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On a method of determination of parameters of conceptual models of open channel flow

by

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The basic approaches to modelling of hydrologic phenomena on the example of process of open channel flow was discussed. The problem of synthesis of pulse response for the case of ungauged catchment was studied. The possibility of evaluation of conceptual model parameters on the basis of conditions of systems equivalence was analyzed. A number of lemmas and remarks justifying the method of moment matching of system pulse responces was given. On the basis of this method the values of parameters of conceptual models were calculated in terms of physical characteristics of an open channel flow. The possibility of choice of structure of conceptual models by means of Pearsonian moment ratios analysis was discussed.

1. Introduction

In the last few decades the necessity of mathematical modelling for control purposes came about in the topics initially outside the classical interests of control systems engineers. Examples of such domain are — hydrology and water management. Due to the increasing value of water as a raw material water economy deals more frequently with mathematical optimization problems. Modelling of hydrologic systems becomes thus an important part of the general problem — the control of water resources in the broad sense.

The techniques of mathematical modelling of hydrologic systems have adopted many elements of electric circuits theory and of dynamic systems science. However, modelling of hydrologic phenomena is far more difficult than the description of electronic elements of electrical circuits. In electrical circuits modelling a passive element e.g. capacitor can be described exactly for a wide range of working regimes by means of linear ordinary differential equation of the first order.

In hydrology the same phenomenon (e.g. open channel flow) can be described in a number of ways, depending upon the parameters of the problem (available information concerning system structure, measurement data), the techniques at one's disposal (computational facilities) and the aim of the modelling (required speed and accuracy of simulation).

The basic tool for modelling the relationships between the quantities characterizing hydrologic cycle are equations of mathematical physics. Assumption of a physical model necessitates the solution of equations describing the laws of con-. servation of mass, momentum and energy.

The parameters of such a model have clear physical interpretation. Hence hydrodynamic models can easily be adapted to different catchments, provided measurement, computation or estimation of parameters pertaining to a given catchment is possible. In practice this approach is often unacceptable because of the difficulties in determining the equations parameters and initial and boundary conditions with sufficient accuracy. This is due to the existence of unknown heterogeneities, anisotropics and nonstationarities of the system and to computational difficulties.

Because of the aforementioned difficulties in applying physical models other methods of mathematical modelling are quite frequently used. A widely used method which completely differs from the hydrodynamic approach is the method of black box system modelling. The real system is modelled by a "black box" with a number of inputs and outputs. The black box operator is identified from time series analyses of input and output data. Examples of this type of model are integral operators, for which kernels are identified by means of one of many competing methods (e.g. — matrix inversion with eventual regularization, methods of integral transforms, correlation, approximation by orthogonal polynomials, etc.).

A third type of hydrologic models has proved to be popular is the class of conceptuals models developed from abstract conceptual structures which transform hydrological variables. These structures reflect a portion of hydrologic reality together with empirical simplifying assumptions based on the pearsonal judgment of a modeller. Since these structures are assumed, identification of such models results in identification of the parameters.

2. Conditions of systems equivalence

As there are a number of ways of describing hydrologic systems, the problem of inter-relationships between models is of primary importance. From a practical point of view obtaining such relationships may enable the construction of a conceptual model based on the measurement or evaluation of physical system parameters, without the need for long series of input/output measurements. However, the only method of describing open channel flow which involves actual physical interpretation of parameters is the hydrodynamic approach to modelling.

Parameters of black box system models can be identified by analysing long time series of input and output values measured in the real system. The weakness of the quasi-physical assumptions, on which conceptual models are based does not allow any clear physical interpretation of parameters of these models.

In case of ungauged catchments measurements over a long period of time which are required to identify system black box model or conceptual model do not exist.

In order to synthetise the pulse response for an ungauged catchment a sufficient number of measurements are first made in controlled catchments. Subsequently the parameters of the pulse response are correlated with the physical catchment characteristics. Major problems occur when these relationships are extended to other catchments. The correlation formulae obtained appear to be regional.

