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Discriminatory Prediction and its Relation to Optimum Control of Economic Systems

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The paper presents an attempt at designing a method for the use of econometric models for control of economic systems. If a goal A referring to a future time period T and a vector Y_T is determined, then it is possible to use the model for determining the set Ω_3 of such values of explanatory variables x_T for Y_T which: (a) are practically possible to be achieved before the period T, (b) ensure the fulfillment of goal A with probability equal at least γ, where γ is a predetermined number, close to one. The procedure leading to the determination of Ω_3 is called discriminatory prediction.

1. Introduction

The aim of the present paper is to show how an econometric model can be used for control of an economic system characterized by vector Y of variables which are looked upon as endogeneous to the model. It is assumed that Y depends on vector x of exogeneous variables which, in turn, can be split into two vectors, x_1 and x_2 respectively, the former representing decision variables while the latter one being composed of purely exogeneous components.

Since econometric models are stochastic their use provides a means for stochastic control of the system. Any vector x ensuring that in a specified fixed future time period T vector Y_T will belong to a predetermined set A will be called an admissible variant of action. The process of finding the set of admissible vectors will be referred to as discriminatory prediction of x's given A. Once such set has been found it becomes then possible to look for the optimum variant of action.

Since with passing of time one usually can get a better insight into the real situation which will exist in time T there arises the problem of periodic corrections of the set of admissible vectors and also — eventually — that of changing the adopted variant of action. Decisions about changing the adopted variant of action must stem from new discriminatory predictions and from classical predictive informa-

tion, i.e. from information derived from direct forecasts or predictions of Y_T, x_{1T}, and x_{2T}.

As decisions about eventual changes of vector x should occur periodically until the occurrence of period T, stochastic control of an economic system by using its econometric model consists in periodic pooling of three types of information: discriminatory predictions, direct predictions of Y_T and predictions referring to the behaviour of exogeneous variables of the model in time T.

The approach presented in this paper is based on this author's papers [2] and [3]. One should add that in econometric literature one can find related, but not similar, approaches advocated for instance by Tinbergen [4] or Chow [1].

2. The Concept of Discriminatory Prediction

Let us assume that Y is a $G\times 1$ vector of variables which are thought of as good characteristics of an economic system[1]). Furthermore, we assume that the behavious of Y_t in time is adequately represented by an econometric model[2]) such that its explanatory variables form vector x, which, in turn, can be split into two subvectors x_1 and x_2, such that x_1 is composed of decision variables and x_2 has as its elements only purely exogeneous variables. By ξ will be denoted $G\times 1$ vector of random components of the model with ξ_i denoting the random component pertaining to the i-th equation of the model, i.e. random variable representing this part of variation of Y_i which cannot be explained by the explanatory variables of the model. The distribution function of ξ will be assumed to be known.

For the time being nothing is assumed about the functional form of the model, although to get any operational results the linear form of the model together with additivity of random components will usually have to be explicitely considered.

Since Y is said to provide a good characteristic of the economic system under analysis one may reasonably suppose that the variables entering Y will be target variables. Without loss of generality one can assume that for a fixed future time period T the target consists in $Y_T \in A$, where A is a known set, the specific form of A depending on the shape of utility function. If, for instance, utility is an increasing function of each component-variable of Y_T, then a likely shape of A is that when A will be composed of points $(y_{1T}, y_{2T}, \ldots, y_{GT})$ such that simultaneously for all i there will be $y_{iT} \geqslant a_i$, where a_i are known real numbers[3]).

Let the econometric model relating Y with x_1, x_2 and ξ be of the form

$$Y_t = F(x_{1t}, x_{2t}, \xi_t) \tag{1}$$

[1]) We deter until the last section the discussion what can be such system composed of, i.e. whether it is formed of a number of firms, regions, sectors or whether it refers to the macro level.

[2]) By using the expression "adequately represented" we mean that errors of estimation of the model are small enough to be left out of further considerations.

