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Optimization of demographic policy in socio-economic growth model

by

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The paper deals with the socio-economic growth model, which includes three main feedbacks:
1) accumulation of capital due to the investments
2) accumulation of labor force resulting from the government expenditures in demographic policy, and
3) technical progress represented by government expenditures in education, health service, research and development, etc.

Using an optimization technique called "the factor coordination principle", the optimum strategy of factor endowments has been derived. In particular, the optimum strategy of government expenditures in population policy was derived (in an explicit form) and analyzed for the case of short and long planning horizons.

1. Introduction

In classical economic growth models, the labor force is usually regarded as an exogeneous factor. It is, however, well-known that demographic policy has a considerable impact on the population growth and future labor force availability [6]. Since the implementation of a demographic policy involves direct and indirect costs (e.g. the stimulation of fertility requires that a system of social benefits for families with many children be implemented, the growth of population requires in turn that a program of new schools, housing, medical care etc. be implemented), it is important to find out what demographic strategy maximizes a given utility function.

In the present paper it shall be shown (at least in the simple growth model) that such a strategy exists and that it can be derived in an explicit form. For that purpose, an optimization technique based on the factor coordination principle will be used.

The author feels very much obliged to Dr. A. Rogers for all his remarks and comments.

2. The Model

Consider the socio-economic growth model shown in Fig. 1.

The model employs a normative approach to planning and management, and is therefore characteristic of planned economies. The output production $Y(t)$, is assumed to be dependent on the number of production (development) factors $Y_\nu(t)$, $\nu=1, ..., m$, which represent the stock of capital (Y_2), labor (employment Y_1), education, research and development, health services, etc. (Y_ν, $\nu=3, ..., m$).

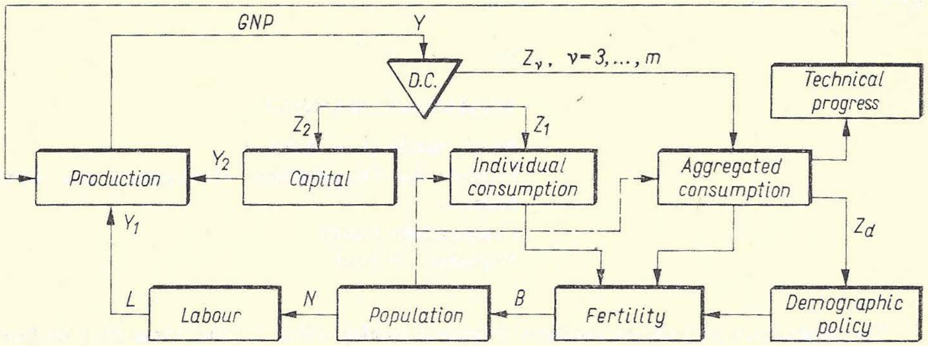

Fig. 1

The decision center (D.C.) allocates the GNP Y among the different activities, $\nu=1, ..., m$, [i.e. Y is spend on investments (Z_2), wages (Z_1) and other government expenditures (Z_ν, $\nu=3, ..., m$)] in such a way that the given utility function is maximized.

As a consequence, growth is a result of the three main feedback effects:

1) accumulation of capital Y_2 due to the investments Z_2;

2) accumulation of labor force Y_1 resulting from the government expenditures in demographic policy, Z_d, which changes fertility B, and population N;

3) technical progress represented by government expenditures Z_ν, $\nu=3, ..., m$, in education, research and development, etc.

Using the general methodology developed in Ref. [2–4] to the model of Fig. 1, one can describe the mapping $Z_\nu \to Y_\nu \to Y$ by the generalized Cobb—Douglas production function:

$$Y(t)=Ke^{\mu t}\prod_{\nu=1}^{m}[Y_\nu(t)]^{\beta_\nu}, \tag{1}$$

$$Y_\nu(t)=\delta_\nu\int_{-\infty}^{t}e^{-\delta_\nu(t-\tau)}[z_\nu(\tau-T_\nu)]^{\alpha_\nu}\,d\tau, \tag{2}$$

$$\sum_{\nu=1}^{m}\beta_\nu=1,\ 0<\alpha_\nu<1,\ K,\ \mu,\ \delta_\nu>0,\ \nu=1, ..., m$$

where $z_\nu(\tau)$ represents the factor endowment intensity.

