Control
and Cybernetics
VOL. 3 (1974) No. 3/4

Analysis of the net composed of neuron-like elements and investigation of the edges influences

by

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The analysis of neuron-like nets with feedback loops, as an element of pattern recognition system, is the subject of this paper. The analysis is based on the results of theoretical and neurophysiological investigations of the nervous system. The difference equation describing the net is investigated by means of modified Z transform method. The properties and behaviour of the net are given, the stability conditions are determined. There are given methods of compensation of the net edges influences. The obtained results are illustrated by comparison of behaviour of compensated and uncompensated nets.

1. Introduction

The structure composed of neuronlike elements presented in this paper are very interesting for engineers and technicians because of possibilities of their applications in technical arrangements for data processing and pattern recognition. Because of parallel serial work neuronlike nets are more effective for the purpose of information processing then digital computers. The idea of neuronlike nets and their applications for information processing is based on the results of morphological and neurophysiological investigations of the nervous system [3, 5, 6].

The general purpose of this work is recognition and explaining of phenomena taking place in nets composed of neuronlike elements from the point of view of technical applications.

2. Some problems of investigations of structures composed of neuron-like elements

Investigated system is composed of many mutually interconnected elements. It is strongly nonlinear one, includes integrating and inertial elements. The most frequently methods of discussed nets investigations are:

(i) physical modelling — this method makes it relatively simple to take into consideration numerous properties of neuron but on the other hand the interpretation of results is rather difficult;

(ii) mathematical modelling — possibilities of this method are in fact strongly limited by computers minimal execution time.

It is of course impossible to analyze the net the model of which would take into consideration all known properties of the nervous system. The appropriate choice of properties of element is a kind of compromise. On one hand all properties necessary from the point of view of application should be taken into consideration (in our case the properties connected with the transmission and processing of information) but on the other hand the model should not be too complicated.

The general task of a group of works on the layer nets [2—4, 7, 9] was to elaborate the methods of net analysis and the method of synthesis of systems for informative points detection. The problem was studied by several authors using different methods of structure description. It appeared unfortunatelly that proposed structures were not optimal for the purpose of recognition, they were redundant and very sensitive to disturbances.

The basic task of this work is to reconstruct the nets proposed as so far so as to minimize their dimensions and minimize the time necessary for detection of informative points.

There are three basic problems and difficulties connected with the analysis of the net:

(i) problem of mathematical description of the net and its elements (the description depends of course on the modelled phenomena and should be as simple as possible);

(ii) choice of method of stability region determining;

(iii) the compensation of edges influences.

Originally the problem of compensation was not the foreground problem but the results of investigations [8, 9] indicated that the influences of the edges were so significant that the information about recognized picture could be lost and furthermore the method of moving operator is more frequently used in pattern recognition systems what made it necessary to look for the methods of edges influences compensation (the moving operator is a little net searching the picture so a very important problem is a problem of its dimension).

The net is composed of neuron-like elements arranged in layers. The first problem which should be solved is the problem of mathematical description of the net element and the problem properties which should be taken into considerations. The choice of model depends upon the phenomena which are to be modelled. Because we consider neuron like element as an element of pattern recognition system so the most important are the properties which are connected with the information processing and transmission.

As a result of considerations given above we choosen a model of neuron like element (Fig. 1) with following properties:

(i) element is many input sumator (it corresponds to spatial summation in neuron) and resultant signal is applied to the inertial element of the first order (it corresponds to biological effect of temporal summation);

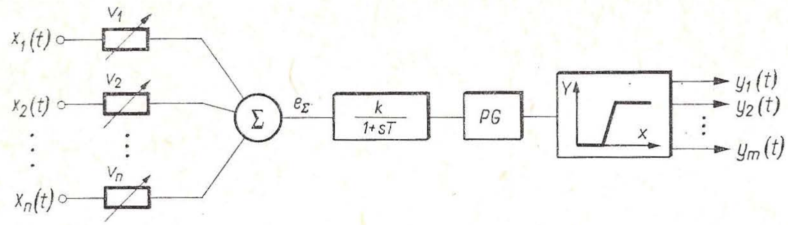
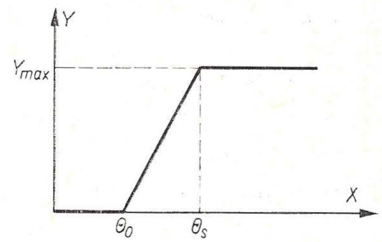


Fig. 1. Functional diagram of neuron model
x_i — input signals, v_i — weights, e_Σ — summary signal, PG — pulse generator, y_i — output signals

(ii) the output value (frequency of the output impulsation) depends on the input signal is a way shown by characteristic given in Fig. 2;

Fig. 2. The simplified characteristic of the threshold element with saturation
Θ_0 — threshold value of input signals, Θ_s — value of saturation



(iii) there are two kinds of outputs: inhibitory and excitatory.