[3]) An example of such a situation is provided by assuming that the variables Y_i either represent outputs or are defined as monotonously increasing measures of efficiency of production process (for instance labour productivity, output-capital ratio, etc.).

where F is a known function. Model (1) can be used to provide an answer in present time t_1 to the following question:

What are the values of explanatory variables in time T which guarantee, with probability at least equal to a predetermined number γ (where $0<\gamma<1$) that in time T there will be $y_T \in A$? Formally, this can be put as

$$P\{Y_T=F(x_{1T}, x_{2T}, \xi_T) \in A|x\} \geqslant \gamma. \tag{2}$$

If F can be solved with respect to ξ_T the answer is provided by the subsequent relation

$$P\{\xi_T \in F^{-1}(A, x_{1T}, x_{2T})|x\} \geqslant \gamma. \tag{3}$$

Since, by assumption, the distribution function of vector ξ_T is known, it is in principle[4]) possible to find the set of all vectors (x_{1T}, x_{2T}) such that the condition imposed on achieving by Y_T the target A is fulfilled. The set of vectors (x_{1T}, x_{2T}) resulting from (2) will be denoted by Ω_2.

It must be observed, however, that not all elements of Ω_2 will be in practice possible to achieve. First, it must be noted that since x_{2T} represents the vector of purely exogeneous variables one is not able to choose freely the values of such variables but must reckon with what the true state of endogeneous world will be like in time T. Second, not all values of decision variables belonging to Ω_2 are really possible because of either technological, economic or other constraints. These arguments lead to the conclusion that — instead of Ω_2 — one must use a narrower set Ω_3 of vectors (x_{1T}, x_{2T}), such that $\Omega_3 \subset \Omega_2$ and its elements are practically achievable in the T.

Set Ω_3 in hence obtained from Ω_2 by dropping all such vectors (x_{1T}, x_{2T}) for which either direct prediction[5]) of purely exogeneous variables in time T shows x_{2T} to be very unlikely or x_{1T} must be considered unrealistic because of existing constraints. Obtained in this way, the set Ω_3 will henceforward be referred to as the set of practically admissible vectors of explanatory variables in time T. Alternatively, elements of this set will be also called practically admissible variants of action. To justify this name one should note that — especially when T lies in a rather distant future — the admissible values of decision variables will usually significantly differ from the values these variables have assumed in the present time period t_1. Since to change the level of a decision variable means undertaking some action[6]) this justifies the term "admissible variants of action".

The process of finding the set Ω_3 will be referred to as discriminatory prediction, the term "discriminatory" resulting from the fact that in this case the desi-

[4]) Practically, this is feasible when the model is linear (or easily transformed into linear) and the corresponding random component is additive.

[5]) By direct prediction of a variable Z we mean the process of inference into the future when a model of variable Z is used to provide an estimate of its future value in time T.

[6]) This is especially true of the situations when some decision variables are defined as "production capacity", "fixed productive assets", "technical labour equipment", when to change the level of such variable one needs to engage in some investment and construction activity.

red level of endogeneous variables of the model is specified and one is interested in finding the appropriate values of "independent" variables[7]).

The shape of set Ω_3 depends obviously on the preassigned probability level γ which in practical situations must be close to unity since otherwise there would be significant level of risk that, because of random fluctuations, even when using a vector $(x_{1T}, x_{2T}) \in \Omega_3$, the target A would not be achieved. It is interesting in this context to compare Ω_3 with the set Ω_1 composed of practically admissible variants of action derived from model (1) when it is treated as a deterministic one, i.e. when ξ_T has been put identically equal to zero.

For values of γ close enough to 1 there will be $\Omega_3 \subset \Omega_1$ and $\Omega_1 - \Omega_3 = \emptyset$ which means that allowing for random variation of ξ_T restricts the set of practically admissible vectors). The set defined by the difference $\Omega_1 - \Omega_3$ will be referred to as the safety margin.

We shall assume henceforward that Ω_3 is a non-empty set. Should it be otherwise this would mean that under the existing conditions target A cannot be reached with probability γ.

