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Modelling and optimum control of complex environment systems

by

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The paper deals with the modelling of complex nonlinear and dynamic systems such as economic, industrial, ecological, social etc. systems. The model consists of a given number of sectors. Each sector has a hierarchical structure with decentralized management systems. It produces a given product and consumes parts of output production of the remaining sectors. The sector input-output relation has been assumed in the form of a nonlinear dynamic operator. The sector decision centers optimize the allocation of input resources in such a way that the output production is maximalized. The supervisory controller optimizes the intersector cooperation links. In the first part of the paper the general methodology of the model construction and optimization has been discussed. The second part containes an analysis of specific problems connected with modelling the social, environment, education and research and development systems.

1. Introduction

There has recently been a considerable increase of interest in modelling of complex production, economic, ecological, social etc. systems, which shall be called environment systems. In particular the econometric macromodel building on a country wide basis has been growing rapidly in many countries (see [1—4]). Some of these models (as for example the model being developed in Project LINK [1]) have over 1000 equations relating to different sector of economics and regions of the world. However, many of these models are not accurate enough and they dont take into account some important real system phenomena. Among them one should mention:

(i) the nonlinear and dynamic input-output processes of the individual production plants;

(ii) the organizational structure and decisions taken at different management and control levels;

(iii) the regional structure of production, supply, demand and local goals and decisions;

(iv) random phenomena and disturbances.

An attempt to increase the model accuracy by taking into account more variables and equations complicates the model analysis and model understanding. Besides, the econometric identification of the model parameters becomes more complicated.

An essential difficulty arise when one wants to take into account the decisions taken by management centers. For the majority of the existing models which are descriptive in character rather the decisions should be treated as random phenomena unless they are known beforehand. This is not, however, the case for a longterm planning. Then, if we want to have a normative model, which could be used for effective planning purposes, it is necessary to build into the model the management and control structure. It is also necessary to formulate the system welfare or goal functional which depends on the decision and control actions. Solving the corresponding optimization problems, i.e. finding the decisions which maximize the welfare functional one can also determine the effect of nonoptimum decisions on the system output and the resulting decrease of welfare functional.

The present paper pursues that last approach. It is assumed that the model consists of n given sectors. Each sector produces a given product and consumes parts of output production of the remaining sectors. The sector input-output relation has been assumed in the form of a nonlinear, dynamic operator. Each sector consist of many independent production units organized in the form of a multilevel hierarchic structure and controlled by decentralized system of decision centers (controllers).

The econometric identification (estimation) of model parameters is carried out at the lowest level of the organizational structure only and at each higher level the aggregated production function is being determined in a purely analytic form by a process of aggregation. The sector controllers are responsible for the optimum allocation of input resources which should ensure the maximum sector output.

A higher level (central or supervisory) controller tries to maximize the goal functional by determining the best sectors level of activity and intersector exchange (cooperation links). That requires in turn the optimization of the investment policy. The optimum investment policy has been implemented in the decentralized form. The central controller allocates the investment resources among the sectors, which in turn allocate them among individual production units in such a manner that the resulting global output is maximum.

The model can be constructed in such a form that the regional structure of the country can be incorporated into the model structure.

The motivation for investigation of the model under consideration is to improve the large scale and long range planning and decision making. For that purpose the optimum strategies have been derived in the explicite form when possible and the effect of the nonoptimum decosions on the system performance has been investigated.

The paper consists of two parts and conclusions. In the first part the general methodology (based mainly on the previous author's publications [5—15]) of the model construction and optimization has been described. The second part containes an analysis of specific problems in the social, and environmental sector including, in particular, the optimization of the model composed of education, research and development, and economy.

2. General methodology

2.1. Production operator

The production plant can be treated as an input-output device which for the given input variables $x_1, ..., x_m$ (such as raw materials, manpower, energy, financial funds etc.) produces the output

$$z = A(x_1, ..., x_m).$$

A is usually a nonlinear dynamic operator acting from the space X of input functions into the space Z of the output functions, i.e. $A: X \to Z$. Neglecting the plant dynamics the typical input-output relation for a single input assumes the s

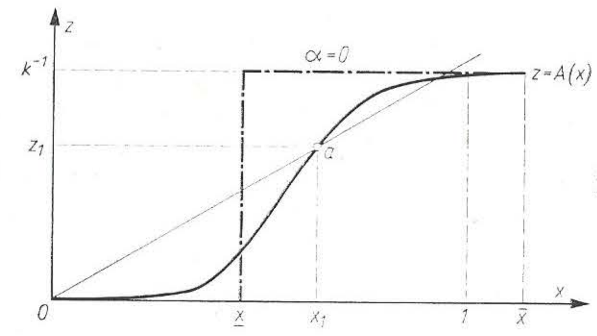

Fig. 1

form, shown in Fig. 1. An approximation of $z = A(x)$, sufficient for practical problems, can be written

$$z = K\varphi[x],$$

where K — positive constant and

$$\varphi[x] = \begin{cases} 0 & \text{for } x \in [0, \underline{x}], \\ [x - \underline{x}]^{\alpha} & \text{for } x > \underline{x}, \end{cases}$$

$\alpha, \underline{x}$ — given nonnegative constants.

It should be observed that for $\alpha = 0$ the function $\varphi[x]$ assumes the rectangular form typical for some production processes where the input increase beyond the threshold value $\underline{x}$ yields the full production capacity. It should be also mentioned that the linear approximation of production function (based on the past observed values of x_1, z_1, and shown in Fig. 1 by the line $0a$) which is typical for many econometric models will give considerable errors when the production resources change.

Besides the nonlinearities the dynamic effects, caused by inertia and delays in production processes, should be introduced.

The main idea is to use as A the product of φ with a linear Volterra operator

$$z(t) = L(\varphi) = \int_{-\infty}^{t} k(t, \tau)\, \varphi(\tau)\, d\tau, \tag{1}$$

where $k(t, \tau)$ is a given function which satisfies the causality condition $k(t, \tau)=0$ for $t<\tau$.

When $k(t, \tau)=k(t-\tau)$, the production plant is stationary in time. Stationarity means that the system does not change in time. The systems which develop in time are nonstationary.

In the case when the production has been originated at $t=0$ instead of (1) one can write

$$z(t)=\int_0^t k(t, \tau)\, \varphi(\tau)\, d\tau. \tag{1a}$$

For a pure delay process one can write

$$k(t, \tau)=K(t)\, \delta(t-T-\tau),$$

where $\delta(t)$ — Dirac's function, and obtain

$$z(t)=K(t)\, \varphi(t-T).$$

In the economic literature it is customary to denote the present values of $z(t)$ by z_t and express it by the past discreate values of φ denoted by φ_i.

In that case one can write

$$k(t, \tau)=\sum_{i=-\infty}^{t} K_{t-i}\, \delta(t-iT-\tau),$$

and obtain instead of (1)

$$z_t=\sum_{i=-\infty}^{t} K_{t-i}\, \varphi_i. \tag{2}$$

For the sake of uniformity in future calculations we shall use mainly the continuous representation (1) of A.

Now we can write down the general form of the production operator (P.O.)

$$z(t)=L\left[\prod_{j=1}^{m} \varphi_j(x_j)\right], \tag{3}$$

where

$$\varphi_j(x_j)=\begin{cases} 0 & \text{for } x_j \in [0, \underline{x}_j] \\ [x_j-\underline{x}_j]^{\alpha_j} & \text{for } x_j \in [\underline{x}_j, \bar{x}_j], \end{cases}$$

where $\underline{x}_j, \bar{x}_j$ — given positive numbers called the threshold and plant capacity; α_j — given positive numbers.

It should be observed that for $\underline{x}_j=0$, $j=1, \ldots, m$, and noninertial production process the production operator assumes the form of well known Cobb-Douglas production function.

The problem of identification of P.O. parameters can be splitted into two independent stages: the estimation of parameters of $\prod_{j=1}^{m} \varphi_j(x_j)$, for the steady-state form of plant operation, and the identification of transient function $k(t, \tau)$ of linear operator L. Since the both problems have been discussed extensively in the literature (see in particular [17]) we shall not discuss it here.

In the case when the nonlinear (steady state) production function is given in the graphical form if can be approximated, by a method described in Ref. [15], by the $\varphi(x)$ function with an accuracy sufficient for practical problems.

2.2. Optimization and aggregation

Consider a production system shown in Fig. 2 which consists of n plants (P_i, $i=1, \ldots, n$) utilizing m input resources, $X_1, \ldots, X_m$, such as raw material, energy, manpower etc.

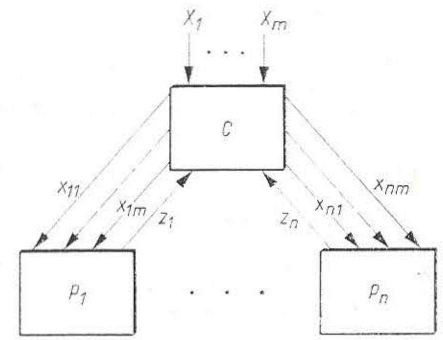

Fig. 2

Let the production operators for P_i be given in the form (3), i.e.

$$z_i(t)=\int_0^t k_i(t, \tau) \prod_{j=1}^{m} \varphi_{ij}[x_{ij}(\tau)]\, d\tau, \quad i=1, \ldots, n. \tag{4}$$

Assume also that the global amounts of resources $X_1, \ldots, X_m$ alloted to the system be given and

$$\sum_{i=1}^{n} \int_0^T x_{ij}(\tau)\, d\tau \leqslant X_j, \quad j=1, \ldots, m, \tag{5}$$

where $x_{ij}(\tau)$ — intensity of allocation of j-th resource to the i-th system; T — given time interval, and

$$X'_j=X_j-T\sum_{i=1}^{n} \underline{x}_{ij} \geqslant 0, \quad j=1, \ldots, m. \tag{6}$$

The problem of optimum allocation of resources can be formulated as follows. Find the nonnegative strategies $x_{ij}(t)=\hat{x}_{ij}(t)$, $t \in [0, T]$, $i=1, \ldots, n$, $j=1, \ldots, m$, such that the integrated output production

$$\Phi(x)=\sum_{i=1}^{n} \int_0^T z_i(t)\, w_i(t)\, dt, \tag{7}$$

where $w_i(t)$ are given nonnegative weight functions, attains in admissible region Ω its maximum value, i.e.

