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Digital stabilization of a linear stochastic system

by

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This paper discusses the methods of digital control of a linear stochastic system. The sampled-time model is considered. The goal of control is to minimize the variance of the output signal. A simple model is proposed for which the least-squares method of identification gives the consistent estimators of parameter. The control policy is also included.

Next the simultaneous identification and control is discussed. It is shown, that it is possible to find the optimal control policy for the one-step-ahead criterion. The on-line identification algorithm is also described. As it converges and produces consistent estimates, the control policy goes to the known parameter case for increasing amount of measurements.

However, in the initial period of control, overshoots due to bad initial estimates are very likely to occur. Some modifications of control in this period are discussed.

Then the extension of the identification algorithm to the time-varying case is attempted. The torgetting of the part measurements is considered. Finally some possibilities of divergence of idenfification algorithm are pointed out.

1. Introduction

It is well known that a sampled-time linear dynamic system with lumped parameters can be described by the difference equation. Using the shift operator $z^{-1}, z^{-1} u_n = u_{n-1}$, the one-input one-output stochastic system with k-step pure delay and coloured additive noise can be represented by the following equation

$$A(z^{-1})\, y_n = z^{-k} B(z^{-1})\, u_n + C(z^{-1})\, e_n \tag{1}$$

where

$$\left.\begin{aligned} A(z^{-1}) &= 1 + A_1 z^{-1} + \dots + A_r z^{-r} \\ B(z^{-1}) &= B_0 + B_1 z^{-1} + \dots + B_r z^{-r} \\ C(z^{-1}) &= 1 + C_1 z^{-1} + \dots + C_r z^{-r} \end{aligned}\right\} \tag{2}$$

and $\{e_n\}$, $n=1, 2, \dots$, is a sequence of stochastic independent normal $\tilde{N}(0, \delta)$ variables (see [1], [2]).

The problem of stabilization of the system is

$$\min_{u_1,\dots u_{N-1}} E\left\{\sum_{i=1}^{N} y_{k+i-1}^2\right\} \tag{3}$$

subject to the relations (1) and (2). This is called N-steps control policy. In the case when the parameters A_i, B_i, C_i in (2) are known the control policy for (3) is equivalent to N steps of one-step control policies

$$\min_{u_n} E\{y_{n+k}^2\}. \tag{4}$$

The solution of the problem (the control policy) for the minimum-phase system was given by Åström [3] in the form

$$u_n = -\frac{G(z^{-1})}{B(z^{-1})F(z^{-1})} y_n \tag{5}$$

where

$$\left.\begin{aligned} F(z^{-1}) &= 1 + F_1 z^{-1} + \dots + F_{k-1} z^{-k+1}, \\ G(z^{-1}) &= G_0 + G_1 z^{-1} + \dots + G_{r-1} z^{-r+1}. \end{aligned}\right\} \tag{6}$$

Satisfy the polynomial equation

$$C(z^{-1}) = F(z^{-1}) A(z^{-1}) + z^{-k} G(z^{-1}) \tag{7}$$

and $A(z^{-1})$, $B(z^{-1})$ are defined in (2). The output of the system subject to control policy (5) is equal to

$$y_n = F(z^{-1}) e_n. \tag{8}$$

Furthermore, the solution for the nonminimum-phase system [1], [4] was also provided.

In this paper only the minimum-phase case is considered. It is also assumed that the zeros of polynomial C lie strictly outside the unit circle.

When dealing with the unknown parameters system the case of time-separated identification and control is usually considered. That is on the basis of some data the parameters of the model are estimated and then the optimal control policy is evaluated. The assumption that the estimates lie near the true values of parameters is made. Unfortunately, the bias-free methods of estimation of the equation (1) parameters are recursive, time-consuming and difficult for on-line applications.

This paper presents a model for which the least-squares identification produces consistent estimators. The resulting control policy is approximately equivalent to the Aström policy. Based on these results the stimultaneous identification discussed.

2. "Least-squares" model

As a result of definite arithmetic operations it is possible to find a model

$$y_n = R(z^{-1}) y_{n-k} + S(z^{-1}) u_{n-k} + F(z^{-1}) e_n \tag{9}$$

where

$$\left.\begin{aligned} R(z^{-1}) &= R_0 + R_1 z^{-1} + \ldots + R_r z^{-r} + \ldots, \\ S(z^{-1}) &= S_0 + S_1 z^{-1} + \ldots + S_r z^{-r} + \ldots, \\ F(z^{-1}) &= 1 + F_1 z^{-1} + \ldots + F_{k-1} z^{-k}. \end{aligned}\right\} \tag{10}$$

It may be noticed that the output of the model at step n depends only on the inputs and outputs at steps $n-k$ and earlier. The disturbance $v_n = F(z^{-1})\, e_n$ is a moving average and depends only on the values of $e_n, \ldots, e_{n-k+1}$.

