Control
and Cybernetics
VOL. 5 (1976) No. 1

Controller Synthesis for Nonlinear Plants with Conditions on Stability Region

by

ANNA LEWANDOWSKA
Institute for Organization,
Management and Control Sciences
Polish Academy of Science
Warszawa, Poland

Synthesis method of a controller, which reduces small disturbances and at the same time ensures maximization of the stability region for dynamical system described by ordinary differential equations is considered.

Two tasks are solved; the first — choosing proper structure of the controller, which performs two functions mentioned above and the second — estimating and maximizing of stability region.

The first function is realized in form of the decomposition. In order to reduce small disturbances the classical method with linearization of the system and solving linear-quadratic problem is used. In order to maximitze the stability region, paralelly to linear controller the nonlinear one is used. The output of this last controller depends on the second and the higher powers of error signal. In consequence of that the linearized system is not changed.

The problem of the stability region estimation is solved using the second Liapunov method. Practical algorithm for determination of the optimal quadratic Liapunov function is presented. The Liapunov function parameters are determined in such a way to maximize the measure of stability region for fixed system parameters. Later on, the parameters of nonlinear controller are chosen in such a way to maximize the measure of stability region estimate for closed-loop system. Computation results for autorefrigerated chemical reactor are also presented as an example.

1. Introduction

In the analysis and design of control systems one must recognize the virtual inevitability of known or unknown disturbances on the system. It is the function of the controller to maintain the system in the desired state in the face of such disturbances. The present work is a study of control-system behaviour for:

1) small disturbances using typical linear theory methods;

2) large disturbances where the disturbances cause large changes in one or more of the state variables.

The need for considering such large disturbances arises from the fact that quite large disturbances do occur in real control systems. Therefore if a controller cannot maintain the desired state of a system when subjected to large disturbances it may often be considered a failure. The need for considering nonlinear phenomena in control systems is obvious from their universal occurrence. Linear approximations fail to predict many types of system behaviour which could only be classified as failures of the control system. Usually in such situations one must use the hand-controlling. However it is very important to neutralize the results of large disturbances as soon as possible. Hence there is the need of a particular process-controller combination, which assures proper work of the system when the large disturbances occur.

In the present work the controller synthesis method is proposed in such a way, as to maintain the system state in the desired point. The function of the controller is decomposed. The state stabilization task is realized by the controller as a linear one. The neutralizing of the results of large disturbances is realized by the controller as a nonlinear one. In that way from theoretical point of view the control system realizing the state stabilization function as a matter of fact works as a linear one near the desired point and the control system counteracting the results of large disturbances works as a nonlinear one. Such a controller can be named the controller with internally changing structure or the adaptive controller with respect to the kind of the disturbances.

2. Problem statement

The disturbances from 1 and 2 group interacting in finite time and causing finite changes of the system state are considered. The changes of the system state after the end of disturbance acting are considered. The function of the controller is to lead the system to desired state.

Let the process be described by a set of ordinary differential equations of the form

$$\dot{x}=f(x, u),\ x(t) \in R^n,\ u(t) \in R^m,\ f\colon R^n \times R^m \to R^n . \tag{1}$$

(Notice that, the Eq. (1) describes the process considering the system state after the end of disturbance acting).

It is assumed that the function $f(x, u)$ has continuous second partial derivatives with respect to x and u.

For $x_0 \in R^n$, $u \in C^0\,[t_0, T]$ the unique solution of the system (1) exists.

There is an equillibrium point (x_e, u_e), for the system (1), which is a solution of the algebraic vector equation

$$f(x_e, u_e)=0 . \tag{2}$$

The control-system synthesis which ensures the state of the system (1) near desired equillibrium point (x_e, u_e) is wanted to be done.

It is assumed that the form of the controller function $u=u(x, p)$, where $p \in R^s$ is the controller parameter vector, is known.

It is also assumed that the function $u=u(x, p)$ is such that ensures the stability of the closed system

$$\dot{x}=f\big(x, u(x, p)\big).$$

Then there is the neighbourhood S of the equillibrium point, named asymptotic stability region or attraction region of the equillibrium point, which is defined in the following way

$$S[p]=\{x_0 \in R^n | \lim_{t\to+\infty} x(t; t_0, x_0, p)=x_e\}, \tag{3}$$

where $x_0=x(t_0; t_0, x_0, p)$.