There is a simple interpretation of $Y_2(z_2)$. The capital stock Y_2 can be regarded as the accumulated investments $z_2(\tau)$, $\tau \leq t$; δ_2 represents the depreciation (aging) of capital stock over time, while T_2 is the construction delay, i.e. the time required for an investment fund to materialize in the form of new production capacity. The inertial effect of investments on plant capacity $Y_2(z_2)$ is illustrated by Fig. 2.

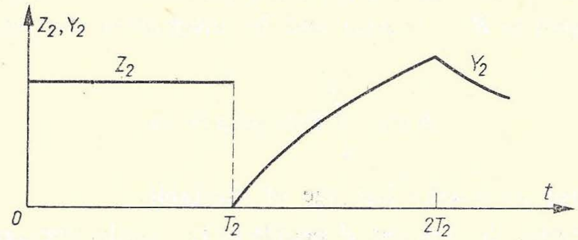

Fig. 2

It is possible to observe that plant capacity decreases for $t > 2T_2$ if no investments are made after $t > T_2$. A similar interpretation (except for labor) can be given for the rest of the $Y_\nu(z_\nu)$, $\nu = 3, \ldots, m$, factors [2, 4].

Assuming that the supply of labor is greater than the demand and taking into account that labor does not depend on past salaries ($z_1(\tau)$, $\tau < t$) one can set $\delta_1 \to \infty$ and $T_1 = 0$ in (2) so that

$$Y_1(t) = [z_1(t)]^{\alpha_1}.$$

In the case when the demand for labor is greater than the supply, it is necessary to investigate the effect of demographic policy on the labor force supply. In particular, it is important to find the effect of government expenditures connected with the implementation of a demographic policy (z_d) on fertility, F. It is well-known that in many developed countries, fertility decreases over time is a result of the change of GNP per capita, increasing health-service level, family planning, etc. In order to stimulate fertility growth, a broad program of social benefits is usually proposed. For example, in 1960 about 1.85 per cent of GNP was spent in Poland on additional monthly allowances, that rose in proportion to the number of children. Since fertility continued to decrease up to 1972 the present system of social benefits in Poland (supplemented with the Acts of January 19, 1972 and December 17, 1974) includes many additional benefits.

From the point of view of systems analysis, it is important to known the elasticity α_d of fertility with respect to the expenditures z_d:

$$\alpha_d = \frac{dF}{F} : \frac{dz_d}{z_d}.$$

Then one can try to construct a model of the general form

$$F = F[X, z_d]\, z_d^{\alpha_d} \qquad (3)$$

where X — the vector of exogenous variables including such factors as GNP per capita, health-service level, etc.

The next step is to find the relation between fertility and the labor force variable L, which enters as the production factor in (1). In order to do that, it is necessary to employ a model of population growth. Following Ref. [1], assume that the births of the country concerned have gone through a certain trajectory, described by $B(t)$ (the density of births) and assume a fixed life table that gives the number of surviving to age a on radix unity, $p(a)$ say. Then the number of persons at each age a at time t is equal to $B(t-a)\,p(a)$ and by integration the total population at time t must be

$$N(t)=\int_0^\infty B(t-A)\,p(a)\,da, \tag{4}$$

where $p(a)=0$ for $a>w=$the last age of life table.

In order to derive the amount of people in the productive ages one has to set

$$\bar{p}(a)\simeq\begin{cases}p(a), & a\geqslant T_d,\\ 0\ , & a<T_d,\end{cases}$$

where $T_d=18$ years — the entering age of the labor market.