As in the case of model (1) the N-steps and one-step control policies are equivalent and have the from

$$u_n = -\frac{R(z^{-1})}{S(z^{-1})}\, y_n \tag{11}$$

or, in time domain

$$u_n = -\frac{1}{S_0}\left(\sum_{i=1}^{\infty} S_i\, u_{n-1} + \sum_{i=1}^{\infty} R_i\, y_{n-i}\right). \tag{12}$$

Let us show that essentially the policy (11) and Aström policy (5) are equivalent. Applying the control policy (11) to the system (1) and inserting (8) for y_n the following polynomial identity can be found

$$A(z^{-1})\, F(z^{-1}) + z^{-k} \frac{B(z^{-1})\, R(z^{-1})\, F(z^{-1})}{S(z^{-1})} = C(z^{-1}). \tag{13}$$

Since $C(z^{-1})$ and $F(z^{-1})\, A(z^{-1})$ are the polynomial of finite degree then

$$G(z^{-1}) \triangleq \frac{B(z^{-1})\, R(z^{-1})\, F(z^{-1})}{S(z^{-1})} \tag{14}$$

is also a polynomian of finite degree. Thus (13) and (14) form an identity

$$C(z^{-1}) = F(z^{-1})\, A(z^{-1}) + z^{-k}\, G(z^{-1}). \tag{15}$$

It is seen that the degree of $G(z^{-1})$ have to be $r-1$ and then the identity (15) is the same as (7). Thus $G(z^{-1})$ in both identities are equal.

However, multiplying and deviding by $B(z^{-1})\, F(z^{-1})$ and using definition (14) the policy (11) can be expressed in the form

$$u_n = -\frac{R(z^{-1})}{S(z^{-1})}\, y_n = -\frac{\dfrac{B(z^{-1})\, R(z^{-1})\, F(z^{-1})}{S(z^{-1})}}{B(z^{1})\, F(z^{-1})}\, y_n = -\frac{G(z^{-1})}{B(z^{-1})\, F(z^{-1})}\, y_n \tag{16}$$

which is the Aström policy (5).

Moreover it may be seen that

$$\left.\begin{aligned} R(z^{-1}) &= G(z^{-1})\, D(z^{-1}), \\ S(z^{-1}) &= B(z^{-1})\, F(z^{-1})\, D(z^{-1}), \end{aligned}\right\} \tag{17}$$

where $B(z^{-1})$, $F(z^{-1})$, $G(z^{-1})$ were defined earlierd and $D(z^{-1})$ is an polynomial of infinite degree.

In practice the polynomials $R(z^{-1})$ and $S(z^{-1})$ have to be truncated. After possible cancellation of equal zeros it is possible in this way to find a good approximation of the policy (5). See Fig. 1 and Table 1 for comparison of control results for the model (1) with maximum likelihood and model (9) with least-squares identification method.

Table 1. Comparison of input nad output signal variances for models (1) with maximum likelihood and model (9) with least-squares identification methods

	Variance of input signals $\sigma_u^2=\frac{1}{N}\sum_{i=1}^{N} u_i^2$	Variance of output signals $\sigma_y^2=\frac{1}{N}\sum_{i=1}^{N} y_i^2$
No control, $u_n=0$, $n=1, 2, \ldots$	0.00000	1.7197
Model (1)	0.08200	1.0284
Model (9)	0.08636	1.0383
Actual system: $y_n-1.3y_{n-1}+0.4y_{n-2}=2u_{n-1}-1.6u_{n-2}+0_n-0.8e_{n-1}+0.15e_{n-2}$.		

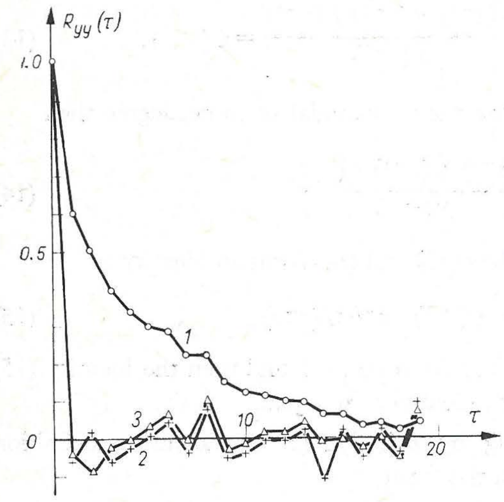

Fig. 1. Comparison of the autocorrelation function for the same case as in Table 1

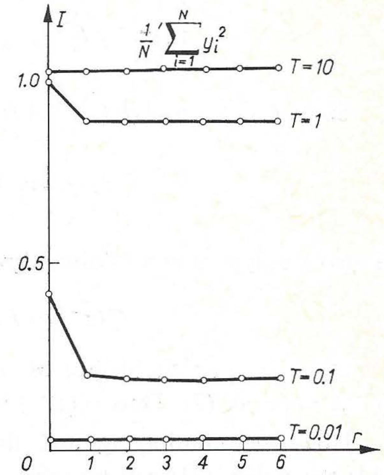

Fig. 2. Estimates of the variances of output signals versus number of model parameters for different sampling intervals. $T_p=1$, $T_n=10$, $\delta_e=\delta_e=1$

When $R(z^{-1})$ and $S(z^{-1})$ are finite then the least-squares method of identification provides consistent estimators of parameters. The proof is a slight generalization of the proof given by Goldberger [5] for one step delay and can be found in [6].

