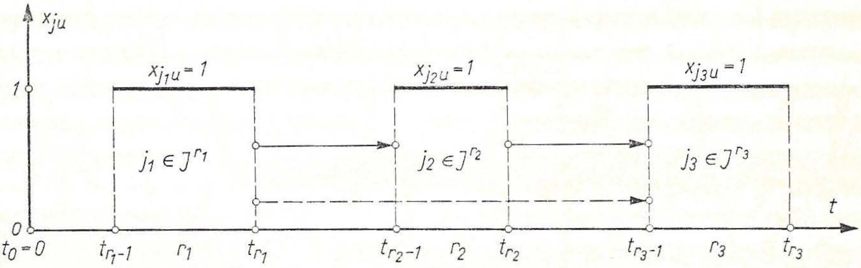


Fig. 1. Non-uniqueness of station change operation

Taking into account the above remarks we shall obtain an expression for a total cost of a network's spatial structure design that is to be minimized

$$\sum_{j \in \left(J \setminus \bigcup_{e=1}^{e=v} J^e\right)} c_j(\eta_j) + \sum_{e=1}^{e=v} c_e(\eta_e) + \sum_{j_1, j_2 \in J} \sum_{u=1}^{w} c_{j_1 j_2 u}(x_{j_1 j_2 u})\, x_{j_1 u}\, x_{j_2 u} \to \min.$$

### 2.4. Example of constraints

A detailed discussion of different conditions that any planned network's spatial structure should satisfy has been given in [12]. Those conditions have been expressed in a form of sets of nonlinear and linear constraints. The former comprise constraints that represent some detection demands being imposed at every point of a region $Q$. The following system of inequalities, requiring every subregion $Q_i$ of $Q$ to be observed by at least $\alpha$ devices each belonging to a different group, may be a typical example of this kind of constraints

$$\sum_{\mu=1}^{\binom{h}{\alpha}} \prod_{g \in \Omega_\mu} \sum_{j \in J^r} \sum_{u \in U^g} a_{ij}^g\, x_{ju} \geqslant 1$$

where $r = 1, 2, \ldots, s$, $i = 1, 2, \ldots, m_r$, and $\Omega_\mu$ is a set composed of $\alpha$ different numbers of groups of devices ($\alpha \leqslant h$) and $a_{ij}^g$ are the elements of the matrix $A^g$.

The second group comprises constraints that guarantee fulfillment of station change conditions resulting from the matrix $T^g$ and conditions due to a limited number of devices. The inequalities representing station change conditions are of the form

$$x_{j_1 u} + x_{j_2 u} \leqslant 1$$

where $j_1 \in J^r$, $j_2 \in J^{(r+k)}$, $r=1, 2, \ldots, s$, $u=1, 2, \ldots, w$, $k=1, 2, \ldots,$, $K$ and $K$ is the least natural number that satisfies the inequality

$$t_{r+k-1} - t_r \geqslant t_{j_1 j_2}.$$

The inequalities due to a limited number of devices are of the form

$$\sum_{j \in J^r} \sum_{u \in U^g} x_{ju} \leqslant w_g$$

where $r=1, 2, \ldots, s$, $g=1, 2, \ldots, h$.

The inequalities

$$\sum_{j \in J^r} x_{ju} \leqslant 1$$

where $r=1, 2, \ldots, s$, $u=1, 2, \ldots, w$, express the evident fact that at ane stage any device can be placed at the most at one station.

Inequalities of the form

$$\sum_{u \in U^g} x_{ju} \leqslant \beta_j^g, \qquad \sum_{u \in U} x_{ju} \leqslant \gamma_j$$

where $j \in J^r$, $r=1, 2, \ldots, s$, $g=1, 2, \ldots, h$, limiting the number of devices allowed at one station at the same time can be included in this group as well.

## 3. Some methods of changing a form of programming problems

There exists a number of possibilities of transforming integer and particularly binary programming problems into other equivalent problems that can be solved by various available algorithms. Some of these methods will be presented in this section. It is easy to find that transformation of a problem into a simpler form, for instance by means of linearization, leads to a considerable increase in number of variables and constraints.

