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State variable approach to the study of hydraulic systems*

by

ANTONIO LEPSCHY
University of Padea
Institute of Electrical Engineering and Electronics

SERGIO MILO, DARIO TORRIANO
University of Trieste
Institute of Electrical Engineering and Electronics

This paper suggests a procedure for the derivation of state models for the most interesting hydraulic systems.

The aims of such a work are similar, from some point of view, to the ones taken into consideration in other fields, mainly in that of electrical networks.

In this paper only systems formed by free level elements are taken in consideration. Particularly hydraulic systems formed by catchment areas, stretches of rivers and canals, storage ponds, etc. are considered. For the study of these systems a previous work of discretization is needed and thereafter the mathematical models of various elements are derived; usually by utilizing the storage equation.

The models obtained in such a way may be used directly for simulation purposes or may be manipulated in order to obtain other realizations more suitable for other aims. By means of these models it is possible to study such characteristic problems of hydraulic system engineering as forecasting of the behaviour under different environmental conditions and under different conduction policies, forecasting of the effects of network modifications, solution of optimization problems concerning the best exploitation of the system for by hydroelectrical or for agricultural uses, for inland navigation etc.

1. Preliminary considerations

Our report deals with the construction of a state variable model for the study of hydraulic, free level systems, with special regard to networks of canals.

It may be of some interest to discuss beforehand the convenience of studying the hydraulic systems in such a way, with reference to the following points:

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(i) advantages of the adopted approach (i.e. efficiency of the use of state variables for studying technical problems in the field of the hydraulic systems and/or conceptual interest of some aspects of such a study);

(ii) relations between the adopted approach and the conventional one and between the use of the concepts of systems theory in the field of hydraulic networks and in the field of electrical networks.

1.1. Advantages of the state variable approach

Concerning the first problem, the following considerations have to be kept in mind.

1.1.1. Advantages of the suggested model for optimization and simulation purposes. The hydraulical resources are exploited more and more intensively and the problem of their exploitation has to be considered within the more general question of land use and system engineering. In such a situation complex optimization problems arise.

For instance let us consider the problems arising for the simultaneous exploitation of the waters for purposes of power generation, irrigation and supply of networks of navigable canals and also for regulating the natural water system (control and prevention of floods etc.); to these usual purposes of water exploitation has often to be added today the one regarding the fields of tourism and of amusement activities (for instance in the case of the use of artificial basins also for water sports and for the touristic utilization of the surrounding areas).

Each one of the mentioned utilizations exhibits its own objectives and its own contraints (in the last mentioned case, for instance, strict constraints on the excursion of the level have to be respected).

Moreover, by facing the problem of resources allocation corresponding to the above mentioned water exploitations, it is necessary to keep in mind that the different subsystems are also to be integrated into other large scale systems (i.e. the system of power generation, in which thermoelectrical and nuclear plants are also present, the system of the agricultural exploitations, the transporation system, the system concerning the hydrogeological soil defense, the one of the touristic activities etc.).

Optimization problems of such a type cannot be studied satisfactorily and in quantitative terms without a suitable model.

For many hydraulic works physical models based on the similitude theory have been largely adopted and their use it still of technical interest: but, in general, they are not suitable for the study of the above mentioned optimization problems: hence the interest for the mathematical models and for their implementation on digital computers.

A mathematical model to be used for the above mentioned purposes might also be set up without explicit reference to state variable techniques; at present, however, it may be stated that the use of state variables for setting up the mathematical model of a plant is a very efficient tool for purposes of optimization and simulation (i.e. for the implementation of the model on a digital computer, with special reference to the methods of digital simulation of continuous processes and to the use of special languages).

Simulation problems are of a considerable interest even if a theoretical approach to the optimization problem is not possible. Often, indeed, the problem to be faced cannot be considered, strictly speaking, an optimization problem but, more precisely, the problem of comparing some conduction policies of the plant and of choosing the best one (because the few policies under consideration are technically the only possible ones or because they may be considered the best ones, among those of the same class, on the basis of empirical considerations).

With reference to problems of such a type, let us consider a case different from the ones previously mentioned, that is the case of a mechanical hoisting reclamation. Usually the water is delivered from the basin by means of punping stations equipped with one or more fixed trim, constant speed pumps. The conventional solution adopted for the control problem is therefore the one of adopting an on-off or step by step control, according to the number of the pumps. Other techniques are also possible, for instance using more expensive but more efficient variable trim and/or variable speed pumps; moreover it is possible to resort to other conduction policies (for instance evacuating the basin before the flood reaches its maximum effect or taking into account the particular fares for the delivery of electrical supply). In such a condition it is evident that the problem of choosing the more convenient solution may hardly be approached by means of optimization techniques, but the use of a good mathematical model of the reclamation plant and in general of suitable techniques of simulation is a very efficient tool for comparing, from a technical point of view, the effects of different control techniques and of different devices and, ultimately, for choosing the best ones.

Considerations of the same type are valid, on the other hand, also for the other problems previously mentioned.