The numerical tests indicate that very often the 4—8 parameters model is sufficiently good, see Fig. 2 which portrays the plot of the criterion values versus number of parameters for the first order system described formally by the equation

$$y(s)=\frac{1}{T_p s+1} u(s)+\frac{1}{T_n s+1} e(s) \tag{18}$$

and sampled at equal time intervals T. The stepwise control was used.

3. Simultaneous identification and control

The case of "simultaneous" identification and control is now considered. This means that at every step of control the control policy with adjusted parameters is used. An adjustment is made on the basis of on-line identification. The method uses the dual control approach of Feldbaum [7] and derivations follows these given by Aoki [8].

To make the formule more concise the following definitions are made.

$$\left.\begin{aligned} \boldsymbol{a}^T &= [a_0, a_1, \ldots, a_{2r+1}] \triangleq [S_0, \ldots, S_r, R_0, \ldots, R_r], \\ \boldsymbol{S}_n^T &= [u_n, \ldots, u_{n-r}, y_n, \ldots, y_{n-r}]. \end{aligned}\right\} \tag{19}$$

Then the equation (9) can be put in the form

$$y_{n+k}=a_0 u_n+\ldots+a_r u_{n-r}+a_{r+1} y_n+\ldots+a+_{2r+1} y_{n-r}+v_n=\boldsymbol{a}^T s_n+v_n. \tag{20}$$

Unfortunaly, due to some mathematical problems the N-steps simultaneous control policy cannot be obtained in closed analytical form (in connection with this see example in Aoki [8] pp. 111—116). Thus in every step n, the one-step control which is now called one-step-ahead control, is considered. The optimal control policy can be expressed in the form

$$u_n=-\frac{\sum_{i=1}^{r} [\hat{a}_0 \hat{a}_i+\operatorname{cov}(a_0 a_i)] u_{n-1}+\sum_{i=1}^{r} [\hat{a}_0 \hat{a}_{r+i+1}+\operatorname{cov}(a_0, a_{r+i+1})] y_{n-1}}{\hat{a}_0^2+\operatorname{var}(a_0)} \tag{21}$$

where a_i and cov a_0, a_i, $i=0, \ldots, 2r+1$, are the estimates of the parameters a_i and the covariance respectively. The deduction of the policy [9] can be found in [9]. Actually the policy (21) is a generalization of the policy (12) and is equal to (12) when the parameters estimates are equal to parameters and covariances estimates are equal to zero.

It is assumed that the disturbance variance $\delta^2 v$ is known and a priori distribution of parameters estimates is normal with known mean $\boldsymbol{a}_0$ and variance P_0, the estimates satisfy the following equations

$$\left.\begin{aligned} \hat{\boldsymbol{a}}_n &= \hat{\boldsymbol{a}}_{n-1}+\boldsymbol{q}_{n-1}(y_n-\hat{\boldsymbol{a}}_{n-1} s_{n-r}), \\ P_n &= P_{n-1}-g_{n-1} s_{n-r}^T P_{n-1}, \\ g_{n-1} &= P_{n-1} s_{n-r}(\delta_v^2+s_{n-1}^T P_{n-1} s_{n-r})^{-1}, \end{aligned}\right\} \tag{22}$$

with initial values P_0, a_0 and $\{u_0, u_{-1}, \ldots, u_{-(k+r-1)}, y_0, y_1, \ldots, y_{-(k+r-1)}\}$.

The above estimation algorithm provides the consistent estimators of parameters even for the closed-loop system, see [10]. Moreover, the estimates satisfy the equations

$$\left.\begin{aligned} \hat{a}_i &= a_i + o\left(\frac{1}{n}\right), \\ \operatorname{cov}(a_0, a_i) &= o\left(\frac{1}{n}\right), \\ \operatorname{var}(a_0) &= o\left(\frac{1}{n}\right), \qquad i=0, \ldots, 2r+1. \end{aligned}\right\} \tag{23}$$

and plim $n \cdot o\left(\frac{1}{n}\right) = 0$.

Thus as $n \to \infty$ the control policy (21) goes to the known parameters policy (12). Since the policy (12) is optimal for arbitrary number of step N in the criterion (3) then in the limit the policy (21) equals to the N-steps one.