After many authors [1, 2, 4, 10] we assume that the structure of the net is homogeneous and symmetrical. This assumption is based on the results of investigations of visual tract.

An example of the net organization is a layer with local (in one layer) and global (between layers) connections. There are different systems of functional connections between individual elements and groups of elements. Especially interesting is a scheme when neurons are connected according to the lateral inhibition rule. In this case the well known, phenomenon of reduction of information is observed [1, 10].

3. One dimensional net — a chain of neurons

Let us consider a simple example of the layer net composed of neuronlike elements — a chain of neurons. Every element of structure affects its neighbours according to the lateral inhibition rule [10]. The range of influence is equal to n as presented in Fig. 3. The relation between outputs Y and inputs X is described by formula

$$X = A \cdot Y, \tag{1}$$

where X and Y are the column matrices with elements determinated by the value of inputs (outputs) and A is a multidiagonal matrix characterizing the connections between the elements (weights) [8]. From formula (1) we are able to determine the

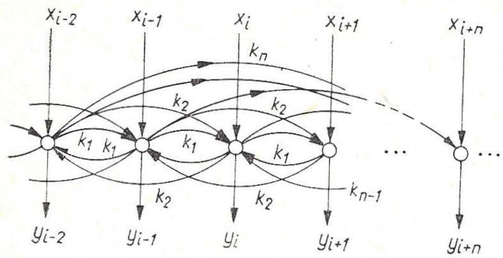


Fig. 3. One dimensional net — a chain of neurons

distribution of output values and define the stability conditions of the net [10], unfortunately when $n>1$ this procedure consisting in solving matrix equation $Y=A^{-1}X$ appears to be very difficult [8].

3.1. Z-transform method

Let us assume that:

(i) the network expands infinitely;

(ii) all elements are identical;

(iii) the operation point is placed in a linear region of the static characteristic (Fig. 2).

The general formula describing the relation between the input $\{x_i\}$ and output $\{y_i\}$ series is described by the difference equation

$$y_i = x_i - \Theta(y_k) \tag{2}$$

(here, the geometry of structure is a discrete variable — not time) where Θ is the linear operation.

For the net shown in Fig. 3 output y_i may be expressed by the equation

$$y_i = x_i - \sum_{j=1}^{n} k_j (y_{i-j} + y_{i+j}), \tag{3}$$

where k_j are the weights of the lateral influences.

After Z-transformation we obtain

$$Y(z) = F(z) \cdot X(z), \tag{4}$$

where $F(z)$ is a transfer function of the net (equation (7)).

3.2. Synthesis and decomposition of the network

However, the analysis of a net with a great number of connections is troublesome. In the previous paper [8] the following theorems were proved.

THEOREM 1. The cascade (or parallel) connection of two networks with feedbac loops between each element of the network and his left neighbours only (maximal ranges of feedbacks are equal to m and n respectively) is equivalent to the network of the same type with the range of feedback equal to $k=m+n$ (the same holds for the systems with the right connections only).

THEOREM 2. Te cascade (or parallel) connection of two networks; the first with the feedbacks to the right and the second with the feedbacks to the left (ranges of feedbacks are m and n) is equivalent to the network with the connections to the right and left and with the range of feedback equal to k ($k=\max(m, n)$).

Basing on those net properties we can decompose the complicated system for simpler ones or inversely construct the complex system by the synthesis of simpler units.