### 3.1. Elimination of conditional terms from a cost function

We shall obtain a linear form of the expression for the minimum number of devices criterion, described in section 2.2 by introducing one auxiliary binary variable and two constraints for each function $f^u(z_u)$

$$y_u \in \{0, 1\}$$

$$\sum_{j \in J} x_{ju} - y_u \geqslant 0$$

$$-\sum_{j \in J} x_{ju} + n y_u \geqslant 0, \qquad u=1, 2, \ldots, w.$$

It can be easily seen that the auxiliary variable $y_u=0$ if and only if $z_u=0$ and $y_u=1$ if and only if $z_u \geqslant 1$. Making use of this remarks we shall write the minimum number of devices criterion in the following linear form

$$\sum_{u=1}^{w} y_u \to \min.$$

In the case of the minimum cost criterion, terms connected with preparation of a station are of a form similar to one of those appearing in fixed charges problems. The "noncontinuity" appearing in the point $\eta_j$ (or $\eta_e)=0$ is removed in the known and commonly used way (auxiliary binary variables $\zeta_j$, $\zeta_e$ and additional constraints). Conditions appearing in the function expressing cost of station change can be eliminated by introducing one binary variable $y_{j_1 j_2}$ subject to two constraints for each pair $j_1 j_2$ and each $u$.

$$y_{j_1 j_2 u} \in \{0, 1\}$$

$$\sum_{r=r_1+1}^{r^e-1} \sum_{j \in J^r} x_{ju} + y_{j_1 j_2 u} \geqslant 1$$

$$\sum_{r=r_1+1}^{r_2-1} \sum_{j \in J^r} x_{ju} + (r_2-r_1-1)\, y_{j_1 j_2 u} \leqslant (r_2-r_1-1)$$

where $r_1, r_2 = 1, 2, \ldots, s$, $r_2 > r_1$, $j_1 \in J^{r_1}$, $j_2 \in J^{r_2}$, $u=1, 2, \ldots, w$.

It can be easily found that

$$\left[x_{j_1 j_2 u} = \sum_{r=r_1+1}^{r_2-1} \sum_{j \in J^r} x_{ju} = 0\right] \Leftrightarrow y_{j_1 j_2 u} = 0$$

and

$$(x_{j_1 j_2 u} \geqslant 1) \Leftrightarrow y_{j_1 j_2 u} = 1.$$

With the use of the auxiliary variables the minimum cost criterion takes the form

$$\sum_{j \in \left(J \setminus \bigcup_{e=1}^{v} J^e\right)} (c_j \eta_j + d_j \zeta_j) + \sum_{e=1}^{v} (c_e \eta_e + d_e \zeta_e) + \sum_{j_1, j_2 \in J} \sum_{u=1}^{w} c_{j_1 j_2 u}\, y_{j_1 j_2 u}\, x_{j_1 u}\, x_{j_2 u} \to \min.$$

In this case we have obtained a third order polynomial.

### 3.2. Cost function and constraint linearization

If in the binary programming problem considered above a cost function and constraints are of the form of polynomials of order greater than one, then the problem can be reduced to a linear one. An equivalent linear problem is obtained if each term of form $\prod_{j \in \Theta} x_j$, where $\Theta \subseteq \{1, 2, \ldots, n\}$ and $k$ is the number of elements of $\Theta$, is replaced by a new binary variable $x_\Theta$ and two constraints

$$\sum_{j \in \Theta} x_j - x_\Theta \leqslant k-1, - \sum_{j \in \Theta} x_j + k x_\Theta \leqslant 0.$$

It is evident that

$$\left[(x_\Theta=0)\Leftrightarrow\left(\prod_{j\in\Theta} x_j=0\right)\right],$$

$$\left[(x_\Theta=1)\Leftrightarrow\left(\prod_{j\in\Theta} x_j=1\right)\right].$$

If a form of constraints is simpler, for instance such as that of observation constraints presented in section 2.4 then a somewhat different method of linearization can be applied. Any $i$-th constraint will be written in a form of a sum of conjunctions for the sake of simplicity where one subscript of a decision variable is used

$$\sum_{k=1}^{k=p_i} \prod_{j\in\Theta_{k_i}} x_j \geqslant 1,\ i=1, 2, \ldots, m.$$

The above inequality can be replaced by equivalent set of inequalities $A\ \boldsymbol{x}\geqslant 1$, where $A=\|a_{ij}\|_{m'\times n}$ and $\boldsymbol{x}$ is $n$-dimensional column vector $a_{ij}=0$ or $1$, $m'\geqslant m$.

Let the $i$-th inequality be regarded as a logical expression which is said to have value one when the inequality is satisfied and value zero otherwise and let algebraic sum and product be replaced by disjunction and conjunction respectively.