In conclusion, it may be assumed that for the solution of many important technical problems the advantages of the use of state variable models are relevant.

1.1.2. Theoretical relevance of the model. The situation is slightly different regarding the conceptual interest of the problem.

Indeed the state variable approach of the study of hydraulic systems does not face the system theory with new problems. On the other hand, however, such a study must not be considered as devoid of conceptual interest because it is not an obvious extension of the electrical network theory, the importance of nonlinear phenomena is always considerable and some problems arise that may be considered interesting also from a theoretical point of view, such as the one of the dimension variation of the state vector in the non autonomous systems (in the hydraulic meaning of the term).

1.2. Relations between the suggested approach and the conventional ones

Concerning the second problem mentioned in Sec. 1.1, the question has to be examined if the state variable approach of the study of hydraulic systems allows us to obtain only results strictly corresponding to the ones obtained in other fields of study, and consists only in the use of a different terminology for the presentation of such results.

From this point of view the approach here considered has to be compared with conventional technical hydraulics, with the topics if analytical mechanics referring to hydraulic problems and with the system approach of the study of electric networks.

1.2.1. Purposes and characteristics of the suggested approach. The approach here considered aims at constructing mathematical models linking the time behaviour of some relevant quantities of the system (i.e. their outputs, for instance flows at given sections or levels at given points) to the input quantities (and ultimately to the rainfall) through the state variables. The relations that represent these links must be as simple as possible and their parameters have to be connected with measurable physical quantities.

1.2.2. Problems and methods here considered versus the ones of technical hydraulics. Technical hydraulics may be regarded as a branch of the engineering sciences, the main aim of which is the one of designing, by resorting chiefly to empirical methods, hydraulic works (for instance canals) suitable for a satisfactory operation even in the most dangerous situations which use has decided to take into account (for instance, in the case of canals, aptitude to deliver given values of flood flows). It is evident, therefore, that aims and methods of conventional technical hydraulics greatly differ from the ones connected with the approach here adopted. It may also be considered, however, that from our approach methods might be derived which are suitable also for design problems (for new plants or for the modification of existing ones) because of the physical (geometrical, geological etc.) meaning of the parameters of the derived equations.

1.2.3. The state variable approach versus the one of analytical mechanics. Analytical mechanics supplies general equations for the study of the motion of fluids; by integrating such equations it would be possible to evaluate the behaviour of the quantities here considered. But this methods, generally speaking, does not appear the most convenient one. Indeed the integration of the partial derivatives equations supplied by analytical fluid mechanics very seldom is obtainable in closed form, and elsewhere it is too cumbersome mainly because this approach, by its very nature, supplies directly a mass of data greater by far than the one needed for the purposes here relevant. In conclusion, analytical mechanics by means of its general laws supplies the bases for the approach here adopted, but the latter cannot be considered as an obvious extension of the procedures of the former.

1.2.4. The state variable approach in the field of hydraulics and in the field of electrical engineering. Let us consider, finally, the relations between the system approach to the study of electrical networks and that of hydraulic networks. At the time of the first developments of electrical engineering the usual procedure was to present the concepts of the connected topics by resorting to electro-hydraulic analogy, because hydraulic notions (i.e. flow and head) seem to be more evident than electric ones (i.e. current and voltage); today, on the contrary, electrical engineers (and also system and control engineers formed in the field of electrical and electronics engineering) like to resort to an electrical interpretation of hydraulic problems for a better understanding of them. From this point of view it would seem that the state variable approach to the study of electrical networks, which forms by now a consistent and widespread topic, might supply a good basis for an "obvious" utilization of the same concepts in the case of hydraulics: in this case, indeed, the problems may appear more simple than in the electrical one because the phenomena corresponding to selfinduction are almost always negligible and the ones corresponding to mutual induction are not present; moreover, the hydraulic networks are usually simpler than the electric ones and often there are no meshes, i.e. there is a convergent or dirvengent tree structure.

In fact, the situation is not so; indeed for the study of canal networks and if one adopts a lumped parameter model the basic elements (see later) cannot be interpreted like resistances or like capacities; let us consider, for instance, a very simple but relevant case, that is the one of a stretch of canal in an autonomous condition; in this case the flow cannot be evaluated by means of a law corresponding to the Ohm law (linking-by means of nonlinear relation, if needed the flow to the head difference across the stretch) but through relations of a completely different type (for instance an exponential law linking the flow to the wetted section and, then, to the volume stored in the considered stretch or to its mean level). In conclusion, it is not a case of transferring directly the results of electrical networks theory to hydraulic networks not only because of the relevance of nonlinear phenomena but also, because of the quite different characteristics of the basic elements of the lumped parameter models.