3.3. Distribution of the output values

Application of modified and adapted Z-transform method to the difference equation describing the net behaviour allowes to define the distribution of the output values (formula (5)) of chain elements

$$y_i = \sum_{n=-\infty}^{+\infty} a_n x_{i+n}, \tag{5}$$

where: a_n — coefficients of the Laurent expansion of the function $F(z)$ determined by formula

$$a_n = \frac{1}{2\pi_j} \oint F(z) z^{-(n+1)} dz. \tag{6}$$

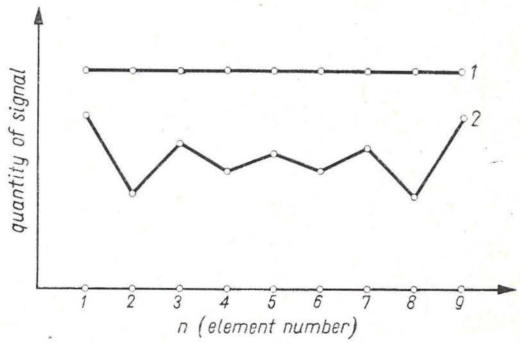


Fig. 4. The distribution of the output values
1 — input signals, 2 — output signals

Of course these values are approximative ones (because the real chain has a finite dimension) but comparison of them with the experimental values (obtained by modelling at digital computer) shows a coincide to a high degree (Fig. 4).

3.4. Stability conditions

One important problem which should be solved before the application of the network are the study of their stability. It was found [7] that we can determine the stability conditions putting $z = \exp(j\varphi)$ into the formula (7) describing a transfer function — for the set from Fig. 3 we obtain

$$F(z) = \left(1 + \sum_{j=1}^{n} k_j (z^j + z^{-j})\right)^{-1} \tag{7}$$

— and demanding $F(\varphi)$ be always positive for any values of φ $(0 \leqslant \varphi < 2\pi)$.

As an example we shall consider a net when $n=1$. The inhibitory influence appears only between the output of each element and its two nearest neighbours, the weights are denoted by k_1. The stability condition is determined by inequality

$$0 < k_1 < 1/2. \tag{8}$$

Computations and results for nets with $n > 1$ may be found in [8].

3.5. Compensation of the edges influence

It appears, however, that the chain of neurons as a detector of informative points (like edges of excitation or variations in the distribution of excitation) sometime gives a false information [8].

Let us consider an example. The distribution of the excitation is applied to the chain of neurons as shown in Fig. 5a. When the step of the excitation appears near

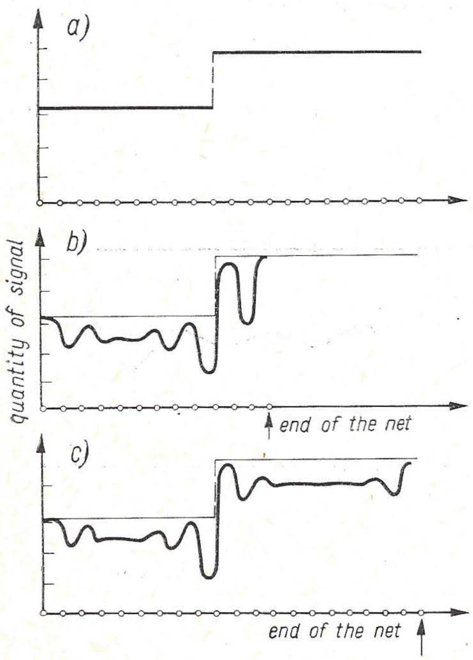


Fig. 5. The detection of informative point (step in the excitation). Comparison of long and short neuron chains

A — input signals, B — output signals at the short net, C — output signals at the long net the end of the net then at the outputs we receive the fluctuations which make the detection impossible (Fig. 5b). On the other side, when this step is in the middle part of the net then the edges influence is not so destructive and we can recognize the informative point (Fig. 5c).

The edges of the structure give some ,,reflections", some oscillations and make impossible identification of the informative points. Because that noxious fact is caused by edges, the structure of the net should be reorganized in such a way so as to compensate the influences of the edges and to make the finite net to behave like the infinitly expanding net (the net without edges).

