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Synthesis of suboptimal linear systems with output regulators

by

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The paper deals with the synthesis of suboptimal linear systems with constant output feedback. The approach presented is based on a modified linear-quadratic problem.

Using the mutual observation property of linear systems a suboptimal output regulator is proposed which guarantees the coincidence of the trajectories of optimal and suboptimal systems for a certain subspace of initial states.

1. Introduction

The problems of synthesis of linear systems with inaccessible states have been treated by many authors (see references).

At least three basic approaches for linear control law realization are known.

In the first a state observer is utilized for obtaining an estimation of the inaccessible states. It must be pointed out that the use of an observer leads to a considerable increase of the performance index for certain initial states [1].

The second approach is based on the realization of the control law by means of compensators [2, 3, 4]. However the use of compensators and observers complicates the system.

The third approach, in which the control law is formed as an output feedback seems to be the most rational one [5, 6, 7].

It is well known that the optimal output regulator matrix depends on the initial state [8].

This is usually overcome supposing that the statistics of the initial state is known [9, 6] (this approach is also used in the synthesis of observers and compensators [10, 3]). Unfortunately information about the initial state is not available in general. That is why it is useful to introduce a modified criterion with independent on the initial state output regulator matrix [11].

These methods lead to an approximative realization of the optimal control and are considered as methods for synthesis of suboptimal linear systems under incomplete state information.

The paper presented deals with synthesis of suboptimal linear systems with output regulator. Statement of the problem is given in Sec. 2. In Sec. 3 a modified linear-quadratic problem (MLQP) is introduced and an unified approach to the synthesis of suboptimal output regulators is proposed.

The problems of existance and uniquenes of the solution are considered and appropriate methods for obtaining the suboptimal control are presented.

In Sec. 4 we deal with the synthesis of suboptimal control law such that the trajectories of the optimal and soboptimal systems coincide for a given subspace of the initial state space.

2. Statement of the problem

Consider the linear system

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t),\ x(0) = x_0, \\ y(t) &= C\,x(t), \end{aligned} \tag{1}$$

with the performance index

$$J = \int_0^\infty [x'(t)\,Q\,x(t) + u'(t)\,R\,u(t)]\,dt \to \min, \tag{2}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$ ($R^{n \times m}$ denotes the space of real $(n \times m)$-matrices; $R^{n \times 1} = R^n$); $Q \geqslant 0$, $R > 0$. It is also assumed that the pair $[A, B)$ is stabilizable, the pair $(Q^{1/2}, A]$ is completely observable and rank $C = r$. It is well known that the optimal control law is

$$u^o[x] = -K^o x, \tag{3}$$

where $K^o = R^{-1} B' P^o$ and $P^o > 0$ is the unique positive definite solution of the Riccati equation

$$A' P^o + P^o A + Q - P^o B R^{-1} B' P^o = 0.$$

The optimal closed loop system with control law (3) is

$$\dot{x}^o(t) = F^o x^o(t),\ F^o = A - BK^o \tag{4}$$

and the performance index is

$$J^o = x_0 P^o x_0.$$

If only the output y is available for measurement the control law is to be synthesised as an output feedback:

$$u[y] = -Ly,\ L \in R^{m \times r}. \tag{5}$$

Further we assume that the set $\Omega \subset R^{m \times r}$ of all matrices L for which the closed loop system

$$\dot{x}(t) = F\,x(t),\ F \triangleq A - BLC \tag{6}$$

is stable, is non-empty.

For $L \in \Omega$ the value of the performance index (2) is

$$J = x_0' P x_0 \tag{7}$$

where $P = P(L) > 0$ is a solution of the Lyapunov equation

$$F' P + PF + Q + C' L' RLC = 0. \tag{8}$$

Further we consider the general case when the equation $LC = K^o$ may have no solution with respect to L.

Since the performance index depends on the initial state: $J = J(u, x_0)$, the direct minimization of (7) by means of L yields the relation $L = L(x_0)$ [8] which is undesirable.

There are two approaches to determine the suboptimal control law so that the output regular matrix to be independent on the initial state.

According to the first (3) the criterion (2) is modified as $J(u, x_0) \to I(u)$ where the functional I doesn't depend on x_0; for instance [11]

$$I(u) = \sup \left\{ \frac{J - J^o}{J^o} : x_0 \in R^n \right\}.$$

The second approach utilizes the fact that there exist matrix L and subspace $\Pi \subset R^n$ of the initial states x_0 such that the systems (4) and (6) trajectories coincide.

