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An application of a tense-logic system to formal planning*

by

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There are considered the basic assumption of a so called time-interval logic being a modification of a tense-logic introduced by A. N. Prior. The time-interval logic makes it possible to formalize the description of the basic time-relations between the technological operations considered as the elements of an operational net being planned. The proving of the internal consistency of the assumptions concerning the causal relations between the operations is thus possible on the basis of some time-interval logic inference rules. Some formal logical properties making it possible to prove the consistency of assumptions of a plan have been derived. The paper contributes to the theory of problem-oriented formal languages for the solution of formal planning tasks using digital computers.

1. Genera emarks

The role of modern mathematical tools in economy and in technology is still increasing. Formal models can not describe the reality perfectly, however, the gap between the reality and its formal description can be diminished by an iterative process of specialization of the mathematical concepts.

Our attention will be paid to the problems of formal planning of composite operational nets in technology and in economy. A formal plan is an approximating model of a real process that consists of a set of mutually related actions or operations located in time. It will be here supposed that an ordered triple

$$P = [A, R, T] \tag{1}$$

where A is a finite set of "actions" a_i, $i \in \overline{[1, I]}$, R is a formal relation described on the set A and corresponding to the causal relationships between actions and T is a time-order introduced into the set A, is the most simplified example of a formal plan under consideration.

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It will be also supposed that R is a quasi-ordering (reciprocal and transitive) relation. Thus, for any two elements a_i, $a_j \in A$ one and only one of the following situations is possible:

— a_i and a_j are mutually irrelative,

— a_i and a_j are relative, that means that at least one of the following situations holds:

$a_i \overset{R}{\to} a_j$ (read: "a_i causes a_j") or $a_j \overset{R}{\to} a_i$.

It is also possible that both $a_i \overset{R}{\to} a_j$ and $a_j \overset{R}{\to} a_i$ are true.

More rigorous restrictions will be imposed on the time-ordering relation T: this will be in addition supposed to be antisymmetrical, that is in the case if both $a_i \overset{T}{\to} a_j$ and $a_j \overset{T}{\to} a_i$ hold, the actions a_i and a_j will be supposed to go simultaneously, this will be denoted by

$$a_i \overset{T}{\leftrightarrow} a_j.$$

However, the time-irrelativeness of two or more actions is also admissible. Thus, T is a specific semi-ordering relations imposed on the set A.

The relations R and T will be called mutually consistent if for any a_i, $a_j \in A$

$$(a_i \overset{R}{\to} a_j) \Rightarrow (a_i \overset{T}{\to} a_j), \tag{2}$$

where $\Rightarrow$ denotes a logical implication.

The formal plan P will be called admissible if the relations R and T are mutually consistent. So as the relation R is usually given in the form of a set of technological or organizational restrictions imposed on the real problem being considered, the problem arises of choising a time-ordering relation T consistent with the given R and satisfying to some criteria of optimality. The criterion of the minimum time-duration of the plan is one of the most important in practice.

Our aim is to propose a formal tool for proving the consistency of plans as well as for generating the plans consistent with any given relation R, more convenient than a point-by-point proving of consistency. The aim can be reached, in particular, if the commonly used terms like "during", "after", "before", "not later than" and so on are formalized up to forming an algebraical system of time-relations. This general idea was firstly proposed in 1929 by R. Carnap and then developped by A. N. Prior and others, who investigated the so called tense-logic systems. We shall start with a very brief description of the Prior's system, before going to more detailed considerations on the application of tense-logics in formal planning.

2. Introduction to tense-logic systems

Let x_i denote a proposition; for the sake of concreteness the proposition "the action a_i is performed" will be here meant. A logical value 1 can be prescribed to the proposition if it is true and the logical value 0 otherwise. The fact that the logical values are time-invariant is a source of some disadvantages if the classical propositional calculus is to be used for the description of changing states of a dynamical system. This difficulty can be passed by introducing a so called date-operator (A. N. Prior [2], see also A. A. Ivin [1]): $D_t x$ for $t \in \mathcal{T}$; $\mathcal{T}$ being a real time-axis, satisfying to the following assumptions:

$$D_t(\neg x) \Rightarrow \neg D_t x, \tag{3a}$$

the symbol $\neg$ being used for the logical negation,

$$D_t(x' \Rightarrow x'') \Rightarrow (D_t x' \Rightarrow D_t x''), \tag{3b}$$

$$[\forall(t) D_t x] \Rightarrow x, \tag{3c}$$

$$D_{t''}(D_{t''} x) \Rightarrow D_{t'+t''} x. \tag{3d}$$

The operator D_t transforms any proposition x into a time-variant one read as "x at the time t" and having the logical value 1 if and only if x is true at the time t. Let us remark, that $D_t x$ does not change its logical value in the time.