1.3. Conclusions

By summarizing the above considerations, the state variable approach to the study of hydraulic free level networks:

— is a very efficient tool to face practical problems especially with regard to simulation and optimization techniques;

— may present also some interesting theoretical aspects;

— pursues aims different from the ones of technical hydraulics and resorts to procedures that are not conventional in this field;

— bases itself on the general laws of analytical fluid mechanics but adopts methods quite different from the ones considered in this topic and oriented towards different purposes;

— does not correspond to an extension of the modern circuit theory because of the quite different characteristics of the basic elements it is convenient to consider.

2. Inputs and outputs

2.1. Relevant quantities of hydraulic systems

As said before, the model to be set up shall allows to link the time behaviour of some quantities to be considered as outputs of the system, to the one of the input quantities, taking into account the behaviour if state variables (and in particular their unitial values); the input quantities may be actual control inputs (if any) or disturbances or simply influencing quantities.

The output variables are usually flows in given sections of the system or levels at relevant points; also the volumes stored in certain basins may be considered as outputs: this is the case of hydroelectric basins and also of ponds for irrigation; in this last case, however, the delivered flows are usually considered as the outputs of the most relevant interest; in the case of shipping canals and of reclamation plants the outputs usually considered are instead the levels.

The input variables are generally flows that, according to the case, may be lumped or distributed, entering or delivered by a punping station.

2.1.1. Natural hydraulic systems. For the study of a natural hydraulic system (the basin of a river etc.) one obviously considers as inputs the distributed rainflow and the lumped inflows of the tributaries, if any; the quantities that it is convenient to consider as outputs are generally the flows or the levels at certain points (hydrometers, reachpoints etc.). Considerations of the same type may be valid for the study of a lagoon: in this case, the output vector is generally a set of levels and the inputs are the various inflows; among these the tide flows are of great importance[1].

2.1.2. Irrigation plants. In an irrigation plant, the input is the set of the inflows into the network of canals utilized for delivering the water; alternatively the quantities may be considered as inputs on which the above inflows depend: positions of sluice gates, control parameters of pumps etc.

The output is the set of the outflows delivered to the users.

2.1.3. Reclamation plants. In a mechanical hoisting reclamation plant the control input of the system is the flow delivered by the pumping station and the output is the level at the hydrometer, but the behaviour of the system is chiefly determined by the inflows entering the reclaimed area, and primarily by the rainfall.

2.2. Characterization of the inputs

The first problem to be considered for the study of a hydraulic system is the one of characterizing the time behaviour of the inputs. Usually the solution of such

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[1] In fact, these flows, that may be lumped or distributed according to the configuration of the coast and change direction periodically, may depend also on the state of the lagoon and therefore the level of the sea at a certain distance from the coast should be considered as the actual input; this distance shall be sufficiently small to make the level itself depend on the presence of the coast, but sufficiently great to make the flow not depend on the state of the lagoon.

a problem is obvious in the case of lumped (of tributaries, of pumping stations etc.); instead for the distributed inflow of the rainfall it is convenient to resort to the following considerations.

2.2.1. Space distribution and time behaviour of the rainfall. The rainfall may be characterized through a function r, depending on the time and on the space, the dimensions of which are the ones of a specific flow (flow per unit area), usually measured in milimeters per hour.

Regarding the space distribution, the rainfall presents a high scattering (important intensity variations from one point to another, even if close); but because of the cumulation effects of the phenomenon and of the statistical compensation of space and time variations, it may be assumed that the rainfall is spatially uniform (at least on the areas corresponding to the parts into which the system has been subdivided for the construction of a lumped model).

Regarding the time behaviour two different positions may be adopted, by referring to real events (deduced by pluviometric records supplying samples of the integral of $r(t)$ at intervals of 6 hrs) or by adopting hydrological criteria and referring to ideal rainfall events assumed as constant at their maximum value and ordered in statistical series. The criterion to be adopted depends on the problem to be tackled.

2.2.2. Rainfall and surface runoff. Moreover, the function $r(t)$ represents the rainfall while the actual input of the system is only its contribution to the surface runoff, that may be represented by a function $p(t)$ having the same dimensions as $r(t)$; $p(t)$ fiffers from $r(t)$ because only a part $k_1\, r(t)$ of the rainfall contributes directly to the surface runoff, another part $k_2 r(t)$ first seeps into the ground and reaches the runoff system at a later time, while a third part $k_3 r(t)$ is definitively absorbed or evaporates.

Concerning the evaluation of the part p_2 of $p(t)$ corresponding to the second part of $r(t)$, usually hydrologists assume that the rain is constant for an interval t_p and that the amount of the second part of $p(t)$ is zero up to t_p when it jumps to a definite value and then decreases according to an exponential law. This procedure is simple and efficient in the field of the statistical methods of hydrology but for simulation purposes, especially considering variable rainfalls for which t_p cannot be evaluated a priori, it is preferable to adopt a different description, according to which the second part of $p(t)$ may be obtained via the equation

$$p_2 = r_2 \frac{1}{1+pT} = r \frac{k_2}{1+pT}. \tag{2.1}$$

This description has a physical meaning; indeed the suggested transference corresponds to a lumped model of the drainage through a porous soil. On the other hand the difference between the results of the conventional procedure and those of the one here suggested is small; in fact, this difference is nonzero only during the time interval t_p that is very small if compared with the queue of the rain ($t_p \ll T$); besides, during t_p, the main part of the surface runoff is formed by $p_1(t) = k_1 r(t)$ because $k_1 \gg k_2$ and that $p_2(t)$ increases slowly ($T \gg t_p$).