It is easy to notice (Fig. 1), that if there is larger stability region S, then longer time-period of large disturbance acting and larger disturbance will be admitted without failure of the system. On the Fig. 1 there is shown the trajectory in the situation when large disturbance acts, the first (dashed line) with time-period $\tau'=t_2'-t_1$ and the second (continuous line) with time-period $\tau''=t_2''-t_1$. In the first case at the moment of the end of disturbance acting the system state is in the asymptotic stability region S. The controller can lead the system state to the equillibrium point x_e. In the second case — the time-period τ'' is too long to allow to do it and the disturbance will lead the system state outside the asymptotic stability region S. Then the state stabilization will be impossible. In order to avoid such a situation the control-system should be designed to assure sufficiently large stability region near the desired equilibrium point.

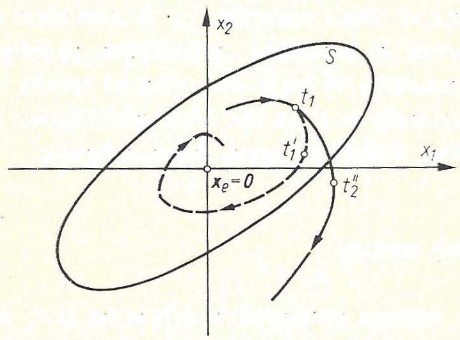

Fig. 1

Since for the closed loop system the stability region (3) depends on the controller parameters, the problem of the parametric synthesis can be formulated as follows: choose the parameter vector p in such a way, to get the largest possible stability region in the sense of a given measure of that region.

The measure of the set is difined as follows.

Definition. The number $m \in R^+$ is called the measure of the set $Z \subset R^n$, if

$$m=\xi(Z), \qquad \xi: 2^{R^n} \to R^+, \qquad \xi(\emptyset)=0, \tag{4}$$

$$\bigwedge_{Z_1 \subset Z_2 \subset R^n} \xi(Z_1) \leqslant \xi(Z_2). \tag{5}$$

Since the stability region depends on the controller parameters, then the measure of that region also depends on that parameters

$$m = m(p) = \xi(S[p]).$$

The task of the parametric controller synthesis algorithm, constructed in order to extend the stability region, is to choose the parameter vector p so as, to receive maximum of the stability region measure:

$$\hat{m} = \max_{p \in P} \{m(p)\}, \tag{6}$$

wher P is the set of the physically realized parameters.

It is known that the extending of the stability region can undesirably change the dynamical properties of the closed-loop system. The system can become "slowly working" and consequently the disturbances will be ineffectively damped. That fact is of great importance in the case of the small disturbances, which act more frequently than the large disturbances.

To the controller synthesis one can use the fact, that the amplitudes of disturbances of 1 and 2 group differ very much.

It can be used one of generally known linear theory methods to ensure the proper work of the control system near the equilibrium point. It means, that one searches the solution of local optimization problem (in such neighbourhood of the equilibrium point, in which linearization of the system is justified) in sense of chosen criterion. It is easy to notice, that such method of solving of the optimization problem is sufficient for damping of small disturbances. It also allows to avoid the system complication, which arises in nonlinear optimization problem.

In the face of the necessity of large disturbances results reduction at the same time it would be conveniently to solve the local optimization problem and the task of maximization of the stability region independently.

3. Decomposed synthesis method

The purpose of the method presented in this point is to decompose the task of local optimization and the task of maximization of the stability region. It means that the control function must be chosen in such a way as to allow to solve the local optimization problem and the task of maximization of the stability region independently.

Consider the control function represented by equation

$$u(x, p) = u_e + M_1 (x - x_e) + (x - x_e)^T M_2 (x - x_e) + \dots, \tag{7}$$

where the $m \times n$ matrices M_1, M_2 have the elements, which are constant with respect to the state x, but depend on the parameters p.

Consider the parameter vector p which is represented as follows:

$$p = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}. \tag{8}$$

It is assumed that the elements of the matrix M_1 depend on the parameters p_1, and the elements of the matrix M_2 depend on the parameters p_2.

For the control function defined above, the parameters p_1 can be chosen in order to solve local optimization problem and the parameters p_2 can be chosen in order to maximize the stability region.