4. Initial period of control

Although in the steady state period the control policy (19) produces satisfying results, the initial period depends strongly on initial conditions. Moreover, at the end of this period large overshoots are likely to happen, see Fig. 3 where the control policy for the system (18) was evaluated on the basis of an arbitrary chosen model

$$y_{n+1} = a_0 u_n + a_1 u_{n-1} + a_0 y_n + v_n \tag{24}$$

with initial values for the regulator

$$a_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad s_0 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad P_0 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}.$$

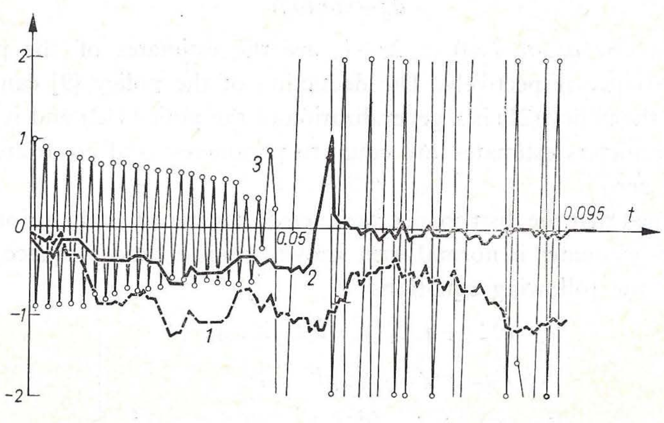

Fig. 3. Input and output runs. One-step-ahead control policy. $T_p=1$, $T_n=2$, $T=0.01$

See [10] for more examples. The overshoots appeare when estimates of the covariancies decrease to small values. In this time parameters estimates are usually still comparatively far from exact values and the regulator (21) is usually unstable. This effect especially likely happens when the zeros of the nominator $S(z^{-1})$ in (9) lie near the unit circle. Then the regulator can easily be unstable for even small changes of parameters in $S(z^{-1})$.

Thus at the initial period a different control policy is needed. The simple idea of finding better initial values is practically difficult to perform. Comparatively good results can be obtained after bounding the input signal, see Fig. 4, where the control of the same system (18), (24) is achieved. However, the slow convergence of parameters and long initial period can be produced, see Fig. 5.

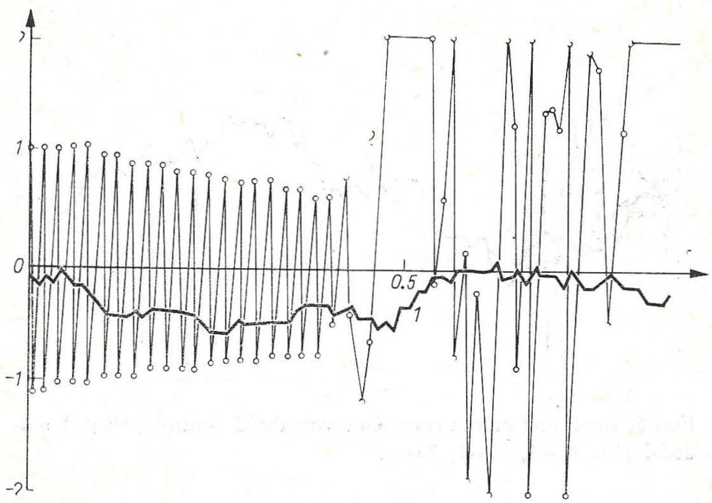

Fig. 4. Input and output runs. One-step-ahead control policy, bounded input. $T_p=1$, $T_n=2$, $T=0.01$

The other possible solution could be the application of N-steps-ahead control, policy where in every step n, the control policy minimizes the criterion

$$\min_{u_n, u^*_{n+1}, \dots, u^*_{n+N-1}} E\left\{\sum_{i=1}^{N} y^2_{n+k+i-1}\right\}. \tag{25}$$

In the step $n+1$ the policy results from similar minimization and u_{n+1} is in general different from u_{n+1}. As in the case of N-steps control policy the N-steps-ahead control policy cannot be found in closed analytical form and therefore an approximation is necessary. An example of suboptimal openloop feedback control policy ([9] pp. 241—246) is included. Due to complicated arithmetical transformations involved, the two-steps-ahead open-loop feedback control policy, only, is presented. The input signal u_n for the system (18), (24) is evaluated in every step n from the equation

$$\begin{bmatrix} \langle a_0^2\rangle & \langle a_0^2\, a_2+a_0\, a_1\rangle \\ \langle a_0^2\, a_2+a_0\, a_1\rangle & \langle a_0^2+a_1^2+a_0^2\, a_2^2+2a_0\, a_1\, a_2\rangle \end{bmatrix} \begin{bmatrix} u_{n+1}^* \\ u_n \end{bmatrix} =$$