Using the general quantifier $\forall$ some new logical operators can be defined on the basis of the date-operator as follows:

$$G_t x = \forall (t' < t) D_{t'} x \quad \text{for} \quad t', t \in \mathcal{T} \tag{4}$$

can be interpreted as a "strong-past operator" changing a proposition x into a one having value 1 if and only if $D_{t'} x$ is true for all $t' < t$. In a similar way,

$$H_t x = \forall (t' > t) D_{t'} x \quad \text{for} \quad t, t' \in \mathcal{T} \tag{5}$$

can be interpreted as a "strong-future operator".

Similarly, the "weak-past" and "weak-future" operators can be defined using the particular quantifier:

$$P_t x = \exists (t' < t) D_{t'} x \quad \text{for} \quad t', t \in \mathcal{T}, \tag{6}$$

transforms the proposition x into a one being true if and only if there exists such a $t' < t$ that $D_{t'} x$ is true. Similarly,

$$F_t x = \exists (t' > t) D_t x \quad \text{for} \quad t', t \in \mathcal{T} \tag{7}$$

acts in the same way on x if and only if there exists such a $t' > t$ that $D_{t'} x$ is true. It is clear that the expressions $G_t x$, $H_t x$, $P_t x$ and $F_t x$ can be read as "it was always x before t", "it will be always x after t", "it was sometimes x before t" and "it will be sometimes x after t", correspondingly.

The past-tense and the future-tense logics are in formal sense mutually symmetrical. According to A. N. Prior, the future-tense logic is based on the following assumptions:

(a) on the axioms of the classical propositional calculus,

(b) on the definition (7) of the "weak-future" operator,

(c) on the following additional assumptions:

$$F_t F_t x \Rightarrow F_t x, \tag{8a}$$

$$F_t(x' \vee x'') = F_t x' \vee F_t x'', \tag{8b}$$

$$[\rightarrow U] \Rightarrow [\rightarrow (H_t U)], \tag{8c}$$

$$[\rightarrow (U \equiv V)] \approx \Rightarrow \rightarrow (F_t U \equiv F_t V), \tag{8d}$$

where $\vee$, $\equiv$ stand for the logical alternative and equivalence, correspondingly, and $\rightarrow U$ denotes any logically deducible expression U.

By changing the operators F_t by P_t and H_t by G_t the "past-tense logic" can be transformed into a "future-tense" one. However, a general "tense logic" in addition to the rules govering in the "past-tense" and in the "future-tense" logics, according to A. N. Prior, should satisfy to a.set of reduction formulae:

	H_tG_t	P_tG_t	G_t	F_tG_t	F_t	H_tF_t	P_tF_t	G_tF_t
P_t	H_tG_t	H_tG_t	P_tG_t	F_tG_t	P_tF_t	P_tF_t	P_tF_t	G_tF_t
H_t	H_tG_t	H_tG_t	H_tG_t	F_tG_t	H_tF_t	H_tF_t	P_tF_t	G_tF_t
F_t	H_tG_t	F_tG_t	F_tG_t	F_tG_t	F_t	F_t	P_tF_t	G_tF_t
G_t	H_tG_t	G_t	G_t	F_tG_t	G_tF_t	G_tF_t	P_tF_t	G_tF_t

Thus, for example, an expression

$P_tH_tG_tG_tx$ = "it was sometimes before t that it was always before t that it will be always after t that it will be always after t that x" can be reduced as follows:

$$P_t\big(H_t(G_t(G_tx))\big) \equiv P_t(H_t(G_tG_tx)) \equiv P_t(H_t(G_tx)) \equiv P_t(H_tG_tx) \equiv H_tG_tx$$

and finally we obtain a proposition: "it was always before t that it will be always after t that x".

