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Binary relations, flou relations and application

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Binary relations on a set A are characterized by means of the families of their classes (semiblocks). Compatible binary relations are also characterized and corresponding mappings described. After these purely algebraic considerations, a class of flou relations are reduced to a class of binary relations and vice versa. Certain compatible flou relations are of importance in rounding mappings of computer arithmetic.

1. Introduction

There is a great need for tools to handle formally ill-defined objects e.g. in computer arithmetic, in social research, in decision processes e.t.c. Fuzzy sets and flou sets (also called partial sets) are formal concepts directed for solving some problems concerning the handling of ill-defined objects. Another approach are tolerance relations, their generalizations and related mappings. The purpose of this paper is to combine these two lines. We start off with considering binary relations and show thereafter that these relations serve a simple way to control some flou binary relations and related mappings. An application is the screen and related rounding mapping studied in computer arithmetic.

For flou sets the reader is referred to the monograph [5] of Negoita and Ralescu as well as Klaua's paper [3] and references therein. Screens and rounding mappings are considered by Kulisch e.g. in [4]. A simple view in problems of computer arithmetics is given by Ratschek in [7] and Ratschek's article [6] is also of interest for getting a good picture about problems of interval arithmetics. As a model for constructing binary relations and their classes we have used Chajda's paper [1] on blocks (classes) of binary relations as well as the paper [2] of Chajda, Niederle and Zelinka, where one can find models of proofs given in this paper.

2. Binary relations

Let R be a binary relation on a non-empty set A and M_s and M_t two subsets of A. A pair $(M_s, M_t)\subset A\times A$ is called a semiblock of R whenever

$$(M_s, M_t)\neq(\emptyset, \emptyset); \tag{1}$$

$$(M_s, M_t)\subset R; \tag{2}$$

$$\text{if } (M_s, M_t)\subset(N_s, N_t) \text{ and } (N_s, N_t)\subset R, \quad \text{then } (M_s, M_t)=(N_s, N_t). \tag{3}$$

We have above substituted $M_s\times M_t$ by the notation (M_s, M_t).

Let $\mathcal{M}=\{(M_{is}, M_{it}) \mid i\in I\}$, where I is a set of indices, a non-empty family of subsets of (A, A). The family $\mathcal{M}$ is called seminormal, if

$$\text{if } k\in I \text{ and } J\subset I, \text{ then } (M_{ks}, M_{kt})\subset\bigcup\{(M_{js}, M_{jt}) \mid j\in J\}$$
$$\text{implies that } \bigcap\{(M_{js}, M_{jt}) \mid j\in J\}\subset(M_{ks}, M_{kt}); \tag{4}$$

if $(N_s, N_t)\subset(A, A)$ and (N_s, N_t) is not contained in any pair (M_{is}, M_{it}) of the family $\mathcal{M}$ then there is a pair (a, b) of elements such that $(a, b)\in(N_s, N_t)$ and (a, b) is not contained in any pair of $\mathcal{M}$. (5)

As shown in [2], when $\mathcal{M}$ is seminormal, one can see that $(M_{is}, M_{it})\not\subset(M_{js}, M_{jt})$ for $i, j\in I$, $i\neq j$. Similarly, $(\emptyset, \emptyset)\notin\mathcal{M}$. Theorem 1 of [2] suggests now to prove

THEOREM 1. *Let A be a non-empty set. Then there exists a one-to-one correspondence between binary relations R on A and seminormal families $\mathcal{M}$ such that, if R is a relation on A and $\mathcal{M}_R$ is the seminormal family corresponding to R, then $(a, b)\in R$ if and only if there is a $(M_{is}, M_{it})\in\mathcal{M}_R$ such that $(a, b)\in(M_{is}, M_{it})$.*

Proof. Let R be a binary relation on A and let $\mathcal{L}_R=\{(L_s, L_t) \mid L_s, L_t\subset A$ and $(a, b)\in R$ whenever $(a, b)\in(L_s, L_t)\}$. A new family $\mathcal{M}_R$ is constructed as follows: $\mathcal{M}_R$ contains all pairs of $\mathcal{L}_R$ which are maximal with respect to the set inclusion. The family $\mathcal{M}_R$ such obtained is denoted as $\mathcal{M}_R=\{(M_{is}, M_{it}) \mid i\in I\}$. In the following we show that $\mathcal{M}_R$ is seminormal by proving the validity of (4) and (5).