$$= -\left(\begin{bmatrix} \langle a_0\, a_1\, a_2\rangle \\ \langle a_0\, a_1\, a_2^2+a_1^2\, a_2+a_0\, a_1\rangle \end{bmatrix} u_{n-1} + \begin{bmatrix} \langle a_0\, a_2^2\rangle \\ \langle a_0\, a_2^3+a_0\, a_2+a_1\, a_2^3\rangle \end{bmatrix} y_{n-1} \right) \tag{26}$$

where $\langle\cdot\rangle$ is the conditional mean value operator, that is $\langle\cdot\rangle=\int_a \{\cdot\}\, p\,(\boldsymbol{a}|u^{n-1}, y^n)\, d\boldsymbol{a}$. The performance of the control is presented at Fig. 6 and Fig. 7. The latter indicates

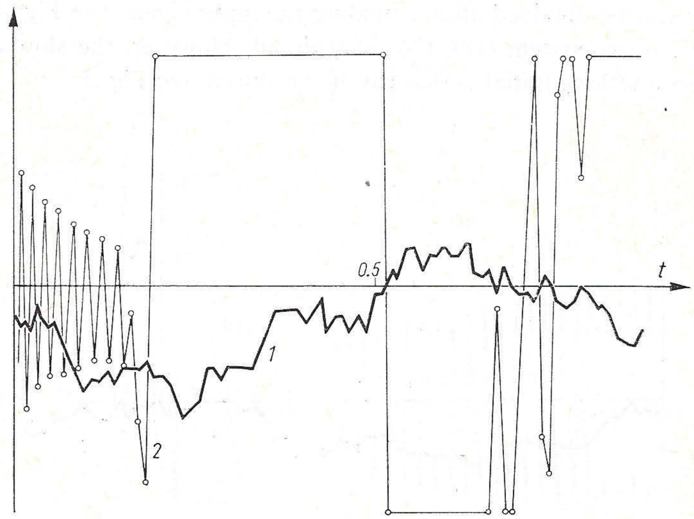

Fig. 5. Input and output runs. One-step-ahead control policy, bonuded input. $T_p=1$, $T_n=1$, $T=0.01$

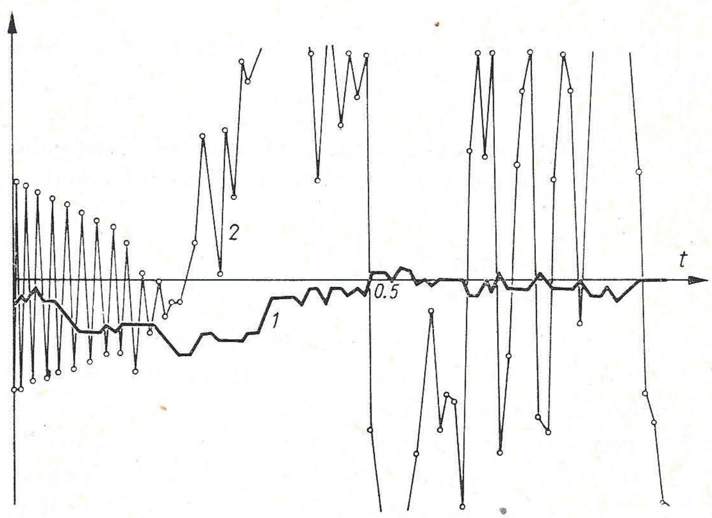

Fig. 6. Input and output runs. Two-steps-ahead open-loop-feedback control policy. $T_p=1$, $T_n=2$, $T=0.01$

that for the regulator with the poles lying near the stability boundary the overshoots are still possible. In this case the use of three (or more) — steps-ahead policy is necessary.

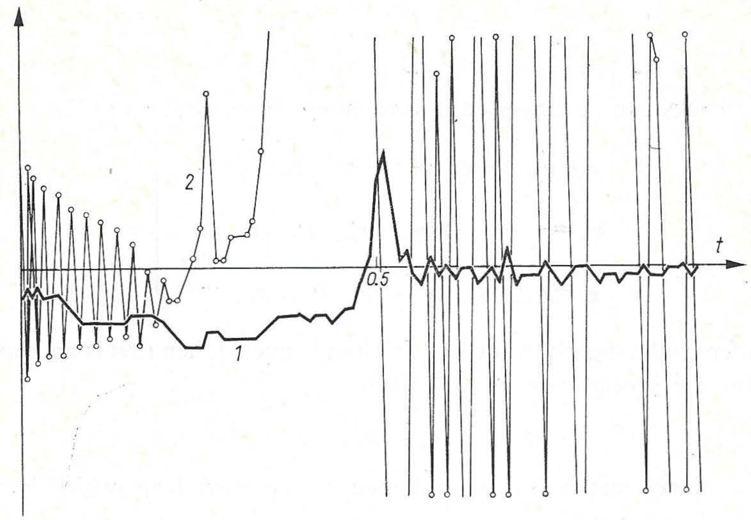

Fig. 7. Input and output runs. Two-steps-ahead open-loop-feedback control policy. $T_p=1$, $T_n=0.001$