We shall not go into more detailed considerations of the Prior's tense logic, because it is no more but a starting point for our purposes.

3. Time-interval logic

It is desirable for the description of operational nets to have a logical system that prescribes to the propositions some logical values during finite time-intervals.

For any $t', t'' \in \mathcal{T}$, $t' < t''$, we shall define the logical operators:

$d_{t',t''}\, x$ — a "weak time-interval operator" that transforms any proposition x into a one being true if and only if there is such a $t \in \mathcal{T}$, $t' \leqslant t \leqslant t''$, that x is true at the time t and it is false for all $t < t'$ and for all $t > t''$; $D_{t',t''}\, x$ — a "strong time-interval operator" transforming any proposition x into a one being true if and only if for all $t \in \mathcal{T}$, $t' \leqslant t \leqslant t''$, x is true at the time t and is false for $t < t'$ and for $t > t''$.

It is clear that the Prior's operators can be derived in particular cases:

$$d_{t,t}x = D_{t,t}x = D_tx, \tag{9a}$$

$$D_{-\infty,t}x = G_tx, \tag{9b}$$

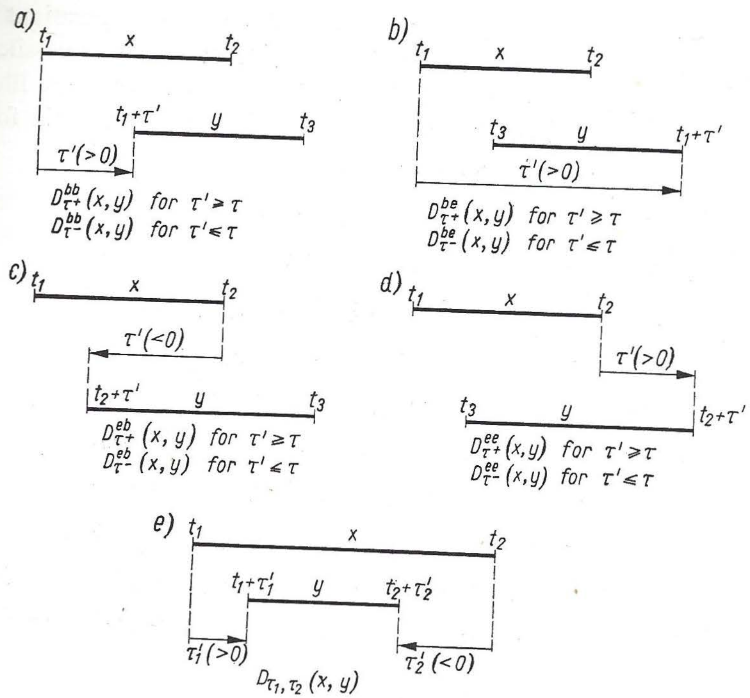
$$D_{t,+\infty}x = H_tx, \tag{9c}$$

$$d_{-\infty,t}x = P_tx, \tag{9d}$$

$$d_{t,+\infty}x = F_tx \tag{9e}$$

and in general

$$D_{-\infty,+\infty}x \Rightarrow x \Rightarrow d_{-\infty,+\infty}x. \tag{9f}$$



Possible mutual positions of two time-intervals

The following assumptions concerning the "time-interval operators" will be made:

(a) the axioms of the classical propositional calculus;

(b) the rules of the classical logical inference;

(c) for any $t' \leqslant t''$ and for any propositions x, y

$$D_{t',t''}x \Rightarrow d_{t',t''}x, \tag{10a}$$

$$\neg d_{t',t''}x \Rightarrow \neg D_{t',t''}x, \tag{10b}$$

$$[D_{t_1't_1''}x \wedge D_{t_2',t_2''}y] \Rightarrow D_{t',t''}(x \wedge y), \tag{10c}$$

where

$t' = \max(t_1', t_2')$,

$t'' = \min(t_1'', t_2'')$,

$$[d_{t_1',t_1''}x \vee d_{t_2',t_2''}y] \Rightarrow d_{t',t''}(x \vee y) \tag{10d}$$

where

$t' = \min(t_1', t_2')$,

$t'' = \max(t_1'', t_2'')$;