An interesting method of determination of physical sense of parameters of conceptual hydrologic models seems to be moment (or cumulant) matching approach introduced by Dooge [1, 2]. Basing on the above method one can find analytic formulae linking parameters of conceptual models and parameters of linear physical model.

The starting point of the moment matching method is analysis of equivalence of linear dynamic systems in the sense of input/output relation. By term "systems equivalence" it is understood here, that both compared systems have identical responses to the same input signal (in particular — signal belonging to a given class of functions).

Let us consider two linear systems described by equations

$$y_j(t)=\int_0^{\infty} h_j(\tau)\, x(t-\tau)\, d\tau = y_{j_0}(t)+\int_0^{t} h_j(\tau)\, x(t-\tau)\, d\tau,\; j=1,2 \tag{1}$$

where: x — input signal, y — output signal, h — pulse response, y_{j_0} — term responsible for the impact of initial condition upon the present output signal.

Physical sense enables assumption of finite system memory T, very convenient from computational point of view. One obtains

$$y_j(t)=\int_0^{T} h_j(\tau)\, x(t-\tau)\, d\tau,\; j=1,2 \tag{2}$$

whereas for initially relaxed systems

$$y_j(t)=\int_0^{\min(t,T)} h_j(\tau)\, x(t-\tau)\, d\tau,\; j=1,2. \tag{3}$$

Equivalence of two linear systems in the sense of input/output relation means, under the assumption of finite system memory T [1)], that for every time instance t and for any input signal $x(t)$ the following relationship holds

$$y_1(t)-y_2(t)=\int_0^{T} [h_1(\tau)-h_2(\tau)]\, x(t-\tau)\, d\tau=0. \tag{4}$$

In a similar way one can define equivalence of infinite memory systems ($T\to\infty$).

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1) In the case of different memory lengths of the two compared systems (i.e. $T_1 \neq T_2$), the common memory length $T=\max(T_1, T_2)$ will be assumed here.

Statement, that the expression (4) vanishes because of identity of impulse responses of both systems at every time instance is of no practical meaning. The input/output equivalence of two linear systems can be formulated in another way — through impulse response moments of the type

$$\langle h(t)\rangle^{n}_{0,T}=\int_{0}^{T} t^{n} h(t)\,dt \tag{5}$$

or

$$\langle h(t)\rangle^{n}_{0,\infty}=\int_{0}^{\infty} t^{n} h(t)\,dt \tag{6}$$

for finite or infinite memory systems respectively.

The basic theorem used in the moment matching method is

THEOREM 1 (Lerch). If a function $f(t)$ is continuous in the interval $t \in [0, T]$ and $\int_{0}^{T} t^{n} f(t)\,dt=0$ for $n=1, 2, \ldots$ then $f(t)=0$ in the whole interval $[0, T]$.

The proof can be found in the book by Mikusiński [4].

Basing on this theorem one can formulate conditions for input/output system equivalence, different from (4).

LEMMA 2. If impulse responses of two linear systems with finite memory T are such, that

$$\int_{0}^{T} \tau^{n} h_{1}(\tau)\,d\tau=\int_{0}^{T} \tau^{n} h_{2}(\tau)\,d\tau,\quad n=0, 1, 2, \ldots .$$

then both systems response identically to any input signal.

The proof is obvious, since from the Theorem 1 it results, that $h_1(t)=h_2(t)$ in the whole interval $t \in [0, T]$ i.e. $y_1(t)-y_2(t)=0$ for every time instance and for all classes of input signals. The above Lemma holds also for $T\to\infty$. Matching infinite number of moments of two impulse responses is rather of academic importance. However matching a finite number of moments also enables drawing some conclusions concerning systems equivalence.

LEMMA 3. If the N first impulse response moments of the type (5) of two linear dynamic systems with finite memory T are respectively equal, then both systems response identically to input signals being polynomial functions of time.