3. An Example

For the sake of example we shall assume in this section that Y consists of a single variable whose model is linear and contains but two explanatory variables, namely X_1 (a decision variable) and X_2 (a purely exogeneous one). The random component ξ_T is normally distributed $N(0, \sigma)$ with σ known. Under these assumptions the econometric model of Y_t is

$$Y_t = \beta_1 X_{1t} + \beta_2 X_{2t} + \xi_t . \tag{4}$$

Now let the target be defined as achieving by Y_T in time T a value at least equal to y_0. The set Ω_2 will be composed of values of X_{1T} and X_{2T} derived from the following condition:

$$P\{Y_T = \beta_1 X_{1T} + \beta_2 X_{2T} + \xi_T \geqslant y_0\} \geqslant \gamma \tag{5}$$

which is equivalent to

$$P\{\xi_T \geqslant y_0 - \beta_1 X_{1T} - \beta_2 X_{2T}\} \geqslant \gamma . \tag{6}$$

Once γ is chosen, it is possible to find such number z_γ that $P\{\xi_T \geqslant z_\gamma\} = \gamma$ and Ω_2 will be composed of all values of explanatory variables which fulfill the condition $y_0 - \beta_1 x_{1T} - \beta_2 x_{2T} \leqslant z_\gamma$.

If it is assumed further that because of technological constraints the decision variable X_1 must in time T belong to the interval $[a, b]$ while the direct forecasts of the

[7]) In mathematical statistics discrimination analysis, roughly speaking, denotes inference about causes underlying a given (known) formation of observed random variable.

[8]) It can be shown that if ξ_T is a random vector with symmetric distribution and zero expectation then $\Omega_1 - \Omega_3$ is a nonempty set when $\gamma > 0.5$.

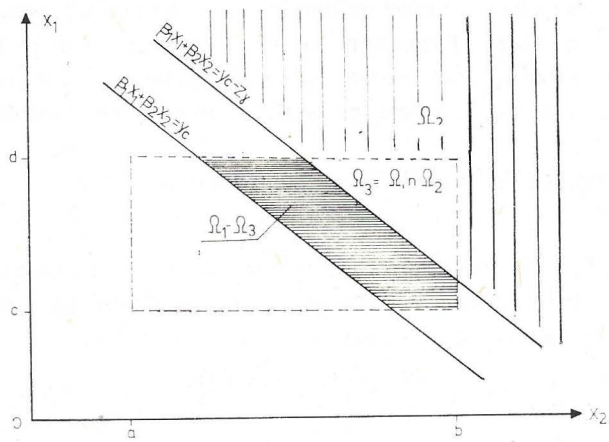
purely exogeneous variable X_2 show that in time T will hold true the double inequality $c \leqslant x_{2T} \leqslant d$, then the resulting set Ω_3 of practically admissible variants of action will be jointly determined by the relations

$$\beta_1 x_{1T} + \beta_2 x_{2T} \geqslant y_0 - z_\gamma, \tag{7}$$

$$a \leqslant x_{1T} \leqslant b, \tag{8}$$

$$c \leqslant x_{2T} \leqslant d. \tag{9}$$

The set Ω_3 is shown on graph.



4. The Choice of the Optimum Element in Ω_3

Once the set Ω_3 has been found and has proved to be nonempty, there arises the problem which one of practically admissible variants of action should be adopted.

Since all the variants in Ω_3 are equivalent with respect to target A one must look for another criterium of discrimination. Having this in mind let us observe that if the target A differs from the value of Y observed at the present time t_1 then variants belonging to Ω_3 will usually be characterized by such values of explanatory variables of Y which are substantially different from their values at $t = t_1$. Since for some of the decision variables to change their values means incurring some costs (e.g. investment outlays[9])) it seems reasonable to adopt costs as criterium for discrimination of different variants in Ω_3.