There are following methods of compensation of the edges influences:

(i) a) discrete change of weights in the feedback loops of the edges elements (Fig. 6);

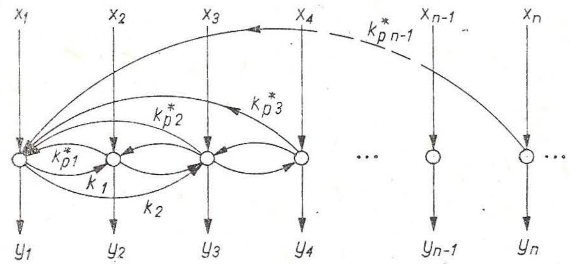


Fig. 6. A compensation by a discrete change of weights
k_j — connection coefficients, k_j^* — new values of coefficients

(ii) a additional self feedback loop for these elements (Fig. 7);

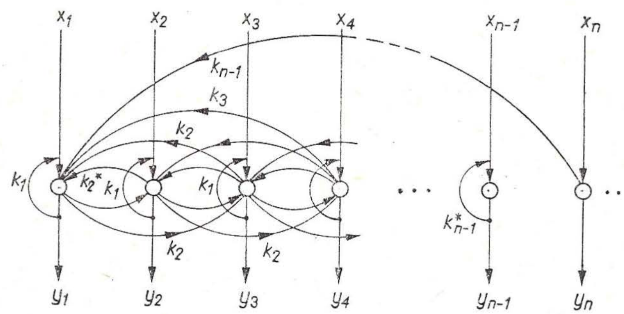


Fig. 7. A compensation by the additional self feedback loops
k_j — connection coefficients, k_j^* — coefficients in the self feedbacks

(iii) a continuous change of weights (Fig. 8).

A very difficult problem arises when the third method is applied — the problem of choice of coefficients assuring the extremal output values at the edges of the chain.

For the chains shown in Figs. 6 and 7, taking into account symmetry and homogeneity of the structure, new values of the coefficients of connections can be determined by the relationships

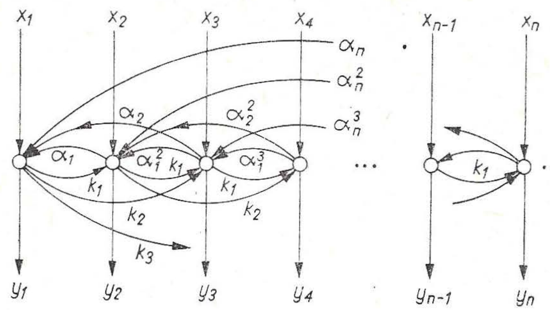


Fig. 8. A compensation by a continuous change of weights k_j — connection coefficients, α_j — compensatory coefficients

$$k_j^* = 2k_j \qquad \text{for the net shown in Fig. 6,} \tag{9}$$

and

$$k_i^* = \sum_{j=1}^{n} k_j \qquad \text{for the net shown in Fig. 7.} \tag{10}$$

3.6. Neuronlike network as a detector of informative points

A behaviour of the net being an element of the identifying and classifying structure we examined modelling the chain on digital computer. Advantage of compensation is shown in Figs. 9, 10, 11, by comparison of results obtained for the net

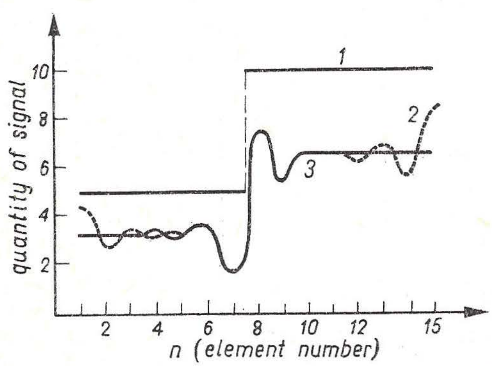


Fig. 9. Detection of the step in the excitation *1* — input signals, *2* — output signals in the uncompensated net, *3* — output signals in the compensated net

with and without the compensation. From these examples we receive the following conclusions:

(i) fluctuations at the edges disappeared after the compensation;
(ii) a compensated net reachs the steady state much faster;
(iii) the compensated net is invariant to the position of the picture.

Examples of behaviour of one dimensional structure used as a detector of the informative points like a step in excitation or the narrow impulse are presented in Figs. 9—11.