Induction proof. Let us assume, that only zero-th moments of impulse responses of both linear dynamic systems are respectively equal, i.e.

$$\int_{0}^{T} e(\tau)\,d\tau=0$$

where

$$e(\tau)=h_1(\tau)-h_2(\tau)$$

then responses of both systems to a constant input signal of the amplitude X are identical, i.e.

$$E(t)=y_1(t)-y_2(t)=X\int_0^T e(\tau)\,d\tau.$$

Matching of zero-th moments is guaranteed by the assumption of finite memory of passive nondissipative systems (both moments are equal to 1).

Let us assume now, that from the relationship

$$\int_0^T \tau^i e(\tau)\,d\tau=0 \quad i=0, \ldots, n$$

results, that for the input signal $x_n(t)=\sum_{i=0}^{n} a_i t^i$ the following holds

$$E(t)=y_1(t)-y_2(t)=\int_0^T x_n(t-\tau)\,e(\tau)\,d\tau=0.$$

Let it be

$$\int_0^T \tau^{n+1} e(\tau)\,d\tau=0.$$

Then, for $x_{n+1}(t)=\sum_{i=0}^{n+1} a_i t^i$ one obtains

$$E(t)=\int_0^T x_{n+1}(t-\tau)\,e(\tau)\,d\tau=\int_0^T x_n(t-\tau)\,e(\tau)\,d\tau+$$
$$+\int_0^T a_{n+1}(t-\tau)^{n+1} e(\tau)\,d\tau=0. \qquad \text{q.e.d.}$$

The above Lemma remains valid also for the case $T\to\infty$.

LEMMA 4. If the first N moments of the type (6) of impulse response of two linear dynamic systems (described by convolution integral, i.e. initial relaxed) are respectively equal and the input signal is common to both systems, then output signals in both systems have identical first N moments.

The proof results directly from the interpretation of formula describing transformation of moments in linear dynamic systems.

$$\langle y_i(t)\rangle_{0,\infty}^n=\langle h_i(t)^* x(t)\rangle_{0,\infty}^n=\sum_{r=0}^{n}\binom{n}{r}\langle h_i(t)\rangle_{0,\infty}^{n-r}\langle x(t)\rangle_{0,\infty}^{r};\ i=1,2.$$

It is easy to see that the N-th moment of the type (6) of the output signals in each of the two systems depends on moments of the input signal and of the impulse response up to the N-th order. Since, by assumption first N moments of the impulse response and all the moments of the input signal are respectively equal for both systems, moments of the output signal for both systems are identical up to the

N-th order inclusive. The above statement is valid for all kinds of input signals. It is thus extension of the consideration out of the class of input signals being polynomial functions of time.

REMARK 5. If the N first impulse response moments of the type (5) of two linear dynamic finite memory systems are respectively equal and the input signal common to both systems reads

$$x(t)=\begin{cases} 0 & t\leqslant 0 \\ \sum_{i=0}^{N} a_i\, t^i & t\geqslant 0 \end{cases}$$

then both systems response identically starting from the time instance T, where T — memory of the system.

EEMARK 5a. If the N first impulse response moments of the type (5) of two linear dynamic finite memory systems are respectively equal and the input signal common to both systems reads

$$x(t)=\begin{cases} \sum_{i=0}^{N_1} a_i\, t^i & t\in(t_1,\, t_2) \\ \sum_{i=0}^{N_k} b_i\, t^i & t\in(t_k,\, t_{k+1}) \qquad N_i\leqslant N\forall_i \\ \sum_{i=0}^{N_{n-1}} z_i\, t^i & t\in(t_{n-1},\, t_n) \end{cases}$$

where n is any given integer number, then both systems response identically in the following time intervals

$$(t_i+T,\, t_{i+1}) \text{ if } t_{i+1}-t_i\geqslant T,\ i=1, 2, \ldots, n.$$

REMARK 6. If the N first impulse response moments of the type (5) of two linear dynamic finite memory systems are respectively equal and the input signal common to both systems is the polynomial function of time of the order $M>N$, then the difference between output signals depends on the weighted sum of differences of $N+1$-th, ..., M-th moments of impulse responses in both systems.

