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Finite dimensional approximation for stabilization of discrete time linear system

by

ANDRZEJ SOSNOWSKI

Technical Warsaw University
Institute of Mathematics
Warszawa, Poland

Stabilization problem for discrete time linear system is solved by approximation method. Sufficient conditions for stabilization by finite dimensional feedback operator are obtained. Under the relatively weak conditions the existence of a stabilizing operator is proved.

1. Introduction

A general discrete time infinite dimensional system is defined by the equation

$$x_{k+1}=Ax_k+Bu_k, \quad k=0, 1, \ldots \tag{1.1}$$

where $x_k \in X$, $u_k \in U$, X is a Banach space, U is a linear normed space and $A \in \mathscr{L}(X)$, $B \in \mathscr{L}(U, X)$ are linear bounded operators. The equation (1.1) can be used as a discrete model for many linear dynamic processes, including those with continuous time. The discretization procedure for continuous time linear system described by strongly continuous semigroup of operators is presented in [5], [6].

For stabilization problem of system (1.1) we use finite dimensional approximations. Approximation problem for infinite dimensional system was considered by Banks and Burns [2], Osborn [4], Trotter [7]. Banks and Burns used finite dimensional approximations to minimization problem for nonlinear time delay system with quadratic cost functional. They used semigroup model of time delay system. Similar approximation procedure was used by Trotter for control problem of distributed parameter systems. Osborn considered problem of eigenvalues and eigenvectors approximation of compact operators in Banach spaces. His results can be basically used in stabilization problem but spectral projections onto eigenspaces are very difficult to compute in concrete cases.

The approximation technique used in this paper to stabilization problem does not use spectral projection and the constrution of approximating systems is simple.

2. Finite dimensional approximations

We construct a sequence of approximating systems for discrete model (1.1).

DEFINITION 2.1. The sequence $\{X^N, P_N, A_N, U^N, Q_N, B_N\}_{N=1}^{\infty}$ is an approximating sequence iff the following hypotheses are satisfied:

($H1$) X^N is a finite dimensional subspace of X for each N.

($H2$) $P_N: X \to X^N$ are continuous projections onto X^N such that $\lim_{N\to\infty} \|P_N x - x\| = 0$ for all $x \in X$.

($H3$) $A_N: X^N \to X^N$ are linear operators such that $\lim_{N\to\infty} \|A_N P_N x - Ax\| = 0$ for all $x \in X$.

($H4$) U^N is a finite dimensional subspace of U for each N.

($H5$) $Q_N: U \to U^N$ are continuous projections onto U^N such that $\lim_{N\to\infty} \|Q_N u - u\| = 0$ for all $u \in U$.

($H6$) $B_N: U^N \to U^N$ are linear operators such that $\lim_{N\to\infty} \|B_N Q_N u - Bu\| = 0$ for all $u \in U$. ■

The definition of an approximating sequence given above is related to one employed for continuous linear systems by Banks and Burns in [2] and by Trotter in [7].

The approximating systems will be described for each $N = 1, 2, \ldots$ by equations

$$x_{k+1}^N = A_N x_k^N + B_N u_k^N, \qquad k = 0, 1, 2, \ldots \tag{2.1}$$

where $x_k^N \in X^N$, $u_k^N \in U^N$ and $\{X^N, P_N, A_N, U^N, Q_N, B_N\}_{N=1}^{\infty}$ is an approximating sequence.

3. Stabilization of infinite dimensional discrete time systems

Stabilization problem for a general discrete time linear system of the form (1.1) will be solved by computing finite dimensional feedback gain from approximating systems (2.1). First we define the notion of r-stability.