3. Modified linear-quadratic problems. Suboptimal control 1

Consider the following modification of the linear-quadratic problem (1), (2): find a new functional $I = I(u)$ independent on x_0 and such that the minimization of $I(u)$ in the class of linear state feedback controls yields the optimal control law (3).

For the linear control law (9) $u[x] = -Kx$ the functional $I(u)$ may be considered as a function in K: $I(u) = \eta(K)$.

The function η defined on the set ω of all $(m \times n)$-matrices which stabilize the system $\dot{x}(t) = (A - BK)x(t)$, may be chosen on the basis of the following natural assumptions [12]:

1. The function η is continuous on $K \in \omega$.
2. $\eta(K) \geqq \eta(K^o)$, $K \in \omega$ and $\eta(K) = \eta(K^o)$ if and only if $K = K^o$.

Consider the functions

$$\eta_1 = \eta_1(K) = \max \left\{ \frac{J - J^o}{J^o} : x_0 \in R^n \right\} =$$
$$= \rho(P_1) = v_1' P_1 v_1; \; P_1 = P_1(K) = S(K) P^{o-1},$$

$$\eta_2 = \eta_2(K) = \frac{\max \{J : \|x_0\| = 1\}}{\max \{J^o : \|x_0\| = 1\}} - 1 =$$
$$= \frac{\rho(P_2)}{\rho(P^o)} - 1 = \frac{v_2' P_2 v_2}{\rho(P^o)} - 1; \; P_2 = P_2(K) = S(K),$$

$$\eta_3 = \eta_3(K) = \frac{\max\{J - J^o : \|x_0\| = 1\}}{\max\{J^o : \|x_0\| = 1\}} =$$

$$= \frac{\rho(P_3)}{\rho(P^o)} = \frac{v_3' P_3 v_3}{\rho(P^o)}; \quad P_3 = P_3(K) = S(K) - P^o,$$

where $\|\cdot\|$ is an Euclidean norm, $\rho(P_h)$ is the spectral radius of the matrix P_h and v_h is the normed eigen vector ($\|v_h\| = 1$) of the matrix P_h corresponding to the eigenvalue $\rho(P_h)$. The matrix $S = S(K)$ satisfies the Lyapunov equation

$$(A - BK)'S + S(A - BK) + Q + K'RK = 0.$$

The functions η_h express the relative growth of the performance index (2) with control law (9).

Denote by $\eta = \eta(K)$ whichever of the functions η_h, $h = 1, 2, 3$.

It may be shown that the function η has a unique minimum $\eta = 0$ for $K = K^o$ [12].

The synthesis of output regulator (5) may be transformed into the problem of conditional minimization of η with respect to the restriction $K = LC$, $L \in \Omega$. This is equivalent to the unconditional minimization of the function

$$\mu = \mu(L) = \eta(LC), \quad L \in \Omega.$$

Since

$$\lim_{L \to L_1 \in \partial\Omega} \mu(L) = \infty$$

and

$$\lim_{\|L\| \to \infty} \mu(L) = \infty$$

where $\partial\Omega$ is the boundary of Ω, then a compact $\bar{\Omega} \subset \Omega$ exists such that $L \in \bar{\Omega}$ and $\tilde{L} \in \Omega \setminus \bar{\Omega}$ implies $\mu(L) \leqslant \mu(\tilde{L})$. Hence according to Weierstrass theorem the function $\mu(L)$ has an absolute minimum in Ω: $\mu(L^o) \leqslant \mu(L)$, $L \in \Omega$.

It must be pointed out that in the general case the matrix L^o is not unique.

The control law

$$u^o[y] = -L^o y$$

may be considered as a suboptimal control law.

It is interesting to note that for every suboptimal control law $u^o[y] = -L^o y$ there exist a region $\Sigma \subset R^n$ and a control law $u[y] = -Ly$, $L^o \neq L = \text{const.}$, such that $J(u[y], x_0) < J(u^o[y], x_0)$ for $x_0 \in \Sigma$.