The logical expression thus obtained can be then transformed into a canonical normal form by multiple application of the following known tautology

$$p\vee(q\wedge r)\equiv(p\vee q)\wedge(p\vee r).$$

An expression in a canonical normal form is true if and only if its every elementary alternative is true. This means that every inequality corresponding to each of these alternatives must be satisfied and this leads to the set of constraints mentioned above that have a form occuring in the well known covering problems. For example inequality

$$x_1\,x_2+x_1\,x_3+x_2\,x_3+x_2\,x_5+x_3\,x_4+x_3\,x_5+x_1\,x_4\,x_5+x_2\,x_4\,x_6\geqslant 1$$

is equivalent to the following set of linear inequalities

$$x_1+x_2+x_3\geqslant 1$$

$$x_2+x_3+x_4\geqslant 1$$

$$x_2+x_3+x_5\geqslant 1$$

$$x_1+x_3+x_4+x_5\geqslant 1$$

$$x_1+x_3+x_5+x_6\geqslant 1$$

$$x_1+x_2+x_4+x_5\geqslant 1.$$

Those inequalities that correspond to alternatives absorbed by other expressions in the sense of Boolean algebra have been left out.

## 4. Linear constraints reduction algorithm

It has been shown in section 2.4 that in order to satisfy the predetermined detection conditions one nonlinear inequality is required to be satisfied for each subregion $Q_i$. The total number of these inequalities is $\sum_{r=1}^{s} m_r$ and takes the values of several hundred up to several thousand according to a method of discretization. This number is increased considerably as a result of transformation of nonlinear into linear form of inequalities as shown in the previous section. It often occurs in practice that the set of constraints can be considerably reduced by removing those inequalities that are absorbed by others. For the purpose of computer reduction of a set of inequalities a simple algorithm is proposed below.

Assume that the inequalities are of the form $\boldsymbol{Ax} \geqslant \boldsymbol{b}$ with additional assumption $b_i > 0$ for $i = 1, 2, \ldots, m$. The coefficients in the $i$-th inequality will be treated as binary $n$-element sequences $A_i = \{a_{i_1}, a_{i_2}, \ldots, a_{i_n}\}$. A set of all sequences corresponding to the original system of inequalities will be denoted by $A$, $A = \{A_1, A_2, \ldots, A_n\}$. We shall introduce the definition of inclusion of sequences belonging to the set $A$.

$$A_{i_1} \subseteq A_{i_2} \overset{\text{def}}{\Leftrightarrow} [(A_{i_1} \dot{-} A_{i_2}) \wedge A_{i_1} = \bar{0}]$$

where $\dot{-}$ is a symbol of symmetrical difference and $\bar{0}$ is a sequence composed of zeros.

Example 1.

$$\begin{aligned} A_1 &= 10010 \\ A_2 &= 11010 \\ A_1 \dot{-} A_2 &= 10010 \\ &\phantom{=} \underline{11010} \\ &\phantom{=} 01000 \\[1ex] (A_1 \dot{-} A_2) \wedge A_1 &= 01000 \\ &\phantom{=} \underline{10010} \\ &\phantom{=} 00000 \end{aligned}$$

We have obtained a sequence composed of zeros, hence $A_2$ included $A_1$.

Example 2.

$$\begin{aligned} A_1 &= 11110 \\ A_2 &= 11010 \\ A_1 \dot{-} A_2 &= 11110 \\ &\phantom{=} \underline{11010} \\ &\phantom{=} 00100 \\[1ex] (A_1 \dot{-} A_2) \wedge A_1 &= 00100 \\ &\phantom{=} \underline{11110} \\ &\phantom{=} 00100 \end{aligned}$$

This time we have obtained a sequence that is not a zero sequence, hence $A_2$ does not include $A_1$.

Let us denote $A^s$ a sequence that is a logical sum of sequences for which $\sum_{j=1}^{n} a_{ij}=b$, that is sequences in which the sum of ones on the left hand side is equal to the value of the right hand side.

$$A^s=\bigcup_{i\in I_1} A_i, \text{ where } I_1=\left\{i \,\middle|\, \sum_{j=1}^{n} a_{ij}=b\right\}.$$

The matrix of the reduced set of constraints should contain those elements of $A$ only that do not include one another and are not included by $A^s$. It is evident that all sequences formed by $A^s$ should belong to the reduced set. We shall denote $W$ the matrix of the reduced set and $W$ the corresponding set of sequences. On the basis of what has been said above

$$W\stackrel{\text{def}}{=}\left\{A_i \,\middle|\, \bigwedge_{\substack{i,k\in I\\ i\neq k}} [(A_i \dot{-} A_k)\wedge A_i]\wedge[(A_i \dot{-} A^s)\wedge A_i]\neq\bar{0}\right\}.$$

The reduced set of inequalities is uniquely determined by matrix $W$ and the two will be identified.