In conclusion the input flow $p(t)$ may be evaluated from the rainfall $r(t)$ via the equation

$$p(t) = p_1(t)+p_2(t) = r_1(t)+r_2(t)\frac{1}{1+pT} = r(t)\left(k_1+\frac{k_2}{1+pT}\right). \tag{2.2}$$

The coefficients k_1 and k_2 depend upon the climatic and geological characteristics of the region and upon the period of the year; the time constant T depends primarily upon the soil permeability. Typical values for these parameters in Northen Italy in February (month of reference for many hydraulic problems) are the following

$$k_1 = 0.5\text{–}0.5, \quad k_2 = 0.06\text{–}0.07, \quad T \approx 15 \text{ days}.$$

3. Some conventional input-output linear models

The input-state-output nonlinear model of hydraulic systems we have set up will be presented in the next section. Beforehand it may be convenient to examine some linear input-output models considered in conventional hydraulics. These models have been derived by resorting to different considerations and therefore they are usually presented in an independent way and by noticing the differences among them. On the contrary they will be presented here as special cases of a general linear model.

3.1. General approach

Let us consider the surface runoff system on a drainage area; the input is the flow $p(\tau, \xi, \eta)$ that is a function of the time and of the point ξ, η on which the rain falls; the output is the outflow $q_o(t)$ through a fixed length section centered on the point x, y. Because of the linearity assumption, the relation between q_o and p may be presented in the form

$$q_o(t) = \int_{-\infty}^{t} \iint_{A} w(t, x, y, \tau, \xi, \eta),\ p(\tau, \xi, \eta)\, d\tau\, d\xi\, d\eta, \tag{3.1}$$

where the double integration in space refers to the whole drainage area A. The physical meaning of w may obviously be understood by referring to the conventional considerations about the impulse response of a linear lumped parameter system; w represents the delay and the distorsion (corresponding to the passage from the point ξ, η to the output section) of the elementary inflow $p(\tau, \xi, \eta)\, d\tau\, d\xi\, d\eta$ that falls on the area $d\xi\, d\eta$, surrounding the point ξ, η during the interval $\tau - \tau + d\tau$.

The system may be considered as time invariant; therefore w depends only on the difference $t - \tau$ and not on t and τ separately.

The situation is not the same concerning the dependency of w on the spatial coordinates; in general, indeed, the effect $q_o(t)$, at a given point of the system, of the input $p(t)$ acting on another point does not depend only on the distance between the points under consideration but also on the position of the first one. On the other hand, however, we are usually interested in the time behaviour of the outflow crossing a considered section but not in its dependency on the position of this section. In other words, for a given section, x and y are fixed and we are interested only in dependency of $q_o(t)$ on the input point of each elementary inflow $p(\tau, \xi, \eta)d\tau d\xi d\eta$.

In conclusion the general expression (3.1) may be rewritten in the form

$$q_p(t) = \int_{-\infty}^{t} \iint_{A} w(t-\tau, \xi, \eta)p(t, \xi, \eta)d\tau d\xi d\eta, \tag{3.2}$$

whereas the spatial integration is still extended to the whole area of the system, but w is zero for the points from which p does not reach the considered section, and in particular, obviously, further downstream.

The above relations may certainly be considered of theoretical interest but their use depends on the validity of the linearity assumption and on the possibility of a correct determination of w.

Regarding this point, a good evaluation of w as a function of space is a very cumbersome problem.

It is possible, indeed, to subdivide the area of the system in subareas A_i in which p may be considered as uniform (with a time behaviour $p_i(t)$) and in this case the expression

$$q_o(t) = \sum_i A_i \int_{-\infty}^{t} w_i(t-\tau)p_i(\tau)d\tau. \tag{3.3}$$

may be adopted.

But also in this case a good evaluation of each w_i is very cumbersome, because of the complexity of the regression methods connected with the separation of the whole outflow into the components originating from each subarea, but especially because the conventional pluviometric records are not suitable for such a problem (the sampling interval of six hours being too high for the purposes here considered).

3.2. Conventional input-output models

Two procedure are then possible: to consider one subarea only or to adopt a suitable assumption regarding the form of w.

The first procedure is the one of the unit hydrograph, the second one corresponds to the socalled cinematic methods.

The first method adopts the expression

$$q_o(t) = A \int_{-\infty}^{t} w(t-\tau)p(\tau)d\tau. \tag{3.4}$$

This one allows us to determine q_o for each assumed $p(t)$ once $w(t)$ is known but it may also be used for determining $w(t)$ by referring to corresponding recorded behaviours of $p(t)$ and $q_o(t)$. This method may give satisfactory results if, besides the linearity hypothesis, $p(t)$ may be assumed as uniform all over the considered area or, more in general, the effects of its uniformities are negligible.