We shall assume hence that the best practically admissible variant of action can be chosen using the function

$$k = f(x, \tau, p), \qquad x \in \Omega_3, \tau > 0, 0 < p < 1, \tag{10}$$

[9]) Such situations will occur, for instance in situation, when Y denotes some output and among the explanatory variables for Y there are such as production capacities or technical labour equipment whose levels can be changed only by engaging in some investments.

where k denotes the cost necessary to set the values of explanatory variables in time T at levels corresponding to variant $x \in \Omega_3$ and τ is time span between the predicted and the present time periods, i.e. $\tau = T - t_1$. The introduction of this second variable is understandable when one takes into account that rush actions, spread over short time intervals, are usually more costly that actions started with adequate time lead. Variable p is defined as the probability of successful achieving until time T the programme of action represented by x_1 and referring to decision variables. It is to be noted that while to actions consisting purely in administrative steps there will usually correspond probability one, this is not so when x_1 induces some investment activity. Since such activity is often subject to delays there results $p < 1$. We shall assume p to be a decreasing function of τ and an increasing one of k.

If the analytical form of function f is known the problem of finding the best variant reduces to finding such variant x_0 which minimizes k for given τ and under the assumption that $p \geqslant p_0$, where p_0 is the minimum permissible level of reliability of variants.

Another approach is possible when the function

$$p = g(x, k), \; x \in \Omega_3 \tag{11}$$

is known. This function relates probability p to variants of action and costs and will be referred to as function of reliability of variants of action. By using (10) and (11) variable p can be eliminated to give

$$k = f\big(x, \tau, g(x, k)\big). \tag{12}$$

Let now

$$k = G(x, \tau) \tag{13}$$

by the solution of (12) with aspect to k. It is obvious that now the problem of the best variant of action reduces to minimizing G with respect to $x \in \Omega$ for given τ.

Finally, let us assume that the decision-maker has in mind but a few practically admissible variants of action which means that — instead of Ω_3 — he considers a set $\Omega_3^* \subset \Omega_3$ containing a finite (and small) number of elements. In this case the choice of the best variant can be made by making comparisons of pairs of variants, consecutively eliminating less efficient ones.

Let there be two variants, i-th and j-th, say, and let be known the corresponding probabilities and costs. Besides, let U and $U(p)$ denote respectively the total utility connected with full achievement in time T of target A and the expected utility of achieving target A when the probability connected with variant x is equal to $p < 1$. This author has shown [2] that in this case the following rule of comparison can be adopted: variant number i is better than variant j in the sense of net expected utility[10]), if for values of γ close to 1, there is

$$p_i U + (1 - p_i) U(p_i) - k_i > p_j U + (1 - p_j) U(p_j) - k_j. \tag{14}$$

It is reasonable to assume that $U(p)$ is a monotonously increasing function of p, such that $U(0) = 0$ and $U(1) = U$.

[10]) I.e. variants are compared by taking into account expected utility connected with each one minus the corresponding cost of action.

5. Consequences of Random Variation of Vectors x_{1T} and x_{2T}

So far it has been assumed that when building a discriminatory prediction at time t_1 values of variables forming vector x could be locked upon as fixed (see formulas (2) and (3)). In practice, however, this is usually an approximation to the real state of things. First, it must be remembered that the chosen variant of action can be achieved in time T with respect to decision variables with probability p which will normally be less than 1. Second, it must not be forgotten that at time t_1 only forecast values of purely exogeneous variables are known with respect to time T. Since such forecasts are subject to errors it follows that some vectors x_T — which at time t_1 were thought to belong to Ω_3 — will in fact in time T fall outside that set[11]) and, consequently, the probability of achieving target A will no longer be equal or higher than γ.