(4): Let $k\in I$, $J\subset I$ and $(M_{ks}, M_{kt})\subset\bigcup\{(M_{js}, M_{jt}) \mid j\in J\}$.
On the other hand, let $(P_s, P_t)=\bigcup\{(M_{js}, M_{jt}) \mid j\in J\}$ and we assume that $(P_s, P_t)\not\subset(M_{ks}, M_{kt})$. Then there exists a pair (a, b) of elements of A such that $(a, b)\in(P_s, P_t)\setminus(M_{ks}, M_{kt})$. Let $(z, w)\in(M_{ks}, M_{kt})$ and so $(z, w)\in\bigcup\{(M_{js}, M_{jt}) \mid j\in J\}$ whence there is an index $n\in J$ such that $(z, w)\in(M_{ns}, M_{nt})$. Because $(a, b)\in(P_s, P_t)=\bigcap\{(M_{js}, M_{jt}) \mid j\in J\}$, also $(a, b)\in(M_{ns}, M_{nt})$. But then $(a, w), (z, b)\in R$, too, and because (z, w) was an arbitrary pair of elements from (M_{ks}, M_{kt}), $(a, w)\in R$ for every $w\in M_{kt}$ and $(z, b)\in R$ for every $z\in M_{ks}$. In particular, this implies that $(M_{ks}, M_{kt})\bigcup(a, b)\in\mathcal{L}_R$ and so (M_{ks}, M_{kt}) were not a maximal pair in $\mathcal{L}_R$, which is a contradiction. Hence $\bigcap\{(M_{js}, M_{jt}) \mid j\in J\}\subset(M_{ks}, M_{kt})$ and the validity of (4) follows.

(5): If (N_s, N_t) of (A, A) is not contained in any pair of $\mathcal{M}_R$, then $(N_s. N_t) \notin \mathcal{L}_R$, and thus $(a, b) \in (N_s, N_t)$ such that $(a, b) \notin R$. Hence (a, b) is not contained in any pair of $\mathcal{M}_R$, and so (5) holds for $\mathcal{M}_R$.

Clearly, when $\mathcal{M}$ is a seminormal family, it determines a binary relation R on A as follows:

$$(a, b) \in R \Leftrightarrow \text{there is an index } i \in I \text{ in } \mathcal{M} \text{ such that } (a, b) \in (M_{is}, M_{it}).$$