DEFINITION 3.1. A homogeneous discrete time system

$$x_{k+1} = Ax_k, \qquad k = 0, 1, 2, \ldots \tag{3.1}$$

where $x_k \in X$, $A \in \mathscr{L}(X)$, is r-stable iff for each $x_0 \in X$ solution $\{x_k\}_{k=0}^{\infty}$ is such that $\|x_k\| = o(r^k)$ as $k \to +\infty$. ■

LEMMA 3.1. *If the spectral radius of an operator A is smaller than r $(\rho(A) < r)$ then system* (3.1) *is r-stable.*

Proof.

$$\|x_n\| = \|A^n x_0\| \leqslant \|A^n\| \, \|x_0\|.$$

For each linear bounded operator A we have

$$\rho(A) = \lim_{n\to\infty} \sqrt[n]{\|A\|}.$$

Hence

$$\forall \varepsilon>0\ \exists n_0\ \forall n\geq n_0\ \|A^n\|<(\rho(A)+\varepsilon)^n.$$

Choosing ε such that $\rho(A)+\varepsilon<r$ we obtain for sufficiently large n

$$\|x_n\|\leq\|A^n\|\,\|x_0\|<(\rho(A)+\varepsilon)^n\,\|x_0\|=o(r^n). \qquad \blacksquare$$

Notice that in general r-stability does not imply that $\rho(A)<r$.

Example 3.1. Let $X=L^2(0,1;R)$, $(Ax)(t)\stackrel{\text{df}}{=}tx(t)$ and $r=1$. Since $\|x_n\|=\|A^n x_0\|\leq \leq\|x_0\|/\sqrt{2n+1}\to 0$ as $n\to+\infty$ then system (3.1) with such operator A is 1-stable. However, the spectrum of the operator A is $\sigma(A)=[0,1]$ hence $\rho(A)=1$. $\blacksquare$

LEMMA 3.2 (Anselone [1]). *If $A\in\mathscr{L}(X)$ and there exists a positive integer k such that $\|A^k\|<1$ then there exists $(I-A)^{-1}\in\mathscr{L}(X)$ and*

$$\|(I-A)^{-1}\|<\frac{\left\|\sum\limits_{i=0}^{k-1}A^i\right\|}{1-\|A^k\|}. \qquad \blacksquare$$

Now we are able to prove the main theorem on stabilization of discrete time linear systems.

THEOREM 3.1. *If for some N there exists an operator $F_N\in\mathscr{L}(X^N,U^N)$ such that $\rho(A_N+B_N F_N)<r$ and there exists a positive integer k such that for each $\lambda\in\Lambda_r=\{\lambda\colon|\lambda|\geq r\}$ the inequality $\|L_N^k(\lambda)\|<1$ holds, where*

$$L_N(\lambda)\stackrel{\text{df}}{=}\big(T_N-(A_N+B_N F_N)P_N\big)\big(\lambda I-(A_N+B_N F_N)P_N\big)^{-1},$$

$$T_N\stackrel{\text{df}}{=}A+BF_N P_N,$$

then system

$$x_{k+1}=T_N x_k,\quad k=0,1,2,\ldots \tag{3.2}$$

is r-stable.

Proof. Let $\lambda\in\Lambda_r$ be arbitrary chosen. We shall prove that λ is a regular value of the operator $(A_N+B_N F_N)P_N$. If λ is not a regular value, i.e., $\lambda\in\sigma((A_N+B_N F_N)P_N)$, then λ is an eigenvalue of this operator, since $(A_N+B_N F_N)P_N$ is a finite dimensional operator. Hence there exists a non zero vector $x_0\in X$ such that $[\lambda x_0-(A_N+B_N F_N)\times\times P_N x_0]=0$. But

$$\begin{aligned}\lambda x_0-(A_N+B_N F_N)P_N x_0&=\lambda\big(P_N x_0+(I-P_N)x_0\big)-(A_N+B_N F_N)P_N x_0\\&=P_N(\lambda P_N x_0)+\lambda(I-P_N)x_0-P_N(A_N+B_N F_N)P_N x_0\\&=P_N\big(\lambda I-(A_N+B_N F_N)\big)P_N x_0+(I-P_N)(\lambda x_0).\end{aligned}$$

Since $P_N\big(\lambda I-(A_N+B_N F_N)\big)P_N x_0\in X^N$ and $(I-P_N)(\lambda x_0)\in(X^N)$ where $X=X^N\oplus(X^N)'$ then $P_N\big(\lambda I-(A_N+B_N F_N)\big)P_N x_0=0$ and $(I-P_N)x_0=0$.