Allowing for random variation of decision and purely exogeneous variables which appear as explanatory variables in model (1) it is easy to find the probability of $Y_T \in A$ under the new circumstances. Obviously, there is now[12])

$$P\{Y_T \in A\} = P\{Y_T \in A | x_T \in \Omega_3\} P\{x_{1T} \in \Omega_3^{(1)}\} P\{x_{2T} \in \Omega_3^{(2)}\} \tag{15}$$

where $\Omega_3 = \Omega_3^{(1)} \times \Omega_3^{(2)}$ — the partitioning of vector space corresponding to partitioning of x into subvectors x_1 and x_2. By assumption, the first term on the right hand side of equation (15) is equal to $\gamma + \varepsilon$ where there is[13]) $\varepsilon \geqslant 0$, the second one equals p, so that denoting the third probability[14]) by q, we get

$$P\{Y_T \in A\} = \gamma pq + \varepsilon pq. \tag{16}$$

Formula (16) shows that the resulting probability of achieving the target may be much lower than the originally assumed number γ if there is $pq < (\gamma/(\gamma+\varepsilon))$ and either p or q differ substantially from unity. This setback can be remedied in two ways.

The first one consists in increasing p. From definition of function (11) it follows that for a fixed variant of action one can increase p by increasing k, i.e. by increasing outlays aimed at fixing the values of decision explanatory variables at levels corresponding to the chosen vector $x_T = [x_{1T}\, x_{2T}] \in \Omega_3$. Any increase of outlays aimed at increasing the probability of achieving in time T the values of decision variables corresponding to the chosen practically admissible variant of action will be referred to as safety margin of the second type[15]).

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[11]) In turn, because of random variation of purely exogeneous variables some vectors which at time t_1 did not appear to belong to Ω_3 may now fall within Ω_3.

[12]) In formula (15) vectors x_{1T} and x_{2T} are assumed to be independent. This is not necessary and one could use a more general expression in which the second term on the right side would represent the conditional probability of $x_{1T} \in \Omega_3^{(1)}$ when $x_{2T} \in \Omega_3^{(2)}$.

[13]) This results from (2). Since we wish P to be equal of least γ, an equivalent way of expressing it is to assume this probability to be $\gamma + \varepsilon$, ε being the excess value over γ.

[14]) Let us observe that q is the probability of prediction concerning vector x_{2T} being right.

[15]) Let us remind the reader that the first safety margin, introduced in Section 2, was defined as $\Omega_1 - \Omega_3$.

The second way to increase the probability of success as defined by (15) is that of increasing q. There are at least three possible approaches with this respect and these will be discussed now.

The first way to counteract the negative consequences of random variation of purely exogeneous variables consists in attempting to build as accurate as possible a forecast (or prediction[16]) of vector x_{2T}. Although this way of proceeding is obvious, it may, nevertheless, not always be successfully pursued. To see this let us observe first that in some cases the extent of random variation of variables forming vector x_{2T} may be such as to preclude the probability of building a good forecast (or prediction) for period T. Second, one must not forget that if the distance between time t_1 and T is large — which will be mostly the case when making discriminatory prediction[17]) — one may not be able to know with enough accuracy the real conditions of formation of vector x_{2T} or one may even have quite a false idea about them. Building a prediction under wrong assumptions obviously will lead to an erroneous result of inference into the future.

Since the mechanism of formation of purely exogeneous variables is beyond control of the econometrician involved in discriminatory prediction a reasonable rule of choice of variants of action may consists in adopting the following rule: if the degree of random variation is so large that inference about x_{2T} is not very reliable, then the final choice of variant of action should be based on the two following conditions:

(a) The cost associated with a chosen variant must not exceed a preasigned level k_0.

(b) The variant finally chosen must be highly robust with respect to random variation of variables forming vector x_{2T}, i.e. the difference

$$d=P\{Y_T\in A|x_{1T}\in\Omega_3^{(1)}\wedge x_{2T}\in\Omega_3^{(2)}\}-P\{Y_T\in A|x_{1T}\in\Omega_3^{(1)}\wedge x_{2T}\notin\Omega_3^{(2)}\} \tag{17}$$

must be minimum for variants obeying condition (a).