$$\max_{x\in\Omega} \Phi(x)=\Phi(\hat{x}).$$

The admissible region can be specified as follows

$$\Omega=\left\{x_{ij}(t):\ \sum_{i=0}^{n}\int_{0}^{T} x_{ij}(t)\,dt\leqslant X_j,\ x_{ij}(t)\geqslant 0\right.$$

$$\left. j=1,\ldots,m,\ i=1,\ldots,n,\ t\in[0,T]\right\}. \tag{8}$$

Assume also that

$$\sum_{j=1}^{m}\alpha_j<1. \tag{9}$$

In order to solve the present problem introduce new variables

$$x_{ij}^{*}(t)=x_{ij}(t)-\bar{x}_{ij}(t),\qquad i=1,\ldots,n,\ j=1,\ldots,m,$$

and change the integration order in (6), (4). Then

$$\Phi(x^{*})=\sum_{i=1}^{n}\int_{0}^{T} f_i^{q}(\tau)\,\varphi_i^{*}(\tau)\,d\tau, \tag{10}$$

where

$$\varphi_i^{*}(\tau)=\prod_{j=1}^{m} x_{ij}^{*\,\alpha_j}\,a_j(\tau),\qquad i=1,\ldots,n,$$

$$q=1-\sum_{j=1}^{m}\alpha_j.$$

Applying to (8) the Hölder inequality for integrals and sums (for details see Ref. [10]) one gets

$$\Phi(x^{*})\leqslant F^{q}\prod_{j=1}^{m}X_j^{\alpha_j}, \tag{11}$$

where

$$F=\sum_{i=1}^{n}\int_{0}^{T} f_i(\tau)\,d\tau, \tag{12}$$

$$f_i(\tau)=\left\{\int_{\tau}^{T} w_i(t)\,k_i(t,\tau)\,dt\right\}^{1/q}. \tag{13}$$

The equality sign in (11) holds if and only if

$$x_{ij}(t)-\bar{x}_{ij}=f_i(\tau)\frac{X_j}{F},\qquad i=1,\ldots,n,\ j=1,\ldots,m.$$

Then the result obtained can be formulated in the form of

THEOREM 1. Under the assumptions (4)—(9) the unique allocation strategy

$$x_{ij}(\tau)=\hat{x}_{ij}(\tau)=f_i(\tau)\frac{X_j}{F}+\bar{x}_{ij},\qquad i=1,\ldots,n,\ j=1,\ldots,m, \tag{14}$$

exists, such that

$$\max_{x\in\Omega}\Phi(x)=\Phi(\hat{x})=F^{q}\prod_{j=1}^{m}\varphi_j[X_j]. \tag{15}$$

where

$$\varphi_j[X_j]=\begin{cases}\left[X_j-\sum_{i=1}^{n}\bar{x}_{ij}\right]^{\alpha_j} & \text{for } X_j\geqslant\sum_{i=1}^{n}\bar{x}_{ij},\\ 0 & \text{for } X_j<\sum_{i=1}^{n}\bar{x}_{ij}.\end{cases}$$

When it is desirable to take into account the influence of $x_{ij}(\tau)$ acting in the past, i.e. for $\tau<0$ (which are not optimized in the present time interval), one should replace (4) by

$$z_i(t)=\int_{-\infty}^{t} k_i(t,\tau)\prod_{j=1}^{m}\varphi_i[x_{ij}(\tau)]\,d\tau, \tag{4'}$$

and consequently get instead of (12)

$$F=F'=\sum_{i=1}^{n}\int_{-\infty}^{T} f_i(\tau)\,d\tau.$$

A number of further extensions of Theorem 1 is possible. First of all consider the situation when a part of production outputs z_i starting with $i=N+1,\ldots,n$, should reach at the given time intervals $t=T_i$, $0\leqslant T_i\leqslant T$, the given values y_i, i.e.

$$z_i(T_i)=y_i,\qquad i=N+1,\ldots,n. \tag{16}$$

Such a situation happens when the given specified projects costing y_i each should be realized in the given time $t=T_i$.

The present problem can be formulated as follows:

Find the strategy $x_{ij}=\hat{x}_{ij}$, $i=1,\ldots,n$, $j=1,\ldots,m$, such that the functional

$$\bar{\Phi}(x)=\sum_{i=1}^{N}\int_{0}^{T} z_i(t)\,w_i(t)\,dt \tag{17}$$

attains maximum value subject to the assumptions and conditions (4)—(6), (8), (9) and (16).

A possible way of solving the present problem is to inco rporate the condition (16) into (17), i.e.

$$\Phi(x)=\bar{\Phi}(x)+\sum_{i=N+1}^{n} z_i(T_i),$$

and assume that

$$w_i(t)=\begin{cases} w_i & \text{for } 0\leqslant t\leqslant T_i \\ 0 & \text{for } t>T_i,\ i=N+1,\dots,n.\end{cases}$$

The unknown numbers w_i should be determined by using the equations (16). Using that approach (for details see Ref. [13]) one can use also the already obtained results specified by Theorem 1. In order to ensure the realization of the set of projects (16) one have to assume that a sufficient amount of resources exist. That condition specifies a class of physically realizable problems.

Now the following theorem can be formulated.

THEOREM 2. Assume the problem (4)—(6), (8), (9) and (16) to be physically realizable. Then the unique allocation strategy

$$x_{ij}(\tau)=\hat{x}_{ij}(\tau)=\begin{cases} f_i(\tau)\dfrac{X_j}{F}+\underline{x}_{ij}, & i=1,\dots,N, \\ w_i\,k_i(T_i,\tau)\dfrac{X_j}{F}+\underline{x}_{ij}, & i=N+1,\dots,n,\end{cases} \tag{17}$$

$$\text{where}\quad f_i(\tau)=\left\{\int_\tau^T w_i(t)\,k_i(t,\tau)\,dt\right\}^{1/\alpha},\qquad F=\sum_{i=1}^n\int_0^T f_i(\tau)\,d\tau$$

$$w_i(t)=w_i\,\delta(t-T_i),\qquad i=N+1,\dots,n, \tag{18}$$

exists, such that

$$\max_{x\in\Omega}\bar{\Phi}(x)=\bar{\Phi}(\hat{x})=F^\alpha\prod_{j=1}^m\left[X_j-\sum_{i=1}^n\underline{x}_{ij}\right]^{\alpha_j}-\sum_{i=N+1}^n y_i.$$

The numbers w_i, $i=N+1,\dots,n$, can be derived from the following set of $n-N$ eq.

$$b_i\,w_i^{1/\alpha}-\sum_{j=N+1}^n (a_j\,w_j)^{1/\alpha}=F,\qquad i=N+1,\dots,n, \tag{19}$$

where

$$a_j=\int_0^{T_j}k_j(T_j,\tau)\,d\tau,\quad b_i=\left[\frac{a_i}{y_i}\prod_{j=1}^m X_j^{\alpha_j}\right]^{(1-\alpha)^{-1}}.$$

The physical realizability coincides here with the condition of positive solution of (19).

Indeed solving the equation

$$z_i(T_i)=\int_0^{T_i}k_i(T_i,\tau)\prod_{j=1}^m[\hat{x}_{ij}(\tau)-\underline{x}_{ij}]^{\alpha_j}\,d\tau=$$

$$=\int_0^{T_i}k_i(T_i,\tau)^{1+\Sigma\alpha_j}\prod_{j=1}^m\left(\frac{w_i X_j}{F}\right)^{\alpha_j}d\tau=y_i,\qquad i=N+1,\dots,n$$

where

$$F=\sum_{i=1}^N\int_0^T f_i(\tau)\,d\tau+\sum_{i=N+1}^n\left\{w_i\int_0^{T_i}k_i(T_i,\tau)\,d\tau\right\}^{1/\alpha}$$

one gets (19).

Another possible extension of the problem being discussed concerns the situation when the integrated output of certain production plants should reach the given values, i.e.

$$\int_0^{T_i}z_i(\tau)\,d\tau=y_i,\quad i=N+1,\dots,n. \tag{20}$$

Obviously, the present problem differs, as compared to (4)—(6), (8), (9) and (16) by the condition (20). Then in order to find the optimum allocation strategy we can use the Theorem 1. However, the weight functions (18) should be replaced by

$$w_i(t)=\begin{cases} w_i & \text{for } 0\leqslant t\leqslant T_i \\ 0 & \text{for } t_i>T_i,\quad i=N+1,\dots,n.\end{cases}$$

As a result the optimum strategy becomes

$$x_{ij}(\tau)=\hat{x}_{ij}(\tau)=\begin{cases} f_i(\tau)\dfrac{X_j}{F}+\underline{x}_{ij}, & i=1,\dots,N,\ j=1,\dots,m, \\ w_i\displaystyle\int_0^{T_j}k_i(T_i,\tau)\,d\tau\,\dfrac{X_j}{F}+\underline{x}_{ij}, & i=N+1,\dots,n,\end{cases} \tag{21}$$

and w_i can be determined by solving the eqs.

$$\int_0^{T_i}\int_0^t k_i(t,\tau)\prod_{j=1}^m\left[w_i\frac{X_j}{F}\int_0^{T_i}k_i(T_i,\tau)\,d\tau\right]^{\alpha_j}d\tau\,dt=y_i,\qquad i=N+1,\dots,n, \tag{22}$$

where

$$F=\sum_{i=1}^N\int_0^T f_i(\tau)\,d\tau+\sum_{i=N+1}^n\left\{\int_0^T w_i\int_\tau^T k_i(t,\tau)\,dt\right\}^{1/\alpha}. \tag{23}$$

It is interesting to observe that the aggregated production function, under optimum control, for the three problems formulated above can be written in the form

$$z=\Phi(\hat{x})=F^\alpha\prod_{j=1}^m\varphi_j[X_j], \tag{24}$$

where

$$\varphi_j[X_j]=\begin{cases}\left[X_j-\displaystyle\sum_{i=1}^n\underline{x}_{ij}\right]^{\alpha_j} & \text{for } X_j\geqslant\displaystyle\sum_{i=1}^n\underline{x}_{ij}=\underline{X}_j \\ 0 & \text{for } X_j<\underline{X}_j,\end{cases}$$

and $w_i(t)$ for $i=N+1,\dots,n$ are given or determined by the eqs. (19) or (22).