Let $\mathcal{M}$ be a given seminormal family, R the relation determined by $\mathcal{M}$, and $\mathcal{M}_R$ the family determined by R above. We will now show that $\mathcal{M} = \mathcal{M}_R$ from which the asserted one-to-one correspondence of seminormal families and binary relations R follows. This will be done by proving that every pair of $\mathcal{M}$ is a maximal pair of $\mathcal{L}_R$ (i.e. a pair of $\mathcal{M}_R$), and conversely. Let us assume that $(M_{ns}, M_{nt}) \in \mathcal{M}$ for some $n \in I$ is not maximal in $\mathcal{L}_R$, i.e. there is in $\mathcal{L}_R$ a pair (L_s, L_t) containing properly (M_{ns}, M_{nt}), and let $(a, b) \in (L_s, L_t) \setminus (M_{ns}, M_{nt})$. Because $(L_s, L_t) \in \mathcal{L}_R$, then $(a, w), (z, b) \in R$ for every $(z, w) \in (M_{ns}, M_{nt})$. This means that for every $(z, w) \in \in (M_{ns}, M_{nt})$ there is an index $i(zw)$ in $\mathcal{M}$ such that $(a, w), (z, b) \in (M_{i(zw)s}, M_{i(zw)t})$. Then, in particular, $(M_{ns}, M_{nt}) \in \bigcup \{(M_{i(zw)s}, M_{i(zw)t} \mid (z, w) \in (M_{ns}, M_{nt})\}$. Because $\mathcal{M}$ is seminormal, then $\bigcap \{(M_{i(zw)s}, M_{i(zw)t} \mid (z, w) \in (M_{ns}, M_{nt})\} \subset (M_{ns}, M_{nt})$. But $(a, b) \in (M_{i(zw)s}, M_{i(zw)t})$ for every $(z, w) \in (M_{ns}, M_{nt})$, whence (a, b) belongs to the intersection above and thus to (M_{ns}, M_{nt}), too. This is a contradiction, and hence every pair of $\mathcal{M}$ is a maximal pair of $\mathcal{L}_R$, i.e. $\mathcal{M} \subset \mathcal{M}_R$. Let us assume now that there were $(L_s, L_t) \in \mathcal{M}_R \setminus \mathcal{M}$. Because $\mathcal{M} \subset \mathcal{M}_R$, then (L_s, L_t) is not contained in anyone of the pairs of $\mathcal{M}$, and thus there is a pair (a, b) of elements of A such that $(a, b) \in (L_s, L_t)$ and no-one of the pairs of $\mathcal{M}$ contains (a, b). Thus $(a, b) \notin R$, whence $(L_s, L_t) \in \mathcal{L}_R$ and so also $(L_s, L_t) \notin \mathcal{M}_R$, which is a contradiction. Thus $\mathcal{M}_R \subset \mathcal{M}$, too, whence $\mathcal{M} = \mathcal{M}_R$. This completes the proof., ∎

If R is symmetric, then $M_{is} = M_{it}$ for some $i \in I$ and then M_{is} is a block (see Chajda [1]) of R. If R is reflexive and symmetric, then $\mathcal{M}$ cotains a τ-covering of A (see Chajda, Niederle and Zelinka [2]).

Let $\mathcal{A} = \langle A, F \rangle$ be an algebra with the carrier set A and with the set F of fundamental operations, where the arity of every $f \in F$ is $n \geq 1$. We say that R is compatible with $\mathcal{A}$, if for every $f \in F$ and for every n pairs $(a_j, b_j) \in R$, $j = 1, \ldots, n$, also $(f(a_1, \ldots, a_n), f(b_1, \ldots, b_n)) \in R$.

As expected, the following theorem is a generalization of Theorem 3 in [2]. The generalization contains essential new aspects so that we will also present the proof which follows the general lines of the corresponding proof in [2].

THEOREM 2. *Let $\mathcal{A} = \langle A, F \rangle$ be an algebra, R a binary relation on A and $\mathcal{M}_R$ the corresponding seminormal family. R is compatible with $\mathcal{A}$ if and only if there are two algebras $\mathcal{B}_1 = \langle B_1, G \rangle$ and $\mathcal{B}_2 = \langle B_2, G \rangle$ as follows*

(i) *there is a one-to-one correspondence $\varphi : F \dashrightarrow G$ such that for any positive n and for every $f \in F$ the operation φf is n-ary if and only if f is n-ary;*

(ii) *there is a one-to-one correspondence $\chi_1 : \{M_{is} | i \in I\} \dashrightarrow B_1$ such that for every n-ary operation $f \in F$ and for any $n+1$ elments $M_{0s}, M_{1s}, M_{2s}, \ldots, M_{ns}$ from $\mathcal{M}_R$*

the equality $\varphi f(\chi_1(M_{1s}), \ldots, \chi_1(M_{ns})) = \chi_1(M_{0s})$ *implies that for any* n *elements* $a_1, \ldots, a_n \in A$ *with* $a_j \in M_{js}$*, where* $j=1, \ldots, n$*, the element* $f(a_1, \ldots, a_n) \in M_{0s}$*;*