If the function μ is differentiable in L then the necessary condition for minimizing μ is $M(L^o) = 0$,

$$M(L) = \frac{d\mu(L)}{dL} = [M_{ij}(L)] = \left[\frac{\partial\mu(L)}{\partial L_{ij}}\right]; \quad L = L_{ij},$$

where for $\mu=\mu_h$, $h=1, 2, 3$ one obtaines

$$M_{ij}(L)=v_h' \frac{\partial P_h(L)}{\partial L_{ij}} v_h = $$

$$= \frac{\operatorname{tr}\left[(P_h(L)-\mu_h I_n)^a \frac{\partial P_h(L)}{\partial L_{ij}}\right]}{\operatorname{tr}[P_h(L)-\mu_h I_h)]^a \lambda}, \qquad \lambda=\begin{cases} 1, h=1 \\ \rho(P^o), h=2, 3 .\end{cases}$$

Here V^a is the matrix adjoint to $V \in R^{n\times n}$: $VV^a=I_n \det V$.

Another way to determine the suboptimal control law is based on the simultaneous calculation of L^o and $\rho^o=\rho(P_h(L^o))$.

In fact the relations

$$\frac{\partial \mu_h(L^o)}{\partial L_{ij}}=0 \Leftrightarrow \frac{\partial \rho(P_h(L^o))}{\partial L_{ij}}=0$$

and

$$\Delta(L, \rho)=\det(P_h(L)-\rho I_n)=0, \quad \rho=\rho(P_h(L))$$

yield

$$T_{ij}(L^o, \rho)=\operatorname{tr}\left[(P_h(L^o)-\rho I_n)^a \frac{\partial P_h(L^o)}{\partial L_{ij}}\right]=0 .$$

Hence the calculation of L^o is reduced to the minimization of the function $\chi=\chi(L, \rho)$ of $mr+1$ variables:

$$\chi(L, \rho)=\Delta^2(L, \rho)+\sum_{i=1}^{m}\sum_{j=1}^{r} T_{ij}^2(L, \rho)$$

where the matrix $\partial P_h(L)/\partial L_{ij}$ is expressed by means of $\partial P(L)/\partial L_{ij}$:

$$F' \frac{\partial P}{\partial L_{ij}}+\frac{\partial P}{\partial L_{ij}} F+C' E_{ji}(RLC-B' P)+(C' L' R-PB) E_{ij} C=0 .$$

Here $F=A-BLC$ and $E_{ij} \in R^{m\times r}$ is a single non-zero element matrix with 1 on (i, j)-th position.

The use of the procedures presented is connected with the determination of eigenvalues and eigenvectors which complicates the calculations. Moreover, in general the spectral radius of a matrix is only piecewise differentiable as a function of its elements. Hence the gradient matrix $M(L)$ may not exist and it is necessary to derive another methods for calculating the matrix L^o.

An auxiliary differentiable function $\bar{\mu}=\bar{\mu}(L)$ can be introduced in order to obtain an approximation $\bar{L}$ of L^o. As an example consider the functions

$$\bar{\mu}_1=\bar{\mu}_1(L)=\|P(L) P^{o-1}\|-1=\|P_1(L)\|-1 ,$$

$$\bar{\mu}_2=\bar{\mu}_2(L)=\|P(L)\|/\|P^o\|-1=\|P_2(L)\|/\|P^o\|-1 ,$$

$$\bar{\mu}_3=\bar{\mu}_3(L)=\|P(L)-P^o\|/\|P^o\|=\|P_3(L)\|/\|P^o\| .$$

In this case the evaluation of the spectral radius is reduced to the evaluation of the euclidean norm. Furthermore the matrices $\bar{L}$ (minimizing $\bar{\mu}$) and L^o are usually close enough.