It should be also noted that the subsystem of aggregated processes with the production function of the general form given by (24) can be aggregated again within a class of p subsystems described by

$$Z_i = F_i^a \prod_{j=1}^{m} \varphi_{ij}[X_{ij}], \quad i=1, \dots, p.$$

In other words one have to find a strategy $X_{ij} = \hat{X}_{ij}$ such that the aggregated output

$$Z^* = \sum_{i=1}^{p} Z_i \tag{25}$$

attains the maximum value subject to the constraints

$$\sum_{i=1}^{p} X_{ij} \leqslant X_j^*, \quad j=1, \dots, m, \tag{26}$$

$$X_{ij} \geqslant 0, \quad i=1, \dots, p, \tag{27}$$

Using the Hölder inequality for sums it is possible to show that under the optimum strategy

$$\hat{X}_{ij} = \frac{F_i}{F} X_j^* + \underline{X}_j^*, \quad j=1, \dots, m, \quad i=1, \dots, p, \tag{28}$$

where

$$F = \sum_{i=1}^{p} F_i, \quad \underline{X}_j^* = \sum_{i=1}^{p} \underline{X}_{ij}, \quad (X_j^* \geqslant \underline{X}_j^*).$$

The aggregated function becomes

$$Z^* = F^a \prod_{j=1}^{m} \varphi_j[X_j^*], \tag{29}$$

where

$$\varphi_j[X_j^*] = \begin{cases} [X_j^* - \underline{X}_j]^{\alpha_j} & \text{for } X^* \geqslant \underline{X}_j^* \\ 0 & \text{for } X_j^* < \underline{X}_j^*, \quad j=1, \dots, m. \end{cases}$$

Then the optimization and aggregation process can be repeted yielding at each stage the production function of the same analytic form (24), (29) but generally with the increased values for F and $\underline{X}_j$. It should be also observed that the greater F and the smaller $\underline{X}_j$ are the greater is the output of the resulting aggregated system.

Using the optimization formulae (14), (28) it is possible to implement the optimum allocation of resources using a decentralized system of decision centers. At the lower level the controlling center C (see Fig. 2) allocates the input resources, in a dynamic manner, using the strategy described by (14). A higher level controller which allocates the resources among the group of subsystems uses the formulae (28) which are static in the sense that they allocate the lumped amount of resources for the whole planning interval $[0, T]$. It should be observed that using the present aggrega-

tion approach one can determine the macromodel for a complicated sector of the economy (which consists of a hierarchic organization of decision centers and production plants) starting with simple micromodels of production units. This approach is also usefull in the case of systems composed of independent regions which are organized in the form of an administrative spatial structure.

A number of extensions of the results obtained in the present section is possible. First of all the transportation losses, which take place during the allocation of resources, can be taken into account. For that purpose one should replace the constraint (26) by

$$\sum_{i=1}^{p} X_{ij} \lambda_{ij} \leqslant X^*, \quad j=1, \dots, m, \tag{26'}$$

where $\lambda_{ij} \geqslant 1$ — given numbers.

As a result of losses the system performance index F decreases.

It is also possible to synthesize the best organizational structure of multilevel control which minimizes the global losses [15].

Another extension concerns the aggregation of variables at the higher level decision centers and a corresponding expansion of variables at the lower decision levels. Suppose that at the given level the aggregated production function of the form

$$Z = F^a \prod_{j=1}^{m} X_j^{\alpha_j}$$

is given. Generally speaking, the resources X_j can be splitted into different groups of activity at the lower decision levels. For example, the total budget of an institution can be subdivided into the main enance cost and investments, salaries etc., which are important for the operation of individual departments but may not be of immediate interest at the global analysis.

Let us formulate the optimization of expansion of variables problem. It consists in replacing a choosen factor, say $X_v^{\alpha_v}$ by the aggregate of M variables

$$Z_v = \prod_{i=1}^{M} X_{vi}^{\alpha_v \beta_i},$$

where β_i are positive given numbers such that

$$\sum_{i=1}^{M} \beta_i = 1, \quad \beta_i > 0, \quad i=1, \dots, M,$$

in such a way that $Z_v(X_{vi})$ attains maximum value in the set

$$\Omega = \left\{ X_{vi} : \sum_{i=1}^{M} X_{vi} \leqslant X_v, \ X_{vi} \geqslant 0, \ i=1, \dots, M \right\}.$$

THEOREM 3. An optimum expansion of variables strategy

$$X_{vi} = \hat{X}_{vi} = \beta_i X_v, \quad i=1, \dots, M, \tag{30}$$

exists, such that

$$\max_{X_{vi}\in\Omega} F^q \prod_{i=1}^{M} X_{vi}^{\alpha_v\beta_i} \prod_{\substack{j=1\\ j\neq v}}^{m} X_j^{\alpha_j} = F^{*q} \prod_{i=1}^{M} \hat{X}_{vi}^{\alpha_v\beta_i} \prod_{\substack{j=1\\ j\neq v}}^{m} X_j^{\alpha_j},$$

where

$$F^* = F^q \left\{ \prod_{i=1}^{M} \beta_i^{\alpha_v\beta_i} \right\}^{1/q}.$$

Proof. Indeed, maximizing the function

$$Z_v = \left[X_v - \sum_{i=1}^{M-1} X_{vi} \right]^{\alpha_M\beta_M} \sum_{i=1}^{M-1} X_{vi}^{\alpha_v\beta_i}$$

by solving the eqs.

$$\partial Z_v / \partial X_{vi} = 0, \quad i=1, \dots, M-1,$$

one obtain

$$X_{vi} = \hat{X}_{vi} = \beta_i X_v, \quad i=1, \dots, M.$$

Since

$$Z_v(\hat{X}_{vi}) = X_v^{\alpha_v} \prod_{i=1}^{M} \beta_i^{\alpha_v\beta_i}$$

it is necessary to change F to F^*, when replacing X^{α_v} by $Z_v(\hat{X}_{vi})$.

It is also possible to extend theorem 3 to the case of non-zero thresholds.

2.3. Optimization of cooperation links

Consider the system of n cooperating sectors shown in Fig. 3. Each sector, say j, consumes a part X_{ij} of the remaining sectors production ($i \neq j$) and produces the output X_{ji}.

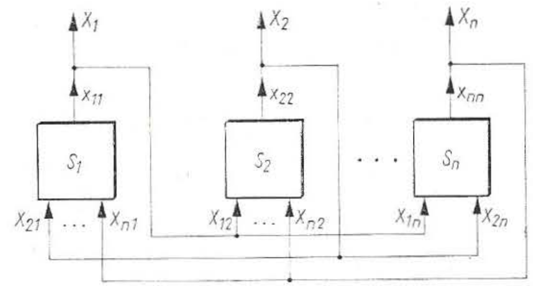

Fig. 3

Assume that the thresholds in the aggregated production functions (29) can be neglected. Then the set of production functions, which describe the model can be written as

$$\prod_{i=1}^{n} X_{ij}^{-\alpha_{ij}} = F_j^{q_j}, \quad j=1, \dots, n, \tag{31}$$

where: F_i = positive numbers, $q_j = 1 - \sum_{i=1}^{n} \alpha_{ij} > 0$; α_{ij} = positive numbers; $\alpha_{ii} = -1$, $i, j = 1, \dots, n$.

The cooperation between the sector i and j can be specified by the coefficients

$$c_{ij} = X_{ij}/X_{ii}, \quad j, i = 1, \dots, n.$$

The equations (31) can be written (by taking logarithms from both sides) as

$$\sum_{i=1}^{n} \alpha_{ij} \ln X_{ii} = -\ln \left\{ F_j^{q_j} \prod_{i=1}^{n} c_{ij}^{-\alpha_{ij}} \right\} = K_j, \quad j=1, \dots, n. \tag{32}$$

Assuming that the system is regular in the sense that

$$D = \mathrm{Det}\, |\alpha_{ij}| \neq 0$$

it is possible to see that for each admissible cooperation strategy $c_{ij} \in \Omega$, where

$$\Omega = \left\{ c_{ij}:\ \sum_{\substack{j=1\\ j\neq i}}^{n} c_{ij} \leq 1,\ c_{ij} \geq 0, \quad i, j = 1, \dots, n \right\},$$

the nonnegative solution $\tilde{X}_{jj}(c_{ij})$, $j=1, \dots, n$, of (32) exists.

The global net output (or income) generated in the optimization time T

$$I(c) = \sum_{i=1}^{n} w_i \tilde{X}_{ii} \left(1 - \sum_{\substack{j=1\\ j\neq i}}^{n} c_{ij} \right)$$

is a continuous function of c_{ij}. According to the well known Weierstras theorem $I(c)$ attains its extremum points in the compact set Ω.

Then the following theorem can be formulated [5].

THEOREM 4. In the regular cooperative system with the aggregated production functions (31) there exist:

(i) the unique nonnegative production $\tilde{X}_{ii}$, $i=1, \dots, n$, for each admissible strategy $c_{ij} \in \Omega$, $i, j = 1, \dots, n$;

(ii) the optimum cooperation strategy $c_{ij} = \hat{c}_{ij} \in \Omega$, $i, j = 1, \dots, n$, such that $I(c)$ attains maximum value at $\hat{c}_{ij}$, $i, j = 1, \dots, n$.

The solution of concreate cooperation problems can be achieved for simpler cases in the explicit form. Consider, as an example, the two sector system shown in Fig. 4. The electric power generating sector produces E units of electric power and uses cF units of coal. The production function of that sector (given usually in the graphic form) can be approximated by a function of the form

$$E = K_1 (cF)^{\delta}, \quad 0 < \delta < 1, \tag{33}$$

where K_1, δ = given positive constants.

The full energy E production is being used at the coal mines for driving excavation and transportation systems. Assuming that DC electric motors are used mainly for that purpose one can write the relation between the output F (i.e. the amount of transported coal) and the electric energy used for that purpose in the form [5]

$$F=K_2\, E^{1/2}, \tag{34}$$

where K_2=positive constant.