(iii) *there is a one-to-one correspondence* $\chi_2 : \{M_{it} | i \in I\} \dashrightarrow B_2$ *such that for every* n*-ary operation* $f \in F$ *and for any* $n+1$ *elements* $M_{0t}, M_{1t}, \ldots, M_{nt}$ *from* $\mathcal{M}_R$ *the equality* $\varphi f(\chi_2(M_{1t}), \ldots, \chi_2(M_{nt})) = \chi_2(M_{0t})$ *implies that for any* n *elements* $b_1, \ldots, \ldots, b_n \in A$ *with* $b_j \in M_{jt}$*, where* $j=1, \ldots, n$*, the element* $f(b_1, \ldots, b_n) \in M_{0t}$*;*

(iv) *the mapping* $\lambda : B_1 \dashrightarrow B_2$*, where* $\lambda(\chi_1(M_{is})) = \chi_2(M_{it})$ *for every* $i \in I$*, is an isomorphism between* $\mathcal{B}_1$ *and* $\mathcal{B}_2$*.*

Proof. Let R be compatible with $\mathcal{A}$ and $\mathcal{M}_R$ the seminormal family corresponding to R. Let $M_{1s}, \ldots, M_{ns}$ be n elements from $\mathcal{M}_R$, a_j, $c_j \in M_{js}$, $j=1, \ldots, n$, and $f \in F$ an n-ary operation. Then there are n sets M_{jt} from $\mathcal{M}_R$ with elements $d_j \in M_{jt}$, $j=1, \ldots, n$, such that $(a_j, d_j), (c_j, d_j) \in R$, $j=1, \ldots, n$. According to the compatibility of R, $(f(a_1, \ldots, a_n), f(d_1, \ldots, d_n))$, $(f(c_1, \ldots, c_n), f(d_1, \ldots, d_n)) \in R$. Because the elements a_j and c_j were choosen arbitrarily from M_{js}, the result above shows that all elements $f(x_1, \ldots, x_n)$, where $x_j \in M_{js}$, have the property $(f(x_1, \ldots, x_n), f(d_1, \ldots, d_n)) \in R$, and thus are contained in a set M_{0s} from $\mathcal{M}_R$. Hence we may put $\{M_{is} | i \in I\} = B_1$ and the mapping χ_1 is the identical mapping in $\{M_{is} | i \in I\}$. Now the operation φf is defined for any $f \in F$ so that $\varphi f(\chi_1(M_{1s}), \ldots, \chi_1(M_{ns})) = \chi_1(M_{0s})$ if and only if $f(a_1, \ldots, a_n) \in M_{0s}$ when $a_j \in M_{js}$ and $j=1, \ldots, n$. This proves (i) and (ii); (iii) is proved analogously.

According to the one-to-one correspondence between the families $\{M_{is} | i \in I\}$ and $\{M_{it} | i \in I\}$ determined by the family $\mathcal{M}_R = \{(M_{is}, M_{it}) | i \in I\}$ and according to the identity mappings $\chi_1 : \{M_{is} | i \in I\} \dashrightarrow B_1$ and $\chi_2 : \{M_{it} | i \in I\} \dashrightarrow B_2$, the mapping $\lambda : \mathcal{B}_1 \dashrightarrow \mathcal{B}_2$ is a one-to-one correspondence. The isomorphism property of λ follows now from the compatibility of R with $\mathcal{A}$.

Conversely, let (i)–(iv) hold for R, $(d_j, e_j) \in R$ for $j=1, \ldots, n$, and let $f \in F$ be an n-ary operation on $\mathcal{A}$. Because $(d_j, e_j) \in R$, $(d_j, e_j) \in (M_{js}, M_{jt})$ for some $j \in I$. Let M_{0s} be the set such that $\varphi f(\chi_1(M_{1s}), \ldots, \chi_1(M_{ns})) = \chi_1(M_{0s})$. Now, according to the isomorphism $\lambda : \mathcal{B}_1 \dashrightarrow \mathcal{B}_2$, the set M_{0t} from $(M_{0s}, M_{0t}) \in \mathcal{M}_R$ has the property: $\varphi f(\chi_2(M_{1t}), \ldots, \chi_2(M_{nt})) = \chi_2(M_{0t})$. But then $(f(d_1, \ldots, d_n), f(e_1, \ldots, e_n)) \in (M_{0s}, M_{0t})$, whence $(f(d_1, \ldots, d_n), f(e_1, \ldots, e_n)) \in R$, which shows the compatibility of R with $\mathcal{A}$. This completes the proof. ∎

There is a pair of relations, that can be derived from R, having some interest: the symmetric derivations $[R_{sd}, R_{td}]$ of R. These relations are obtained from R as follows:

$(a, b) \in R_{sd} \Leftrightarrow$ there is an index $i \in I$ such that $a, b \in M_{is}$.