(d) for any $A \in \{D, d\}$ and for any $B \in \{D, d\}$

$$A_{t_1',t_1''}B_{t_2',t_2''}x = B_{t_2',t_2''}x. \tag{10e}$$

The "time-interval operators" make it possible to consider the operations located in an absolute time. A lot of practical problems lead us to the relative time-allocations of the operations. The main mutual positions of two time-intervals are illustrated in the Figure. According to this, if two propositions x and y are given, the following eight "relative-time operators" can be defined:

$$D^{be}_{\tau+}(x, y) = \exists\,(\tau' \geqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_3,t_1+\tau'}y), \tag{11a}$$

$$D^{be}_{\tau-}(x, y) = \exists\,(\tau' \leqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_3,t_1+\tau'}y), \tag{11b}$$

$$D^{bb}_{\tau+}(x, y) = \exists\,(\tau' \geqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_1+\tau',t_3}y), \tag{11c}$$

$$D^{bb}_{\tau-}(x, y) = \exists\,(\tau' \leqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_1+\tau',t_3}y), \tag{11d}$$

$$D^{eb}_{\tau+}(x, y) = \exists\,(\tau' \geqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_2+\tau',t_3}y), \tag{11e}$$

$$D^{eb}_{\tau-}(x, y) = \exists\,(\tau' \leqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_2+\tau',t_3}y), \tag{11f}$$

$$D^{ee}_{\tau+}(x, y) = \exists\,(\tau' \geqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_3,t_2+\tau'}y), \tag{11g}$$

$$D^{ee}_{\tau-}(x, y) = \exists\,(\tau' \leqslant \tau)\,\exists\,(t_1, t_2, t_3)(D_{t_1,t_2}x \wedge D_{t_3,t_2+\tau'}y), \tag{11h}$$

The operator $D^{be}_{\tau+}(x, y)$ transforms the propositions x, y into a one being true if and only if x is true inside a time-interval $[t_1, t_2]$ and y is true inside a time-interval $[t_3, t_4]$ where

$$t_4 \geqslant t_1 + \tau. \tag{12}$$

Thus, $D^{be}_{\tau+}(x, y)$ can be read as "the beginning of the time-interval of x is at least τ earlier than the end of the time-interval of y". Similarly, the operator $D^{be}_{\tau-}(x, y)$ can be read as "the beginning of the time-interval of x is at most τ earlier than the end of the time-interval of y". The meaning of the others operators can be explained in a similar way.

The more complicated situations can be described using the above-given "relative-time operators". For example, an expression

$$D_{\tau_1,\tau_2}(x, y) = D^{bb}_{\tau_2^+}(x, y) \wedge D^{ee}_{\tau_2^-}(x, y) \tag{13}$$

for some $\tau_1 > 0$, $\tau_2 < 0$, describe a logical operator that transforms the propositions x, y into a one which is true if and only if x is true inside a time-interval whose beginning is at least τ_1 earlier than the beginning of the time-interval of y and whose end is at least τ_2 later than the end of the time-interval of y.

The stronger forms of the "relative-time operators" can be also defined:

$$D^{be}_{\tau}(x, y) = D^{be}_{\tau+}(x, y) \wedge D^{be}_{\tau-}(x, y), \tag{14a}$$

$$D^{bb}_{\tau}(x, y) = D^{bb}_{\tau+}(x, y) \wedge D^{bb}_{\tau-}(x, y), \tag{14b}$$

$$D^{eb}_{\tau}(x, y) = D^{eb}_{\tau+}(x, y) \wedge D^{eb}_{\tau-}(x, y), \tag{14c}$$

$$D^{ee}_{\tau}(x, y) = D^{ee}_{\tau+}(x, y) \wedge D^{ee}_{\tau-}(x, y). \tag{14d}$$

The weak analogues of the "relative-time operators" based on the "weak time-interval operator" $d_{t',t''}$, probably, would be of less practical importance.