Hence $P_N x_0 \neq 0$ and $(\lambda I-(A_N+B_N F_N)) P_N x_0=0$ so λ is an eigenvalue of the operator $A_N+B_N F_N$ and it contradicts the assumption that $\rho(A_N+B_N F_N)<r$. Thus we obtained that $(\lambda I-(A_N+B_N F_N) P_N)^{-1} \in \mathscr{L}(X)$, $L_N(\lambda)$ is well defined and $L_N(\lambda) \in \mathscr{L}(X)$.

Since $\|L^k(\lambda)\|<1$ then by Lemma 3.2 there exists $(I-L_N(\lambda))^{-1} \in \mathscr{L}(X)$. Simple computation shows that

$$(\lambda I-T_N)^{-1}=(\lambda I-(A_N+B_N F_N) P_N)^{-1}(I-L_N(\lambda))^{-1}$$

so $(\lambda I-T_N)^{-1} \in \mathscr{L}(X)$. Hence λ is a regular value of the operator T_N. Since λ was arbitrary in $\Lambda_r=\{\lambda: |\lambda| \geqslant r\}$ then $\rho(T_N)<r$ (spectrum of a linear bounded operator is a closed set) and by Lemma 3.1 we obtain that system (3.2) is r-stable. ■

The Theorem 3.1 shows that finite dimensional approximating systems (2.1) can be used to compute stabilizing feedback operator for system (2.1). In the next section we prove under additional assumptions that there exist operators stabilizing approximating systems and satisfying assumptions of Theorem 3.1.

4. Sufficient conditions for the existence of a stabilizing finite dimensional feedback operator

We shall formulate sufficient conditions for existence of operator F_N such that $\rho(A_N+B_N F_N)<r$. The following notions will be used.

Definition 4.1. System (1.1) is r-stabilizable iff for each $x_0 \in X$ there is a sequence of controls $\{u_k\}_{k=0}^{\infty}$ such that $\|u_n\|=o(r^n)$ and $\|x_n\|=o(r^n)$ as $n \to +\infty$. ■

The notion of r-stabilizability and relations between this property and other controllability properties were discused in [3].

Definition 4.2. Operator $T \in \mathscr{L}(X, Y)$ is compact iff the set $T\mathscr{B}_1=\{y \in Y: y=Tx, \|x\| \leqslant 1\}$ is relatively compact, i.e. the set $cl\ T\mathscr{B}_1$ is compact. ■

Definition 4.3. The set $\mathscr{K} \subset \mathscr{L}(X, Y)$ is collectively compact iff the set $K\mathscr{B}_1 =\{Kx: K \in \mathscr{K} \subset \mathscr{L}(X, Y), \|x\| \leqslant 1\}$ is relatively compact. The sequence of operators is collectively compact iff the set of the elements of this sequence is collectively compact. ■

Properties of collectively compact sets and sequences can be found in Anselone [1]. We mention here only one theorem.

Theorem 4.1 (Anselone [1]). *Let $T, T_n \in \mathscr{L}(X)$, $n=1, 2, \ldots$. Assume that for each $x \in X$, $\lim_{n \to \infty} \|T_n x-Tx\|=0$ and the sequence $\{T_n-T\}_{n=1}^{\infty}$ is collectively compact. Then for each open set $\Omega \supset \sigma(T)$ there exists N such that $\Omega \supset \sigma(T_n)$ for each $n \geqslant N$.* ■

Sufficient conditions for r-stabilizability for approximate systems (2.1) can be formulated in the following theorem.