Example 1. For the system

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u(t),$$

$$y(t)=x_1(t)$$

with a performance index

$$J=\int_0^\infty [6x_1^2(t)+3x_2^2(t)+u^2(t)]\,dt\to\min$$

the optimal control law is

$$u^o[x]=-2\sqrt{2}x_1-x_2$$

and the performance index has minimal value

$$J^o=2\sqrt{2}x_{1o}^2+2x_{1o}x_{2o}+4\sqrt{2}x_{2o}^2.$$

The solution $P(L)$ of equation (8) for the output feedback

$$u[y]=-Lx_1,\ L>0$$

is

$$P(L)=\frac{1}{2}\begin{bmatrix}(9+L^2)/L & 3\\ 3 & (9+4L^2)/L\end{bmatrix}.$$

The implementation of the modified criterion

$$\mu_2(L)\to\min$$

yields

$$L^o=1.617;\ \mu_2(L^o)=0.127.$$

The minimization of the auxiliary function $\bar{\mu}_2(L)$ gives

$$\bar{L}=1.760;\ \mu_2(\bar{L})=0.130.$$

4. Coincidence of the trajectories of systems with state regulator and output regulator. Suboptimal control 2

Consider the linear systems

$$\dot{z}_1(t)=G_1z_1(t),\ z_1(0)=z_0 \tag{10}$$

and

$$\dot{z}_2(t)=G_2z_2(t),\ z_2(0)=z_0, \tag{11}$$

where $z_1, z_2\in R^n$ and $G_1\neq G_2$.

We shall set down the problem of finding a subspace $\Pi \subset R^n$ such that $z_0 \in \Pi$ implies $z_1(t) \equiv z_2(t)$ and vice versa.

From the condition

$$\left(\sum_{k=0}^{\infty} \frac{G_1^k t^k}{k!}\right) z_0 = \left(\sum_{k=0}^{\infty} \frac{G_2^k t^k}{k!}\right) z_0$$

and from Hamilton—Cayley theorem it follows

$$(G_1^k - G_2^k) z_0 = 0, \quad k = 1, \ldots, n$$

or

$$(G_1 - G_2) G_1^k z_0 = (G_2 - G_1) G_2^k z_0 = 0; \quad k = 0, \ldots, n-1.$$

Hence the necessary and sufficient condition for coincidence of the trajectories of (10) and (11) is

$$z_0 \in \bigcap_{k=1}^{n} \operatorname{Ker} (G_1^k - G_2^k) = \bigcap_{k=0}^{n-1} \operatorname{Ker} (G_1 - G_2) G_1^k =$$

$$= \bigcap_{k=0}^{n-1} \operatorname{Ker} (G_2 - G_1) G_2^k \triangleq \langle G_1, G_2 \rangle = \langle G_2, G_1 \rangle \subset R^n.$$

For $G_1 \neq G_2$ the inequalites $\dim \langle G_1, G_2 \rangle \leqq n-1$ or $\dim \langle G_1, G_2 \rangle \leqq n-2$ are valid.

Under certain sufficiently general assumptions for G_1 and G_2 the relation $\dim \langle G_1, G_2 \rangle \geqq 1$ holds true.

Consider again the systems (4) and (6):

$$\dot{x}^o(t) = F^o x^o(t), \quad F^o = A - BK^o, \tag{12}$$

$$\dot{x}(t) = Fx(t), \quad F = A - BLC. \tag{13}$$

In accordance with the above statements the trajectories of (12) and (13) coincide if and only if

$$x_0 \in \langle B(K^o - LC), F^o \rangle = \bigcap_{k=0}^{n-1} \operatorname{Ker} B(K^o - LC) F^{ok} = \Pi(L) \subset R^n.$$

This result shows that the uniqueness of the optimal control must be interpreted in the following way:

1. For each $x_0 \in R^n$ the optimal control is unique as a timeprogram

$$u^o(t, x_0) = -K^o e^{F^o t} x_0.$$

2. The control law

$$u^o[x] = -K^o x$$

is the unique linear control law which generates the optimal program

$$u^o[x^o](t) \equiv u^o(t, x_0)$$

for all $x_0 \in R^n$.

3. For a given x_0 the optimal programm may be generated also by means of other control laws, for instance

$$u[y]=-Ly;\ x_0\in \Pi(L).$$

Consequently an alternative approach to the synthesis of suboptimal control laws consists in determining an output feedback

$$u[y]=-L_0 y$$

for which

$$\dim \Pi(L_0)=\max\{\dim \Pi(L): L\in \Omega\}.$$

It will be shown that a control law

$$u[y]=-Ly$$

can be found such that $\dim \Pi(L)\geqq r$.