5. Time varying systems

When dealing with the varying parameters system it is often assumed that parameters form the Gauss-Markov sequence

$$a_n = \emptyset a_{n-1} + w_n . \qquad (27)$$

The control policy for this case is the same as for the constant parameters system, i.e. policy (21). The suitable identification algorithm can be produced provided $\emptyset$ is a known matrix [11]. As practically, the matrix $\emptyset$ is scarcely known the algorithm relying on forgetting the past measurements is considered. As an example, the exponential forgetting (weighted least-squares) algorithms is presented. However, it is believed that argumentation also matches other forms of forgetting, like moving--window algorithms etc.

In the exponential forgetting method the minimization of the following criterion is considered

$$\min_a (y_n - x_n a)^T w_n (y_n - x_n a) \qquad (28)$$

where

$$y_n^T = [y_1, \ldots, y_n] \text{ — vector of outputs}$$

$$x_n = \begin{bmatrix} u_0, \ldots, & u_{-r}, & y_0, \ldots, & y_{-r} \\ u_1, \ldots, & u_{-r+1}, & y_1, \ldots, & y_{-r+1} \\ \cdots & \cdots & \cdots & \cdots \\ u_{n-1}, \ldots, & u_{n-r-1}, & y_{n-1}, \ldots, & y_{n-r-1} \end{bmatrix} \text{ — matrix of measurements}$$

$$w_n = \begin{bmatrix} \alpha^n & & & \\ & \ddots & & 0 \\ & & \ddots & \\ 0 & & \alpha & \\ & & & 1 \end{bmatrix} \text{— weighting matrix, } 0 \leqslant \alpha \leqslant 1.$$

The estimates can be expressed in a reccurent from similar to (22)

$$\left.\begin{aligned} \hat{a}_n &= \hat{a}_{n-1} + g_{n-1}(y_n - a_{n-1}\, s_{n-1}), \\ P_n &= \frac{1}{\alpha}(P_{n-1} - g_{n-1}\, s_{n-1}^T\, P_{n-1}), \\ g_n &= P_{n-1}\, s_{n-1}\, (\alpha + s_{n-1}^T\, P_{n-1}\, s_{n-1})^{-1}. \end{aligned}\right\} \tag{29}$$

If α differs only slightly from 1 (as it often happens) then (29) is an approximate solution for the special case of (27) where

$$a_n = \alpha a_{n-1}. \tag{30}$$

The covariance matrix can be evaluated in one more form which is of special interest in the Appendix

$$P_n^{-1} = \alpha^n P_0^{-1} + \sum_{i=0}^{n-1} \alpha^i s_{n-i-1}\, s_{n-i-1}^T. \tag{31}$$

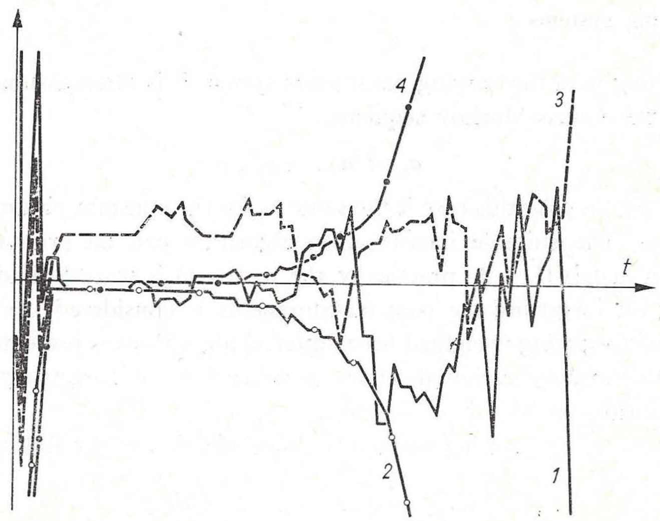

Fig. 8. Estimates of parameters and covariances. One-step-ahead control policy, exponential smoothing. $T_p=1$, $T_n=1$, $T=1$, $\delta=0.83$

Unfortunately, with groving n the matrix P_n^{-1} becomes singular and the identification algorithm diverges, see Fig. 8 and [10] for more details and mathematical derivation. Thus a different solution is necessary.