The third approach to cope with uncontrolled variation of purely exogeneous variables is to further restrict to set of practically admissible variants of action by imposing some safety margins on exogeneous variables and constraints with such variables which appear when determining the set Ω_3. This will result in obtaining a narrower set $\Omega_4\subset\Omega_3$ and the difference $\Omega_3-\Omega_4\neq\emptyset$ will be referred to as safety margin with respect to random variation of purely exogeneous variables[18]). The obvious precondition to establishing this safety margin is either to know the probability distribution of vector x_{2T} or at least to have a good estimate of it.

Without entering into details we shall restrict our argument to two examples how one should proceed to construct the set Ω_4.

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[16]) In this author's opinion the terms forecast and prediction should be distinguished. The first refers to inference into the future made by simple extrapolation or any other noncausal argument, while prediction means the result of inference into the future based on a causal-type model.

[17]) By its very virtue discriminatory prediction must be a long-term one since it preassumes to provide necessary basis for some action which will cause Y_T to fall into region A. The longer is such action the longer will be prediction lead of discriminatory prediction.

[18]) This, let us observe, is the third safety margin introduced in this paper.

First, let us assume that X_2 is an exogeneous random variable and that there exists a constraint of the type $x_{2T}<a$, where a is a known number.

Let now ε be a predetermined significance level close enough to zero[19]), and let us consider two cases, namely when

$$P\{X_{2T}<a\}\geqslant 1-\varepsilon. \tag{18}$$

$$P\{X_{2T}<a\}<1-\varepsilon. \tag{19}$$

In the first case the constraint x_{2T} a is likely enough to be obeyed even though X_{2T} is a random variable, so there is no need to introduce a change into the constraint. If, however, relation (19) holds true one must reckon with the possibility that there will be $x_{2T}\geqslant a$, which means violation of imposed constraint. In this case the original constraint must be replaced by a stronger one. Let now a' be such real number that $a'<a$ and there is

$$P\{X_{2T}<a'\}=1-\varepsilon. \tag{20}$$

Then the new constraint will take the form $x_{2T}<a'$.

As a second example let us suppose that the constraint is of the form

$$\sum_{i=1}^{k} \beta_i x_{iT}<a \tag{21}$$

where the first k_1 variables are exogeneous and random while the remaining ones, i.e. from (k_1+1)-th up to k-th are non-random decision variables. Inequality (21) can be rewritten as.

$$\sum_{i=1}^{k} \beta_i x_{iT}<a-\sum_{i=k_1+1}^{k} \beta_i x_{iT}=c. \tag{22}$$

If the probability distribution of the linear form appearing on the left-hand side is known then the problem can be treated in exactly the same way as it was done in the previous case of a constraint affecting one random variable[20]).

6. The Problem of Periodicity of Inference. Prediction and Control

Building at time t_1 the set Ω_3 and consequently choosing a practically admissible variant of action is but the first step in the sequential inference which must be carried on until time T. As a matter of fact, in a periodical way over all time interval (t_1, T) there is need for further inference about what the situation will be in period T.

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[19]) The value of ε is to be determined by the user of discriminative prediction.

[20]) It is to be observed that this approach could also be used with respect to these decision variables which are characterized by a large variance making thus a successful achievement of the chosen variant of action not very probable.

The consecutive steps of inference must consists in gathering, analysing and combining three different types of information, namely: (a) direct forecasts[21]) of variable Y_T, (b) direct forecasts of vector x_{2T}, (c) information about the state of realisation of the chosen variant of action with special reference to the possibility of fully achieving the values of explanatory decision variables assumed for time T.

This three-fold information provides a basis for analysis whether the system undergoes changes implied by the discriminatory prediction and the chosen variant of action or whether substantial deviations have occurred from its expected path.

Information stemming from the above mentioned three types of information should be supplemented by a new discriminatory prediction, the scope of which is to find out if the chosen variant of action is still in Ω_3 and if — in view of the latest observations of Y_t — some new additional measures should not be taken to ensure attaining target A in the time T.

In this way control of economic system (whose first step is the discriminatory prediction and then the choice of vector $x \in \Omega_3$) is supplemented by direct observation and prediction of the behaviour of this system which, in turn, provide a basis for necessary shifts and corrections of controlling measures. An important problem of how often within time interval (t_1, T) such corrective inference should be made is of great importance. This, however, is another subject which will not be treated in this paper.