It may be noted that this method may be applied for the evaluation of outflows coming from little areas (for any one of which the uniformity assumption holds), and these flows may be considered as lumped inputs of a greater system, the outflow of which may be then ecaluated by means of (3.3).

Among the cinematic methods the more widespread is the one based on the corrivation time; this method makes use of (3.3) by assuming that each $w_i(t)$ may be uniquely defined by a pure delay T_i:

$$w_i(t) = \delta(t-T_i) \leftrightarrow W_i(s) = \exp(-sT_i), \tag{3.5}$$

$$q_o(t) = \sum_i A_i p_i(t-T_i) \leftrightarrow Q_0(s) = \sum_i A_i P_i(s) \exp(-sT_i). \tag{3.6}$$

The intervals T_i may be determined experimentally by using suitable tracing matter (colouring or radioactive matter, for instance) and evaluating the interval between the time at which they have been poured at the center of each subarea and the one at which they appear at the section under consideration.

3.3. Conclusions

The above methods may be usefully adopted in some cases, at least for a first approach; in fact, for many problems of fluvial hydraulics the assumption of linearity may be considered as valid.

In general, however, because of the importance of nonlinear phenomena neither is it possible to adopt directly these methods nor does it seem convenient to look for suitable modifications of them. Indeed if we adopt a correct general equation corresponding to (3.1) for the nonlinear case, because it cannot be presented as a convolution integral., it is impossible to approximate separately the input and the weighting function as in the above procedure.

Therefore it seemed convenient to resort to a completely different approach, that may be valid also for the nonlinear case. This approach bases itself on the socalled hydraulic storage methods and will be presented in the next section.

4. A state variable model

4.1. General considerations about the hydraulic storage method

The method of hydraulic storage refers to the volume stored in the whole system or in a part of it; therefore for an approach using the concept of state and utilizing this method it is convenient to consider the volume stored in each part of the system as the state of the said part[2]; but in order that a single part may be satisfactorily described by an unique dimension state it is necessary that in this part the socalled "synchronism condition" be verified.

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[2] On the other hand this choice is also the most convenient of the Lagrangean energetic approach s adopted.

As it is well known, in a single, constant section stretch of canal, the synchronism condition corresponds to the fact that the free level remains parallel to the bottom; more in general this condition, often expressed in qualitative terms, corresponds to the fact that inside the subsystem the levels in practice vary simultaneously[3].

In this condition it is possible also to characterize the state of the subsystem not only through the volume, which is a lumped variable, but also through variables that in general depend on space but, in this case, assume values that in each position are uniquely linked to the volume stored in the whole part and therefore may be considered as lumped variables; these are, for instance, the level and the wetted section in any point of the network[4].

Under the assumptions of synchronism, for each part of the system the equation of continuity

$$\dot{v} = q_i - q_o \tag{4.1}$$

is to be considered, where v is the volume stored in the part, q_i is the total inflow and q_o is the outflow; usually q_o will be considered as an input to a subsequent part of the system and therefore the evaporation or absorption outflows, if any, will be considered as negative components of q_i.

To equation (4.1) are to be added other equations linking q_o and q_i to variables associated to the same subsystem or to other ones. From this point of view, it is obvious that q_i is always an input to the subsystem (and therefore it must be obtained from the outside) while q_o may depend only on quantities external to the subsystem, or only on internal ones or it may also be a function of both; in this last case, however, q_o may be considered as the sum of a first part, depending uniquely on the state of the subsystem, and of a second one independent from it.

The first situation corresponds, for instance, to the case of a storage pond from which the flow q_o is delivered by a pump.

The second situation corresponds to the case of a subsystem for which the so called "autonomy condition" is valid. From a physical point of view, this condition corresponds to the fact that the outflow depends uniquely on the situation upstream.

The third situation corresponds to the case of backwater.

4.2. Equations of a storage basin without backwater from which the water is delivered by a pumping station

A storage pond without backwater can be described by an equation of the type (4.1) in which the flow q_o is replaced by the flow q_h that is delivered from the hoisting station; for the evaluation of q_h the following considerations are to be made.

The simplest solution and also the one most frequently adopted in practice is that in which fixed trim, constant speed pumps are used. In such a case, the delivery

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[3] In a well designed network of canals the synchronism condition is valid for large zones and therefore the number of components of the state vector of the whole system may be small; general considerations about this point cannot be made with reference to natural systems.

[4] The use of lumped parameters greatly simplifies the structure of the model and the involved computations, while, for a suitably subdivided network of canals, the results obtained in such a way differ only slightly from the actual behaviour.

capacity of the pumps depends only on the value of the pumping head (difference between the level h of the basin and the level h_r of the reservoir) through a nonlinear law that may be approximated by a linear relation of the type:

$$q_o = c_1 - c_2(h_r - h) \tag{4.2}$$

because level variations are small.