Assuming that a part $\xi(t)$ $(0<\xi(t)<1)$ of the total population in the productive age group can be employed and introducing the new variable $\tau=t-a+T_d$ in the integral in (4), one gets the employed labor

$$L(t)=\xi(t)\int_{-\infty}^{t} B(\tau-T_d)\,\bar{p}(t-\tau+T_d)\,d\tau. \tag{5}$$

Since $B(\tau)=\bar{F}z_d^{\alpha}(\tau)\,\bar{N}(\tau)$, where $\bar{N}(\tau)$=female population in the reproductive ages, one can write (5) in the form

$$L(t)=\int_{-\infty}^{t} k(t,\tau)\,z_d^{\alpha_d}(\tau-T_d)\,d\tau, \tag{6}$$

where

$$k(t,\tau)=\xi(t)\,\bar{p}(t-\tau+T_d)\,\bar{F}\bar{N}(\tau-T_d).$$

The lag$\approx$1 year between $B(\tau)$ and $z_d(\tau)$ has been neglected.

In the simplified situation when $\bar{F}$="constant" $p(t)$ decreases exponentially with the time constant T_a (the average duration of life), while $\xi(t)$ is assumed to be decreasing at the same rate $\dot{\xi}/\xi=-\zeta$ as the female population increases (i.e. $\dot{\bar{N}}/\bar{N}=\zeta$). Thus one obtains

$$k(t,\tau)=K_0\,e^{-\delta_d(t-\tau)},$$

$$\delta_d=\frac{1}{T_a}+\zeta,\qquad K_0=\text{a ocnstant},$$

whence

$$L(t)=K_0\int_{-\infty}^{t} e^{-\delta_d(t-\tau)}\,[z_d(\tau-T_d)]^{\alpha_d}\,d\tau \tag{7}$$

which is almost identical to (2). In other words, the supply of labor behaves in a similar way to the supply of capital. There is a constant lag T_d and the labor is aging at the rate $-\delta_d$ as in the depreciation of capital.

When the average wage $\omega_1 = \text{const.}$, the labor cost (Y_1) in (1) should be proportional to (7). The impact of expenditures change z_d on the labor level change is similar to the impact of investment change on the capital stock level. One can "invest" here in the population sector out of the present resources (i.e. GNP) in order to increase the labor force, which is the main production factor, in the future [1].

Thus, from the point of view of optimization of long-term development, it is important to find out what is the best strategy for allocating GNP among investments, demographic expenditures, and individual and aggregate consumption.

We shall investigate this problem in the next section, using the methodology of Ref. [2–4].

3. Optimum Strategies

The optimization problem which faces us can be formulated as follows:

Find the nonnegative functions $z_\nu(t) = \hat{z}_\nu(t)$, $\nu = 1, \ldots, m$, $t \in [0, T]$ which *maximize* the functional

$$Y(z) = \int_0^T (1+\varepsilon)^{-t}\, Y(t)\, dt = K \int_0^T \prod_{\nu=1}^{m} f_\nu(t)\, dt, \tag{8}$$

$$f_\nu(t) = e^{(\mu-\lambda)\frac{t}{\beta_\nu}}\, Y_\nu(z_\nu)^{\beta_\nu}, \qquad \lambda = \ln(1+\varepsilon),$$

where ε = the given discount rate, T = given planning horizon; *subject to* one of the two sets of constraints:

a) the integral-type of constraints

$$\int_0^T w_\nu(\tau)\, z_\nu(\tau - T_\nu)\, d\tau \leqslant Z_\nu, \qquad \nu = 1, \ldots, n, \tag{9}$$

$$\sum_{\nu=1}^{m} Z_\nu \leqslant Z. \tag{10}$$

b) the amplitude-type of constraints

$$\sum_{\nu=1}^{m} z_\nu(t - T_\nu) \leqslant Z(t), \qquad t \in [0, T], \tag{11}$$

where $w_\nu(t) = (1+\varepsilon_\nu)^{T-\tau}$, ε_ν = given positive numbers.

