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Numerical solution of the *c*-observation problem for nonlinear systems

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A practical, effective method by which the current state of nonlinear systems at a present time can be evaluated from a complete knowledge of the system's input and output history on some finite time interval has been developed.

This method requires that only auxiliary systems of differential equations with known initial or terminal conditions and system of algebraic equations are solved in each iteration, so it seems to be well adapted to computations by digital or hybrid computers.

Some constructive sufficient conditions are presented for the convergence of the iterative process.

1. Introduction

Often in control design it is necessary to have sufficiently complete information on the current state of the system in the state space. In many control situations, however, direct measurement of some state coordinates are difficult, and, sometimes, even impossible. In these cases, the problem of determining the system's state from a complete knowledge of the system's input and output data becomes important to be studied.

In Ref. [1], this problem is referred to as the *c*-observation problem.

A dynamic system which permits the reconstruction of the current state from a complete knowledge of the system's input and output history on some finite time interval is called *c*-observable.

Kalman first has considered the problem of determining the conditions which a linear dynamical system must satisfy in order that it be *c*-observable. Assuming that the system's input is zero and that a complete knowledge of the system's output on the finite time interval is available, Kalman has obtained necessary and sufficient conditions of *c*-observability for both continuous and discrete-time linear dynamical systems [2].

Kalman's investigations were further developed and extended by Gilbert [3], Krasovskii [4] and other authors [5, 6]. In Ref. [1], a new computing procedure of solving the *c*-observation problem for linear nonstationary systems is proposed.

Furthermore, from the obtained results the block diagram of the indirect c-observer has been constructed. This observer representing the special purpose hybrid computer connected in parallel with the considered object provides automatical calculation of the current state and, therefore, can effectively surmount the difficulties associated with control design when the state is inaccessible to direct observation.

For nonlinear systems Lee and Markus obtained necessary and sufficient conditions of c-observability in the neighborhood of the origin [7]. Hwang and Steinfeld extended the work of Lee and Markus from local c-observability about the origin to arbitrary point in the entire domain of initial conditions [8]. In Ref. [9] Roitenberg transformed the c-observability problem into the problem of construction of Liapunov function by a model reference method. From this viewpoint, Kostyukovskii [10] and Griffith and Kumar [11] considered the c-observability problem for nonlinear systems. Some global conditions in the form of sufficient conditions for c-observability of nonlinear systems are obtained by Yamamoto and Sugiura in Ref. [12].

Al'brekht and Krasovskii have considered the c-observation problem for nonlinear systems in the neighborhood of a given motion [13]. Assuming a complete knowledge of both the system's input and output on some finite time interval, and in addition, that the motion of the system is close to the given motion, they calculate the state in terms of the system's input and output history.

The present paper derives its inspiration from the work of Al'brekht and Krasovskii and obtains results different from theirs by employing another method for solving this problem.

2. Problem statement

Consider a continuous-time dynamic object described by a system of nonlinear differential equations in the following matrix form

$$\frac{dx}{dt} = A(t)\,x + B(t)\,u(t) + \mu f(x, u(t), t), \tag{2.1}$$

where x is an n-dimensional vector-valued function of time describing the state of the considered object at any time t, $u(t)$ is an r-dimensional real vector-valued function of time that represents the control parameters, $f(x, u(t), t)$ is an n-dimensional real vector-valued function, nonlinear with respect to x and $u(t)$, $A(t)$ and $B(t)$ are known, respectively, $(n\times n)$ and $(n\times r)$-dimensional matrices and μ is a small parameter.

Let us assume that the system's input $u(t)$ is either known a priori or can be measured exactly and that the state $x(t)$ is inaccessible to direct observation, only the system's output

$$z(t) = Q(t)\,x(t), \quad \vartheta_0 \leqslant t \leqslant \vartheta, \quad \vartheta - \vartheta_0 = h > 0 \tag{2.2}$$

is accessible to noise-free measurement, where $z(t)$ is an m-dimensional vector-valued function of time, $m<n$, and $Q(t)$ is a known $(m\times n)$-dimensional matrix.

The vector-valued function $f(x, u(t), t)$ in the right-hand side of the Eq. (2.1) is assumed to be continuous in all arguments in some domain of the space (x, u, t). Furthermore, this function must satisfy certain other restrictions mentioned later.

All the elements of the matrices $A(t)$, $B(t)$ and $Q(t)$ are continuous functions of time and are $(n-1)$ times continuously differentiable on the finite time interval $\vartheta_0\leqslant t\leqslant\vartheta$

The c-observation problem for nonlinear system (2,1) consists in the following: *it is required to find the unknown state $x(t)$ at the present time $t=\vartheta$ from a complete knowledge of the system's input and output history on the finite time interval $\vartheta_0\leqslant t\leqslant\vartheta$, where ϑ_0 is some past time $(\vartheta_0<\vartheta)$.*

3. Solution technique

It is well known that the system of differential equations (2.1) can be written in the following equivalent matrix integral form

$$x(t)=X(t)X^{-1}(\vartheta)x(\vartheta)-\int_t^{\vartheta} X(t)X^{-1}(\xi)\left[B(\xi)u(\xi)+\mu f(x(\xi), u(\xi), \xi)\right]d\xi, \tag{3.1}$$

where $X(t)$ is an $(n\times n)$-dimensional fundamental matrix for the corresponding linear $(\mu=0)$ homogeneous system and $X^{-1}(.)$ is an inverse matrix.