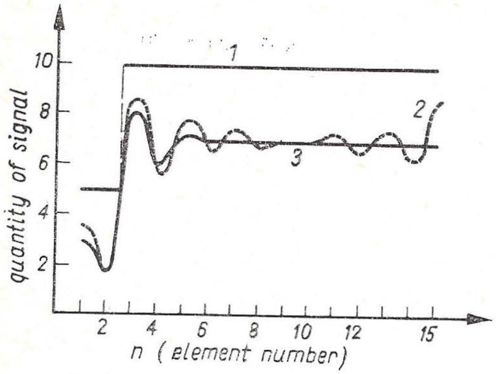


Fig. 10. Detection of the step in the excitation
1 — input signals, *2* — output signals in the uncompensated net, *3* — output signals in the compensated net

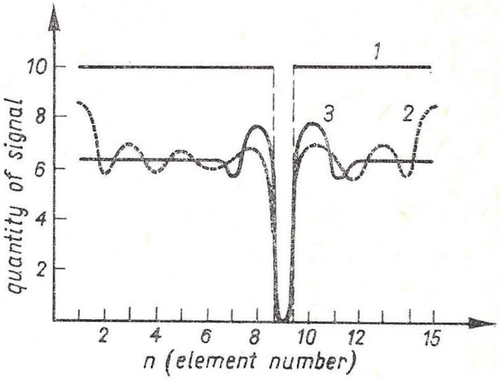


Fig. 11. Detection of the narrow impulse (a narrow interruption in the sequence of impulses)
1 — input signals, *2* — output signals in the uncompensated net, *3* — output signals in the compensated net

3.7. Asymmetrical chain

An interesting example of a neuronlike structure is an asymmetrical chain. The structure characterises the asymmetry of coefficients of the local feedbacks (Fig. 12). Difference equation describing this net has form

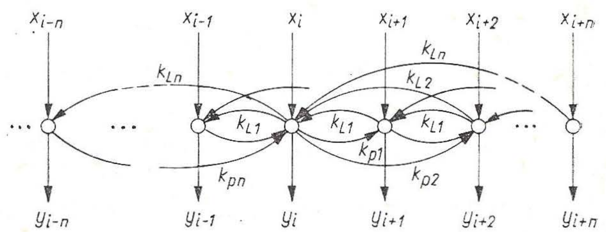


Fig. 12. Asymmetrical chain

$$y_i = x_i - \sum_{j=1}^{n} (k_{Lj}\, y_{i-j} + k_{Pj}\, y_{i+j}). \tag{11}$$

After Z-transformation

$$Y(z) = F_1(z)\, X(z), \tag{12}$$

where

$$F_1(z) = \frac{1}{1 + \sum_{j=1}^{n} k_j\left((\alpha_j z) + (\alpha_j z)^{-1}\right)}, \tag{13}$$

denoting

$$k_j=\sqrt{k_{Lj}\,k_{Pj}} \text{ and } \alpha_j=\sqrt{k_{Pj}/k_{Lj}}.$$

On the ground of comparison of formula (13) with the analogous are for the symmetrical chain [7] arise the following conclusions:

If the relations between the suitable coefficients k_{Pj} and k_{Lj} are constant, i.e. $\alpha_j=\text{const.}$, then the transfer function $F_1(z)$ for the asymmetrical structure is identical with the suitable function for the symmetrical chain. It follows, that we can partly resign from the assumption of symmetry. The investigations of the asymmetrical structures has a great significance. In the paper of Gawroński [4] we find the supposition that the detection of the movement direction (in neural nets) is due to the asymmetry of the network. It may easily be proved by mathematical methods that the asymmetrical networks also may be used for investigation of the nets with local damages.

4. Two dimensional network

The same method of analysis can be applied to two dimensional flat net (Fig. 13). By the method of modified and adapted Z-transform be proved the analogous

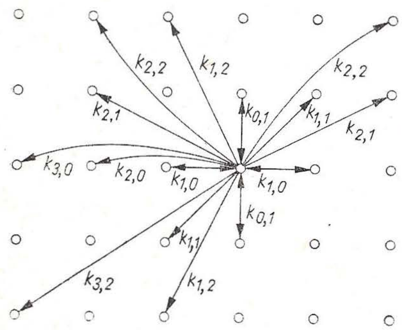


Fig. 13. The domensional net. Examples of connections

theorems as for a chain (theorems about synthesis, decomposition and stability conditions).