In the simple case when $T_\nu = 0$, it is possible to replace $Z(t)$ by $Y(t-1)$. In that case, the constraint (11) has the following meaning. The GNP generated at the end of the year $t-1$ is allocated at the year t among m development factors, i.e.

$$z_\nu(t) = \gamma_\nu(t)\, Y(t-1), \qquad \nu = 1, \ldots, m$$

[1] It is also possible to construct a model in which z_d is assigned to wages (ω_1) and find the best wage-strategy which affects the pertility and determines future labor supply.

where

$$\sum_{\nu=1}^{m} \gamma_\nu(t)=1, \qquad t \in [0, T].$$

In the case when some of the government expenditures, say $Z_0=\gamma_0\, Y(t-1)$, have no productive effect, one should write

$$Z(t)=(1-\gamma_0)\, Y(t-1).$$

In the general case where $T_\nu \neq 0$, one can write

$$Z(t)=(1-\gamma_0) \sum_{\nu=1}^{m} \gamma_\nu(t-T_\nu)\, Y(t-T_\nu-1). \tag{12}$$

Assuming that the average growth $\rho=\dot{Y}/Y$ in $[t-T_\nu, t]$ is constant it is also possible to write (12) in the form

$$Z(t)=\sum_{\nu=1}^{n} Z_\nu(t), \qquad Z_\nu(t)=\tilde{\gamma}_\nu(t)\, Y(t)$$

where

$$\tilde{\gamma}_\nu(t)=\gamma_\nu(t-T_\nu)\, e^{-\rho(T_\nu+1)}. \tag{13}$$

The values Z_ν in (9) can be assumed to be equal

$$Z_\nu=\int_0^T (1+\varepsilon)^{-t}\, Z_\nu(t)\, dt=\tilde{\gamma}_\nu\, Y, \qquad \nu=1, \ldots, m. \tag{14}$$

Usually

$$\sum_{\nu=1}^{m} \tilde{\gamma}_\nu<1.$$

The growth rate under amplitude constraints (11) is characteristic for the closed economy (autarky) in which factor endowments are limited by the GNP currently achieved. In the case of integral constraints (9) and (10), it is possible to make use of international cooperation by taking foreign credits, exchange of labor, etc. The credits should be paid back, however, together with the interest rates.

In order to find the solution $\hat{z}_\nu(t)$, $\nu=1, \ldots, m$ for the integral constraints (9) one can use the generalized Hölder inequality:

$$Y(z) \leqslant K \prod_{\nu=1}^{m} \left(\int_0^T (|f_\nu^{1/\beta_\nu}(t)|\, dt \right)^{\beta_\nu}. \tag{15}$$

The upper bound in (15) is attained if the following conditions hold:

$$C_\nu\, e^{\vartheta_\nu t}\, Y_\nu(z_\nu)=C_1\, Y_1(z_1), \qquad \vartheta_\nu=(\mu-\lambda)\,(\beta_\nu^{-1}-\beta_1^{-1}) \tag{16}$$

$\nu=2, \ldots, m$, $t \in [0, T]$. The functions $Y_\nu(t)$ are integrable, so the conditions (16) should hold almost everywhere in $[0, T]$.

The conditions (16) should be regarded as the necessary conditions of optimality and may be called the "factor coordination principle". According to that "principle", in order to get the maximum of Y it is necessary to spend the z_ν in a way such that the development factors $Y_\nu(z_\nu)$ rise in fixed proportions. It does not pay, for example, to increase the capital stock in production sectors if there is no skilled labor available or if the education level is not adequate.

When a coordinated growth strategy is used (15) can be written as

$$Y=K\prod_{\nu=2}^{m} C_\nu^{-\beta_\nu} \int_0^T dt\, e^{\vartheta_1 t} \int_{-\infty}^{t} e^{-\delta_1(t-\tau)} [z_1(\tau-T_1)]^{\alpha_1}\, d\tau=\bar{Y}+\Delta Y \tag{17}$$

where $\bar{Y}$ is the contribution to GNP resulting from the past decision:

$$z_\nu(t), \quad t<-T_\nu, \quad \nu=1, \ldots, m,$$

and

$$\Delta Y=K\prod_{\nu=2}^{m} C_\nu^{-\beta_\nu} \int_0^t dt\, e^{\vartheta_1 t} \int_{-\infty}^{t} e^{-\delta_1(t-\tau)} [z_1(\tau-T_1)]^{\alpha_1}\, dt \tag{18}$$

represents the contribution to GNP in the planning interval $[0, T]$ resulting from the expenditures $z_\nu(t)$, $t\in[-T_\nu, T-T_\nu]$, $\nu=1, \ldots, m$.