When the pumping station is equipped with only one pump, the control is of an on-off type; when several pumps are present, a step control is employed, acting on the number of the working pumps.

Other solutions can also be considered that imply the use of variable trim and/or variable speed pumps, for instance in order to make the delivery capacity of the station correspond to the input flow and to make it independent of the pumping head; consequently the level may remain practically constant because the dynamics of the control system is very fast in comparison with the dynamics of the hydraulic plant.

Such a solution is very expansive and, at present, it is not frequently used; by resorting to it, however, q_o is the actual input of the system because it depends only on the control parameter of the pumping system. In the other case it depends also on the level of the basin that is linked to the stored volume through a relation depending on the shape of the basin; in such a way a feedback loop is present (external feedback) (Fig. 1).

Usually, however, the variations of the head are small (at least in short periods) and moreover the part of q_o independent from it is greater than the other one. Therefore if the water is delivered by a pumping station, usually it is possible to assume hat feedback effects on the outflow are negligible. This situation corresponds to the first case considered in the preceding section (Fig. 2).

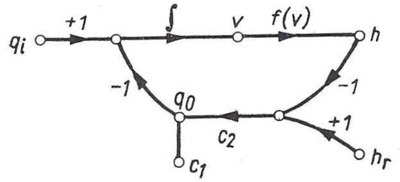

Fig. 1

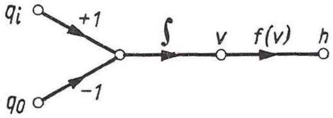

Fig. 2

4.3. Equations of an autonomous stretch of a canal network

Let us consider a single stretch of canal in condition of autonomy, in which the outflow q_o of equation (4.1) is a function of the stored volume v.

In order to determine the link between q_o and v it convenient to resort to be conventional procedure that considers the nonlinear but unique link between outflow and wetted section S:

$$q_o = \mu S^{\alpha} \tag{4.3}$$

(where μ and α are suitable coefficients depending on the geometrical and physical features of the canal).

In the assumed conditions of synchronism the wetted section is directly proportional to the volume

$$S = (1/l^*)v \tag{4.4}$$

(l^* being the "equivalent" length of the stretch, which corresponds to the real length for a stretch of a single constant section canal).

The value h_o of the level in correspondence to the wetted section S at the output of the stretch is a simple function of S depending on the shape of the canal.

The relations between the relevant quantities are represented by the flowgraph in Fig. 3a where the link between q_o and S (eg. (4.3)) is labelled f_1 and the link between h_o and S is labelled f_2. In order to simplify the computations it is better to modify the graph into the form represented in Fig. 3b.

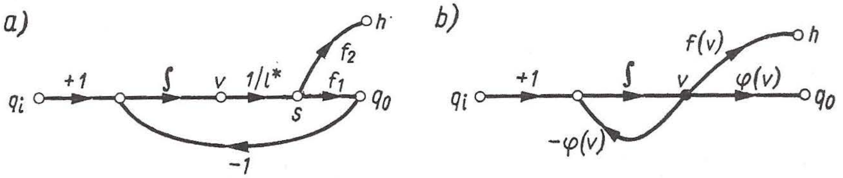

Fig. 3

Usually it is convenient to assume a description of this type for a larger area, with canals that do not joint at the same reach point; in this case the input will be the sum of all inputs of the subsystem under consideration.

This description may be used also for the subsystem formed by each synchronous part of a draining basin; for these subsystems the input is that due to the rain. The situations here considered corresponds to the second case of Section 4.1.

4.4. Equations of a subsystem with backwater at a reachpoint

The third case considered in Sec. 4.1 will be examined with reference to the particular but important case of a main collector pouring into a storage pond. The analysis of such a case is quite similar to that of the case of a tributary pouring into a stretch of canal.

The problem should be examined by integrating the partial derivatives equations of the system or by using other methods suggested by the hydraulic literature (method of characteristics etc.). Such methods are extremely cumbersome even if implemented on a computer; on the other hand the mass of data these methods provide, is far too large for the purposes here considered.

Alternatively we suggest a method that is very simple and suitable for the computations here involved.

This methods may be presented on the basis of the following considerations.

The volume stored in the main collector and in the storage pond is divided into three parts (Fig. 4); the first one is the volume v_a stored in the collector canal in an autonomous condition, the second one is the volume v_b of the backwater upon v_a and the third part is the volume v_p stored in the pond.

Two main conditions are possible: either at the terminal section of the collector the level h_b upstream (i.e. in the canal) is equal to the level h_p downstream (i.e. in the storage pond) or a sharp decrease in level occurs in this section ($h_b > h_p$).

The first condition occurs typically for the terminal pond of a reclamation plant when the pumps are not working and the levels of the canal and of the pond increase slowly together; more precisely this condition occurs when the flow delivered by the pumping station is lower than the one entering from the main collector.