Let the output matrix C be of the form $C=[I_r|0]$. The state vector can be separated as $x=[x_1'|x_2')'$ where $x_1\in R^r$, $x_2\in R^{n-r}$ and the systems (12) and (13) have the form

$$\left[\begin{array}{c}\dot x_1(t)\\ \hline \dot x_2(t)\end{array}\right]=\left[\begin{array}{c|c}F_{11}^o & F_{12}^o\\ \hline F_{21}^o & F_{22}^o\end{array}\right]\left[\begin{array}{c}x_1^o(t)\\ \hline x_2^o(t)\end{array}\right], \tag{14}$$

$$F_{ij}^o=A_{ij}-B_i K_j^o;\ i,j=1,2$$

and

$$\left[\begin{array}{c}\dot x_1(t)\\ \hline \dot x_2(t)\end{array}\right]=\left[\begin{array}{c|c}F_{11} & F_{12}\\ \hline F_{21} & F_{22}\end{array}\right]\left[\begin{array}{c}x_1(t)\\ \hline x_2(t)\end{array}\right], \tag{15}$$

$$F_{i1}=A_{i1}-B_i L,\ F_{i2}=A_{i2};\ i=1,2$$

where

$$A=\left[\begin{array}{c|c}A_{11} & A_{12}\\ \hline A_{21} & A_{22}\end{array}\right],\quad B=\left[\begin{array}{c}B_1\\ \hline B_2\end{array}\right],\quad K^o=[K_1^o|K_2^o].$$

If the matrix $T^o\in R^{(n-r)\times r}$ satisfies the non-symmetric Riccati equation

$$F_{22}^o T^o-T^o F_{11}^o+F_{21}^o-T^o F_{12}^o T^o=0 \tag{16}$$

then the mutual observation relation

$$x_2^o(t)=T^o x_1^o(t)+v^o(t),\ v^o\in R^{n-r}$$

holds in the system (14) [13].

The system (14) is transformed as

$$\left[\begin{array}{c}\dot x_1^o(t)\\ \hline \dot v^o(t)\end{array}\right]=\left[\begin{array}{c|c}F_{11}^o+F_{12}^o T^o & F_{12}^o\\ \hline 0 & F_{22}^o-T^o F_{12}^o\end{array}\right]\left[\begin{array}{c}x_1^o(t)\\ \hline v^o(t)\end{array}\right]. \tag{17}$$

Let the matrix L is chosen from

$$L=\tilde L=K_1^o+K_2^o T^o.$$

Then in (15) there exists the relation

$$x_2(t)=T^o x_1(t)+v(t), \quad v\in R^{n-r}$$

and

$$\left[\begin{array}{c}\dot{x}_1(t)\\ \hline \dot{v}(t)\end{array}\right]=\left[\begin{array}{c|c}F_{11}^o+F_{12}^o T^o & F_{12}\\ \hline 0 & F_{22}-T^o F_{12}\end{array}\right]\left[\begin{array}{c}x_1(t)\\ \hline v(t)\end{array}\right]. \tag{18}$$

According to (17) and (18) the systems (14) and (15) have at least r common poles (the eigenvalues of the matrix $F_{11}^o+F_{12}^o T^o$).

For $x_{2^o}=T^o x_{1^o}$ one obtaines $v^o(t)\equiv v(t)\equiv 0$ and $x_1^o(t)\equiv x_1(t)$. Hence the output regulator $u[y]=-Ly$ generates the optimal program for

$$x_0\in \operatorname{Ker} W, \; W=[-T^o | I_{n-r}]\in R^{(r-r)\times n}.$$

From the other hand $x_0\in \Pi(\tilde{L})$, i.e. $\operatorname{Ker} W\subset \Pi(\tilde{L})$. The last inclusion can be derived directly. In fact we have

$$WF^o=(F_{22}^o-T^o F_{12}^o)W$$

and

$$WF^{ok}=(F_{22}^o-T^o F_{12}^o)^k W, \; k=2, 3, \dots .$$

Hence

$$\Pi(\tilde{L})=\bigcap_{k=0}^{n-1}\operatorname{Ker} B(K^o-LC)F^{ok}=\bigcap_{k=0}^{n-1}\operatorname{Ker} BK_2^o WF^{ok}=$$

$$=\bigcap_{k=0}^{n-1}\operatorname{Ker} BK_2^o(F_{22}^o-T^o F_{12}^o)^k W\supset \operatorname{Ker} W.$$

Thus we have

$$\dim \Pi(\tilde{L})\geqq \dim \operatorname{Ker} W=r.$$

The stability of the system (18) depends on the eigenvalues of the matrix $F_{22}-$ $-T^o F_{12}$. In the general case the Riccati equation (16) has several solutions with respect to T^o corresponding to different matrices $F_{22}-T^o F_{12}$. Hence the system (18) is asymptotically stable if there exists a solution T^o stabilizing the matrix $F_{22}-T^o F_{12}$. Further the choice of T^o among the stabilizing solutions of (16) is to be done in accordance with the modified performance index used.