Changing the order of integration in (18) and using again the Hölder inequality and (9) one gets [3]

$$\hat{z}_1(\tau-T_1)=\frac{\varphi_1(\tau)\, Z_1}{\int_0^T w_1(t)\, \varphi_1(\tau)\, d\tau}, \qquad \tau\in[0, T] \tag{19}$$

where

$$\varphi_1(\tau)=\left\{\frac{Kw_1(\tau)^{-q}}{\delta_1-\vartheta_1} e^{\delta_1\tau} [e^{(\vartheta_1-\delta_1)\tau}-e^{(\vartheta_1-\delta_1)T}]\right\}^{1/q_1}, \qquad q_1=1-\alpha_1 .$$

The remaining $\hat{z}_\nu(t)$ strategies can be derived by (16) and (9) yielding (for $\vartheta_\nu=0$):

$$\hat{z}_\nu(\tau-T_\nu)=\frac{\varphi_\nu(\tau)\, Z_\nu}{\int_0^T w_\nu(\tau)\, \varphi_\nu(\tau)\, d\tau}, \qquad \tau\in[0, T], \qquad \nu=1, \ldots, m \tag{20}$$

where

$$\varphi_\nu(\tau)=\left[\hat{z}_1^{\alpha_1}(\tau-T_1)+(\delta_\nu-\delta_1)\int_0^\tau e^{-\delta_1(\tau-t)}\hat{z}_1^{\alpha_1}(t-T_1)\, dt\right]^{1/\alpha_\nu}. \tag{21}$$

From (20) it follows that in order to have the strategies $\hat{z}_\nu$, $\nu=1, \ldots, m$, which satisfy the conditions $\hat{z}_\nu(t)\geqslant 0$, $\nu=1, \ldots, m$, it is necessary to enumerate the factors in such a way that $\delta_\nu\geqslant\delta_1$, $\nu=2, \ldots, m$, i.e. $\nu=1$, should be assigned to that factor which has the smalest depreciation over time.

As an example, consider the model with two production factors: labor and capital, and find the optimum investments in production $\hat{z}_p(\tau)$ and in demography $\hat{z}_d(\tau)$, $\tau\in[0, T]$.

Assuming $T_2=75$ years, $\zeta=0.01$/year, one gets

$$\delta_d \approx 0.0023/\text{year}$$

while δ_p is usually ca. 0.05. Then $\hat{z}_d(\tau)$ should be derived by (19)

$$\hat{z}_d(\tau-T_d)=\frac{\varphi_1(\tau)\, Z_d}{\int\limits_0^T w_1(T)\,\varphi_1(\tau)\,d\tau}$$

where

$$\varphi_1(\tau)=\left\{\frac{Kw_1(\tau)^{-1}}{\delta_d}\left[1-e^{\delta_d(\tau-T_d)}\right]\right\}^{1/\alpha_1} \tag{22}$$

and

$$Z_d = \text{total expenditures in } [0, T].$$

From (22) we find also

$$\hat{z}_p(\tau-T_p)=\frac{\varphi_2(\tau)\, Z_p}{\int\limits_0^T w_2(\tau)\,\varphi_2(\tau)\,d\tau}$$

where

$$\varphi_2(\tau)=\left[\hat{z}_d^{\alpha_1}(\tau-T_d)+(\delta_p-\delta_d)\int\limits_0^{\tau} e^{-\delta_d(\tau-t)}\,\hat{z}_d^{\alpha_1}(t-T_d)\,dt\right]^{1/\alpha_2} \tag{23}$$

A sketch of $\hat{z}_p$, $\hat{z}_d$ strategies for $T=25$ years, $T_p=2$ years, $T_d=18$ years is shown in Fig. 3.