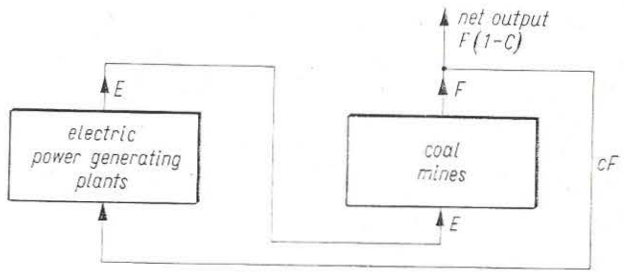

Fig. 4

It can be checked very easily that the net output

$$I(c)=F(1-c)=[K_1^{1/2}K_2]^{\delta_1}\, c^{\delta_2}\,(1-c)$$

$$\delta_1=\frac{2}{2-\delta}, \qquad \delta_2=\frac{\delta}{2-\delta},$$

attains the maximum value for

$$c=\hat{c}=\delta/2$$

and

$$I(\hat{c})=\left[K_1^{1/2}\,K_2\,\frac{\delta}{2}\right]^{\delta_1}.$$

The resulting production levels become

$$F=[K_1^{1/2}\,K_2]^{\delta_1}\left(\frac{\delta}{2}\right)^{\delta_2}, \qquad E=K_1^{\delta}\,K_2^{2\delta_2}\left(\frac{\delta}{2}\right)^{\delta_1}.$$

When c attains the boundary $c=0$ or $c=1$ the net output $I(c)=0$. It is interesting to observe that the net income can be increased by an increase of the parameters K_1, K_2, δ only and not by a change in the level of activity of both sectors (for example, by loading more the existing motors and electric power generators). It means that a technological and technical reconstruction and investment improving the sector parameters can increase the net income only.

The explicite form of the optimum cooperation strategy can be also obtained for a more complicated situation when each sector of the system shown in Fig. 3 cooperates with its neariest neighbour only. In that case the sector production functions (31) can be written in the following form:

$$\alpha_{i,i-1}\ln X_{i-1,i-1}-\ln X_{ii}+\alpha_{i,i+1}\ln X_{i+1,i+1}=$$
$$=-\ln\{F_i^{q_i}\, c_{i,i-1}^{\alpha_{i,i-1}}\, c_{i,i+1}^{\alpha_{i,i+1}}\}=K_i, \qquad i=1,\dots,n,$$

where the values of indices equal 0 or $n+1$ coincide with n or 1 respectively. For example

$$X_{00}=X_{nn}, \quad c_{1,0}=c_{1,n}.$$

Solving for X_{ii} one gets

$$X_{ii}=\prod_{\nu=1}^{n}\{K_\nu\, c_{\nu-1,\nu}^{\alpha_{\nu-1,\nu}}\, c_{\nu+1,\nu}^{\alpha_{\nu+1,\nu}}\}^{\beta_{i\nu}}, \qquad i=1,\dots,n, \tag{36}$$

where

$$\beta_{i\nu}=(-1)^{i+\nu}\, D_{i\nu}/D, \qquad \nu=1,\dots,n,$$

D_i is the determinante obtained by replacing the column $\{K_\nu\}_1^n$ with the i-th column of D, $D_{i\nu}$=subdeterminante obtained by expansion of D_i along the elements of the i-th column of D_i.

Suppose the system is closed so

$$c_{j,j-1}+c_{j,j+1}=1, \qquad j=1,\dots,n \tag{37}$$

and one wants to maximize the output X_{ii}. The corresponding optimum strategies $\hat{c}_{j,j-1}^{(i)}$, $\hat{c}_{j,j+1}^{(i)}$ can be determined by optimizing each factor of (36) of the form

$$[c_{j,j-1}]^{(\alpha_{j,j-1}+\beta_{i,j-1})}\,[c_{j,j+1}]^{(\alpha_{j,j+1}+\beta_{i,j+1})},$$

subject to (37).

That yields the following strategies [9]

$$\hat{c}_{j,j-1}^{(i)}=\frac{1}{1+\gamma_{ij}}, \tag{38}$$

$$\hat{c}_{j,j+1}^{(i)}=\frac{\gamma_{ij}}{1+\gamma_{ij}}, \quad j=1,\dots,n, \tag{39}$$

where

$$\gamma_{ij}=D^{-1}\,[(-1)^{i+j}\,\alpha_{j,j+1}\,D_{i,j-1}+(-1)^{i+j+1}\,\alpha_{j,j+1}\,D_{i,j+1}].$$

It is possible to extend the results obtained to the case of opened systems by assuming that a part of output production goes out of the system. Assume, as an example, that $X_{n,1}$ is being sold on the market (instead of being consumed by sector S_1) with the income proportional to $X_{nn}\,c_{n,1}$. That value should be maximized. However in order to use eqs. (38), (39) one can suppres the variable $X_{n,1}$ in sector S_1 production function by assuming $\alpha_{n,1}=0$. In that case

$$\gamma_{nn}=-\alpha_{n,n-1}\,D_{n,n-1}/D. \tag{40}$$

The optimization of cooperation in the more complicated cases, including the case of nonzero thresholds in production functions, can be solved numerically using the iterative technique of nonlinear programming, such as the gradient projection method. It should be also observed that in the general case of nonzero threshold in production functions the nonzero activity in the model under consideration requires a number of assumptions regarding the positive solutions of X_{ii}, $i=1,\dots,n$.

2.4. Optimization of investment policy

Comparing the optimal strategy of allocation of resources and cooperation with the strategy used in the real systems one might find out that they generally do no coincide. As a result the losses in the performance follow.

As an example consider the loss of output performance in the sector with n optimized processes (7) and the production operators

$$z_i(t)=\int_0^t k_i(t,\tau)\prod_{j=1}^{m}[x_{ij}(\tau)]^{\alpha_j}\,d\tau, \qquad i=1,\dots,n.$$

Assume $\hat{x}_{ij}(t)>0$, $t\in[0,T]$, $i=1,\dots,n$, $j=1,\dots,m$.

Then

$$\Phi(x)=\sum_{i=1}^{n}\int_0^T w_i(t)\int_0^t k_i^*(t,\tau)\prod_{j=1}^{m}[\hat{x}_{ij}(\tau)]^{\alpha_j}\,d\tau,$$

where

$$k_i^*(t,\tau)=k_i(t,\tau)\prod_{j=1}^{m}\left[\frac{x_{ij}(\tau)}{\hat{x}_{ij}(\tau)}\right]^{\alpha_j}. \tag{41}$$

Then the system behaves as if the performance under F has changed to the value

$$F^*=\sum_{i=1}^{n}\int_0^T\left\{\int_\tau^T w_i(t)\,k_i^*(t,\tau)\,dt\right\}^{1/q}d\tau<F.$$

A similar expressions can be derived for the decrease of performance following nonoptimum cooperation strategy.

Obviously one of the main obstacles in implementation of the optimum strategy is the limited plants capacity with respect to the input variables. For example an increase of employment or raw materials consumption may not be realized because there is no space for additional manpower or machines and tools. It is well known that by an investment process the plant capacity can be increased. The investment can also increase the output without changing the input capacity by improving the production technology, automation modernization and new production tools, better management etc. This requires utilization of products made by different sectors of economy, mainly in the proceeding planning interval. In the model of investment optimization we shall assume that M products $Y_1,\dots,Y_M$ were accumulated in previous planning intervals and can be therefore treated as exogeneous variables. It is also assumed that the investment process is nonlinear, dynamic and as a result the sector production operator assumes the form

$$z_i(t)=\int_0^t K_i(t,\tau)\prod_{j=1}^{m}\varphi_i[x_{ij}(\tau)]\prod_{\nu=1}^{M}\varphi_\nu[y_{i\nu}(\tau)]\,d\tau$$

where

$$\varphi_\nu[y_{i\nu}(\tau)]=\begin{cases}[y_{i\nu}(\tau)-\underline{y}_{i\nu}]^{\beta_\nu} & \text{for } y_{i\nu}(\tau)\geq\underline{y}_{i\nu}\\ 0 & \text{for } y_{i\nu}(\tau)<\underline{y}_{i\nu}\end{cases}$$

$\underline{y}_{i\nu}$ = given thresholds of investment processes.

The optimum investment policy which maximizes the integrated output (7) under the additional conditions:

$$\sum_{i=1}^{n}\int_0^T y_{i\nu}(t)\,dt\leq Y_\nu, \qquad \nu=1,\dots,M \tag{42}$$

$$y_{i\nu}(t)\geq 0, \qquad i=1,\dots,n,\ \nu=1,\dots,M \tag{43}$$

can be derived using the formulae (14)

$$y_{i\nu}=\hat{y}_{\nu\nu}=f_i(\tau)\frac{Y_\nu}{F}+\underline{y}_{i\nu}, \qquad i=1,\dots,n,\ \nu=1,\dots,M. \tag{44}$$

The aggregated output production becames

$$F=\Phi(\hat{x},\hat{y})=F^q\prod_{j=1}^{m}\varphi_j[X_j]\prod_{\nu=1}^{M}\varphi_\nu[Y_\nu], \tag{45}$$

where

$$\varphi_\nu[Y_\nu]=\begin{cases}\left[Y_\nu-\sum_{i=1}^{n}\underline{y}_{i\nu}\right]^{\beta_\nu} & \text{for } Y_\nu\geq\sum_{i=1}^{n}\underline{y}_{i\nu},\\ 0 & \text{for } Y_\nu\leq\sum_{i=1}^{n}\underline{y}_{i\nu}.\end{cases}$$

By a number of optimization and aggregation, which correspond to the structure of decision system, we arrive to the n sectors model of Fig. 3, described by the production function

$$\prod_{j=1}^{n}X_{ij}^{-\alpha_{ij}}=F_i'^{\,q_i},$$

where

$$F_i'=F\left\{\prod_{\nu=1}^{M}\varphi_\nu[Y_{i\nu}]\right\}^{1/q_i}.$$

It is also possible (using methods described in Sec. 2.3) to find the optimum inter sector investment strategy. For example if it is necessary to optimize the i-th sector output

$$X_{ii}\left[\prod_{\nu=1}^{M}\varphi_\nu(Y_{i\nu})\right]$$

subject to the constraints

$$\sum_{i=1}^{n}Y_{i\nu}\leq Y_\nu, \qquad Y_{i\nu}\geq 0,\ \nu=1,\dots,M,\ i=1,\dots,n,$$

one can use the nonlinear programming technique. When the optimum values of $Y_{i\nu}=\hat{Y}_{i\nu}$, $i=1,\dots,n$, $\nu=1,\dots,M$, have been determined the corresponding values of investment resources at each level of optimization can be determined in a backward manner. In other words, a decentralized process of investment decisions follows, where the higher-level decision centers allocate the resources for the lower-level (controlled) investment processes.