It is easy to notice, that the control function (7) satisfies the following conditions:

1. The assurance of the maintaining desired equilibrium point.

It means, that the equation, which assures the maintaining of the steady state is satisfied

$$\bigwedge_{p \in P} u_e = u(x_e, p). \tag{9}$$

2. The assurance of the possiblity of solving of local optimization problem.

For the small state and control deviations from equilibrium point, which can be characterized by

$$\delta x = x - x_e, \quad \delta u = u - u_e \tag{10}$$

the linearized system state equation of the system is the following

$$\delta \dot{x} = A\,\delta x + B\,\delta u, \tag{11}$$

where

$$A = \left.\frac{\partial f(x, u)}{\partial x}\right|_{\substack{x = x_e \\ u = u_e}}, \quad B = \left.\frac{\partial f(x, u)}{\partial u}\right|_{\substack{x = x_e \\ u = u_e}}. \tag{12}$$

For the example one can take the quadratic criterion, which is obvious for the small state and control deviations from the equilibrium point:

$$J(\delta u) = \frac{1}{2} \int_{t_0}^{+\infty} (\delta x^T Q\, \delta x + \delta u^T U\, \delta u)\, dt, \tag{13}$$

where $Q_{n \times n}$ — the symetric semi-positive defined matrix, $U_{m \times m}$ — the symetric positive defined matrix. There are no constrains on δu.

Moreover it is assumed, that the system (11) is completely controlable, or that the uncontrolable part of the system (11) is stable.

As the solution of the linear — quadratic problem (minimalization of the performance (13)) one obtains the analytic form of the local optimal control [2]

$$\delta \hat{u}(t) = -U^{-1} B^T \hat{K}\, \delta x(t) \tag{14}$$

where the matrix $\hat{K}_{n \times n}$ has the constant elements. The matrix $\hat{K}$ is the solution of the nonlinear Riccati matrix equation

$$-\hat{K}A - A^T \hat{K} + \hat{K} B\, U^{-1} B^T \hat{K} - Q = 0. \tag{15}$$

For the linear system, which has m control variables and n state variables the matrix M_1 has m rows and n columns, it means $m \cdot n$ elements. It follows that the sufficient condition of the possibility of the local optimization solution in sense of the criterion (13) is the existence of the solution (with respect to p_1) of the equation:

$$M(\hat{p}_1) = -U^{-1} B^T \hat{K}. \tag{16}$$

For the existence of the solution of the Eq. (16) it is sufficient to assume the conditions:

$$\dim p_1 = m \cdot n \tag{17}$$

and

$$p_1 = \begin{Bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{Bmatrix} \tag{18}$$

where the column vector m_i is the row number i of the matrix M_1.

It must be emphasize, that in order to design the linear part of the control function one can use the other methods of the small disturbance reduction. It may be for example the expanding of the maximal band width frequency of the control system or so on.

3. The possibility of the maximization of the asymptotic stability region without influence on the local optimization.

It means, that the task of maximization of the stability region measure can be solved by choosing vector p_2

$$\hat{m} = \max_{p_2 \in P} \{m(\tilde{p})\}, \qquad \tilde{p} = \begin{Bmatrix} \hat{p}_1 \\ p_2 \end{Bmatrix}, \tag{19}$$

where vector $\hat{p}_1$ is the solution of local optimization problem.

It's easy to see, that if matrix M_1 depends only on parameters p_1, then the values of parameters p_2 do not affect the local dynamical properties of the system.

Hence, the control function (7), which satisfies above mentioned conditions 1, 2, allows to decompose the task of the local optimization and the task of maximization of the stability region. The first task of the controller synthesis can be realized by choosing the parameters p_1 of the linear part of the control function (7). The second task of the controller synthesis can be realized by choosing the parameters p_2 of the nonlinear part of the control function.

The diagram which illustrates the nonlinear controller structure (7) synthesised by the above presented method is shown in the Fig. 2.