The following properties of the "relative-time operators" can be easily proven:

$$D^{be}_{\tau+}(x, y) = D^{eb}_{-\tau-}(y, x), \tag{15a}$$

$$D^{be}_{\tau-}(x, y) = D^{eb}_{-\tau+}(y, x), \tag{15b}$$

$$D^{bb}_{\tau+}(x, y) = D^{bb}_{-\tau-}(y, x), \tag{15c}$$

$$D^{ee}_{\tau+}(x, y) = D^{ee}_{-\tau-}(y, x). \tag{15d}$$

There will be also useful the following "time-interval duration" operators: for any $\tau > 0$

$$\vartheta^{+}_{\tau}x = \exists\,(t', t'')[d_{t',t''}x \wedge (t''-t') > \tau], \tag{16a}$$

$$\vartheta^{-}_{\tau}x = \exists\,(t', t'')[d_{t',t''}x \wedge (t''-t') < \tau], \tag{16b}$$

$$\Theta^{+}_{\tau}x = \exists\,(t', t'')[D_{t',t''}x \wedge (t''-t') > \tau], \tag{16c}$$

$$\Theta^{-}_{\tau}x = \exists\,(t', t'')[D_{t',t''}x \wedge (t''-t') < \tau]. \tag{16d}$$

For proving the consistency of the formal plans the following semi-group properties of the above-defined operators will be available:

$$[D^{bb}_{\tau_1^+}(x, y) \wedge D^{bb}_{\tau_2^+}(y, z)] \Rightarrow D^{bb}_{\tau_1+\tau_2^+}(x, z), \tag{17a}$$

$$[D^{bb}_{\tau_1^-}(x, y) \wedge D^{bb}_{\tau_2^-}(y, z)] \Rightarrow D^{bb}_{\tau_1+\tau_2^-}(x, z), \tag{17b}$$

$$[D^{ee}_{\tau_1^+}(x, y) \wedge D^{ee}_{\tau_2^+}(y, z)] \Rightarrow D^{ee}_{\tau_1+\tau_2^-}(x, z), \tag{18a}$$

$$[D^{ee}_{\tau_1^-}(x, y) \wedge D^{ee}_{\tau_2^-}(y, z)] \Rightarrow D^{ee}_{\tau_1+\tau_2^-}(x, z), \tag{18b}$$

$$[D^{be}_{\tau_1^+}(x, y) \wedge D^{ee}_{\tau_2^+}(y, z)] \Rightarrow D^{be}_{\tau_1+\tau_2^+}(x, z), \tag{19a}$$

$$[D^{be}_{\tau_1^-}(x, y) \wedge D^{ee}_{\tau_2^-}(y, z)] \Rightarrow D^{be}_{\tau_1+\tau_2^-}(x, z), \tag{19b}$$

$$[D^{be}_{\tau_1^+}(x, y) \wedge D^{bb}_{\tau_2^+}(y, z)] \Rightarrow D^{eb}_{\tau_1+\tau_2^+}(x, z), \tag{20a}$$

$$[D^{eb}_{\tau_1^-}(x, y) \wedge D^{bb}_{\tau_2^-}(y, z)] \Rightarrow D^{eb}_{\tau_1+\tau_2^-}(x, z), \tag{20b}$$

$$[D^{bb}_{\tau_1^+}(x, y) \wedge D^{be}_{\tau_2^+}(y, z)] \Rightarrow D^{be}_{\tau_1+\tau_2^+}(x, z), \tag{21a}$$

$$[D^{bb}_{\tau_1}(x, y) \wedge D^{be}_{\tau_2^-}(y, z)] \Rightarrow D^{be}_{\tau_1+\tau_2^-}(x, z), \tag{21b}$$

$$[D^{be}_{\tau_1^+}(x, y) \wedge D^{eb}_{\tau_2^+}(y, z)] \Rightarrow D^{bb}_{\tau_1+\tau_2^-}(x, z), \tag{22a}$$

$$[D^{be}_{\tau_1^-}(x, y) \wedge D^{be}_{\tau_2^-}(y, z)] \Rightarrow D^{bb}_{\tau_1+\tau_2^-}(x, z), \tag{22b}$$

$$[D^{eb}_{\tau_1^+}(x, y) \wedge D^{be}_{\tau_2^+}(y, z)] \Rightarrow D^{ee}_{\tau_1+\tau_2^+}(x, z), \tag{23a}$$