3. Specific problems of sector analysis

3.1. Society and welfare

Society is the main sector of the complex environment model under consideration. All the remaining sectors perform services or produce goods which are consumed by social system. Besides productive investments, which increase the production rate of system development, the investments in the social sector should be taken into account. They should increase the economic, social and cultural standard of living by building new houses, transportation, telecommunication, cultural institutions, sport facilities etc. Among social services one should mention: medical and social care, education, recreation, entertainment etc.

The output of the social system in the form of labor or manpower serves as one of the inputs to all the remaining sectors of the model. However, the maximum output is not the main goal of the social sector performance. A welfare function of that sector should be constructed in such a way that it should include the satisfactions of social groups in different geographical regions connected with realization of demands for consumption goods, capital investments etc.

Keeping that in mind an analytic model of regional allocation of services, goods etc. will be introduced in the present section [11].

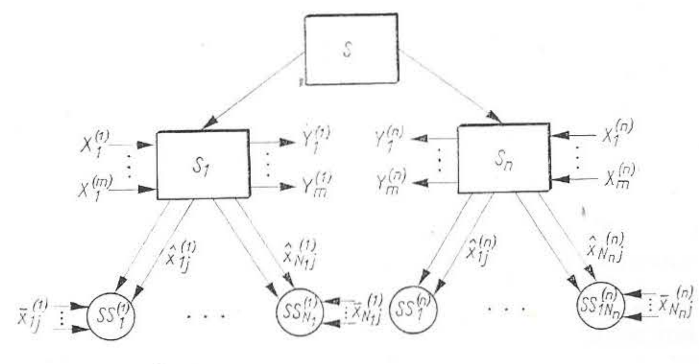

Fig. 5

Consider the model shown in Fig. 5, which consists of n regional decision centers $S_1, \ldots, S_n$, and the higher level (supervisory) decision center S. A given number of goods, services, resources $X_1^{(i)}, \ldots, X_m^{(i)}$ produced by corresponding sectors in region i should be allocated by S_i in an optimum manner among the local subsystems $SS_\nu^{(i)}$, $\nu=1, \ldots, N_i$. The supervisory decision center S controls the amount of goods etc. $Y_j^{(i)}$, $i=1, \ldots, n$, $j=1, \ldots, m$, exchanged between the specific regions S_i.

It can be assumed that for certain number of goods and services the supply equals demand. First of all that will concern all the planned values (y_i) produced by the corresponding sectors (compare (16), (20)). There is, however, a number of goods and services for which the supply is less then the corresponding demand. It concerns such items as: housing, transportation, medical and social care etc. Each sector is trying to put these items on the list of maximalized outputs. But despite that the total production is less than the total demand. In the capitalist economy the process of allocation of scarce goods and services in that situation is regulated by market mainly and to a less extend by gouvernment expenditures. In the socialist countries prices are usually fixed and the state decisions allocate the expenditures for such items as transportation, social and medical care, housing, education etc.

When the amount of allocated goods etc. is less than the demand the subsystems suffer losses. Then the goal of the decision centers is to allocate the goods is such a manner that the global loss or dissatisfaction function is minimum.

Denote the expected demand for the particular item j in the subsystem ν of the region i by $\bar{x}_{\nu j}^{(i)}$. Assume the amount of item j alloted to region i: $x_{\nu j}^{(i)}$ is less than $\bar{x}_{\nu j}^{(i)}$, i.e. $x_{\nu j}^{(i)} < \bar{x}_{\nu j}^{(i)}$, and as a result a dissatisfaction of subsystem follows, which is an increasing convex function $U(\underline{x})$ of the vector variable

$$\underline{x} = \{\bar{x}_{\nu 1}^{(i)} - x_{\nu 1}^{(i)}, \bar{x}_{\nu 2}^{(i)} - x_{\nu 2}^{(i)}, \ldots, \bar{x}_{\nu m}^{(i)} - x_{\nu m}^{(i)}\}.$$

That function determines the priority relation which says $\underline{x} \geqslant 0$ is better than $\underline{y} \geqslant 0$ if and only if

$$U(\underline{x}) > U(\underline{y}).$$

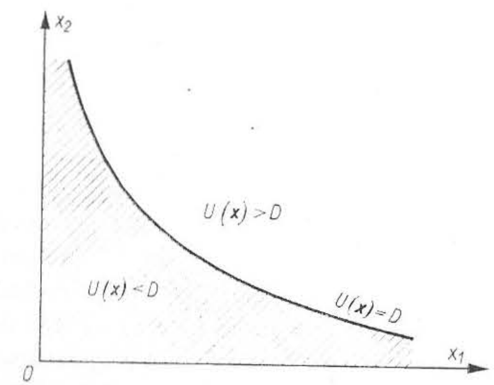

Fig. 6

In Fig. 6 a typical plot of $U(\underline{x}) = D = \text{const.}$ for $m=2$ has been shown. The shaded area under the plot correspond to that values of $\underline{x}$ which create the dissatisfaction less than D.

It should be observed that the plot can be approximated by the function of the form

$$D = \text{const} \cdot x_1^{a_1} x_2^{a_2},$$

where α_1, α_2 — positive constants. In the general multidimensional case the dissatisfaction function can be written as

$$D_\nu^{(i)}(\underline{x}_\nu^{(i)}) = [k_\nu^{(i)}]^q \prod_{j=1}^{m} (\bar{x}_{\nu j}^{(i)} - x_{\nu j}^{(i)})^{\alpha_j}, \tag{46}$$

where $\alpha_j, k_\nu^{(i)}$ = positive constants, $q = \sum_{j=1}^{m} \alpha_j - 1 > 0$.

Assuming that the parameters of (46) are known or determined experimentally (for that purpose one can use the relation $\alpha_j = -\frac{dD_\nu^{(i)}}{D_\nu^{(i)}} : \frac{dx_{ij}}{x_{ij}}, j=1, \ldots, m$) it is possible to formulate the optimum allocation problem. Find the optimum strategy $(\hat{\underline{x}}, \hat{\underline{y}})$ which minimizes the global dissatisfaction

$$D = \sum_{i=1}^{n} \sum_{\nu=1}^{N_i} D_\nu^{(i)}(\underline{x}_\nu^{(i)}) \tag{47}$$

subject to the constraints

$$\sum_{\nu=1}^{N_i} x_{\nu j}^{(i)} \leqslant X_j^{(i)} - Y_j^{(i)}, \tag{48}$$

or

$$\sum_{\nu=1}^{N_i} (\bar{x}_{\nu j}^{(i)} - x_{\nu j}^{(i)}) \geqslant \sum_{\nu=1}^{N_i} \bar{x}_{\nu j}^{(i)} - X_j^{(i)} + Y_j^{(i)} > 0, \tag{48'}$$

$$j=1, \ldots, m, \quad i=1, \ldots, n,$$

$$\bar{x}_{\nu j}^{(i)} - x_{\nu j}^{(i)} \geqslant 0, \quad \nu=1, \ldots, N_i, \quad j=1, \ldots, m, \quad i=1, \ldots, n, \tag{49}$$

$$\sum_{i=1}^{n} Y_j^{(i)} = 0, \quad i=1, \ldots, n. \tag{50}$$

The last equation (50) means that no goods etc. are generated within the system.

Taking into account the existing administrative and social structure it is desirable to formulate the optimum strategy in the decentralized form including at the lower level the decisions concerning $\underline{x}^{(i)}$ and at the higher level the decisions concerning $Y_j^{(i)}$. That can be achieved using the aggregation approach described by (24)—(29) and results obtained can be formulated in the form of the theorem (for details see Ref. [11]).

THEOREM 5. The unique optimum decentralized strategy of allocation

$$x_{\nu j}^{(i)} = \hat{x}_{\nu j}^{(i)} = \bar{x}_{\nu j}^{(i)} - \frac{k_\nu^{(i)}}{\sum_\nu k_\nu^{(i)}} \left[\sum_{\nu=1}^{N_i} \bar{x}_{\nu j}^{(i)} - X_j^{(i)} + \hat{Y}_j^{(i)} \right],$$

$$\nu=1, \ldots, N_i, \quad i=1, \ldots, n, \quad j=1, \ldots, m, \tag{51}$$

$$Y_j^{(i)} = \hat{Y}_j^{(i)} = \frac{\sum_\nu k_\nu^{(i)}}{\sum_i \sum_\nu k_\nu^{(i)}} \left[\sum_{i=1}^{n} \sum_{\nu=1}^{N_i} \bar{x}_{\nu j} - \sum_{i=1}^{n} X_j^{(i)} \right] - \sum_{\nu=1}^{N_i} \bar{x}_{\nu j}^{(i)} + X_j^{(i)},$$

$$i=1, \ldots, n, \quad j=1, \ldots, m, \tag{52}$$

exists, such that

$$D(\hat{\underline{x}}, \hat{\underline{y}}) = \min_{\underline{x}, \underline{y} \in \Omega} D(\underline{x}, \underline{y}) = \left(\sum_{i=1}^{n} \sum_{\nu=1}^{N_i} k_\nu^{(i)} \right)^q \prod_{j=1}^{M} \left[\sum_{i=1}^{n} \sum_{\nu=1}^{N_i} \bar{x}_{\nu j}^{(i)} - \sum_{i=1}^{n} X_j^{(i)} \right]^{\alpha_j}, \tag{53}$$

where the admissible region Ω is specified by eqs. (48)—(50).