The value of the performance index (2) for the stabilizing control law $u[y]=-\tilde{L}y$ is

$$J=x_0' P^o x_0+v_0' Hv_0, \quad v_0=x_{2^o}-T^o x_{1^o},$$

where $H\geqslant 0$ is the solution of the Lyapunov equation

$$(F_{22}-T^o F_{12})' H+H(F_{22}-T^o F_{12})+K_2^{o\prime} RK_2^o=0.$$

Example 2. The matrix of the optimal closed loop system in example 1 is

$$F^o=\begin{bmatrix}-2\sqrt{2} & -2\\ 1 & 0\end{bmatrix}$$

and the Riccati equation (16) has a unique solution $T^o = -\sqrt{2}/2$. Hence the sub-optimal control law $u[y] = -\tilde{L}y$ is $u[y] = -3\sqrt{2}/2x_1$.

For comparison with example 1 note that

$$\mu_2(\tilde{L}) \doteq 0.165 .$$

5. Conclusions

A unified approach to the synthesis of suboptimal control systems with output regulators is considered. The approach presented is based on a modified linear-quadratic problem for which the output regulator matrix is independent on the initial state. The problems of existence of suboptimal linear control as well as gradient calculating procedures are discussed. Approximate methods for sysnthesis of suboptimal controls are considered also.

Necessary and sufficient conditions for coincidence of the trajectories of two linear systems are given. On this basis the problems of uniqueness of the optimal control are discussed.

It is shown that the trajectories of the systems with state regulator and output regulator coincide for a subspace of the initial state space. Using the mutual observation property of linear systems a suboptimal output regulator is proposed which garantees the coincidence of the trajectories of optimal and suboptimal systems for a r-dimensional subspace of initial states where r is the number of outputs.

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Synteza suboptymalnego układu liniowego z regulatorami wyjściowymi

Rozpatrzono syntezę suboptymalnego układu liniowego z stałym sprzężeniem zwrotnym wyjściowym. Podstawą przedstawionego ujęcia jest zmodyfikowany problem liniowo-kwadratowy. Wykorzystując właściwości obserwacji wzajemnych układu liniowego zaproponowano suboptymalny regulator wyjściowy, który zapewnia zgodność trajektorii układu optymalnego i suboptymalnego dla pewnej podprzestrzeni stanów początkowych.

Синтез субоптимальной линейной системы с регулятора-ми на выходе

В работе рассматривается синтез субоптимальной линейной системы, охваченной посто-янной цепью обратной связи. Описан подход к решении этого вопроса, основанный на моди-фицированной линейно-квадратической задачи.

Используя свойства совместных наблюдений линейной системы, предложен субопти-мальный регулятор выхода, который гарантирует согласованность траекторий оптимальной и субоптимальной системы для некоторого подпространства начальных состояний.

Recomendations for the Authors

Control and Cybernetics publishes original papers which have not previously appeared in other journals. The publications of the papers in English is recommended. No paper should exceed in length 20 type written pages (210 × 297 mm) with lines spaced and a 50 mm margin on the lefthand side. Papers should be submitted in duplicate. The plan and form of the paper should be as follows:

1. The heading should include the title, the full names and surnames of the authors in alphabetic order, the name of the institution he represents and the name of the city or town. This heading should be followed by a brief summary (about 15 typewritten lines).

2. Figures, photographs tables, diagrams should be enclosed to the manuscript. The texts related to the figures should be typed on a separate page.

3. Of possible all mathematical expressions should be typewritten. Particular attention should be paid to differentiation between capital and small letters. Greek letters should as a rule be defined. Indices and exponents should be written with particular care. Round brackets should not be replaced by an inclined fraction line.

4. References should be put on the separate page. Numbers in the text identified by references should be enclosed in brackets. This should contain the surname and the initials of Christian names, of the author (or authors), the complete title of the work (in the original language) and, in addition:

a) for books — the place and the year of puylication and the publisher's name;

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Wskazówki dla autorów

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