It is easy to see, that

$$E(t)=\int_0^T \Big[\sum_{i=0}^{M} [a_i\,(t-\tau)^i\Big]\, e(\tau)\, d\tau=\sum_{i=N+1}^{M} \int_0^T a_i\,(t-\tau)^i\, e(\tau)\, d\tau$$

since, by assumption

$$\int_0^T \Big[\sum_{i=0}^{N} a_i\,(t-\tau)^i\Big]\, e(\tau)\, d\tau=0\,.$$

REMARK 7. If the N first impulse response moments of the type (6) of two linear dynamic systems (described by convolution integral — initially relaxed) are respectively equal and the input signal common to both systems has the following properties

$$\langle x_1(t)\rangle^r_{0,\infty} = \langle x_2(t)\rangle^r_{0,\infty} \qquad r=1, ..., N,$$

$$\langle x_1(t)\rangle^r_{0,\infty} \neq \langle x_2(t)\rangle^r_{0,\infty} \qquad r=N+1, N+2, ...$$

then output signals from both systems retain matching of the first N moments of the type

$$\langle y_i(t)\rangle^r_{0,\infty}, \; i=1, 2.$$

It results directly from the Lemma 4.

3. Cumulant matching technique

When comparing differing structurally linear models of the same hydrologic processes one can take into account several parameters of the pulse response (maximum value of the pulse response, time corresponding to this value, time corresponding to the "centre of gravity" of the area under the curve $h(t)$, properties of the rising part of this curve and of its falling part).

More general conclusions can be drawn from analysis of matching the pulse responses by moments — aggregated function characteristics. Even more convenient from computational point of view than moment matching is cumulant matching. The simplest formula for determination of cumulants is

$$C_n = (-1)^n \frac{d^n}{ds^n} [\log F(s)]|_{s=0} \tag{7}$$

where:

n — cumulant order, $F(s)$ — transfer function i.e. Laplace transform of the impulse response function $f(t)$.

The functional relationship exists between moments and cumulants (cf. [1], [3]). Thus it seems, that construction of theorems concerning cumulant matching is not necessary.

Let us assume, that the real physical system is modelled by two different structurally linear models with k_1 and k_2 parameters respectively. Let k_1 denotes number of parameters (of conceptual model) that should be determined in terms of k_2 parameters of physical model.

Then by matching k_1 first moments of impulse responses of both systems one can find the conditions of systems equivalence for input signals being polynomial functions of time of the k_1-th order.

It can happen however, that k^* physical parameters and k_1 parameters of conceptual model (determined by optimization) are known. The problem reads — find lacking $(k_2 - k^*)$ unmeasurable physical parameters. It can also be solved by moment matching method.

A separate problem is connected with existence of solution of equations conditioning systems equivalence in the sense of input/output relation by means of matching impulse responses by moments. One cannot formulate any statements concerning the existence of solution in general case, with no assumption on the class to which impulse responses belong.

4. Examples of application of moment matching method

As an example matching cumulants of impulse responses of linear dynamic wave model and linear conceptual model will be performed. Moment matching allows coarse identification of conceptual model parameters basing upon physical system characteristics.

The linear dynamic wave model has been obtained in the result of linearization of physical equations describing movement of a flood wave in an open channel of semi-infinite length. The equation of linear dynamic wave for prismatic channel with rectangular cross-section reads

$$(gy_0-v_0^2)\frac{\partial^2 q}{\partial x^2}-2v_0\frac{\partial^2 q}{\partial x\,\partial t}-\frac{\partial^2 q}{\partial t^2}-\frac{2gS_0}{v_0}\frac{\partial q}{\partial t}+3gS_0\frac{\partial q}{\partial x}=0 \tag{8}$$

or, when avoiding description by parameters connected explicitely with the reference values

$$(F^6\,g^4\,n^6\,S_0^{-3}-F^8\,g^4\,n^6\,S_0^{3-})\frac{\partial^2 q}{\partial x^2}-2F^4\,g^2\,n^3\,S_0^{-3/2}\frac{\partial^2 q}{\partial t\,\partial x}+$$
$$+\frac{\partial^2 q}{\partial t^2}+2F^{-4}\,g^{-1}\,n^{-3}\,S_0^{-1/2}\frac{\partial q}{\partial t}+3gS_0\frac{\partial q}{\partial x}=0 \tag{9}$$

wheie:

g — gravitational acceleration, y_0 — reference depth, v_0 — reference flow valocity, q — flow rate, x — longitudinal variable, t — time instance, s_0 — channel slope, F — Froude number, $F=v_0/\sqrt{gy_0}$, n — Manning coefficient.