$$[D^{eb}_{\tau_1^-}(x, y) \wedge D^{be}_{\tau_2^-}(y, z)] \Rightarrow D^{ee}_{\tau_1+\tau_2^-}(x, z), \tag{23b}$$

$$[D^{ee}_{\tau_1^+}(x, y) \wedge D^{eb}_{\tau_2^+}(y, z)] \Rightarrow D^{eb}_{\tau_1+\tau_2^+}(x, z), \tag{24a}$$

$$[D^{ee}_{\tau_1^-}(x, y) \wedge D^{eb}_{\tau_2^-}(y, z)] \Rightarrow D^{eb}_{\tau_1+\tau_2^-}(x, z), \tag{24b}$$

$$[D^{bb}_{\tau_1^+}(x, y) \wedge \Theta^{+}_{\tau_2}(y)] \Rightarrow D^{eb}_{\tau_1+\tau_2^-}(y, x), \tag{25a}$$

$$[D^{bb}_{\tau_1^-}(x, y) \wedge \Theta^{-}_{\tau_2}(y) \Rightarrow D^{be}_{\tau_1+\tau_2^-}(x, y), \tag{25b}$$

$$[D^{ee}_{\tau_1^+}(x, y) \wedge \Theta^{-}_{\tau_2}(y)] \Rightarrow D^{eb}_{\tau_1-\tau_2^+}(x, y), \tag{26a}$$

$$[D^{ee}_{\tau_1^-}(x, y) \wedge \Theta^{+}_{\tau_2}(y)] \Rightarrow D^{eb}_{\tau_1-\tau_2^-}(x, y), \tag{26b}$$

$$[D^{be}_{\tau_1^+}(x, y) \wedge \Theta^-_{\tau_2}(y)] \Rightarrow D^{bb}_{\tau_1-\tau_2^+}(x, y), \tag{27a}$$

$$[D^{be}_{\tau_1^-}(x, y) \wedge \Theta^+_{\tau_2}(y)] \Rightarrow D^{bb}_{\tau_1-\tau_2^-}(x, y), \tag{27b}$$

$$[D^{eb}_{\tau_1^+}(x, y) \wedge \Theta_{\tau_2^+}(y)] \Rightarrow D^{ee}_{\tau_1+\tau_2^+}(x, y), \tag{28a}$$

$$[D^{eb}_{\tau_1^-}(x, y) \wedge \Theta_{\tau_2^-}(y)] \Rightarrow D^{ee}_{\tau_1+\tau_2^-}(x, y); \tag{28b}$$

in addition, if we put in (13)

$$D_{\tau_1,\tau_2}(x, y) = D_{-\tau_3,-\tau_4}(y, x), \qquad \tau_1 > 0, \qquad \tau_2 < 0, \tag{29}$$

we also obtain

$$D_{\tau_1,\tau_2}(x, y) \wedge D_{\tau_1,\tau_2}(y, z) \Rightarrow D_{\tau_1+\tau_3,\tau_2+\tau_4}(x, z). \tag{30}$$

4. The application to formal planning

The possible applications of the time-interval logic to the formal planning are based on several rules of logical inference in the time-relations domain. In particular, the transitive property of the implication:

$$(x \Rightarrow y) \wedge (y \Rightarrow z) \Rightarrow (x \Rightarrow z) \tag{31}$$

will be used.

Let us remark that on the basis of the formulae (15 a-d) and (25)–(28) the complementary relations can be derived, as for example, from (25b), (15a) and (15c) we obtain:

$$D^{bb}_{\tau_1^-}(x, y) \wedge \Theta^-_{\tau_2}(y) \Rightarrow D^{be}_{\tau_1+\tau_2^-}(x, y),$$

$$D^{bb}_{-\tau_1^+}(y, x) \wedge \Theta^-_{\tau_2}(y) \Rightarrow D^{eb}_{-\tau_1-\tau_2^+}(y, x),$$

and

$$\Theta^-_{\tau_2}(x) \wedge D^{bb}_{\tau_3^+}(x, y) \Rightarrow D^{eb}_{\tau_3-\tau_2^+}(x, y) \tag{32}$$

which is a formula complementary to the (25b).