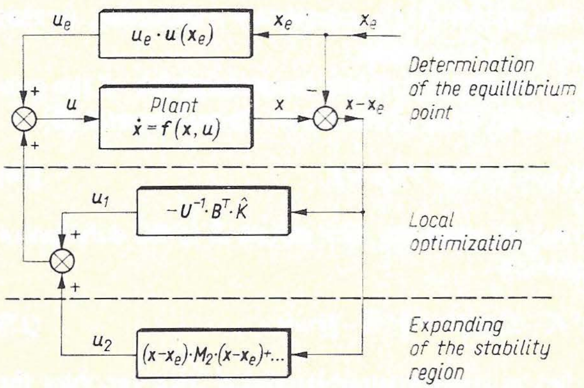

Fig. 2

It must be added, that the control function, which posses above mentioned decomposition property, can be used in more general form

$$u_i = u_{ei} + \sum_{k=1}^{r} \alpha_{ik}\, \varphi_k(x), \tag{20}$$

where u_i — the component number i of the vector u; $\varphi_k(x)$ — arbitrally chosen set of functions of the class C^1 (the functions which have continuous partial derivatives); α_{ik} — constant coefficients.

It was presented in details in [4].

It is easy to show [4], that the proposed method can be used to choose the parameters of PI controller and also to choose the structure of the nonlinear PI controller.

4. Algorithm of approximation of the asymptotic stability region

In practice it is very difficult to find the stability region (3). Using Liapunov's stability theory, one can propose the method of the approximation of stability region.

The set, which approximates the stability region one can call the estimate of that region.

The interest of the method lies in the fact that, for fixed Liapunov function (for example quadratic form), it allows to make the best possible estimate of the stability region with respect to the measure of that region. The algorithm of the asymptotic stability region approximation uses following theorem.

Theorem [3]. Consider the dynamical system $\dot{x} = f(x), f(0) = 0$, Let $\varphi(x): R^n \to \to R^1$, $\psi(x): R^n \to R^1$ be the scalar functions.

Let moreover

1) $V = \varphi(x) \in C^1$;
2) $\varphi(0) = 0$;
3) $\psi(x) = \langle \operatorname{grad} \varphi(x), f(x) \rangle$;
4) $N'(\beta) = \{x \in R^n \mid \varphi(x) < \beta\}$; let $N(\beta)$ be the component of $N'(\beta)$ containing the equilibrium point $x_e = 0$; otherwise let $N(\beta) = \emptyset$;
5) $\bigvee_{\substack{\beta^0 > 0 \\ \beta^0 \in R^1}} \bigwedge_{\{x_n\} \subset N(\beta^0)} (x_n \nrightarrow 0) \cap (\psi(x_n) \to 0) \Rightarrow \varphi(x_n) \to \beta^0$;
6) the equilibrium point $x_e = 0$ is asymptotically stable.

Then

$$S \supseteq N(\beta^0).$$

From the thesis of the theorem one obtains, that the set $N(\beta^0)$ can be used as the estimate of the real stability region S. Note that from the condition 5 of the theorem it follows that for the continuous functions $\varphi(x)$, $\psi(x)$ the number β^0 can be defined in following way

$$\beta^0 = \min_{x \in \Omega} \varphi(x), \qquad \Omega = \{x \in R^n \mid x \neq 0 \cap \psi(x) = 0\} \tag{21}$$

and then the estimate of the stability region is given by

$$N(\beta^0)=\{x \in R^n \mid \varphi(x)<\beta^0\}. \tag{22}$$

It is easy to notice that Liapunov function satisfies the conditions of the Theorem. In other words $\varphi(x)$ can be Liapunov function.

One can introduce the measure of the estimate, defined by (22), using the definition presented in the point 2. If one chooses different Liapunov functions, as function $\varphi(x)$, one can obtain the estimates with different measures. Hence the following problem arises. One finds the optimal estimate in sense of the measure and suiting to that the optimal Liapunov function.

That Problem can be solved as follows:

1. Assume the form of Liapunov function

$$V=\varphi(x, H),$$

where $H \in R^k$ — vector of Liapunov function parameters.

2. Find the value β^0 defined by (21).
3. Calculate the measure of the set $N(\beta^0)$

$$m=\xi(N(\beta^0)). \tag{23}$$

of course the value m depends on the Liapunov function parameters

$$m=m(H). \tag{24}$$

4. Solve the maximization problem

$$\max_{H \in \Theta} m(H), \tag{25}$$

where the set Θ is defined as follows

$$\Theta=\{H \in R^k \mid \bigvee_{\varepsilon>0} \ \bigwedge_{x \in K(0,\varepsilon)} \ \varphi(x, H)>0 \cap \psi(x, H)<0\}. \tag{26}$$

The condition $H \in \Theta$ ensures that $\varphi(x, H)$ is the Liapunov function.