In order to determine the impulse response of linear dynamic wave model the following conditions must be fulfilled:

$$\begin{aligned}
&\text{(i)}\ \ q(x,0)=\dot{q}_t(x,0)=0,\\
&\text{(ii)}\ \ q(0,t)=\delta(t),\\
&\text{(ii)}\ \ q(x,t)<\infty\ \ \forall x\in[0,\infty),\\
&\text{(iv)}\ \ \int_0^\infty q(x,t)\,dt=1\ \ \forall x\in[0,\infty).
\end{aligned} \tag{10}$$

It is easy to calculate the Laplace transform of impulse response (system transfer function) of this model. It reads

$$H(x,s)=\exp\left(\alpha xs+\beta x-x\sqrt{\gamma s^2+\delta s+\varepsilon}\right) \tag{11}$$

where the coefficients α, β, γ, δ, ε are functions of reference values and of constant parameters of the equations (8)–(9) (Dooge [1], Kundzewicz [2])·

The cumulants can be computed directly from the above formula for the transfer function with no necessity of inverting the relationship (11) to the time domain.

Due to the complexity of the above model and to its limited applicability, hydrologists have introduced a number of linear empirical conceptual models.

An easy and popular approach to modelling the open channel flow is based upon conceptual partitioning the channel into a cascade of abstract linear reservoires (characteristic reaches). The basic element of this model is an abstract concept of linear reservoir (characteristic reach) fulfilling the continuity equation

$$q_{x0}(t)-q_{xL}(t)=\frac{ds(t)}{dt} \tag{12}$$

and the storage equation

$$s(t)=Kq_{xL}(t) \tag{13}$$

where:

q_{x0} (inflow to the reservoir (flow rate in the cross-section opening the characteristic reach), q_{xL} — outflow from the reservoir (flow rate in the cross-section closing the characteristic reach), s — storage volume (retention in the characteristic reach), K — storage coefficient.

If the outflow from the i-th linear reservoir in the cascade equals to the $(i+1)$-th reservoir, then the system of N reservoirs can be described by means of the impulse response

$$h(t)=\frac{1}{K(N-1)!}\left(\frac{t}{K}\right)^{N-1} e^{-t/K}. \tag{14}$$

If the concept of partitioning the length of the channel into characteristic reaches is followed, one can deal with real values of N (inconvenient for the reservoir interpretation of the former version).

$$h(t)=\frac{1}{K\Gamma(N)}\left(\frac{t}{K}\right)^{N-1} e^{-t/k}. \tag{15}$$

where $\Gamma(N)$ is the gamma function

The general formula for the R-th cumulant of model of cascade of linear reservoir reads

$$C_R=N(R-1)!\,K^R. \tag{16}$$

The starting point for calculation of cumulants of the complete linearized model is the relationship (11) for Laplace transform of the impulse response of the model (8) subject to the conditions (10).

When differentiating the logarithm of the Laplace transform of impulse response with respect to s in the point $s=0$ one obtains

$$C_1^* = \frac{x}{1.5v_0} = \frac{2}{3}\, x n^{-3}\, s_0^{3/2}\, F^{-4}\, g^{-2} \tag{17}$$

where notation follows the formulae (8)–(9)

$$C_2^* = \frac{2}{3}\left(1-\frac{F^2}{4}\right)\left(\frac{y_0}{s_0\, x}\right)\left(\frac{x}{1.5v_0}\right)^2 = \frac{8}{27}\left(1-\frac{F^2}{4}\right) x s_0^{-1}\, g^{-1}\, F^{-2}. \tag{18}$$