To conclude our arguments let us remark that if at a time $t \in (t_1, T)$ it is found that the variant of action chosen at $t = t_1$ is carried on very unsatisfactorily or that because of unexpected changes of exogeneous variables target A is very unlike to be reached, there may arise the problem of changing the variant of action pursued so far. In some situations such change may not be possible because of some technical or technological constraints but if such change is feasible one must find out if it is economically advantageous. In his paper [3] this author has shown that a new variant (number 2, say) is more efficient that the variant numer 1, carried on until now, if the following inequality holds true:

$$\underset{Y_T x_{1T} x_{2T}}{EEE} U(y, p_2, \alpha_2) - \kappa_{12}(t) > \underset{Y_T x_{1T} x_{2T}}{EEE} U(y, p_1, \alpha_1) + k_{12}(\tau). \tag{23}$$

In the above formula $E[U(y, p, \alpha)]$ denotes the expected utility derived from achieving target A when the expectation is taken with respect to random variation of Y_T and with respect to random events $x_{1T} \in \Omega_3^{(1)}$ and x_{2T} $(\Omega_3^{(2)})$ whose probabilities are equal p and α, respectively. Symbol $\kappa_{12}(t)$ denotes the amount of costs incurred in connection with variant 1 until time t and $k_{12}(\tau)$ stands for costs necessary for shifting from variant 1 to variant 2 and carrying on the new variant 2 when period T is distant from period t by τ units of time. While $\kappa_{12}(t)$ can be assumed to be a monotonously increasing function of t, $k_{12}(\tau)$ is a decreasing function of τ and it can be reasonably supposed that the rate at which $k_{12}(\tau)$ increases when $\tau \to 0$ is a high one. This means that decisions about eventual change of variants of action can be taken only when T is still distant.

[21]) Eventually the three types of information may result from a predictive type of inference.

7. Some Examples of Possible Applications

Finally, it seems worthwhile to add a few remarks about possible applications of the approach presented in the preceeding sections. With this scope in mind we shall give two examples of possible uses of discriminatory predictions.

First, let us assume that Y denotes the vector of outputs of a number of independent industrial sectors for which the target A is defined by setting lower limits to output levels in time T. Let us assume also that these outputs are determined by production functions depending on 3 variables, namely: fixed productive assets, employment and supply of imported (or agricultural) raw materials, the first two being decision variables while the third is considered a purely exogeneous one. Technology implies some constraints on relations of inputs of the decision variables and there will also be some constraints on the rate they may be increased in time and some constraints on supply of raw materials. These constraints, together with knowledge of the distribution of random components of sectorial production functions and the assumed probability γ will determine the shape of set Ω. Since the set Ω_3 has been determined for every sector[22]) it becomes possible to start looking for the best variant of action (i.e. for the best combination of labour and fixed assets in time T) leading to fulfilment of target T.

Let us observe that increasing production capacities will usually imply some investing in order to increase the stock of fixed productive assets. Since, however, investment actions are apt to delays and also since former forecasts about supplies of raw materials may become outdated, it becomes necessary to combine discriminatory prediction (choice of variant of action) with newer direct forecats referring to the chances of completing investments before time T and to the level of supply of raw materials. Pessimistic prospects stemming from such inference into the future may eventually lead to a change of target A or to a shift to another admissible variant of action.

The second example will refer to regional analysis. Let Y denote a vector of variables which are though of as basic characteristics of regional economy and let a target A be imposed on these variables with respect to time T. Let the (regional) model of vector Y depend on vector x_1 of decision variables and on vector x_2 of exogeneous variables which, in turn, can be split into two subvectors x_{21} and x_{22}, such that x_{21} is composed of variables pertaining to other sectors, while x_{22} is built up of macrovariables and of other purely exogeneous ones.