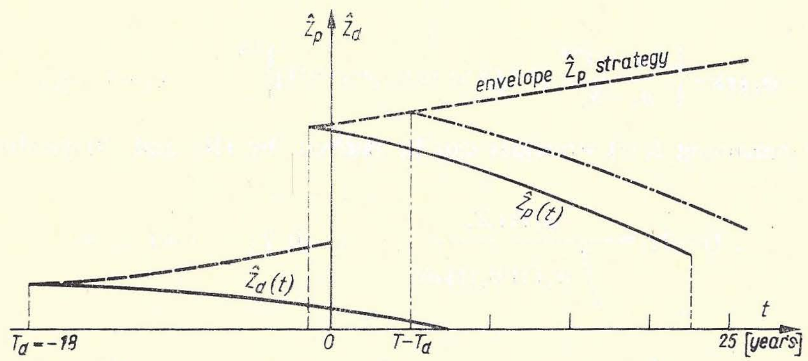

Fig. 3

It is possible to observe that the expenditures $\hat{z}_d(t)$ precede the $\hat{z}_p(t)$ by $T_d-T_p=16$ years. However, the effects $Y_d(t)$ is proportional to $Y_p(t)$, according to the factor coordination principle. When $T<T_d$ the strategy $\hat{z}_d$ shifts outside $[0, T]$, i.e. becomes completely exogeneous.

When a moving horizon technique is used for the planning of $\hat{z}_p$, $\hat{z}_d$ strategies, the values of real expenditures (envelopes of $\hat{z}_p$, z_d) increase over time as shown by the dashed lines in Fig. 3.

As shown in [3] the value of ΔY under the optimum strategy $z=\hat{z}$ becomes

$$\Delta Y(\hat{z})=G\prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu}, \qquad \gamma_\nu=\alpha_\nu \beta_\nu . \tag{24}$$

where G is a constant depending on the parameters K, T, T_ν, δ_ν, and ε_ν.

Now we can derive the optimum values of $Z_\nu=\hat{Z}_\nu$, $\nu=1, \ldots, m$, which maximize (24) subject to (10). Since (24) is strictly concave in the compact set (10), a unique optimum solution exists and can be derived by the formula

$$\hat{Z}_\nu=\frac{\gamma_\nu}{\sum_{\nu=1}^{m}\gamma_\nu} Z=g_\nu Z, \qquad \nu=1, \ldots, m . \tag{25}$$

When the optimum strategy is set in (24), one gets

$$\Delta Y=\Delta \bar{Y}=G\prod_{\nu=1}^{m} g_\nu^{\gamma_\nu} Z^\gamma, \qquad \gamma=\sum_{\nu=1}^{m}\gamma_\nu . \tag{26}$$

As follows from (14), g can be regarded as equal to $\sum_{\nu=1}^{m}\tilde{\gamma}_\nu$, so one can write $Z=gY$. Then

$$Y=\bar{Y}+G\prod_{\nu=1}^{m} g_\nu^{\gamma_\nu} g^\gamma Y^\gamma=\bar{Y}+\bar{G}Y^\gamma, \tag{27}$$

$$\bar{G}=G\prod_{\nu=1}^{m}(g_\nu g)^{\gamma_\nu} .$$

As shown in Fig. 4 a unique solution $Y=Y^*$ of (27) exists, which determines the GNP generated within $[0, T]$ under the optimum strategy.

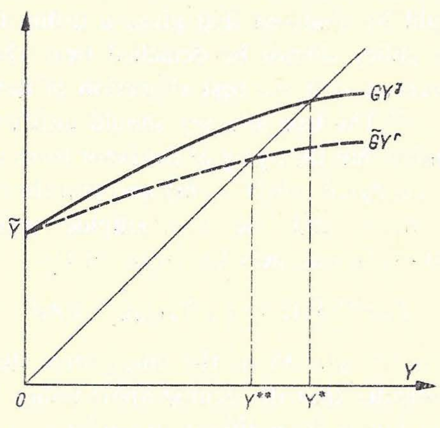

Fig. 4

Since $\gamma<1$, the contraction property of the right side of (27) takes place for any given T or $\bar{G}(T)$. When one sets $Z=gY$ the "open loop solutions" (19) (20) (25) become the "closed loop solutions".