$(a, b) \in R_{td} \Leftrightarrow$ there is an index $i \in I$ such that $a, b \in M_{it}$.

According to the definitions of R_{sd} and R_{td}, they are symmetric relations on A. In the following theorem we consider the compatibility of the couple $[R_{sd}, R_{td}]$; a partial converse is given in Theorem 5.

THEOREM 3. *Let* $\mathcal{A} = \langle A, F \rangle$ *be an algebra and* R *a binary relation on* A*. If* R *is compatible with* $\mathcal{A}$*, then* R_{sd} *and* R_{td} *are compatible with* $\mathcal{A}$*, too.*

Proof. We will show the compatibility of R_{sd} with $\mathscr{A}$; the compatibility of R_{td} is proved analogously.

Let $f \in F$ be an n-ary operation and $(a_j, b_j) \in R_{sd}$, $j=1, \dots, n$. This means that for every j there is a set M_{js} such that $a_j, b_j \in M_{js}$. We can now choose freely an element c_j from M_{jt}, where $(M_{js}, M_{jt}) \in \mathscr{M}_R$ and $j=1, \dots, n$. Thus (a_j, c_j), $(b_j, c_j) \in R$ for every j, and according to the compatibility of R, also $(f(a_1, \dots, a_n), f(c_1, \dots, c_n))$, $(f(b_1, \dots, b_n), f(c_1, \dots, c_n)) \in R$. But then $\mathscr{M}_R$ contains a pair (M_{0s}, M_{0t}) such that $f(c_1, \dots, c_n) \in M_{0t}$ and $f(a_1, \dots, a_n), f(b_1, \dots, b_n) \in M_{0s}$. But then also $(f(a_1, \dots, a_n)$, $f(b_1, \dots, b_n) \in R_{sd}$, from which the compatibility of R_{sd} follows. ■

3. Flou relations and application

Let A be a reference set and $G, H \subset A$ its subsets. A couple $[G, H]$, where $G \subset H$ is called (see [5]) a flou set (or a partial set, see [3]). Accordingly, we call a couple $[R_1, R_2]$ of binary relations on a set A flou binary relation on A whenever $R_1 \subset R_2$. A flou relation $[R_1, R_2]$ is called symmetric if R_1 and R_2 are symmetric on A. A class of symmetric flou relations can be controlled by means of binary relations on A as shown below.

THEOREM 4. *There is a one-to-one correspondence between symmetric flou relations* $[R_1, R_2]$ *of* (6) *and binary relations* R *of* (7) *on a set* A, *where*

$6_1 : R_1 \subset R_2$;

$6_2 : R_1$ *is transitive*;

$6_3 :$ *for every* $(a, b) \in R_2$ *there is an element* c *such that* $(c, c) \in R_1$ *and* $(c, a), (c, b) \in R_2$;

$6_4 :$ *a single block of* R_1 *intersects at most one block of* R_2. (6)

$7_1 :$ *if* $(a, b) \in R$ *then* $(a, a) \in R$;

$7_2 :$ *if* $(a, a), (b, b), (a, b) \in R$ *then* $(b, a) \in R$ *and if* $(a, a), (b, b), (b, a) \in R$ *then* $(a, b) \in R$;

$7_3 :$ *if* $(a, b), (b, a), (b, c), (c, b) \in R$ *then* $(a, c), (c, a) \in R$;

7_4 ; *if* $(a, b), (b, a), (b, d) \in R$ *then* $(a, d) \in R$. (7)

Proof. We will show first that every $[R_1, R_2]$ of (6) determines a relation R of (7) and vice versa. Thereafter we consider a given $[R_1, R_2]$ of (6) and determine by means of this a relation R of (7) and from this $[R_1^*, R_2^*]$ of (6). One sees easily that $R_1 = R_1^*$ and $R_2 = R^*$, from which the theorem now follows.