It is possible to extend theorem 4 to the case when transport or maintenance losses during the allocation process are present. In that case instead of (48') one can write

$$\sum_{\nu=1}^{N_i} x_{\nu j}^{(i)} \lambda_{\nu j}^{(i)} \leqslant X_j^{(i)} - Y_j^{(i)}$$

and the corresponding optimum allocation becomes [11]

$$\hat{x}_{\nu j}^{(i)} = \bar{x}_{\nu j}^{(i)} - \frac{\tilde{k}_\nu^{(i)}}{\lambda_{\nu j}^{(i)} \sum_\nu \tilde{k}_\nu^{(i)}} \left[\sum_{\nu=1}^{N_i} \bar{x}_{\nu j}^{(i)} \lambda_{\nu j}^{(i)} - X_j^{(i)} + \hat{Y}_j^{(i)} \right],$$

$$\hat{Y}_j^{(i)} = \frac{\sum_\nu \tilde{k}_\nu^{(i)}}{\sum_i \sum_\nu \tilde{k}_\nu^{(i)}} \left[\sum_{i=1}^{n} \sum_{\nu}^{N_i} \bar{x}_{\nu j}^{(i)} \lambda_{\nu j}^{(i)} - \sum_{i=1}^{n} X_j^{(i)} \right] - \sum_{\nu=1}^{N_i} \bar{x}_{\nu j}^{(i)} \lambda_{\nu j}^{(i)} + X_j^{(i)},$$

where

$$\tilde{k}_\nu^{(i)} = k_\nu^{(i)} \prod_{j=1}^{m} (\lambda_{\nu j}^{(i)})^{-\alpha_j/q}, \quad \nu=1, \ldots, N_i.$$

Another possible extension [11] concerns the case when we have a multilevel hierarchic structure of regions or towns which differ by the amount of services or allocated items.

It is also possible to consider a dynamic optimization models when the dissatisfaction function changes in time and the functional

$$D(\underline{x}, \underline{y}) = \int_0^T \sum_i \sum_\nu D_\nu^{(i)}(\underline{x}, \underline{y}, t)\, dt,$$

should be minimalized, with respect to the integral constraints

$$\int_0^T \sum_{\nu=1}^{N_i} x_{\nu j}^{(i)}(t)\, dt \leqslant X_j^{(i)} - Y_j^{(i)}, \quad i=1, \ldots, n, \quad j=1, \ldots, m,$$

$$\bar{x}_{\nu j}^{(i)}(t) - x_{\nu j}^{(i)}(t) \geqslant 0, \quad i=1, \ldots, n, \; j=1, \ldots, m, \quad \nu=1, \ldots, N_i, \; t \in [0, T].$$

To solve the present problem one can use the method of Sec. 2.2.

3.2. Pollution influence on the environment

The productive activity performed by the sectors of the environmental system is usually accompanied by side production of waste materials which are generally harmful to the human environment. At the present state of science and technology the most of the waste materials can be purified, utilized or recycled. However, the cost of purifying of waste materials increases rapidly when a high degree of purity is required. Since the environment has on ability of clearning itself with the waste decay ratio (which depends on the waste ingredients) the following approach to the pollution problem has been proposed: minimize the cost of waste and pollution treatment subject to the conditions that the degree of environment pollution is less then a given value. Following that approach consider the pollution control model shown in Fig. 7.

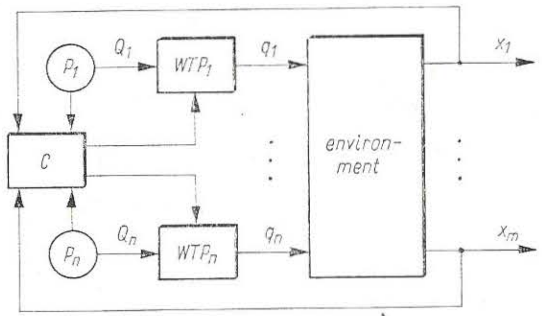

Fig. 7

Assume that the waste with intensity $Q_i(t)$, $i=1, ..., n$, generated by n given polluters P_i (such as factories, power plants, urban centers etc.) is being treated by the waste treatment plants WTP_i and with the intensity q_i it is discharged into the environment (i.e. into air, water or soil). Q_i in turn may depend on the sector productions (X_{ii}). The degree of environment contamination (expressed by such factors as pollutant fall out, dissolvent oxigen (D.O.) concentration, or the biological oxygen demand (B.O.D.)) $x_i(t)$ can be observed by the pollution sensitive devices in the m given points or areas. The information obtained in that way together with the information regarding the wether forecast etc. is being used by the controller C to optimize the decision variables q_i, $i=1, ..., n$.

The performance of pollution control can be measured by the functional

$$\Phi=\sum_{i=1}^{n}\int_{0}^{T} w_i(t)\, x_i(t)\, dt, \tag{54}$$

where $w_{ik}(t)$=given nonnegative continuous weight functions, T=optimization horizont.

The input-output dynamical properties of the environment according to the theoretical and experimental data can be approximated by the Volterra operator

$$x_j(t)=\sum_{i=1}^{n}\int_{0}^{t} K_{ij}(t,\tau)\, q_i(\tau)\, d\tau, \quad j=1, ..., m, \tag{55}$$

where $K_{ij}(t,\tau)$=given nonnegative continuous functions which satisfy the causality condition $K_{ij}(t,\tau)=0$ for $t<\tau$.

A typical example of the cost function of the waste treatment plant has been shown in Fig. 8. It can be approximated by the function

$$C_i(q_i)=k_i^{1-\beta}(Q_i-q_i)^{\beta}, \quad \beta>1,\ k_i>0. \tag{56}$$

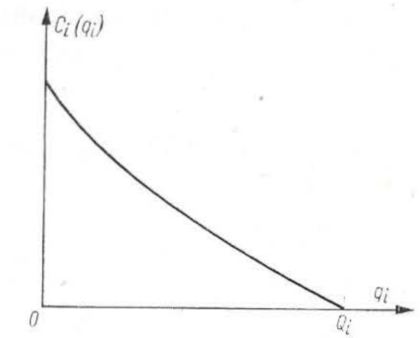

Fig. 8

It is also assumed that the total waste treatment cost is limited, i.e.

$$\sum_{i=1}^{n}\int_{0}^{T} C_i(q_i)\, dt \leqslant C, \tag{57}$$

where C=given positive number.

The pollution treatment optimization problem can be formulated as follows. Find the nonnegative strategy $c_i=\hat{c}_i$, $i=1, ..., n$, such that

$$\Phi(c)=\sum_{j=1}^{m}\int_{0}^{T} w_j(t)\sum_{i=1}^{n}\int_{0}^{T} K_{ij}(t,\tau)\,[Q_i(\tau)+k_i^{1-\alpha}c_i^{\alpha}(\tau)]\, d\tau\, dt,$$

where $\alpha=1/\beta$, attains for $c=\hat{c}$ the minimum value subject to the constraints (57) and

$$Q_i(\tau)-k_i^{1-\alpha}C_i^{\alpha}(\tau)\geqslant 0, \quad t\in[0, T], \tag{58}$$

$$c_i(\tau)\geqslant 0. \tag{59}$$

Since the term

$$\Phi=\sum_{j=1}^{m}\sum_{i=1}^{n}\int_{0}^{T} w_j(t)\int_{0}^{t} K_{ij}(t,\tau)\, Q_i(\tau)\, d\tau\, dt$$

is a constant, the problem boils down to the maximization of

$$F(c)=\sum_{i=1}^{n}\int_{0}^{T} f_i^{q}(\tau)\, C_i^{\alpha}(\tau)\, d\tau,$$

where

$$f_i(\tau)=\left[\int_{\tau}^{T} k_i^{1-\alpha}\sum_{j=1}^{m} w_j(t)\, K_{ij}(t,\tau)\, dt\right]^{1/q}, \quad q=1-\alpha.$$

Subject to the constraints (57)—(59).

It is obvious that when the constraint (58) is not active the optimum control strategy can be derived by using Theorem 1. When (58) is active the optimum strategy can be derived from the eq.

$$Q_i(t)-k_i^{1-\alpha}\, C_i^{\alpha}(t)=0\,.$$

Then the following theorem can be proved (for details see Ref. [12]).

THEOREM 6. An optimum pollution strategy

$$C_i(t)=\hat{C}_i(t)=\tilde{f}_i(t)\frac{C}{F}\,,\qquad i=1,\ldots,n,\ t\in[0,T], \tag{60}$$

where

$$\tilde{f}_i(t)=\begin{cases} f_i(t) & \text{for } t\notin S_i \\ \dfrac{F}{C}\,k_i^{1-1/\alpha}\,Q_i^{1/\alpha}(t) & \text{for } t\in S_i \end{cases},$$

$$S_i=\left\{t: f_i(t)>\frac{F}{C}\,k_i^{1-1/\alpha}\,Q_i^{1/\alpha}(t),\ t\in[0,T],\ i=1,\ldots,n\right\},$$

$$F=\sum_{i=1}^{n}\int_0^T \tilde{f}_i(t)\,dt$$

exists, such that

$$\Phi(\hat{C})=\min_{C=\Omega}\,[\bar{\Phi}-F(C)]=\bar{\Phi}-F^{1-\alpha}\,C^{\alpha} \tag{61}$$

and Ω is the admissible control set defined by (57)—(59).

The optimum waste discharge strategies become

$$\hat{q}_i(t)=k_i^{1-\alpha}\,\hat{C}_i^{\alpha}(t),\qquad i=1,\ldots,n\,.$$

Using the aggregation Theorem 6 it is possible for a given admissible pollution level ε to find the corresponding minimum waste treatment cost $\tilde{C}$ (by solving the eq. $\bar{\Phi}-\varepsilon=F^{1-\alpha}\,C^{\alpha}$)

$$\tilde{C}=\left[\frac{\bar{\Phi}-\varepsilon}{F^{1-\alpha}}\right]^{1/\alpha}. \tag{62}$$

When the function $\bar{\Phi}\,(X_i)$ which expresses the pollution level in terms of sectors production activity X_i increases rapidly it may happens that the corresponding waste treatment cost becomes greater than the production income. In that case a new technology of production or new waste treatment plant should be developed.

That requires capital investment which can be optimized by the methods described is Sec. 2.4.

Using the aggregation formula (60), (61) it is also possible to optimize a complex hierarchic systems of environment pollution control (for details see Ref. [12]).

Education, research and development (R+D) constitute the main factors determining the development rate of the economy. In the simple economic development models these factors are considered as contributing mainly to the technical progress in production processes. It is argued that the research activity generates new ideas, technological and technical solutions which by a process of concreate projects development produce new technology and increase the efficiency of existing production processes.

The research and development could not be able to exist without the financial and material support from the economy. The both sectors in turn depend on the supply of skilful labor which is produced by the education sector. The education sector is also supported by the economy, and to a less extend — by the (R+D) sector (mainly in the form of training of the teaching staff which is angaged part time in research activity).