It should also be noticed that the expression (24) can be regarded as the utility function. For that purpose one can write

$$\Delta U=\tilde{G}\prod_{\nu=1}^{m}\bar{Y}_{\nu}^{\gamma_{\nu}}, \tag{28}$$

where

$$\bar{Y}=Z_{\nu}/\omega_{\nu}, \quad \tilde{G}=G\prod_{\nu=1}^{m}\omega_{\nu}^{\gamma_{\nu}}$$

and

$$\omega_{\nu}=\text{prices attached to the } \bar{Y}_{\nu} \text{ factors.}$$

The problem of choosing the optimum values of $\bar{Y}_{\nu}=\hat{Y}_{\nu}$, $\nu=1, \ldots,$ which maximize (28) subject to the constraint

$$\sum_{\nu=1}^{m}\omega_{\nu}\,\bar{Y}_{\nu}\leqslant Z,$$

is obviously equivalent to (24) (25).

As shown in Ref. [4], it is also possible to solve the general optimization problem (8)–(11) with the amplitude-type of constraints using the present methodology. The development under amplitude constraints (autarky) is always slower than in the case of integral constraints, i.e. an open economy, which makes possible an exchange of production factors with different regions and countries.

4. Optimum Demographic Policy

Keeping in mind the results of section three, we can now formulate recommendations regarding a demographic policy for the model of Fig. 1.

First of all, it should be observed that given a utility function of the type in (28), the demographic policy cannot be detached from the general development strategy, which is concerned with the best allocation of factor endowments represented by Z_{ν}, $\nu=1, \ldots, m$[2]. The best strategy should satisfy the factor coordination principle in (16). Assuming that the aging of the labor force is slower than the aging of all the other factors, i.e. $\delta_d<\delta_{\nu}$, $d\in[1, \ldots, m]$, and that the planning horizon is long enough (so that $T-T_d>0$ and the demographic policy can be exercised within the planning interval), one gets by virtue of (16)

$$C_{\nu}\,e^{\vartheta_{\nu}\tau}\,Y_{\nu}(z_{\nu})=C_d\,Y_d(z_d), \quad \nu\neq d \tag{29}$$

where $Y_d(z_d)$ represents the growth of the labor force due to z_d expenditures.

The last relation indicates that the demographic policy (in terms of z_d) determines all the rest of the factor expenditures strategies z_{ν}, $\nu=1, \ldots, m$.

That relation also indicates how important the demographic policy is for the long-term planning of development. That policy, concerned with the long-range

[2] $Z_1=\omega_1\,Y_1$ represents here the personal consumption in terms of salaries in $[0, T]$; $Y_1=$ labor supply ($Y_1=Y_d$).

development goals of a community, should not be mixed up with the short-range goals, which are motivated by, for example, the necessity of a fast improvement in the standard of living, consumption per capita, etc.

In the case of a short-horizon policy, $T-T_d<0$, the labor force should be regarded as an exogenous factor in the production function in (1). When one wants to keep full employment (which in many countries, and first of all in the socialist countries, is a necessity), the strategies $z_v(t)$, $v\neq 2$, are determined according to the factor coordination principle in (16), by the supply of labor. In that case, it is necessary to build new factories for the purpose of getting full employment. In other words, the economy should adjust to the random changes in fertility and resulting population and labor force changes. Such a situation takes place in Poland, where the labor force fluctuates over time according to the population-age structures shown in Fig. 5 [8]. As a result, the direct and indirect costs connected with the

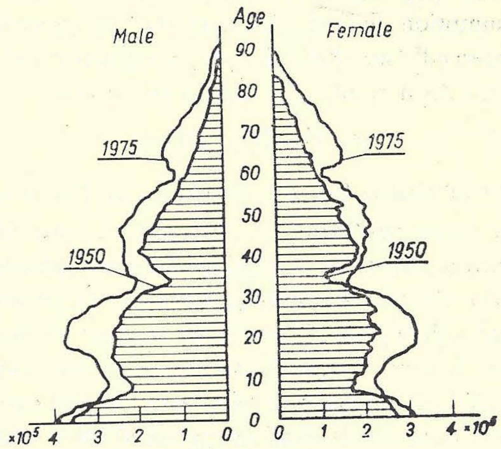

Fig. 5

necessity of building new schools, hospitals, housing and social care programs also change over time. That in turn stimulates the discussion regarding the general question: what should the objectives of a rational policy in demography be? (See Ref. [5]).