In [4] and [5] the practical numerical realization of that algorithm for the Liapunov function in the quadratic form

$$\varphi(x, H)=x^T Hx \tag{27}$$

is presented. In this case the stability region estimate is an ellipsoid. In [4] and [5] as the measure of ellipsoid the following value is taken:

$$m=\prod_{i=1}^{n} h_i, \tag{28}$$

where h_i — the length of the main ellipsoid axis number i. This measure is proportional to the volume of the ellipsoid. The measure (28) is maximized by changing the length of the ellipsoid main axises and the angles of the ellipsoid rotations.

5. Algorithm of finding the nonlinear part of the controller

The theorem presented in the point 4, can be used to design the closed control system with the fixed parameters vector p. Then the set defined as follows

$$\bigwedge_{\substack{p\in P\\ H\in\Theta}} E[H,p]=\{x(t;\,t_0,x_0,p)\in R^n \,|\, \varphi(x,H)<\beta^0\} \tag{29}$$

satisfies the theorem thesis

$$S[p]\supseteq E[H,p].$$

The set $E[H,p]$ can be used as the estimate of the stability region (3).

The measure of the set $E[H,p]$ depends on the controller parameters and on the Liapunov function parameters

$$m=m(H,p)=\xi(E[H,p]). \tag{30}$$

Then the task of the parametric synthesis of the controller (7), which expands the stability region can be written as follows

$$\hat{m}=\max_{\substack{H\in\Theta\\ p_2\in P}}\{m(H,p)\}, \tag{31}$$

where vector p is defined by (8) and vector $\hat{p}_1$ is the solution of the equation (16).

The solution of the problem (31) can be received by choosing the Liapunov function parameters H in such a way to obtain the estimate which approximates the stability region in the best way, in sense of the measure. At the same time the parameters vector p_2 of the controller can be chosen in such a way to receive the maximal expanding of the best estimate of stability region, hence as the consequence to receive the expanding of the real stability region.

Control system synthesis is done in two stages:

In the first the local optimization problem is solved and the stability region estimate (the parameters of Liapunov function H) is found for the locally optimal system with linear controller $p_2=0$. In the next using the found estimate (the parameters of Liapunov function H) as the initial approximation, the measure of that estimate is maximized with respect to the controller and Liapunov function parameters. The diagram of the method is shown in the Fig. 3.

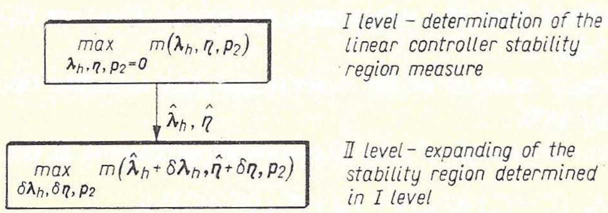

Fig. 3

Using that method one can be assured, that obtained stability region estimate for the system with nonlinear controller will not be less, than the one for the system without that controller. The numerical results confirmed the efficiency of that method.

6. The numerical example

The control system synthesis presented in the work was used for the continuous stirred-tank chemical reactor in which a first order irreversible reaction occurs $A \rightarrow B$. Simple kinetics are chosen for purposes of illustration to make the analysis techniques clear. Extension of the method to more complex reactions is straightforward.

The state equations describing the reactor are respectively the component continuity equation and energy equation:

$$\dot{C} = \frac{F}{V}(C_0 - C) - C\alpha \exp\left[\frac{-E}{R(T+460)}\right],$$

$$\dot{T} = \frac{F}{V}(T_0 - T) + \frac{(-\Delta H_R)}{C_p} C\alpha \exp\left[\frac{-E}{R(T+460)}\right] + \qquad (32)$$

$$- \frac{\Delta H_{v_0}}{VC_p} W_c \sqrt{1 - \frac{T}{T_{CR}}}\,.$$

The state variables C, T are appropriately the concentration and temperature of the substance A inside of the reactor. They are at the same time the output variables. The variables C_s, T_s are respectively the concentration and temperature of the substance A in steady state. The variables C_0, T_0 are respectively the concentration and temperature of the substance A at the input to the reactor. Other parameters in equations (32) are the physical values characterizing the reaction, taken from [7].