Let us take into account a homogeneous chain of pairwise adjusted logical alternatives

$$Q = D^{a\alpha}_{\tau^*}(x, u) \wedge D^{\alpha\beta}_{\tau_\beta^*}(u, v) \wedge \ \dots \ \wedge D^{\varrho r}_{\tau_r^*}(w, y) \tag{33}$$

where

$$a, \alpha, \beta, \dots, \varrho, r \in \{e, b\}, \tag{33a}$$

$$* \in \{+, -\}, \tag{33b}$$

$x, u, v, \dots, w, y$ being some propositions. Using the formulae (17)–(24) we get

$$Q \Rightarrow D^{ar}_{\tau_\alpha+\tau_\beta+\ \cdots\ +\tau_r^*}(x, y) \tag{34}$$

In a more general case the non-homogenous chains containing the operators. $D_{\tau_1,\tau_2}(x, y)$ or $\Theta^*_\tau(x)$ besides the $D^{ab}_{\tau *}(x, y)$ ones can occur when the time-relations in a set of technological actions are considered. The non-homogenous chains will be called pairwise adjusted if they can be transformed into the pairwise adjusted homogenous chains using the formulae (13), (15a–d), (29) or other logical identities

For example, a chain

$$Q = D^{eb}_{\tau_1}(u, x) \wedge D_{\tau_2,\tau_3}(u, v) \wedge D^{be}_{\tau_4^+}(v, y)$$

can be transformed as follows:

$$Q = D^{be}_{-\tau_1^+}(x, u) \wedge D^{ee}_{\tau_3^+}(u, v) \wedge D^{bb}_{\tau_2^+}(u, v) \wedge D^{be}_{\tau_4^+}(v, y)$$

and thus

$$Q \Rightarrow D^{e}_{-\tau_1+\tau_2+\tau_3+\tau_4^+}(x, y).$$

Now, the practical importance of the time-interval logic for the formal plannig can be explained. When a set of time-relations is given in the form of some time-interval logic expressions and some of them can be joined into pairwise adjusted chains, the rules of logical inference can be used in order to prove the consistency of the set of relations. The conclusions can not contradict to each other. On the other hand, for some pairs of actions no time-relation can be deducible on the basis of the time-interval logic rules. Such pairs of actions will be characterized by some degrees of freedom of their mutual allocation in time.

The time-interval logic can be also considered as a formal tool for proving the semantical correctness of the expressions of a language of time-relations description. Therefore, the time-interval logic investigations can be considered as a part of a more general problem of problem-oriented languages for operations planning.

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Zastosowanie logiki następstw czasowych w formalnym planowaniu operacji

Przedstawiono podstawowe założenia tzw. logiki przedziałów czasowych będącej modyfikacją logiki następstw czasowych wprowadzonej przez A. N. Priora. Logika przedziałów czasowych umożliwia sformalizowanie zapisu podstawowych relacji czasowych zachodzących między operacjami technologicznymi stanowiącymi elementy planowanej sieci operacji. Na podstawie reguł wnioskowania logicznego staje się wówczas możliwe formalne sprawdzenie wewnętrznej niesprzeczności układu założeń dotyczących następstw przyczynowo-skutkowych operacji przez sprawdzenie niesprzeczności układu odpowiednich relacji czasowych zachodzących między operacjami. Wyprowadzono ciąg formalnych zależności logicznych mogących ułatwić dokonanie podobnej analizy niesprzeczności wewnętrznej założeń. Praca stanowi przyczynek do teorii sformalizowanych języków zorientowanych problemowo na rozwiązywanie zadań z dziedziny planowania operacji przy użyciu maszyn cyfrowych.

Применение логики наследства времени к формальному планированию операций

Рассматриваются основные предположения т. н. логики времени введеной А. Н. Прио- ром. Логика интервалов времени делает возможной формализацию записи основных временных соотношений между технологическими операциями являющимися элементами планированной сети операций. На основании правил логического вывода становится возможной формальная проверка внутренней непротиворечивости системы предположений относящихся к причинно-следственным соотношениям между операциями, путем проверки соответствующих временных соотношений. Выведен ряд формальных логических правил облеггающих вышеуказанный анализ внутренних непротиворечимостей. Работа относится к теории формализованных языков ориентированных на решение задач из области планирования операций с помощью эвм.