When comparing the first two cumulants calculated according to (16) and (17)–(18) one can obtain the relationship between parameters of conceptual model and physical parameters of linearized hydrodynamical model.

$$K = \frac{2}{3}\left(1-\frac{F^2}{4}\right)\left(\frac{y_0}{s_0\, x}\right)\left(\frac{x}{1.5v_0}\right) = \frac{4}{9}\left(1-\frac{F^2}{4}\right) F^2\, g n^3\, s_0^{-5/2}, \tag{19}$$

$$N = \frac{1}{\frac{2}{3}\left(1-\frac{F^2}{4}\right)\frac{y_0}{s_0\, x}} = \frac{3}{2}\,\frac{s_0^{4}\, x}{F^6\, g^3\, n^6\left(1-\frac{F^2}{4}\right)} \tag{20}$$

Another method of construction of a simple conceptual model is simplification of complete linear dynamic wave equations. This method however, in the contrary to the moment matching approach does not enable exact determination of class of input signals granting full equivalence of both systems in the input/output sense.

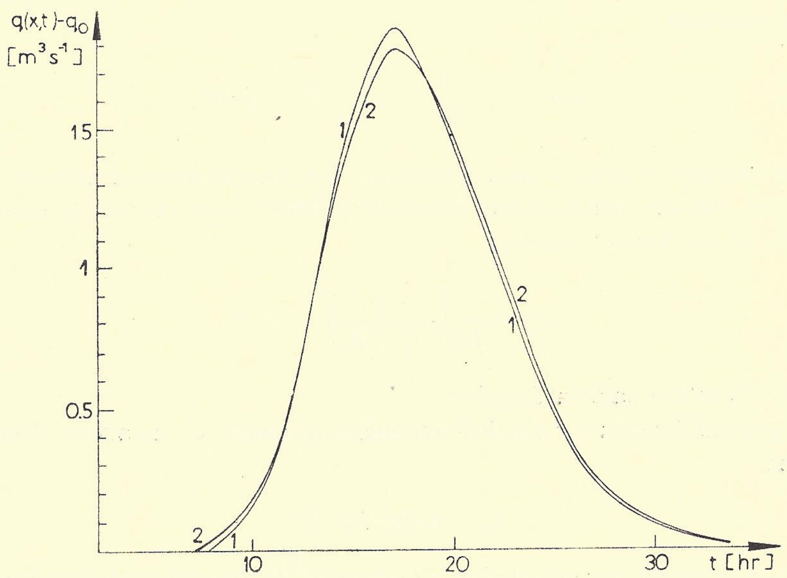

Fig. 1. Comparison of responses of linearized dynamic wave model (*1*) and of cascade of linear reservoir (*2*) to rectangular input signal

In Fig. 1 the comparison of simulation of the flow in the system modelled by eq. (8) and subject to the conditions (10) and in the system described by conceptual model (12)–(13) with parameters evaluated by moment matching method is made. The input signal common to both models of flow in prismatic open channel with rectangular cross-section and length of 100 km is the switched step function

$$x(t)=\begin{cases} 200 m^3 s^{-1}, & t<0,\ t>11 \text{ min.} \\ 210 m^3 s^{-1}, & t\in[0, 11 \text{ min}]. \end{cases}$$

In the similar way one can find the responses of the three-parameters model composed of the cascade of linear reservoirs in series with linear channel are also shown.

Parameters of such models have been obtained also from matching cumulants of impulse response of such a conceptual model and of a linearized hydrodynamic model (cf. [3]).

In the so far presented considerations no real system with an adequate set of measurement data has been taken into account. The moment matching technique has been tested on data generated by the solution of linear dynamic wave equation

Now the discussed method will be applied to a real system of river flow (reach of lower Vistula-Włocławek — dam, Włocławek — flow rate measurement section located 150 m upstream fiom the permanent gauge).