Since the reactor posses the stable equilibrium point for the temperature $T_s = 140°F$ one can use the linear controller

$$W_c = W_0 + k_{pc}(C - C_s) + k_{pt}(T - T_s), \qquad (33)$$

where W_0 — the rate of coolant flow in steady state; k_{pc}, k_{pt} — the gains of the controller.

The concentration C_s and the rate of coolant flow W_0 suitable to the temperature T_s can be calculated from the steady state equations. They are $C_s = 0.253016$ lb/lb, $W_0 = 698.822$ lb/hr.

From the linear analysis of the reactor one can calculate the values of gains of the controller (33).

The nonlinear controller (7) for the considered reactor is the following

$$W_c = W_0 + k_{pc}(C - C_s) + k_{pt}(T - T_s) + R_{11}(C - C_s)^2 +$$
$$+ 2R_{12}(C - C_s)(T - T_s) + R_{22}(T - T_s)^2 . \qquad (34)$$

The parameter values are calculated in such a way as to obtain maximal value of the stability region estimate measure (31).

The numerical results were obtained for two examples.

Example I. Reactor with the linear controller (33) with the parameters

$$k_{pc}=0, \quad k_{pt}=70. \tag{35}$$

As the initial approximation of the Liapunov function matrix it was taken the following one

$$H=\begin{bmatrix} 0.287620, & 0.809765\times 10^{-2} \\ 0.809765\times 10^{-2}, & 0.139491 \end{bmatrix}.$$

For the above mentioned values the measure (28) of the stability region estimate is equal to $m=0.584466\times 10^{-4}$.

As a result of the maximization of the measure (28) with respect to the Liapunov function parameters it was received the following Liapunov function matrix

$$H=\begin{bmatrix} 0.235198, & 0.694158\times 10^{-1} \\ 0.694158\times 10^{-1}, & 0.108509 \end{bmatrix}$$

and the measure equal $m=0.239540\times 10^{-2}$.

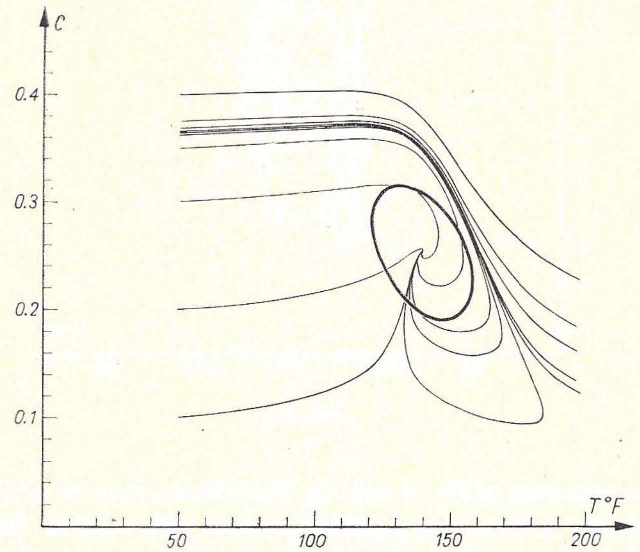

Fig. 4

The phase plane of that reactor with linear controller (33) is presented in the Fig. 4. The optimal estimate of the stability region is also illustrated in the Fig. 4.

Reactor with the nonlinear controller (34), with the parameters of the linear part defined by (35).

As a result of the maximization of the measure (28) with respect to the Liapunov function parameters and at the same time to the parameters R_{11}, R_{12}, R_{22} one received the following matrix

$$H=\begin{bmatrix} 2.10328, & 0.320933 \\ 0.320933, & 1.64465 \end{bmatrix}$$

and the parameters $R_{11}=-0.766377$, $R_{12}=1.81010$, $R_{22}=1.67776$; the measure is equal to $m=0.174019\times10^{-1}$.

Hence by adding the optimal nonlinear controller one obtained 7-times greater measure. The phase plane of the reactor with the received nonlinear controller is shown in the Fig. 5. In the Fig. 5 there is also illustrated: (a) — the stability region estimate for the reactor with the optimal nonlinear controller and (b) — the estimate for the case $R_{11}=R_{12}=R_{22}=0$.