At the stage of analysis of potential possibilities of a detection network when $b$ is not known it may be worth-while to reduce rows of matrix $A$ itself. For this part of a region for which homogeneous detection conditions (i.e. $b_i=\text{const}$ for $i=1, 2, \ldots, m_r$) are assumed the definition of the reduced set $W'$ is the following

$$W'\stackrel{\text{def}}{=}\left\{A_i \,\middle|\, \bigwedge_{\substack{i,k\in I\\ i\neq k}} [(A_i \dot{-} A_k)\wedge A_i]\neq\bar{0}\right\}.$$

An ARO algorithm gives the possibility of obtaining sets $W'$ and $W$ (which is equivalents to obtaining the matrice $W'$ and the system $Wx\geqslant b$). A flow diagram of the algorithm is shown in Fig. 2. The algorithm has been programmed in the ODRA-LJAPAS language for the computer ODRA 1204. The results of matrix $W$ computation are presented in the table.

A sphere of problems connected with the optimal selection of a nonstationary spatial structure of information systems has been presented.

The basic practical difficulty consists in finding effective algorithms for solution of a resulting binary programming problem (0—1). A wide survey of integer programming methods and algorithms covering problems discussed here is given in [13]. Among algorithms that are not presented in [13] but can be used for our purposes we mention pseudo-boolean programming algorithms that are described in [14] and [15].

Works on programming some variants of pseudo-boolean algorithms for ODRA 1204 computer are being carried out.

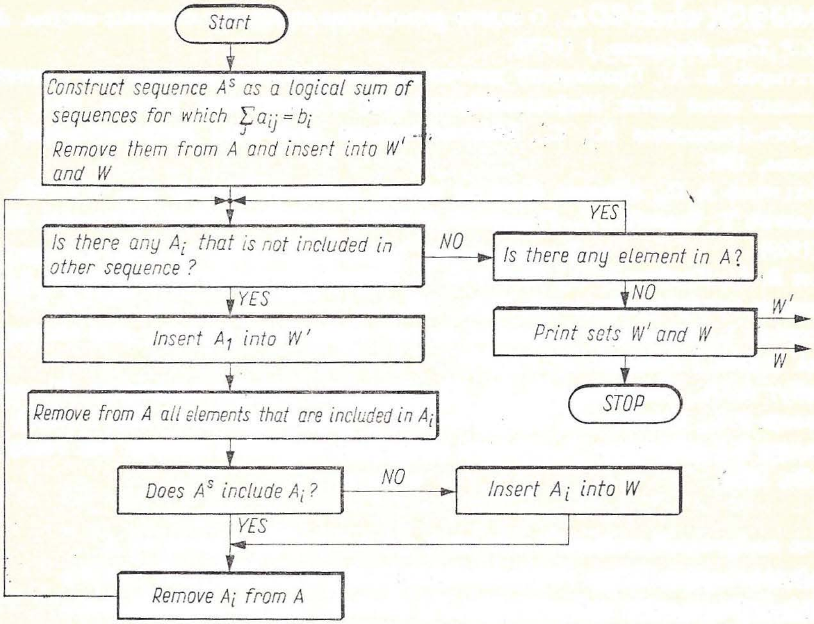


Fig. 2. Flow-diagram of the ARO algorithm

| Number of example | Size of example $m \times n$ | | Computation time computer ODRA 1204, language ODRA-LJAPAS | Comments |
|---|---|---|---|---|
| | Before reduction | After reduction | | |
| 1 | $(225 \times 10)$ | $(9 \times 10)$ | 10 sec. | |
| 2 | $(573 \times 48)$ | $(33 \times 48)$ | — | 2.5 min. ZAM-41 SAS |
| 3 | $(900 \times 40)$ | $(31 \times 40)$ | 8 min. | |
| 4 | $(1064 \times 52)$ | $(60 \times 52)$ | 10 min. | |

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## O zadaniu optymalnego wyboru dynamicznej struktury przestrzennej systemów informacyjnych

Omówiono problemy związane ze sformułowaniem zagadnienia optymalnego wyboru dynamicznych struktur przestrzennych systemów informacyjnych jako zadania programowania dyskretnego. Wskazano na możliwości linearyzacji zadania nieliniowego. Podano maszynowy algorytm ARO redukcji początkowego zbioru ograniczeń liniowych.

## К вопросу поиска оптимальной динамической структуры информационных систем

В статье рассматриваются вопросы, связанные с поиском оптимальной динамической структуры информационных систем, сформулированы на языке целочисленного программирования. Показаны возможностн линеаризации нелинейных задач и приведен алгоритм АРО редукции количества ограничений в исходной задаче.