THEOREM 4.2. *If the system* (1.1) *is r-stabilizable, A and B are compact operators, sequences* $\{A_N P_N\}_{N=1}^{\infty}$, $\{B_N Q_N\}_{N=1}^{\infty}$ *are collectively compact then there exists positive integer* N_0 *such that for each* $N \geq N_0$ *system* (2.1) *is r-stabilizable. The integer* N_0 *depends of approximating sequence*

$$\{X^N, P_N, A_N, U^N, Q_N, B_N\}_{N=1}^{\infty}.$$

Proof. Since the operator A is compact then there exists an operator $F \in \mathscr{L}(X, U)$ such that $\rho(A+BF)<r$, (see [3]), hence $\sigma(A+BF) \subset D_r = \{\lambda : |\lambda| < r\}$. Let $G \overset{\text{df}}{=} A+BF$ and $F_N : X^N \to U^N$, $N=1, 2, \ldots$, $F_N \overset{\text{df}}{=} Q_N F|_{X^N}$, and $G_N \overset{\text{df}}{=} (A_N + B_N F_N) P_N$, $N=1, 2, \ldots$ Then $G_N = A_N P_N + B_N Q_N FP_N$. By Banach-Steinhaus theorem and condition ($H2$) of Definition 2.1 the sequence $\{P_N\}_{N=1}^{\infty}$ is bounded and the sequence $\{FP_N\}_{N=1}^{\infty}$ also is. Hence the sequence of operators $\{B_N Q_N FP_N\}_{N=1}^{\infty}$ is collectively compact (see Anselone [1]) and the sequence $\{G_N\}_{N=1}^{\infty}$ is collectively as compact a sum of the two collectively compact sequences. Similarly the opreator G is compact and hence the sequence $\{G_N - G\}_{N=1}^{\infty}$ is collectively compact. For each $x \in X$ we have

$$\lim_{N\to\infty} \|G_n x - Gx\| = \lim_{N\to\infty} \|(A_N P_N x - Ax) + (B_n Q_N FP_N x - BFx)\|$$

$$= \lim_{N\to\infty} \|(A_N P_N x - Ax) + (B_N Q_N FP_N x - B_N Q_N Fx) + (B_N QFx - BFx)\|$$

$$\leq \lim_{N\to\infty} (\|A_N P_N x - Ax\| + \|B_N Q_N F\| \, \|P_N x - x\| + \|B_N Q_N (Fx) - B(Fx)\|) = 0,$$

by ($H2$), ($H3$), ($H6$) and boundedness of the sequence $\{B_N Q_N F\}_{N=1}^{\infty}$ as a consequence of ($H6$) and Banach-Steinhaus theorem. Hence the sequence $\{G_N\}_{N=1}^{\infty}$ is pointwise convergent to G and by Theorem 4.1 there exists N_0 such that for each $N \geq N_0$, $\sigma((A_N + B_N F_N) P_N) \subset D_r$. Similarly as in the proof of Theorem 3.1 we can show that $\sigma((A_N + B_N F_N) P_N) = \sigma(A_N + B_N F_N)$ and hence the system (2.1) is r-stabilizable for $N \geq N_0$. ∎

REMARK 4.1. If for spaces X^N and operators P_N conditions ($H1$) and ($H2$) of Definition 2.1 hold an A is a compact operator then the sequence $\{A_N P_N\}_{N=1}^{\infty}$, where $A_N P_N \overset{\text{df}}{=} P_N A P_N$, is collectively compact.

Proof. By Banach-Steinhaus theorem and condition ($H2$) we obtain that the sequence $\{P_N\}_{N=1}^{\infty}$ is bounded. Let $\{x_N\}_{N=1}^{\infty}$ be an arbitrary sequence of elements from the unit ball $\mathscr{B}_s = \{x : \|x\| < 1\} \subset X$. Operator A is compact, hence the set of elements of the sequence $\{y_N\}_{N=1}^{\infty} = \{Ax_N\}_{N=1}^{\infty}$ is relatively compact and there exists a convergent subsequence $\{y_{N_i}\}_{i=1}^{\infty}$. Let $y \overset{\text{df}}{=} \lim_{i\to\infty} y_{N_i}$. We shall prove that the sequence $\{P_{N_i} y_{N_i}\}_{i=1}^{\infty}$ converges to y.