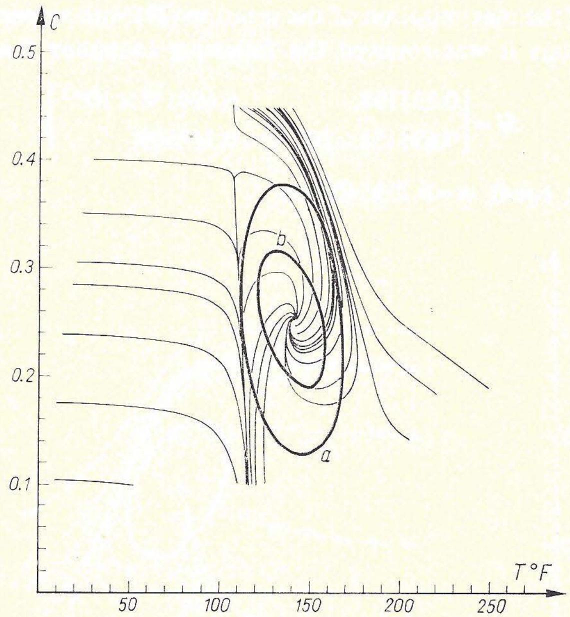

Fig. 5

From the comparison of Fig. 4 and Fig. 5 it follows, that the adding of received nonlinear controller gives a new equilibrium point (the saddle). But at the same time the adding of received nonlinear controller decreases danger of the failure by considerable displacement of boundary of the stability region into direction of the higher temperatures. In the system with the optimal nonlinear controller greater oscillations of the concentration can be admitted.

Example II. Reactor with the linear controller (33), with the parameters

$$k_{pc}=21.3020, \quad k_{pt}=67.5027. \tag{36}$$

The values (36) were obtained as the solution of the quadratic criterion minimization

$$J=\frac{1}{2}\int_{t_0}^{+\infty}[500\,(C-C_s)^2+500\,(T-T_s)^2+(W_c-W_0)^2]\,dt. \tag{37}$$

For that case where the optimal Liapunov function matrix is the following one

$$H=\begin{bmatrix}2.71641 & 0.782905\\ 0.782905 & 0.997712\end{bmatrix}$$

and the measure is equal to $m=0.101436$.

Reactor with the nonlinear controller (34) and with the parameters of the linear part of the controller given by (36).

As the result of the maximization of the measure (28) with respect to the Liapunov function parameters and at the same time to the parameters R_{11}, R_{12}, R_{22} one received the following matrix

$$H=\begin{bmatrix}1.77808 & 0.485215\\ 0.485215 & 0.540283\end{bmatrix}$$

and the parameters $R_{11}=-0.463326\times 10^{-1}$, $R_{12}=0.249916$, $R_{22}=1.36133$; the measure is equal to $m=0.177316$.

In this case, by adding the optimal nonlinear controller one obtained about 2-times greater measure.

The computations were done on CDC 3170 computer. The time of the computations for the cases with linear controller is about 5 min. and for the cases with nonlinear controller is about 10 min.

7. Conclusions

The presented decomposing method of the controller synthesis can be used in practice. If there is the need to satisfy the given conditions with respect to the small disturbance reduction, it can be done and independently it can be expanded the stability region in order to avoid the failure of the system.

It may be disscused if the linear controller synthesis method is sufficient in order to expand the stability region or not. Very often there is such situation that the conditions describing the dynamical properties of the system in the small neighbourhood of the desired equilibrium point are desired. The expanding of the stability region can deteriorate the dynamical properties of the system required with respect to the local optimization criterion. Hence, there is the need of searching of compromised solutions. The decomposing method of the synthesis makes easy such a compromise by assuring the linear controller parameters as the solution of the local optimization problem and by choosing the other parameters (the nonlinear part of the controller) in order to expanding the stability region.

It will be interesting to study decomposing method of the synthesis for the different criterions of the local optimization and for more general form of the control function and also Liapunov function.

In this paper the results of the approximation of the stability region were obtained for the simplest Liapunov function — quadratic form. The extension of the method of the stability region approximation to more complex forms of Liapunov function

$$\varphi(x, H) = \sum_{k=0}^{m-1} (x^T H x)^{2k} \tag{38}$$

is straightforward.

The problem of global minimization, which must be solved in order to find the number β^0 defined by (21) seems difficult. The above problem can be transformed to searching the global minimum with respect to $(n-1)$ variables. But now, there are not effective methods of searching the global minimum with respect to more than 2 variables. Hence the employing of that method for the systems n-dimensional when $n>3$ can be difficult.