Let $[R_1, R_2]$ be a symmetric flou relation of (6). We define a binary relation R on A as follows:

$$(a, b) R \in \Leftrightarrow (a, b) \in R_2 \text{ and } (a, a) \in R_1.$$

R thus defined has the properties of (7) as shown below.

7_1: $(a, b) \in R \Rightarrow (a, b) \in R_2$ and $(a, a) \in R_1$. As shown by Chajda [1], when R_2 is symmetric, then $(a, b) \in R_2$ implies $(a, a), (b, b) \in R_2$. But $(a, a) \in R_2$ and $(a, a) \in R_1$ imply $(a, a) \in R$.

7_2: Let $(a, a), (b, b), (a, b) \in R$. Then $(a, b) \in R_2$ and because R_2 is symmetric, $(b, a) \in R_2$. Now $(b, a) \in R_2$ and $(b, b) \in R_1$ (because $(b, b) \in R$) imply $(b, a) \in R$. The other part is proved similarly.

7_3: Let $(a, b), (b, a), (b, c), (c, b) \in R$. Thus $(a, b), (c, b) \in R_2$ and $(a, a), (b, b), (c, c) \in R_1$. Because $(a, b) \in R_2$, a and b belong to some block B_1 of R_2. Because $(a, a), (b, b) \in R_1$, a belongs to some and b to another block of R_1. But, in fact, a and b belong to a single block of R_1, because in the other case R_1 would intersect two blocks of R_1. Similarly we see that b and c are in a single block of R_1. Thus $(a, b), (b, c) \in R_1$ and according to the transitivity of R_1, also $(a, c) \in R_1 \subset R_2$. This combined with $(a, a) \in R_1$ implies $(a, c) \in R$.

One observes now that $(a, b) \in R_1 \Leftrightarrow (a, b), (b, a) \in R$. Indeed, when $(a, b) \in R_1 \subset R_2$, then $(a, a), (b, b) \in R_1$ and thus $(a, b), (b, a) \in R$ because of symmetry of R_2. When $(a, b), (b, a) \in R$, then $(a, b), (b, a) \in R_2$ and $(a, a) \in R_1$. As in the proof of 7_3 we see that a and b belong to a single block of R_1, whence $(a, b) \in R_1$.

7_4: Let $(a, b), (b, a), (b, d) \in R$. This means that $(a, b) \in R_1$ and $(b, d) \in R_2$. Thus the block of R_2 containing b and d and the block of R_1 containing b intersect, whence a and d should belong to a single block of R_2. Thus $(a, d) \in R_2$, and because $(a, a) \in R_1$, also $(a, d) \in R$.

Let R be a relation of (7). We define $[R_1, R_2]$ by means of R as follows:

$(a, b) \in R_2 \Leftrightarrow$ there is an element c such that $(c, a), (c, b) \in R$, and

$(a, b) \in R_1 \Leftrightarrow (a, b), (b, a) \in R$.

It is now, as above, a routine proof to show the symmetry of R_1 and R_2 and the validity of (6) for $[R_1, R_2]$.

Let now $[R_1, R_2]$ of (6) determine R of (7) and this further $[R_1^*, R_2^*]$. The definitions above show that $R_1 = R_1^*$ $((a, b) \in R_1 \Leftrightarrow (a, b), (b, a) \in R \Leftrightarrow (a, b) \in R_1^*)$ and $R_2 = R_2^*$ $((a, b) \in R_2 \Leftrightarrow$ there is an element c such that $(c, c) \in R_1$ and $(c, a), (c, b) \in R_2 \Leftrightarrow (c, a), (c, b) \in R \Leftrightarrow (a, b) \in R_2^*)$. This completes the proof. ■