In fact,

$$\|P_{N_i} y_{N_i} - y\| = \|P_{N_i} y_{N_i} - P_{N_i} y + P_{N_i} y - y\| \leq$$

$$\leq \|P_{N_i} y_{N_i} - P_{N_i} y\| + \|P_{N_i} y - y\| \leq \|P_{N_i}\| \, \|y_{N_i} - y\| +$$

$$+ \|P_{N_i} y - y\| \to 0 \text{ as } i \to +\infty.$$

Hence the set of elements of the sequence $\{P_N A x_N\}_{N=1}^{\infty}$ is relatively compact. Since this is true for an arbitrary sequence $\{x_N\}_{N=1}^{\infty} \subset \mathscr{B}_1$ then the sequence $\{P_N A\}_{N=1}^{\infty}$ is collectively compact and hence the sequence $\{P_N A P_N\}_{N=1}^{\infty}$ is also collectively compact (see Anselone [1]). ∎

REMARK 4.2. Let U be a Banach space. If for spaces U^N and operators P_N and Q_N conditions ($H4$), ($H2$), ($H5$) of Definition 2.1 hold and B is a compact operator then the sequence $\{B_N Q_N\}_{N=1}^{\infty}$ where $B_N Q_N \overset{\mathrm{df}}{=} P_N B Q_N$, is collectively compact.

Proof. The same as for Remark 4.1. ∎

If operators A and B are compact then by Theorem 4.2 and Remarks 4.1 and 4.2 we obtain the construction procedure for approximating sequence such that approximating systems (2.1) are r-stabilizable for sufficiently large N. Thus we can use this approximations to comuputation of stabilizing feedback operators F_N.

Now we prove that for sufficiently large N it is possible to choose feedback operators F_N such that the closed loop system (3.2) is r-stable. We will use the following theorem.

THEOREM 4.3. *If the sequence of operators $L_N(\lambda) \to 0$ as $N \to +\infty$ uniformly with respect to $\lambda \in \Lambda_r = \{\lambda : |\lambda| \geqslant r\}$ and the set $\{L_N(\lambda) : N = 1, 2, \ldots, |\lambda| \geqslant r\}$ is collectively compact then $\|L_N^2(\lambda)\| \to 0$ uniformly with respect to $\lambda \in \Lambda_r$.*

Proof. We will use the following property of a pointwise convergent sequence. A pointwise convergent sequence in Banach space X is uniformly convergent in each totally bounded subset of X. We say that the set $\mathscr{S}$ is totally bounded iff for each $\varepsilon > 0$ there exists a finite subset $\mathscr{S}_\varepsilon \subset X$ such that for each $x \in \mathscr{S}$ there exists $x_\varepsilon \in \mathscr{S}_\varepsilon$ such that $\|x_\varepsilon - x\| < \varepsilon$. In Banach space the set is totally bounded iff it is relatively bounded [1].

The condition that $L_N(\lambda) \to 0$ uniformly with respect to $\lambda \in \Lambda_r$ can be written as

$$\forall \varepsilon > 0\ \exists x \in X\ \exists N_0\ \forall \lambda \in \Lambda_r\ \forall N \geqslant N_0\ \|L_N(\lambda)\, x\| < \varepsilon.$$