References

1. Arienti G., Sutti C., Szegö G. P., On the numerical construction of Liapunov functions. Proc. IV IFAC Congr. Warszawa 1969. Pap. 20.1.
2. Athans M., Falb., Optimal control. An introduction to the theory and its applications. New York 1966.
3. Bathia N. P., Szegö G. P., Stability theory of dynamical systems. Berlin 1970.
4. Lewandowska A., Controller synthesis for nonlinear plants with conditions for stability region (in Polish). Ph. D. Thesis. Warszawa 1975 Institute for Organization, Management and Control Sciences Polish Academy of Sciences.
5. Lewandowska A., Practical algorithm for approximation of the stability region (in Polish) — in preparation.
6. Lewandowska A., Methods of the optimal controller synthesis with conditions for stability region (in Polish). Proc. VI Polish Conf. on Automatic Control, Poznań 1974.
7. Luyben W. L., Stability of autorefrigerated chemical reactors. *AICHE J.* **12**, 4 (1966).

Received September 1975

Synteza regulatorów dla obiektów nieliniowych przy warunku na obszar stabilności

Przedstawiono metodę syntezy regulatora dla układu dynamicznego opisywanego równaniem różniczkowym zwyczajnym, który tłumi zakłócenia o małej amplitudzie i zapewnia jednocześnie maksymalizację obszaru stabilności asymptotycznej układu. W celu dokonania takiej syntezy rozwiązano dwa zadania: zadanie doboru właściwej struktury regulatora spełniającego obie funkcje oraz zadanie estymacji i maksymalizacji obszaru stabilności.

Pierwsze zadanie zostało rozwiązane przez dekompozycję. Dla realizacji tłumienia zakłóceń stochastycznych o małej amplitudzie stosuje się klasyczne podejście z linearyzacją obiektu i syntezą regulatora liniowego maksymalizującego funkcjonał kwadratowy. Natomiast dla maksymalizacji

obszaru stabilności równolegle do regulatora liniowego wprowadza się regulator nieliniowy, którego wyjście zależy od drugiej lub wyższych potęg uchybu, a więc nie zmienia układu zlinearyzowanego. Dzięki temu można dokonać niezależnej syntezy każdej części regulatora.

Problem estymacji obszaru stabilności został rozwiązany przy użyciu drugiej metody Lapunowa. Wprowadzono funkcję Lapunowa w postaci formy kwadratowej. Następnie macierz formy kwadratowej dobrano tak, aby zapewnić maksimum miary obszaru aproksymującego obszar stabilności dla ustalonych parametrów układu. Z kolei parametry części nieliniowej regulatora dobrano tak, aby miara estymaty obszaru stabilności układu zamkniętego była maksymalna.

Ponadto przedstawiono wyniki obliczeń dla egzotermicznej reakcji wymiany w reaktorze chemicznym z idealnym mieszaniem.

Синтез регуляторов для нелинейных систем при условиях на область устойчивости

В работе представлен новый метод синтеза регуляторов для динамических систем, описываемых обылновенными дифференциальными уравнениями, который гарантирует подавление помех с малой амплитудой, и одновременно гарантирует максимизацию области асимптотической устойчивости системы.

Для реализации этой задачи решены следующие проблемы: задача подбора структуры регулятора и задача максимизации области устойчивости. Первая задача была решена при использовании принципа декомпозиции. Для решения задачи подавления стохастических помех с малой амплитудой применяется классический подход — линеаризация уравнений системы и синтез линейного регулятора, оптимального в смысле квадратичного функционала. Для релшения задачи максимизации области вводится нелинейный регулятор, работающий параллельно с линейным. Выход налинейного регулятора зависит от второй и высших степеней сигнала ошибки, и не изменяет вида линеаризованных уравнений системы. Благодаря этому является возможным произвести независимо синтез обеих частей регулятора. Задача определения оценки области устойчивости решена при использовании второго метода Ляпунова. Вводится функция Ляпунова в виде квадратичной формы; матрицу этой формы подбирается так, чтобы максимизировать некоторую меру области устойчивости, для постоянных параметров регулятора. Затем параметры нелинейной части регулятора подбираются таким образом, чтобы мера области устойчивости замкнутой системы была максимальной.

Представлены результаты расчётов для случая химического реактора с экзотермической реакцией обмена.