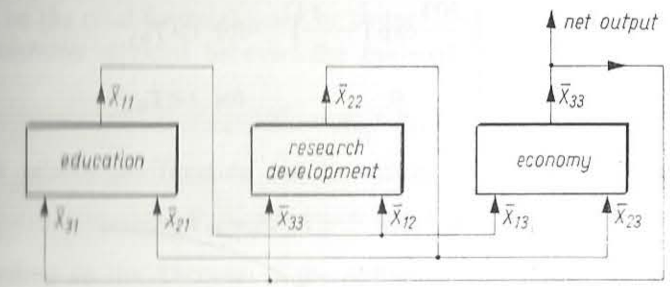

Fig. 9

Then the model of the system analysed in the present section assumes the form shown in Fig. 9. The model of the education sector for the two-level structure (the undergraduate and graduate studies) has been shown in Fig. 10.

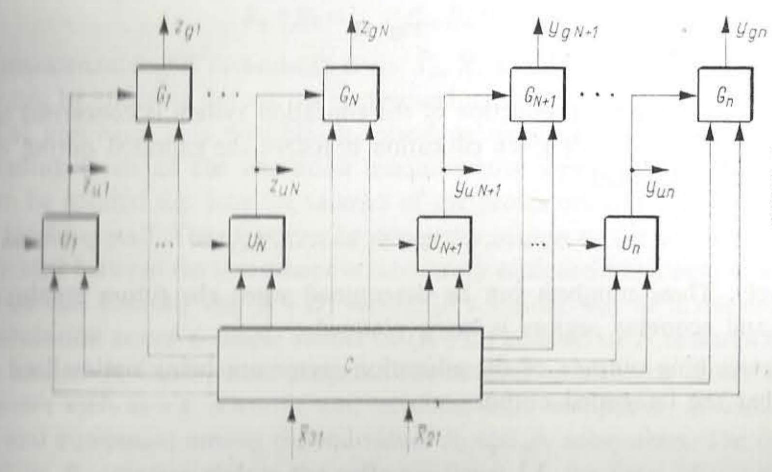

Fig. 10

The resources $\bar{X}_{31}, \bar{X}_{21}$ can be allocated among the individual education institutions $U_1, \ldots, U_n, G_1, \ldots, G_n$ by using the Theorem 2.

The input-output production model of the education institution can be assumed in the form (4) or (4′). The threshold and maximum capacity $(\underline{X}_{ij}, \bar{X}_{ij})$ for different inputs can are determined directly from the minimum budget, staff, the size and capacities of the education facilities. The values of the exponents α_j can be determined if the different input-output data for the past activity are known.

The functions $k_i(t, \tau)$ which determine the dynamics of education processes can be determined by input-output observations. Suppose for example that at the year 1967 hundred students in the form of a unitary step had been admitted to the 3 years post graduate studies. There is usually a delay $(T_0 \approx 3$ years) of the output production and an exponential growth with the time constant $\tau = 2-3$ years, as shown in Fig. 11. As a result the corresponding stationary $k_i(t)$ function can be approximated by the function

$$\tilde{k}_i(t) = \begin{cases} \dfrac{100}{\tau_i} \exp\left(-\dfrac{t}{\tau_i}\right) & \text{for } t \geq T_0, \\ 0 & \text{for } t < T_0. \end{cases}$$

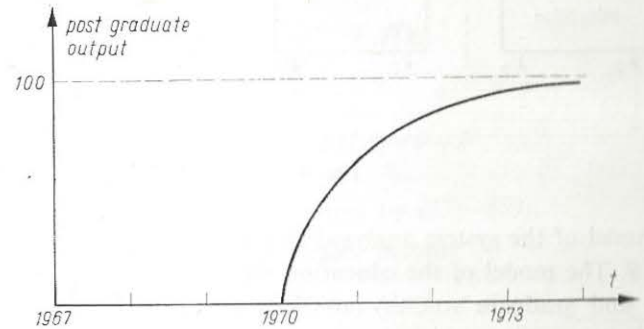

Fig. 11

As far as the output production of the education system is concerned it can be assumed that in the $n-N$ given education branches the expected output demands are known and equal

$$y_{u,N+1}, \ldots, y_{u,n}, y_{g,N+1}, \ldots, y_{g,n};$$

respectively. These numbers can be determined when the future employment in (R+D) and economy sectors is being planned.

The remaining outputs of the education sector are being optimalized in such a way that the integrated output

$$\Phi(x) = \sum_{i=1}^{N} \int_0^T [z_{ui}(t)\, w_{ui}(t) + z_{gi}(t)\, w_{gi}(t)]\, dt$$

is maximum. That concerns first of all these branches of education where the demands is much greater than the possible supply of trained specialists. The weight functions $w_i(t)$ can be assumed in the form

$$w_i(t) = w_i \exp - \gamma_i t, \tag{63}$$

where w_i, γ_i increase along with the respective demands. It is assumed that the numerical values of these numbers are being determined by the higher planning center while the allocation of the outputs can be realized by the methods described in Sec. 3.1.

It should be observed that the sector controller C, which obtains the resources $\bar{X}_{11}, \bar{X}_{31}$ from the cooperating sectors of the model of Fig. 9 can split the existing inputs into a number of new inputs for the lower level subsystems. Assume for example the aggregated sector production function in the form

$$\bar{X}_{11} = F_3^q \bar{X}_{31}^{\alpha_{31}} \bar{X}_{21}^{\alpha_{21}}, \quad q = 1 - \alpha_{31} - \alpha_{21} > 0. \tag{64}$$

Let $\bar{X}_{31}$ be the total financial grant or budget obtained from the economy which may be arbitrarily devided between the maintenance (X_m) and the investments (X_i), i.e.

$$\bar{X}_{31} = X_m + X_i.$$

The new production function can be written in the form

$$\bar{X}_{11} = F_3^{*q} X_m^{\alpha_{31}\beta_1} X_i^{\alpha_{31}\beta_2} X_{21}^{\alpha_{21}}, \tag{65}$$

where according to the Theorem 3 the optimum strategies for X_m, X_i become

$$\bar{X}_m = \beta_1 \bar{X}_{31}, \quad \bar{X}_i = \beta_2 \bar{X}_{31} \tag{66}$$

and

$$F_3^* = [\beta_1^{\beta_1} \beta_2^{\beta_2}]^{\alpha_{31}/q} \tag{67}$$

where

$$\beta_1 + \beta_2 = 1, \quad \beta_1, \beta_2 > 0. \tag{68}$$

The maintenance and investment costs $\bar{X}_m, \bar{X}_i$ should be treated now as fixed and they can be allocated among the corresponding subsystems by the formula (28). Using that approach it is possible to introduce more decision variables at the corresponding levels of the education administrative structure. The maintenance funds can be splitted e.g. into the salaries of the professors and the salaries of the younger teaching staff. The same can be done with respect to the investments which can be divided between the investment in laboratory equipment and new building etc.

Now we can consider the (R+D) sector. In a similar way as if has been done for the education sector a simple model of (R+D) consists of N research fields or activities and $n-N$ development projects. The control or planning center allocates the resources such as e.g. scientific and technical staff, manpower, financial funds, material and equipment among the individual R_i and D_i subsystems. The output z_i generated by R_i assumes mainly the software form, i.e. the form of scientific and technical information, patents, reports, computer algorithms, etc. There are many possible forms of evaluation of z_i intensity. It may be characterized e.g. by the volume of scientific information (measured by the amount of reviewed publications) generated in unit time etc.

The planning center evaluates the research results z_i (obtained mainly in the preceeding planning intervals) and selects a number $(n-N)$ of research projects, costing y_i, $i=N+1, \ldots, n$, respectively, which should be realized within the present planning interval $[0, T]$. The time intervals (T_i) necessary for realization of each project y_i are also determined. Besider the optimum allocation of resources the planning center should also determine the best time shedule for realization of individual projects and research activities.

It should also determine the best relation between research and development activity. It should be observed that most of the existing (R+D) and production systems are nonlinear and dynamic. The output of the industrial production $P(t)$ originated by R and D projects increases along with time and generally speaking it is delayed with respect to the resources cost function $C(t)$ used by (R+D) and production, as shown for a special case in Fig. 12.

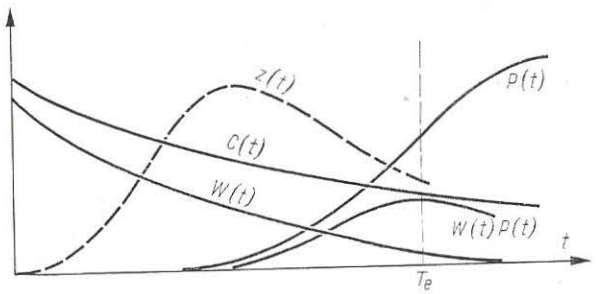

Fig. 12

At the same time the price or weight $w(t)$ attached to the production output decreases monotoneously in time. As a result a time moment $t=T_e$ exists such that the production income $w(T_e)\, P(T_e)$ equals the production cost $C(T_e)$. Arround that time a new production process based on recent (R+D) projects should be originated. The value of research output alone, measured by the number of publications, patents etc. and shown in Fig. 12 by the dotted line, reaches usually a peak value and then decreases in time.

Since the inertial and nonlinear effects are present in the (R+D) systems the input-output operator can be approximated by the operator of the general form (4), (4′). The R system performance measure can be assumed in the form of the integrated weighted research activities $\Phi(x)$ of the form (7). Then the optimum allocation of input resources strategy, which maximizes the performance measure subject to the constraints (5), (16) can be determined by using Theorem 2 and 4. It should be observed that the planning center can change the relation between resources alloted to R and D as well as the individual research fields and projects by changing the weight functions $w_i(t)$.

As follows from (18) the maximum output of (R+D) depends on the parameters of production operator. In particular it is desirable to get small research thresholds and good dynamic characteristic $k_i(T, \tau)$, i.e. the maximum gain and short delays. When the development program (Σy_i) increases the research gain $\Phi(x)$ decreases. Since the new projects can be originated only by using results of research previously done the proportion between research and development should be kept within given limits.

It should be also observed that the global amount of projects and fields of research should be choosen from a larger possible set of projects $n'>n$. That approach enables the selection of the best program of research and development. As shown in the Ref. [13] by neglecting the dynamic effects and assuming $\alpha_j \approx 0$ one can reduce the problem of choosing the best (R+D) program to discreate programming (which can be treated as the first approximation in choosing the optimum planning strategy). It is also possible [13] to extend the method under consideration to the case of multistage and multilevel decentralized (R+D) planning systems.