About the compatibility of R corresponding to $[R_1, R_2]$ of (6) one can prove

THEOREM 5. *Let $\mathcal{A} = \langle A, F \rangle$ be an algebra, $[R_1. R_2]$ a symmeteric flou relation of* (6) *on A, and let R_1 and R_2 be compatible with $\mathcal{A}$. Then the corresponding R of* (7) *is also compatible with $\mathcal{A}$.*

Proof. Let $f \in F$ be an n-ary operation and $(a_j, b_j) \in R$, $j = 1, \ldots, n$. As shown above, $(a_j, b_j) \in R$ implies the existence of an element c_j such that $(c_j, a_j), (c_j, b_j) \in R_2$ and $(c_j, c_j) \in R_1$, $j = 1, \ldots, n$. According to the compatibility of R_2 and R_1, respectively, $(f(c_1, \ldots, c_n), f(a_1, \ldots, a_n)), (f(c_1, \ldots, c_n), f(b_1, \ldots, b_n)) \in R_2$ and $(f(c_1, \ldots, c_n), f(c_1, \ldots, c_n)) \in R_1$. Thus $(f(a_1, \ldots, a_n), f(b_1, \ldots, b_n)) \in R$, and the theorem follows. ■

In [4] Kulisch defined a screen S of a complete lattice L as a complete sublattice of L such that the least elements of L and S, as well as the greatest elements of L and S, coincide. The rounding mapping φ of L preserves the operations of L and maps L onto S such that $\varphi(s)=s$ for every $s \in S$. Thus $\varphi(a \vee b)=\varphi(a) \vee \varphi(b)$ and $\varphi(a \wedge b)=\varphi(a) \wedge \varphi(b)$ for every two elements $a, b \in L$. This means that φ determines a reflexive, symmetric and compatible relation R_2 on L as follows:

$$(a, b) \in R_2 \Leftrightarrow a, b \in L \text{ and } \varphi(a)=\varphi(b).$$

The screen determines another symmetric and compatible relation R_1 on L with the definition

$$(u, s) \in R_1 \Leftrightarrow u, s \in S \subset L \text{ and } u=s.$$

Thus every block of R_1 consists of a single element; this implies the transitivity of R_1. As easily seen, $R_1 \subset R_2$, and because $\varphi(s)=s$, a block of R_1 intersects at most one block of R_2. Further, when $(a, b) \in R_2$, $\varphi(a)=\varphi(b)=c \in S$, where $c=\varphi(c)$, whence $(c, a), (c, b) \in R_2$ and $(c, c) \in R_1$. Thus the couple $[R_1, R_2]$ obtained from a screen and the corresponding rounding mapping is a symmetric flou relation of (6) compatible with L. By substituting now L by an algebra $\mathscr{A}=\langle A, F\rangle$ and the screen and the rounding mapping by a symmetric flou relation $[R_1, R_2]$ of (6) compatible with $\mathscr{A}$, one obtains a couple of relations considered by Ratschek in [7] when he sketched a model for rouding mappings in computer arithmetic.

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Relacje binarne, relacje rozmyte blokowe i ich zastosowania

Relacje binarne na zbiorze A są określone za pomocą rodzin ich klas (półbloków). W artykule są również przedstawione niesprzeczne relacje binarne oraz opisane odpowiednie odwzorowania. Po tych rozważaniach czysto algebraicznych klasa relacji rozmytych blokowych jest sprowadzona do klasy relacji binarnych i odwrotnie. Niektóre niesprzeczne relacje rozmyte blokowe mają znaczenie dla odwzorowań zaokrąglania w arytmetyce maszyn cyfrowych.

Бинарные соотношения, размытые блочные соотношения и их применение

Бинарные соотношения на множестве A определяются с помощью семейств их классов (полублоков). В статье представлены также непротиворечивые бинарные соотношения и описаны соответствующие отображения. После этих чисто алгебраических рассуждений, класс размытых блочных соотношений сводится к классу бинарных соотношений и наоборот. Некоторые непротиворечивые размытые блочные соотношения играют существенную роль для отражений приближенных вычислений в арифметике цифровых машин.