A simple and convienient algorithm is obtained after fixing the first parameter is the vector $\boldsymbol{a}$, i.e. a_0. Then the control policy has the form similar to the known parameters case (12)

$$u_n = -\frac{1}{a_0}\left(\sum_{i=1}^{r} \hat{a}_i u_{n-1} + \sum_{i=0}^{r} \hat{a}_{r+1+1} y_{n-i}\right) \tag{32}$$

and the identification algorithm has the form (29) exept that $\hat{\boldsymbol{a}}_n$ is now $(2r+1)$ — vector instead $(2r+2)$ like previously, that is $\hat{\boldsymbol{a}}_n^T = [\hat{a}_1, \ldots, \hat{a}_{2r+1}]$. The regulator working on the basis of this policy is called self-tunning regulator [12], [13]. The convienient properties of the regulator and industrial applications prove it could be a very useful tool in control practice.

However, as in the optimal one-step-ahead control policy the initial period of control has to be treated with some care as the overshoots are likely to happen. The bounding of the input signals often gives good results unless the situation pictured on Fig. 5 occurs. In such a case a prolonged constant input may once more cause the singularity of the covariance matrix and dovergence of the identification algorithm. The mathematical argumentation for this case is presented in Appendix.

6. Conclusions

This paper motivates the use of a simple recursive algorithm for automatic adaptive control of unknown linear system. Although the algorithm is not optimal for the canonical model of the system, the advantages of recursive form, which include the small amount of computer store needed and high speed of computation, make the algorithm attractive for practical applications. The behaviour of control in the steady state period is satisfactory and only the initial period needs a more detailed theoretical argumentation and possibly better control policy.

Further research can include: generalization of the algorithm to multi-input multi-output systems, the more practically relevant solution for nonminimum-phase systems and also a discussion of the choice of sampling interval. All these problems are very important for practical application of the algorithm. Moreover, the numerical examples indicate that the sampling interval has a strong influence on the closed-loop characteristics such as the input and output signal variances or the gain in output variance in closed-loop in comparison with the same variance subject to zero input signal.

7. Appendix

Let us look at the structure of matrix $\boldsymbol{s}_i \boldsymbol{s}_i^T$ for soma $i>r$. It has the form

$$\boldsymbol{s}_i \boldsymbol{s}_i^T = \begin{bmatrix} u_i^2, & \ldots, & u_i u_{i-r}, & u_i y_i, & \ldots, & u_i y_{i-r} \\ \cdots & & & & & \cdots \\ u_{i-r} u_i, & \ldots, & u_{i-r}, & u_{i-r} y_i, & \ldots, & u_{i-r} y_{i-r} \\ y_i u_i, & \ldots, & y_i u_{i-r}, & y_i^2, & \ldots, & y_i y_{i-r} \\ \cdots & & & & & \cdots \\ y_{i-r} u_i, & \ldots, & y_{i-r} u_{i-r}, & y_{i-r} y_i, & \ldots, & y_{i-r}^2 \end{bmatrix} \tag{33}$$

Assuming that at least for r steps the input was constant and equal to u, that is $u_j=u$, $j=i-r, ..., i$, then

$$s_i\, s_i^T = \begin{bmatrix} u^2 & ,..., & u^2 & , & uy_i & ,..., & uy_{i-r} \\ \cdots & & & & & & \cdots \\ u^2 & ,..., & u^2 & , & uy_i & ,..., & uy_{i-r} \\ uy_i & ,..., & uy_i & , & y_i^2 & ,..., & y_i\, y_{i-r} \\ \cdots & & & & & & \cdots \\ uy_{i-r}, & ..., & uy_{i-r}, & & y_{i-r}\, y_i, & ..., & y_{i-r}^2 \end{bmatrix} \tag{34}$$

As r first rows are equal, the matrix $s_i\, s_i^T$ is singular. Let j be the step from which the input is constant and $n>j+r$. Then from (31)

$$P_n^{-1} = \alpha^n P_0^{-1} + \sum_{i=n-j-r}^{n-1} \alpha^i s_{n-i-1}\, s_{n-i-1}^T + \sum_{i=0}^{n-j-r-1} \alpha^i s_{n-i-1}\, s_{n-i-1}^T . \tag{35}$$

But

$$\sum_{i=0}^{n-j-r-1} \alpha^i s_{n-i-1}\, s_{n-i-1}^T =$$

$$= \begin{bmatrix} u^2\Sigma\alpha^i & ,..., & u^2\Sigma\alpha^i & , & u\Sigma\alpha^i y_i & ,..., & u\Sigma\alpha^i y_i \\ \cdots & & & & & & \cdots \\ u^2\Sigma\alpha^i & ,..., & u^2\Sigma\alpha^i & , & u\Sigma\alpha^i y_i & ,..., & u\Sigma\alpha^i y_i \\ u\Sigma\alpha^i y_i & ,..., & u\Sigma\alpha^i y_i & , & \Sigma\alpha^i y_i^2 & ,..., & \Sigma\alpha^i y_i\, y_{i-r} \\ \cdots & & & & & & \cdots \\ u\Sigma\alpha^i y_{i-r}, & ..., & u\Sigma\alpha^i y_{i-r}, & & \Sigma\alpha^i y_{i-r} y_i, & ..., & \Sigma\alpha^i y_{i-r} \end{bmatrix} \tag{36}$$

and the matrix is singular. Now

$$\left.\begin{aligned} &\alpha^n P_0^{-1} \xrightarrow[n\to\infty]{} 0, \\ &\sum_{i=n-j-r}^{n-1} \alpha^i s_{n-i-1}\, s_{n-i-1}^T \xrightarrow[n-j\to\infty]{} 0. \end{aligned}\right\} \tag{37}$$

When the two first terms on the right hand side of equation (35) are negligible then P_n^{-1} is almost singular and its inverse in (29) diverges.