Inserting the values of physical parameters to the formulae (17)–(20) one obtains

$$K=13226.197\ s,\ n=.43188.$$

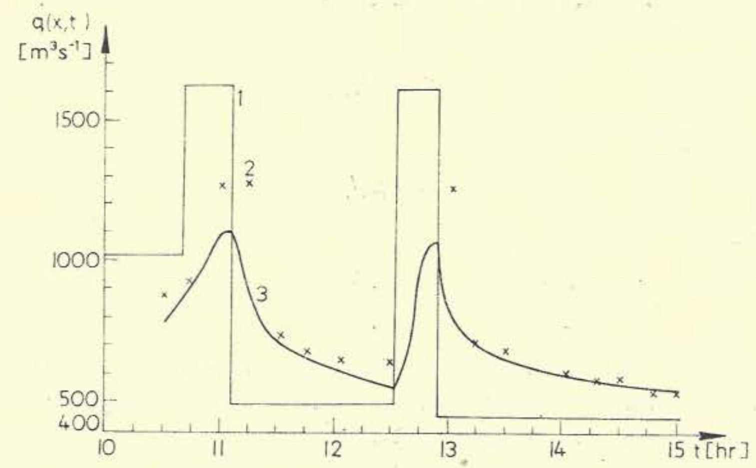

Fig. 2. Simulation of the flow rate by means of the model of cascade of linear reservoirs (Vistula at Włocławek, Sept. 25th 1975); $x=3770$ m
1 — discharge from the power station, *2* — measured flow rate in the cross-section x, *3* — simulated flow rate in the cross-section x

In Fig. 2 simulation of flow on the basis of conceptual model of cascade of linear reservoirs with parameters obtained by moment matching method is presented. The records of discharge from Włocławek dam and measured flow rate in the cross-

-section located 3770 m downstreams are compared with the flow rate simulated by the model (12)–(13) with parameters computed by moment matching method.

Relatively poorer simulation in the range of greater flow rates can be explained by evident nonlinearity of the system. In the example a simplified assumption of steady state was accepted for step input signals presented in Fig. 2. In fact, the last section of input function occured when the steady state corresponding to reference discharge has not been reached yet in the system what is one of the sources of inaccuracy.

Linearization was performed under assumption of small deviations from the reference point. The time variability of discharges from the dam violates seriously the above assumption. The amplitude of flow rate at the input to the river reach modelled ranks in the example considered from 450 to 1650 $m^3 \sec^{-1}$. The errors obtained are not catastrophic, even for big variations of input signal.

5. Choice of structure of conceptual model basing on Pearsonian moment characteristics

The further extension of application of the concept of functional moments in modelling hydrologic processes is determination of criterion of choice of structure of conceptual model. As such criterion one can accept the assessment of similarity of impulse responses of conceptual model and of linearized hydrodynamic model expressed by Pearsonian moment characteristics, i.e.
measure of skewness

$$\beta_1=\mu_3^2/\mu_2^3 \tag{21}$$

measure of curtosis

$$\beta_2=\mu_4/\mu_2^2 \tag{22}$$

and generally

$$\beta_{2n}=\frac{\mu_{2n}}{\mu_2^n}, \qquad \beta_{2n+1}=\frac{\mu_{2n+1}^2}{\mu_2^{2n+1}}, \qquad n=1,2,\ldots, \tag{23}$$

where μ_i denotes the central moment of the impulse response of the i-th order.

The above characteristics give image of shape of impulse response function independently on scale and location parameters.

Assume, that several three-parameters conceptual models are at one's disposal. One can (provided the solution of adequate set of equations exists) make the three first moments for conceptual models considered and for linearized dynamic wave model respectively equal. Then the difference of values of moment characteristics can be a measure of quality of fitting the structure of given conceptual model to the structure of linearized hydrodynamical model and can constitute the criterion of choice. By determination of the loci corresponding to different conceptual models and to the linear dynamic wave model one can illustrate the applicability of conceptual models considered for different flow regimes. More extensive discussion of this problem can be found elsewhere [6]. These considerations help to explain the world-wide career of conceptual model in the form of serial cascade of linear reservoirs with or without linear channel. It has been proved, that for the Froude number equal to .54 such model and linear dynamic wave model (8) have identical form of the relationship between β_1 and β_2