In the first condition the volume v_p of the basin and the one v_b of the backwater are to be considered as unique volume v_t. For this one, equation (4.1) assumes the form

$$\dot{v}_t = q_a - q_o, \tag{4.5}$$

where $q_a = \dot{v}_a$ is the outflow of the collector canal in an autonomous condition and has to be computed via equation (4.3). Volume v_t is linked univocally to the level h_p in the storage pond through an equation

$$h_p = f(v_r + v_b). \tag{4.6}$$

As far as the form of f is concerned, it is possible, as a first approximation, to refer to a linear profile of the backwater; nevertheless it is quite easy to consider a backwater of different shape (as in Fig. 4); the results based on the assumption of a linear profile with a suitable slope have been very satisfactory.

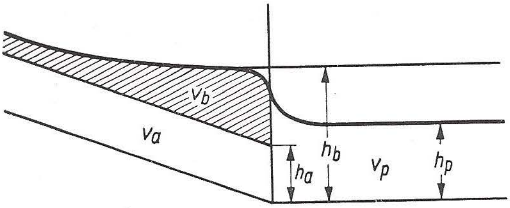

Fig. 4

In the second condition, the laws of variation of v_p and v_b are to be considered separately. The volume v_b decreases because of the flow it pours into the storage pond, according to the laws of the phoronomy

$$\dot{v}_l = g(h_l, h_a, h_p), \tag{4.7}$$

where h_b is the level of the backwater at the reachpoint, h_a is the level of the canal upstream (in autonomous conditions) and h_p is the level of the pond.

The variations of v_p depend on the flow poured from the canal (sum of the autonomous term and of $\dot{v}_b$) and on the outflow q_o (for instance the one delivered by the pumps); equation (4.1) assumes here the form

$$\dot{v}_p = q_a + \dot{v}_l - q_o. \tag{4.8}$$

In the first condition the system is represented by the diagram of Fig. 5a; in the second one by the diagram of Fig. 5b.

The dimension of the state vector of the system formed by the collector and by the pond, including the backwater, is variable because of the existence of two different behaviour conditions; indeed in the second case the state vector if of order 3 and in the first one it is of order 2, the third state variable being not controllable (in autonomy conditions the state vector of the same system is of order 2).

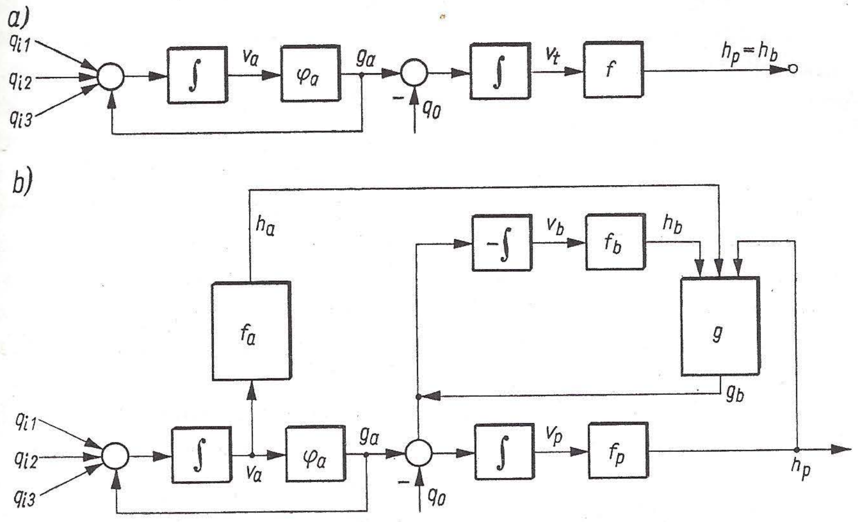

Fig. 5

4.5. Setting up the model of a hydraulic network

The analysis of the cases considered in the Sections 4.2 and 4.3, is simple but complete; the analysis of the third case, as said before, refers only to a particular but important case of nonsynchronous subsystems, further work has to be done for the analysis of more complex nonsynchronous subsystems.

Nevertheless, by using only the above discussed procedure it is possible to analyze many important hydraulic systems and, in particular, almost all cases of practical interest for artificial systems (networks of canals for inland navigation, irrigation, reclamation etc.).

The first step of the procedure for setting up the model corresponds to the evaluation of the inputs (cf. Sec. 2.2).

The second step corresponds to the subdivision of the whole system into subsystems. For this purpose it is convenient to identify all subsystems which may be considered synchronous (i.e. the subsystems of order (state dimension) 1, that may be described through the diagrams of Fig. 2 or of Fig. 3). If each of the remaining parts may be described using the method of Sec. 4.4, the analysis of the whole system may be carried out.

After the subdivision of the system, the equations of each part have to be considered, and suitable values have to be chosen for the coefficients of such equations.

The coefficients c_1 and c_2 of equation (4.2) may be directly obtained by referring to the actual characteristics of the pumps.