Much what has been said so far with respect to the education and (R+D) sectors concerns as well the economic sector. We shall not repeat, however, that analysis and concentrate on the determination of the optimum cooperation strategy among the corresponding sectors, shown in Fig. 9.

The problem of optimum cooperation consists in finding $\bar{X}_{rc}=\hat{\bar{X}}_{rc}$, $c, r=1, 2, 3$, $c \neq r$, such that

$$\hat{I}=\max_{\bar{X}_{rc}\in\Omega} (\hat{\bar{X}}_{33}-\bar{X}_{31}-\bar{X}_{32}), \tag{69}$$

where

$$\Omega=\left\{\begin{array}{ll} \bar{X}_{rp}: \bar{X}_{rp}+\bar{X}_{re} \leqslant \hat{\bar{X}}_{rr} & r, p=1, 2, 3, \quad r\neq p \\ \bar{X}_{re}: X_{rp}, X_{re} \geqslant 0, & (p, r, e)\in P \end{array}\right\}$$

$$\bar{X}_{rr}=F_r^{q_r}\, \bar{X}_{cr}^{\alpha_{cr}}\, \bar{X}_{er}^{\alpha_{er}} \quad (p, e, r)\in P \tag{70}$$

$$P=\{(p, e, r): p\neq e,\ e\neq r,\ p\neq r,\ p, r, e=1, 2, 3\}.$$

In other words we want to find the optimum cooperation strategy between sectors with the production function (70) which will produce the maximum net output of the economy.

By introducing the variables $c_{rp}=\bar{X}_{rp}/\hat{\bar{X}}_{rr}$ (69) can be written in the form

$$\hat{I}=\max_{c_{rp}\in\Omega'} \hat{\bar{X}}_{33}(c_{rp})\,(1-c_{31}-c_{32}),$$

$$\Omega'=\left\{ c_{rp}, c_{re}: \begin{array}{ll} 0\leqslant c_{rp}\leqslant 1, & r, p=1, 2, 3, \quad p\neq r \\ c_{rp}+c_{re}\leqslant 1, & (p, e, r)\in P \end{array}\right\}.$$

After simple calculations one gets [14]

$$I=A c_{12}^{-\alpha_{12} D_{23}/D}\, c_{13}^{\alpha_{13} D_{33}/D}\, c_{21}^{\alpha_{21} D_{13}/D}\, c_{23}^{\alpha_{23} D_{33}/D}\, c_{31}^{\alpha_{31} D_{13}/D}\, c_{32}^{-\alpha_{32} D_{23}/D}$$

$$A=F_1^{q_1 D_{13}/D}\, F_2^{-q_2 D_{23}/D}\, F_3^{q_3 D_{33}/D}\,(1-c_{31}-c_{32})$$

where

$$D=\begin{vmatrix} 1 & -\alpha_{21} & -\alpha_{31} \\ -\alpha_{12} & 1 & -\alpha_{32} \\ -\alpha_{13} & -\alpha_{23} & 1 \end{vmatrix} \neq 0,$$

D_{ik}=subdeterminante of the (i, k) element of D.

The optimal values of c_{ij} which maximize I become

$$\hat{c}_{12}=\frac{-\alpha_{12} D_{23}}{d_1}, \qquad \hat{c}_{13}=\frac{\alpha_{13} D_{33}}{d_1},$$

$$\hat{c}_{21}=\frac{\alpha_{21} D_{13}}{d_2}, \qquad \hat{c}_{23}=\frac{\alpha_{23} D_{33}}{d_2},$$

$$\hat{c}_{31}=\frac{\alpha_{31} D_{13}}{D+d_3}, \qquad \hat{c}_{32}=\frac{-\alpha_{32} D_{23}}{D+d_3},$$

$$d_1=\alpha_{13} D_{33}-\alpha_{12} D_{23},\ d_2=\alpha_{21} D_{13}+\alpha_{23} D_{33},\ d_3=\alpha_{31} D_{13}-\alpha_{32} D_{23}.$$

Besides

$$\hat{I}=\frac{\hat{X}_{33} D}{D+d_3}.$$

Given $\hat{c}_{pr}$ and $\hat{X}_{rr}$ the values $\hat{X}_{rp}$, $p, r=1, 2, 3$, $p \neq r$, can be determined by the relation

$$\hat{X}_{rp}=\hat{c}_{rp} \hat{X}_{rr}.$$

It should be noted that after computation of the numerical values of optimum cooperation strategy it may happen that the real cooperation strategy differs from the optimum values. In that case a process of cooperation improvement based, generally speaking, on the subsequent optimum investment strategy can be proposed.

4. Conclusions

As stated in the Introduction the motivation for the research done and presented in the present paper was the improvement of the large-scale and long range planning and decision making using the nonlinear and dynamic modelling approach.

It has been shown that the methodology based on the aggregation concept make it possible to construct an analytic model dealing at each decision level with the amount of variables which corresponds to the information important and available at that level.

That model is not based on the observations of macroprocesses (which is a common practice in the most econometric macromodels) but it is an aggregated structure of concreate production processes. Since all the decision centers are incorporated into the model structure one can investigate the effects of these decisions on the future system development and improve the future forecasting.

The model can be easily extended, if necessary, to incorporate more production plants and decision centers at each sector. The corresponding change in the parameters of the aggregated production function makes it possible to investigate the influence of a concreate investment process on the system output.

The derivation of optimum allocation strategy within each sector is quite simple and the calculation effort does not depend on the number of processes or variables. In order to derive that strategy a number of planned outputs (y_i) should be sent from the control planning center to the sector controllers what is a standard procedure at least in socialist countries.

According to that procedure the outputs proposed by the sector management are being confronted with the corresponding demands for y_i and an eventually corrected set of numbers y_i is accepted as the planned system production.

The weight functions which determine the maximalized sector outputs should be also supplied to the sector controller. They should take into account the social demands for the scarce products existing within the model. The social model welfare function determines also the best allocation strategy of scarce products within the social and regional structures. Changing the weights attached to the maximalized sector outputs it is possible to investigate the corresponding changes in the social dissatisfaction.

The weight coefficients may incorporate as well the prices, which are considered to be given exogeneously (as it is usually assumed in the models of socialist countries). They can be treated as decision variables and the influence of price changes on the system growth can be investigated. A corresponding optimization problem for the best price strategy can be also formulated.

The decentralization of decision atrategies corresponds to the existing administrative and regional management structure. Many of the existing systems of long-range planning can be therefore modeled and incorporated into the model structure.

The derivation of the existing and optimal cooperation strategies among the corresponding sectors may reveal many possible ways of acceleration of the system development. That requires, generally speaking, evaluation of the cost connected with new investments, reemployment and retraining of manpower, automation etc. The complex model under considerations enables to investigate such problems in an analytic and efficient way.

Among the problems which can be investigated by using the present model there are also:

(i) the changes in system growth resulting from redistribution of gross national product among the consumption and investments in different sectors;

(ii) the influence of environment pollution (resulting from the industrial development) and waste treatment costs on the system growth;

(iii) the relation between the education research and development costs and their influence on the system growth, the intersector flow of skilled labor etc.;

(iv) optimization of decisions in the situations when one should decide whether to import new technology, patents and licences or develop them within the existing (R + D) system;

(v) optimization of employment and education within the sectors and regional structure;

(vi) optimization of the processes of exploatation and utilization of natural resources;

(vii) the influence of changes in organization and structure of the social sector on the system performance;

(viii) the effects of changes caused by the redistribution of investments and labor among the sectors (e.g. the industry and agriculture) on the system growth, etc.

The flexibility of the model structure with respect to aggregation of the new sectors and processes enables also the cooperation with the other existing models of a more specific character such as e.g. the econometric, demographic, fuel and energy models etc. The methodology presented here may be also used for further development of these models.

From the analysis carried out it is obvious that the model discussed is especially usefull for investigation of the sectors with highly inertial and nonlinear properties, such as e.g. education research and development, agriculture and forestry, investments etc.

The paper did not consider the influence of the competetive, antagonistic environment, such as e.g. the competition on the capitalistic market etc. on the system development. These problems can be, however, investigated using the same methodology (see Ref. [6]).

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Modelowanie i sterowanie optymalne złożonych systemów reprezentujących środowisko

Praca dotyczy modelowania złożonych systemów nieliniowych dynamicznych, takich jak systemy ekonomiczne, przemysłowe, ekologiczne, społeczne itd. Model składa się z kilku sektorów. Przyjmuje się, że każdy sektor ma strukturę hierarchiczną z zdecentralizowanym systemem zarządzania. Sektor wytwarza określony produkt, a także zużywa część produkcji wyjściowej pozostałych sektorów. Zakłada się, że zależność wyjścia od wejścia sektora ma postać nieliniowego operatora dynamicznego.

Ośrodki decyzyjne sektora dokonują optymalizacji rozdziału zasobów wejściowych w taki sposób, aby maksymalizować produkcję wyjściową. Regulator nadrzędny optymalizuje połączenia kooperacyjne między sektorami.

W pierwszej części pracy omówiono ogólną metodologię budowy modelu i optymalizacji. Druga część pracy zawiera analizę wybranych zagadnień związanych z modelowaniem systemów kształcenia, badań i rozwoju oraz systemu reprezentującego problemy społeczne.

Моделирование и оптимальное управление сложных систем отражающих среду

Работа касается моделирования сложных нелинейных и динамических систем, таких как экономические, промышленные, экологические, общественные и др. системы. Модель состоит из нескольких секторов. Предполагается, что каждый сектор имеет иерархическую структуру с децентрализованной управленческой системой управления. Сектор производит определенный продукт, а также использует часть выходного продукта остальных секторов. Предполагается, что зависимость между выходом и входом имеет див нелинейного динамического оператора.

Центр принятия решений сектора производит оптимизацию раздела входных ресурсов таким образом, чтобы максимизировать выходное произвоство. Регулятор высшего уровня оптимизирует связи кооперации между секторами.

В первой части работы рассмотрена общая методология построения модели и оптимизации. Вторая часть работы содержит анализ некоторых вопросов связанных с моделированием систем образования, исследований и развития а также системы отражающей общественные проблемы.