$$\beta_2 = 1.5\beta_1 + 3. \tag{24}$$

It means, that matching the first three moments results in matching of the fourth moments of both system. It is wondering, that in spite of completely different formulae for impulse responses of both models, for the above value of Foude number also a very close fit of moment characteristics β_3 can be observed, with identical structure of the relationship

$$\beta_3 = \beta_1 (a\beta_1 + 10)^2 \tag{25}$$

where: $a=3$ for the model of cascade of linear reservoirs, $a=3.0556$ for the model of linear dynamic wave.

The characteristics β_2 and β_3 are simple measure of differences in responses of both models to input signals in the form of polynomial functions of time of fourth and fifth order respectively.

One can create also another criterion of choice of structure that does not impose the validity of linear dynamic wave model, which proves to be the model of limited applicability. When comparing Pearsonian plots corresponding to different conceptual model which gives the best fit to given conditions.

6. Concluding remarks

In the paper the method of moment matching and its application to approximation of conceptual models parameters (on the example of hydrologic model of open channel flow) on the basis of known physical system characteristics is discussed. The "quality" of simulation obtained by conceptual models with parameters determined by moment matching method in comparison ot the pattern behaviour of linear physical model is analyzed. The accuracy of simulation is referred to classes of input signals. The material presented forms the foundation to further reserach on extension of the abovementioned method by matching pseudomoments with weighting function of the type

$$\langle f(t) \cdot \varphi(t) \rangle^n = \int t^n f(t) \, \varphi(t) \, dt. \tag{26}$$

where $\varphi(t)$ is a weighting function of the form following the heavily damped nature of real flow systems.

Extension of this methodology to the case of system black box models is straight forward. System models in the identification stage (basing on the approximation of impulse response by means of orthogonal polynomials) can be treated similarly to conceptual models discussed in this paper. The coefficients of particular terms of orthogonal polynomials can be related to the physical system characteristics.

The authors are aware of the nonlinearity of the complete real physical system. The moment matching method can be easily generalized to cover the case of systems governed by nonlinear integral Volterra series equations. It would enable performing the approximation of parameters of conceptual models of the Volterra integral series structure upon the basis of development of physical model in the form of Volterra series.

The authors do hope, that the moment matching method and the analysis of Pearsonian moment characteristics will prove to be a tool easing the application of big variety of existing linear hydrologic models.

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O pewnej metodzie określania parametrów koncepcyjnych modeli przepływu w korycie otwartym

Omówiono podstawowe sposoby modelowania zjawisk hydrologicznych na przykładzie procesu przepływu w korycie otwartym. Postawiono problem syntezy odpowiedzi impulsowych dla zlewni nie kontrolowanych. Zbadano możliwość szacowania parametrów modeli koncepcyjnych na podstawie warunków równoważności systemów. Przedstawiono twierdzenia i spostrzeżenia uzasadniające metodę dopasowania momentów odpowiedzi impulsowych systemów. Stosując tę metodę określono wartości parametrów równoważnych modeli koncepcyjnych na podstawie fizycznych charakterystyk przepływu w korycie otwartym. Omówiono możliwość wyboru struktury modeli koncepcyjnych na podstawie analizy pearsonowskich charakterystyk momentowych.

О некотором методе определения параметров концептуальных моделей течения в открытом русле

Рассмотрены основные методы моделирования гидрологических явлений на примере процесса течения в открытом русле. Поставлена задача синтеза импульсной характеристики для неконтролируемых водосборных бассейнов. Исследована возможность оценки параметров концептуальных моделей на основе условий эквивалентности систем. Представлены теоремы и замечания обосновывающие метод согласования моментов импульсных характеристик систем. Используя этот метод определены значения параметров эквивалентных концептуальных моделей на основе физических характеристик течения в открытом русле. Рассмотрена возможность выбора структуры концептуальных моделей на основе анализа пирсоновых характеристик моментов.