This convergence is uniform with respect to x from any totally bounded set. Since the set $\{L_N(\lambda) : N = 1, 2, \ldots, |\lambda| \geqslant r\}$ is collectively compact then $\mathscr{S} \overset{\mathrm{df}}{=} \{L_N(\lambda)\, x : N = 1, 2, \ldots, |\lambda| \geqslant r, \|x\| < 1\}$ is relatively compact and therefore totally bounded. Hence

$$\forall \varepsilon > 0\ \exists N_0\ \forall x \in \mathscr{S}\ \forall \lambda \in \Lambda_r\ \forall N \geqslant N_0\ \|L_N(\lambda)\, x\| < \varepsilon$$

and this is equivalent to

$$\forall \varepsilon > 0\ \exists N_0\ \forall x \in \{x : \|x\| < 1\}\ \forall \lambda \in \Lambda_r\ \forall N \geqslant N_0\ \|L_N^2(\lambda)\, x\| < \varepsilon.$$

We see that the sequence $\{L_N^2(\lambda)\, x\}_{N=1}^{\infty}$ converges to zero uniformly with respect to $\lambda \in \Lambda_r$ and $x \in \{x : \|x\| < 1\}$. Hence $\|L^2(\lambda)\| \to 0$ uniformly with respect to $\lambda \in \Lambda_r$. ∎

Let $R(T)$ denote the set of regular values of operator $T \in \mathscr{L}(X)$, $R(T) = \{\lambda \in C : \exists\, (\lambda I - T)^{-1} \in \mathscr{L}(X)\}$ and let $\bar{R}(T)$ denote the extended set of regular values, i.e. the smallest compact set in $\bar{C} \overset{\mathrm{df}}{=} C \cup \{\infty\}$ in which $R(T)$ is contained. The following property holds.

LEMMA 4.1 (Anselone [1]). *Let $T, T_n \in \mathscr{L}(X)$, $n=1, 2, \ldots, X$ is a Banach space. If $T_n \to T$ pointwise and $\{T_n - T\}_{n=1}^{\infty}$ is collectively compact then for each closed subset $\Lambda \subset R(T)$ there exists N such that the set $\{(\lambda I - T_n)^{-1} : \lambda \in \Lambda, n \geqslant N\}$ is bounded.* ■

THEOREM 4.4. *If the system (1.1) is r-stabilizable, operators A and B are compact, $\{X^N, P_N, A_N, U^N, Q_N, B_N\}_{N=1}^{\infty}$ is an approximating sequence such that $\{A_N P_N\}_{N=1}^{\infty}$ and $\{B_N Q_N\}_{N=1}^{\infty}$ are collectively compact then there exists a finite dimensional feedback operator F_{N_0} which stabilizes approximating system (2.1) such that system (3.2) with the operator $T_N = A + BF_{N_0} P_{N_0}$ is r-stable.*

Proof. We shall construct the sequence of operators $\{F_N\}_{N=1}^{\infty}$ such that the assumptions of Theorem 3.1 hold for sufficiently large N. Since system (1.1) is r-stabilizable there exists an operator $F \in \mathscr{L}(X, U)$ such that $\rho(A+BF) < r$. Let $F_N \stackrel{\text{df}}{=} Q_N F|_{X^N}$. By Theorem 4.2 there exists N_1 such that for each $N \geqslant N_1$, $\rho(A_N + B_N F_N) < r$. Hence the operators F_N r-stabilize systems (2.1).

Let $G_N \stackrel{\text{df}}{=} (A_N + B_N F_N) P_N$, $N = 1, 2, \ldots$, $G \stackrel{\text{df}}{=} A + BF$. In the proof of Theorem 4.2 it was shown that $G_n \to G$ and the set $\{G_N - G\}_{N=1}^{\infty}$ is collectively compact. So assumptions of Lemma 4.1 are satisfied and the set $\{(\lambda I - G_N)^{-1} : |\lambda| \geqslant r, N \geqslant N_2\}$ is bounded for some $N_2 \geqslant N_1$. Let $L_N(\lambda)$ be the operator defined in Theorem 3.1. For $N \geqslant N_2$ we obtain