Let us assume that the net consists of the identical threshold elements and the structure is symmetrical and homogeneous one, the connections between the neighbouring elements are of lateral inhibition type. The relation (difference equation) between the outputs $(y_{i,j})$ and inputs $(x_{i,j})$ of this net is described by the following formula:

$$y_{i,j}=x_{i,j}-\sum_{l=-m}^{+m}\sum_{p=-n}^{+n}k_{|l|,|p|}\,y_{i-l,j-p}, \tag{14}$$

where: $k_{l,p}$ — the weight of the negative feedback between the element (i,j) and the element (i',j'); $l=|i-i'|$; $p=|j-j'|$; $k_{o,o}$ — the weight of the self feedback loop; assuming symmetry we obtain

$$k_{-l,p}=k_{l,-p}=k_{l,p}=k_{p,l}.$$

The problem of the edges influence and its compensation is more important then in the case of one dimensional structure. For example, when at all inputs of

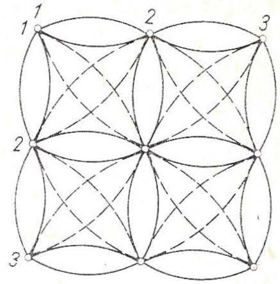


Fig. 14. Example of connection in 9-elements net

the nine-element two dimensional net (Fig. 14) excitations are the same it appears that at the outputs we receive a distribution of signals of quite another type (Fig. 15).

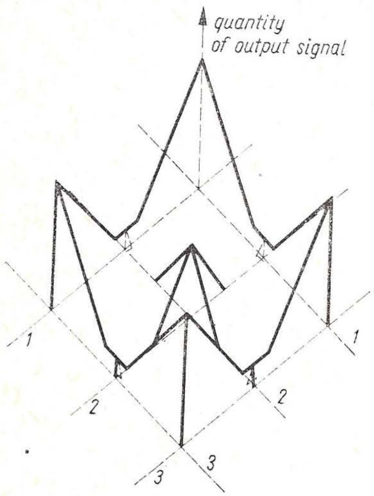
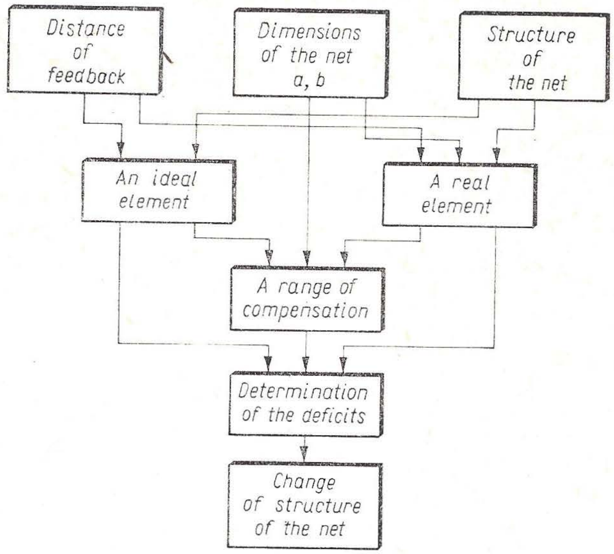


Fig. 15. Distribution of the quantities of the output signal for the 9-elements net (see Fig. 14)

Comparing the description of the infinitely expanding net and the net consisting of $(a \cdot b)$ elements (a difference is only in the initial conditions) we found the definition of the connections which we have to complete to the edges elements of the net [8, 9]. A diagram of compensation is shown in Fig. 16. This problem needs the algorithms, programs and computation to be worked out [8].

A compensation of the influence of the edges can be achieved in two ways. The first one consists in application of additional self feedback loops from the output to the input of each element the conditions of the work of which are different from the typical ones (described by the equation (14)) the second method consisst in a change of the coefficients (weights) of the feedback connections coming to this element.

Two dimensional net used as a detector of the informative points allows us to detect: ends, bends, crossings, ramifications etc. [4]. The obtained results of a detection depend to a high degree upon the contrast between the picture and the background (differences in an illumination). In Fig. 17 a threshold of a discrimina-