The advocates of a curb on population growth argue that a considerable increase of GNP per capita could be obtained by spending the government expenditures z_v on the publicity of family planning (especially in rural areas), contraceptives, etc., which would result in a fertility decrease. The objectives of that policy presumably could be described by a utility function, which in addition to Y also takes into account population growth $N(z_d)$, i.e.

$$U=\int_0^T (1+\varepsilon)^{-t}[Y(Z)]^{1-\beta}[N^{-1}(z_d)]^{\beta}\,dt, \quad 0<\beta<1,$$

where according to (4)

$$N(z_d)=\int_0^\infty B(t-a)\,p(a)\,da\,,$$

$$B(t)=\bar{N}(t)\,\bar{F}z_d^{-\alpha_d}\,,$$

$$\alpha_D=-\frac{dF}{F}:\frac{dz_d}{z_d}>0\,.$$

$$\bar{N}(t)=\text{female population}\,.$$

Using the factor coordination principle the optimum z_d — strategy can be chosen in such a way that $N[z_d]$ becomes inversely proportional to the growth of $Y(Z)$. That strategy obviously would tend to smooth labor-force fluctuations.

In the general discussions regarding the best growth and demographic policy the consumption per head in different population groups (preworking, working, postworking ages) as well as environment-protection costs should be taken also in account (the increased population growth favours the retired, while the faster industrial and consumption growth damages the environment). As far as the environment is concerned (see Ref. [4]) the protection cost function $C(Y)$ is increasing and convex. As a result (27) should be replaced by

$$Y=\bar{Y}+\tilde{G}[Y-C(Y)]^{\gamma}. \tag{30}$$

The solution Y^{**} of (30) is less Y^{*}, as shown in Fig. 4 by the dashed line.

Many extensions of the problem just discussed are possible. First of all, it is important to take into account the migrations between different regions and production sectors [7]. For the Polish economy, the most important are the migrations between the rural and urban areas. There is an outflow of labor of approximately one per cent per year from the agricultural sector to the industrial and service sectors. In order to achieve optimum development, the demographic policy should take into account both the technological changes and labor efficiencies in all of the sectors of the national economy.

For that purpose, it is necessary to employ the multisector-normative model of development. Such a model has been recently constructed at the Polish Academy of Sciences [3]. However, the optimum demographic policy for that model will be described elsewhere.

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Optymalizacja polityki demograficznej w społeczno-ekonomicznym modelu wzrostu

Praca dotyczy społeczno-ekonomicznego modelu wzrostu, w którym występują trzy główne sprzężenia zwrotne:

1) akumulacja kapitału powodowana inwestowaniem,

2) akumulacja siły roboczej wynikająca z nakładów przeznaczanych przez rząd na realizację polityki demograficznej,

3) postępu technicznego reprezentowanego przez nakłady przeznaczane przez rząd na szkolnictwo, służbę zdrowia, działalność naukowo-badawczą itp.

Wykorzystując technikę optymalizacyjną noszącą nazwę ,,zasady koordynacji czynników wyznaczono optymalną strategię rozwoju czynników. Wyznaczono optymalną strategię (w postaci jawnej) wydatków przeznaczanych przez rząd na realizację polityki populacyjnej i przeanalizowano ją uwzględniając zarówno przypadek krótkiego jak i przypadek długiego horyzontu planowania.

Оптимизация демографической политики в общественно-экономической модели роста

Работа касается общественно-экономической модели роста, в которой имеют место три основные обратные связи:

1) аккумуляция капитала в результате инвестирования
2) аккумуляция рабочей силы, вытекающая уз правительственных затрат на реализацию демографической политики
3) технический процесс, вытекающий из правительственных затрат на образование, здравоохранение, научно-исследовательскую деятельность итп.

Используя оптимизирующую методику, называемую ,,принципом координации факторов определена оптимальная стратегия розвития факторов. В частности определена оптимальная стратегия (в явном виде) правительственных затрат на реализацию демографической политики и дан её анализ, учитывая так краткосрочный так и долгосрочный горизонт планирования.