8. Nomenclature

$\boldsymbol{a}$	— vector of system parameters, eg. (19)
$\boldsymbol{a}_i$	— components of $\boldsymbol{a}$
$\hat{\boldsymbol{a}}_n$	— vector of system parameters estimates, eg. (22)
$A, B, C,$	— polynomials in z^{-1}, eg. 2
A_i, B_i, C_i	— coefficients of A, B, C respectively
D	— polynomial in z^{-1}, eg. (17)
e_n	— stochastic normal variable, eg. 2
E	— mean value

F, G	— polynomials in z^{-1}, eg. (6)
F_i, G_i	— coefficients of F, G respectively
g_n	— vector, eg. (22)
k	— pure delay of the system, eg. (1)
n	— present step of control
N	— horizon of control, eg. (3), (25)
P_n	— covariance matrix of estimates, eg. (22)
r	— number of measurements of input and output in the vector s_n
R, S	— polynomials in z^{-1}, eg. (10)
R_i, S_i	— coefficients of R, S respectively
s_n	— vector of measurements, eg. (19)
u_n	— input signal at step n
v_n	— stochastic normal variable, eg. (20)
w_n	— stochastic vector, eg. (27)
W_n	— eweighting matrix, eg. (28)
X_n	— matrix of measurements, eg. (28)
y_n	— output signal at step n
y_n	— vector of output signals, eg. (28)
z^{-1}	— back-shifting operator
α	— weighting factor, eg. (28)
$\emptyset$	— transfer matrix, eg. (27)
δ^2	— variance of the sequence $\{e_n\}$
δ_v^2	— variance of the sequence $\{v_n\}$

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Stabilizacja cyfrowa liniowego układu stochastycznego

Rozpatrzono metody sterowania cyfrowego dla stochastycznego układu liniowego. Rozważono model o próbkowaniu czasowym. Celem sterowania jest minimalizacja wariancji sygnału wyjściowego. Zaproponowano prosty model, dla którego identyfikacja metodą najmniejszych kwadratów daje zgodne estymatory parametrów. Uwzględniono również syntezę sterowania układem. Następnie rozpatrzono jednoczesną identyfikację i sterowanie. Pokazano, że dla takiego postępowania jest możliwe wyznaczenie sterowania optymalnego przy kryterium z jednym krokiem wyprzedzenia. Opisano także algorytm sterowania on-line. Ze wzrostem liczby pomiarów algorytm wykazuje zbieżność. Wyznacza on oceny zgodne, natomiast otrzymane sterowanie optymalne dąży do wyliczonego na podstawie znajomości parametrów. Jednak w początkowym okresie sterowania złe wstępne oceny mogą powodować przeregulowania. Omówiono więc metodę modyfikacji metody sterowania dla okresu początkowego. W dalszej części pracy podano próbę rozszerzenia algorytmu identyfikacji na przypadek zmienności w czasie. Rozważono przy tym możliwość zapominania części pomiarów. Uwagi końcowe zawierają wskazania co do możliwej niezbieżności algorytmu identyfikacji.

Цифровая стабилизация линейной стохастической системы

В статье рассмотрены методы цифрового управления стохастической линейной системой. Рассматриваемая модель является задачей с временными выборками. Целью управления является минимизация дисперсии выходного сигнала. Предложена простая модель, для которой идентификация методом наименьших квадратов даёт доверительные оценки параметров. Учтено также синтез управления системой.

Затем рассмотрены идентификация и управление одновременно. Показано, что в этом случае возможно определить оптимальное управление для показателя с опережением на один шаг. Описан также непосредственный алгоритм управления. С увеличением числа измерений обеспечивается сходимость алгоритма, который определяет доверительные оценки, а полученное оптимальное управление стремутся к рассчитанному на основе известных параметров.

Однако в начальный период управленя плохие предварительные оценки могут привести к отрегулировке. Рассмотрены затем некоторые модификации управления для начального периода.

Далее предложена попытка расштрипь алгоритм идентификации на случай изменений во времени. При этом рассмотрена возможность отбрасывать часть измерений. В заключение предложено замечания касающиеся возможной несходимости алгоритма идентификации.