Because of interdependency of variables entering vector Y the set Ω_3, will have to be determined simultaneously with respect to all endogeneous variables of the regional model, i.e. by jointly considering all the equations of the model. In setting bounds on the set of admissible variants Ω_2 at least three types of constraints must be accounted for: (1) technical or social constraints imposed on decision variables, (2) constraints due to regional interdependency, (3) constraints referring to purely exogeneous factors.

[22]) Since there is no interdependency between different components of vector Y the set Ω_3 can be determined separately for every variable belonging to Y.

The final choice of variant of action may consist in finding such one which will maximize the difference between the achieved level of utility and the amount of financial outlays connected with pursuing this particular variant of regional policy.

Let us note also that in this case of great importance will be the forecasts (or predictions) pertaining to economic and other types of activity of other regions, and especially of neighbouring ones.

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Predykcja dyskryminacyjna i jej związek z optymalnym sterowaniem systemami ekonomicznymi

Przedstawiono próbę opracowania pewnej metody użycia modeli ekonometrycznych do sterowania systemami ekonomicznymi. Jeżeli jest określony cel A odnoszący się do pewnego przyszłego momentu czasowego T oraz pewnego wektora Y_T, to można użyć modelu do określenia zbioru Ω_3 takich wartości zmiennych objaśniających x_T dla Y_T, które: a) są praktycznie osiągalne przed okresem T, b) zapewniają spełnienie celu A z prawdopodobieństwem co najmniej γ, gdzie γ jest pewną z góry ustaloną liczbą zbliżoną do jedności. Procedurę prowadzącą do określenia Ω_3 nazywa się predykcją dyskryminacyjną.

Разложимое предсказывание и его связь с оптимальным упрацлением экономической системой

В работе представлена попытка разработки некоторого метода использования эконометрических моделей для управления экономическими системами. Если определена цель A, относящаяся к некоторому будущему моменту времени T и некоторому вектору Y_T, то можно использовать модель для определения множества Ω_3 таких значений переменных интерпретирующих x_T для Y_T, которые: а) являются практически достигаемы перед истечением периода T, б) обеспечивают удовлетворение цели A с вероятностью по крайней мере γ, где γ является некоторым заранее заданным числом близким единице. Процедура ведущая к определению Ω_3 называется разложимым предсказыванием.

Recommendations for the Authors

Control and Cybernetics publishes original papers which have not previously appeared in other journals. The publications of the papers in English is recommended. No paper should exceed in length 20 type written pages (210×297 mm) with lines spaced and a 50 mm margin on the lefthand side. Papers should be submitted in duplicate. The plan and form of the paper should be as follows:

1. The heading should include the title, the full names and surnames of the authors in aphabetic order, the name of the institution he represents and the name of the city or town. This heading should be followed by a brief summary (about 15 typewritten lines).

2. Figures, photographs tables, diagrams should be enclosed to the manuscript. The texts related to the figures should be typed on a separate page.

3. Of possible all mathematical expressions should be typewritten. Particular attention should be paid to differentiation between capital and small letters. Greek letters should as a rule be defined. Indices and exponents should be written with particular care. Round brackets should not be replaced by an inclined fraction line.

4. References should be put on the separate page. Numbers)in the text identified by references should be enclosed in brackets. This should contain the surname and the initials of Christian names, of the author (or authors), the complete title of the work (in the original language) and, in addition:

a) for books—the)place and the year of publication and the publisher's name;

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Wskazówki dla autorów

W wydawnictwie „Control and Cybernetics" drukuje się prace oryginalne nie publikowane w innych czasopismach. Zalecane jest nadsyłanie artykułów w języku angielskim. W przypadku nadesłania artykułu w języku polskim Redakcja może zalecić przetłumaczenie na język angielski. Objętość artykułu nie powinna przekraczać 1 arkusza wydawniczego, czyli ok. 20 stron maszynopisu formatu A4 z zachowaniem interlinii i marginesu szerokości 5 cm z lewej strony. Prace należy składać w 2 egzemplarzach. Układ pracy i forma powinny być dostosowane do niżej podanych wskazówek.

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