$$L_N(\lambda) = (T_N - G_N)(\lambda I - G_N)^{-1} = \big((A - A_N P_N) + (B - B_N Q_N) F_N P_N\big)(\lambda I - G_N)^{-1} \to 0$$

uniformly with respect to $\lambda \in \{\lambda : |\lambda| \geqslant r\}$ as $N \to +\infty$, because $A_N P_N \to A$, $B_N Q_N \to B$ and the sets $\{F_N P_N : N \geqslant N_2\}$, $\{(\lambda I - G_N)^{-1} : |\lambda| \geqslant r, N \geqslant N_2\}$ are bounded. Since the operators A and B are compact sequences

$$\{A_N P_N\}_{N=1}^{\infty} \text{ and } \{B_N Q_N\}_{N=1}^{\infty}$$

are collectively compact and the sets $\{F_N P_N : N \geqslant N_2\}$ and $\{(\lambda I - G_N)^{-1} : |\lambda| \geqslant r, N \geqslant N_2\}$ are bounded then the set $\{L_N(\lambda) : N \geqslant N_2, |\lambda| \geqslant r\}$ is collectively compact (see [1]). By Theorem 4.3 we obtain that there exists $N_0 \geqslant N_2$ such that $\|L_{N_0}^2(\lambda)\| < 1$ for each $\lambda : |\lambda| \geqslant r$. So assumptions of Theorem 3.1 hold and system (3.2) for $N = N_0$ is r-stable. ■

Using Remarks 4.1 and 4.2 and Theorem 4.4 we obtain

COROLLARY 4.1. *If system (1.1) is r-stabilizable, operators A and B are compact, X and U are Banach spaces, the sequence of approximations satisfy conditions (H1), (H2), (H4), (H5) and operators $A_N P_N$ and $B_N Q_N$ are defined by*

$$A_N P_N \stackrel{\text{df}}{=} P_N A P_N, \qquad B_N Q_N \stackrel{\text{df}}{=} P_N B Q_N, \qquad N = 1, 2, \ldots$$

then there exists a finite dimensional feedback operator F_{N_0} which stabilites, for $N = N_0$, approximating system (2.1) such that system (3.2) with operator

$$T_N = A + BF_{N_0} P_{N_0}$$

is r-stable. ■

5. Concluding remarks

Application of finite dimensional approximation theory to stabilization problem of discrete time system has been presented. Approximation used in solution of stabilization problem require relatively weak assumptions. This makes the method presented much more useful than spectral approximation methods of stabilization.

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Zastosowanie aproksymacji skończenie wymiarowych do stabilizacji dyskretnych układów liniowych

Problem stabilizacji nieskończenie wymiarowego dyskretnego układu liniowego został rozwiązany przy wykorzystaniu metody aproksymacji. Do konstrukcji regulatora stabilizującego układ nieskończenie wymiarowy zostały użyte operatory sprzężenia zwrotnego stabilizujące skończenie wymiarowe układy aproksymujące. Przy stosunkowo słabych założeniach odnośnie skończenie wymiarowych aproksymacji uzyskano warunki dostateczne zapewniające stabilność układu nieskończenie wymiarowego ze skończenie wymiarowym operatorem w pętli sprzężenia zwrotnego. Dla pewnej klasy układów, często spotykanych w zagadnieniach praktycznych, wykazano istnienie skończenie wymiarowego operatora stabilizującego.

Примение конечномерных аппроксимаций к стабилизации дискретных линейных систем

Задача стабилизации бесконечномерной дискретной линейной системы была решена при использовании метода аппроксимации. Для разработки регулятора стабилизирующего бесконечномерную систему использовались операторы обратной связи, стабилизирующие конечномерные аппроксимирующие системы. При относительно слабых предпосылках о конечномерных аппроксимациях получены достаточные условия для обеспечения стабильности бесконечномерной системы с конечномерным оператором в цепи обратной связи. Для некоторого класса систем, часто встречаемых в практике, доказано существование конечномерного стабилизирующего оператора.