The exponent α of equation (4.3) depends upon the geometrical characteristics of the canal (or of the canals forming the stretch of network) and upon the conditions of its (their) walls; the value of α is not critical and remains close to 1.59 for usual shapes and characteristics of canals. The coefficient μ of the same equation may be determined experimentally as the ration between any wetted section and the corresponding measured flow; it may also be determined on the basis of the design data which refer to the maximum flow and to the corresponding wetted section.

For a correct use of equation (4.4) it is necessary to evaluate the equivalent length l^* of the stretch. In the case of a constant section canal, l^* correspond, as said before, to the actual length; for more complex structures, l^* may be evaluated as the ratio between any volume stored in the subsystem and the corresponding terminal wetted section. The mentioned volume may be easily evaluated for the part stored in canals with regular section and only approximatively for the part concerning ditches and draining areas. In this case too, it is possible to refer to design data.

The function f of equation (4.6) depends on the geometrical characteristics of the system.

Regarding the function g of equation (4.7), we suggest the use of the conventional equation for a partially submerged overfall, and in particular of the equation

$$\dot{v}_l = [\sqrt{2g(h_l-h_r)}0.5(h_r-h_a)+0.44(h_l-h_p)]\left(l_0+\frac{2h_r}{m}\right) \qquad (4.7')$$

where l_0 is the width of the canal at its bottom and m is the slope (in percentage) of the walls.

4.6. Conclusions

In this report a method has been presented for the analysis of hydraulic systems with special regard to the case of canal networks.

The approach here adopted is quite different from the one usually adopted in conventional hydraulics, the main aim of which is to design canals suitable for discharging critical rainflows. The aim here considered is instead to set up a model for evaluating the time-behaviour of relevant quantities such as flows in given sections, stored volumes, water levels and so on. Furthermore the model has been set up in terms of state variables to allow us to adopt the system approach for the study of the plant (considering also the possibilities of utilizing techniques of parameter estimation, optimization and so on).

This features assumes today great importance because of recent changes in the policy of soil and hydraulic sources exploitation and also with regard to the use of computers for simulation of hydraulic systems.

By means of the proposed model significant cases of reclamation plant conduction have been studied and the results of the simulation have been very satisfactory.

Further work to be done in this field will be oriented towards the study of more complex cases (subsystems formed by more than two interacting parts in presence of backwater), towards the improvement of computer programs for model implementation and towards use for this model the study of some important problems of hydraulic system engineering (forecasting of the behaviour of a system under different environmental conditions and under different conduction policies; forecasting of the effects of network modifications; solution of optimization problems concerning the best exploitation of hydraulic systems etc.).

Badanie systemów hydraulicznych metodą zmiennych stanu

Przedstawiono procedurę wyprowadzania modeli bazujących na pojęciu zmiennych stanu dla najbardziej interesujących systemów hydraulicznych. Cele tej pracy są podobne z pewnego punktu widzenia do celów podobnych poczynań w innych dziedzinach, zwłaszcza w dziedzinie sieci elektrycznych.

Wzięto pod uwagę tylko elementy o swobodnych poziomach — szczególnie takie systemy hydrauliczne jak zlewnie, sieci rzek i kanałów, zbiorniki wodne itp. Pracę prowadzono w dwu etapach. W pierwszym dokonano dekompozycji, a w drugim wyprowadzono modele matematyczne poszczególnych elementów przeważnie na podstawie równania magazynowego.

Otrzymane w ten sposób modele mogą być zastosowane bezpośrednio do celów symulacji lub mogą być przekształcane tak, aby otrzymać inne ich postacie bardziej przydatne do innych celów. Modele te umożliwiają badanie takich typowych problemów inżynierii systemów hydraulicznych jak prognozy zachowania się systemu przy różnych warunkach zewnętrznych i przy różnych oddziaływaniach sterujących, przewidywanie skutków modyfikacji sieci, rozwiązywanie zadań optymalizacji związanych z najlepszą eksploatacją systemu do celów wytwarzania energii elektrycznej, rolniczych i dla żeglugi.

Исследование водных систем методом переменных состояния

Представлен способ образования моделей, базирующих на понятии переменных состояния, для наиболее интересных водных систем. Цель этой работы аналогична, с некоторой точки зрения, целям работ в других областях, особенно в области электрических сетей.

Принимались во внимание элементы только со свободными уровнями — в частности такие водные системы как бассейны, сети рек и каналов, водоёмы и т. п. Работа проводилась в двух этапах. На первом этапе произведена декомпозиция, а на втором разработаны математические модели отдельных элементов, в основном используя при этом уравнение склада.

Полученные таким образом модели могут применяться непосредственно для целей моделирования, либо можно их преобразовывать таким способом, чтобы получить другие их виды, более пригодные для других цепей. Эти модели позволяют исследовать такие типовые инженерные вопросы водных систем как прогнозирование поведения системы при разных внешних условиях и при разных управляющих воздействиях, предвидение последствий модификации сетей, решение оптимизационных задач, связанных с наилучшей эксплуатацией системы для целей производства электроэнергии, для сельскохозяйственных целей и для плавания.