Fi. 17. A diagram of the compensation

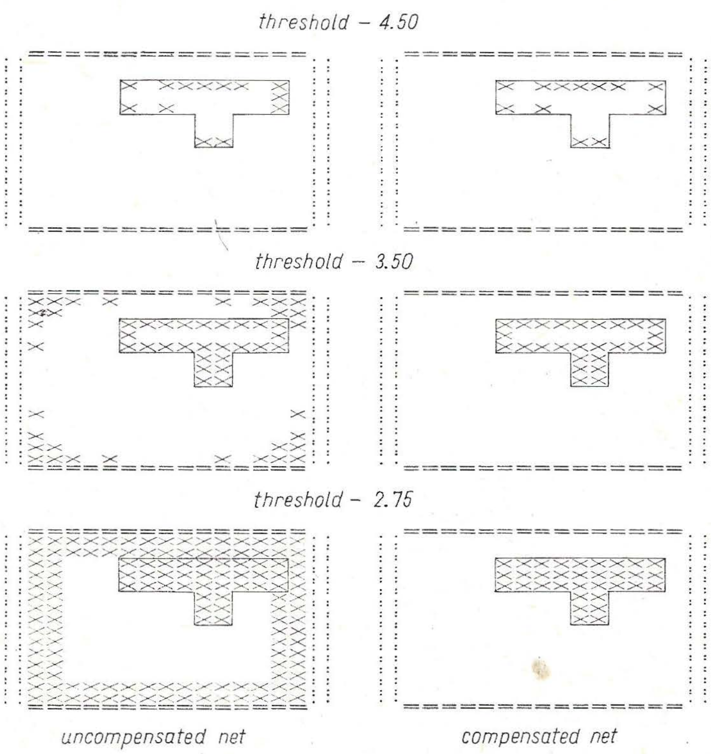


Fig. 17. Comparison of the behaviour of the compensated and uncompensated nets as a detector of informative regions (the picture is laying in a border part of the net)

tion which cuts off the noises from the essential information is marked (thin line indicates outlines of the picture in investigation). When this contrast is a small one then the net (particularly the net without the compensation) can give the false information very easily.

For the typical net with the uncompensated influence of the edges, a position of a picture on the screen is an important problem. When the inquired picture is located at the middle part of the net then the additional impulses given by the edges influence do not blear the picture. On the other side when the inquired picture is located at the border part of the net, even if this is a net with a small range of influence (1 or 2), then the picture is not detected and recognized (as may be seen in Fig. 17).

It is shown that after the compensation detection is practically independent from the position of a picture [8].

5. Conclusions

From the consideration given above and from the examples modelled at the digital computer we obtained the following conclusions:

(i) A two dimensional net, like as a chain of neurons, holds a phenomenon of contrast (for the chain it was obserwed by many authors e.g. Gawroński [2, 4], Reichardt and Mac Gintie [10]) and are suitable for a detection of informative points (or regions) of a pictures.

(ii) The compensation of the influence of edges is a very important problem, because in the uncompensated net a detection of a picture located in a border part is practically impossible.

(iii) The algorithms of the compensation assure the independence of detection on the position of pictures and make their classification more exact and sure [8].

The net composed from the neuronlike elements may be used as a detector of informative points in technical identification arrangements.

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Analiza sieci zbudowanej z neuropodobnych elementów oraz badanie wpływu brzegów na jej działanie

Na podstawie wyników badań neurofizjologicznych i teoretycznych układu nerwowego przeprowadzono badanie sieci neuropodobnej z lokalnymi sprzężeniami zwrotnymi jako elementu układu rozpoznającego. Zastosowanie zmodyfikowanej i odpowiednio zaadaptowanej metody transformacji Z umożliwiło określenie działania sieci, opisanie jej własności i warunków stabilności. Ponieważ skończone rozmiary sieci rzeczywistej powodują "odbicia" sygnałów od brzegów struktury (co utrudnia, a czasem uniemożliwia prawidłowe jej działanie), powstała konieczność opracowania odpowiednich metod kompensacji wpływu brzegów. Przez porównanie działania sieci o skompensowanym i nie skompensowanym wpływie brzegów wykazano korzyści, jakie daje kompensacja.

Анализ сети построенной из нейроподобных элементов и исследование влияния краевых эффектов на ее функционирование

На основе результатов нейрофизиологических и теоретических исследований нервной системы проведено исследование нейроподобной сети с локальными обратными связами, используемой в качестве элемента распознающей системы. Применение модифицированного и соответствующе измененного метода λ-преобразований позволило определить функционирование сети, описать её свойства и условия устойчивости. Поскольку конечные размеры действительной сети приводят к „отражению" сигналов от краев структуры (что усложняет или иногда делает невозможным её правильное функционирование) возникла необходимость разработки соответствующих методов компенсации краевых эффектов. Путем сравнения функционирования с компенсированием и без компенсирования влияния краев показаны преимущества, которые дает компенсация.