Substituting (3.1) into the right-hand side of (2.2) we have the following equation

$$Q(t)X(t)X^{-1}(\vartheta)x(\vartheta)=z(t)-Q(t)\lambda\left(t, x(.), u(.)\right), \quad \vartheta_0\leqslant t\leqslant\vartheta, \tag{3.2}$$

where

$$\lambda\left(t, x(.), u(.)\right)=-\int_t^{\vartheta} X(t)X^{-1}(\xi)\left[B(\xi)u(\xi)+\mu f(x(\xi), u(\xi), \xi)\right]d\xi, \tag{3.3}$$

Multiplying (3.2) by $[X^{-1}(\vartheta)]'X'(t)Q'(t)$ from the left and integrating from ϑ_0 to ϑ we obtain

$$G(\vartheta)x(\vartheta)=\int_{\vartheta_0}^{\vartheta}[X^{-1}(\vartheta)]'X'(t)Q'(t))\left[z(t)-Q(t)\lambda(t, x(.), u(.))\right]dt \tag{3.4}$$

where $G(t)$ denotes the following Gramian matrix:

$$G(t)=\int_{\vartheta_0}^{t}[X^{-1}(t)]'X'(\xi)Q'(\xi)Q(\xi)X(\xi)X^{-1}(t)\,d\xi \tag{3.5}$$

Here the prime designetes transposition.

Thus, for determining the unknown state at present time $t=\vartheta$ we shall have to solve equation (3.4) associated with the original nonlinear matrix integral equation (3.1).

Since it is actually impossible to obtain an analytic solution (in the closed form) of these equations, they must be solved by means of various numerical methods.

It turns out that the numerical technique developed by Nguyen in Refs. [1, 14] can be applied in an iterative fashion to solve the abovementioned equations.

The initial approximation may be defined as

$$G(\vartheta)\,\bar{x}^0(\vartheta)=\int_{\vartheta_0}^{\vartheta}[X^{-1}(\vartheta)]'X'(t)\,Q'(t)\,[z(t)-Q'(t)\,\lambda^0(t)]\,dt, \tag{3.6}$$

$$x^0(t)=X(t)\,X^{-1}(\vartheta)\,\bar{x}^0(\vartheta)+\lambda^0(t),\quad \lambda^0(t)=-\int_t^{\vartheta}X(t)\,X^{-1}(\xi)\,B(\xi)\,u(\xi)\,d\xi \tag{3.7}$$

It is clear from Ref. [1] that the state vector $\bar{x}^0(\vartheta)$ defined by (3.6) represents the solution of the c-observation problem for the corresponding linear ($\mu=0$) system. For this case the following result holds (see Ref. [1]).

PROPOSITION 1. *The corresponding linear* ($\mu=0$) *system is c-observable at time* $t=\vartheta$ *if and only if the row vectors of the matrix* $[X^{-1}(\vartheta)]'X'(t)\,Q'(t)$ *are linearly independent for any t on the interval* $\vartheta_0\leqslant t\leqslant\vartheta$, *where* ϑ_0 *is some past time and* $h=\vartheta-\vartheta_0>0$.

It should be noted that this test for c-observability of the corresponding linear ($\mu=0$) system depends on knowing the fundamental matrix $X(t)$. In many practical cases, however, the problem of the construction of the fundamental matrix is either difficult or time consuming on the computer. Thus, it would be a distinct advantage to have a test for c-observability which does not require a knowledge of the fundamental matrix.

The following proposition proved by Krasovskii [4] gives such a test.

PROPOSITION 2. *Let* $[X^{-1}(\vartheta)]'X'(t)\,Q'(t)$ *be* $(n-1)$ *times differentiable with respect to t, where* $t\in[\vartheta_0,\vartheta]$. *The row vectors of this matrix are linearly independent on the interval* $\vartheta_0\leqslant t\leqslant\vartheta$ *if*

$$\operatorname{rank}P(t)=n,\quad \forall t\in[\vartheta_0,\vartheta] \tag{3.8}$$

where

$$P(t)=(P_1(t),P_2(t),\ldots,P_n(t))$$

$$P_1(t)=Q'(t),\;P_{i+1}(t)=\frac{dP_i(t)}{dt}+A'(t)\,P_i(t),\qquad i=1,2,\ldots,n-1 \tag{3.9}$$

Thus, if the matrix $P(t)$ is of rank n for any t on the interval $\vartheta_0\leqslant t\leqslant\vartheta$, then it is easy to see via proposition 1 that the corresponding linear ($\mu=0$) system is c-observable at time $t=\vartheta$.

Therefore, in what follows we shall assume that the condition (3.8) of c-observability for the corresponding linear ($\mu=0$) system is fulfilled.

Then, it is very easy to show that det $G(\vartheta)\neq 0$ and the state vector $\bar{x}^0(\vartheta)$ will be defined uniquely from algebraic equation (3.6).

Now, suppose that $(j-1)$-th approximation is already found, i.e. $\tilde{x}^{j-1}(\vartheta)$ and $x^{j-1}(t)$ are known. Then j-th approximation for $x(\vartheta)$ is determined as a solution of the following algebraic equation

$$G(\vartheta)\,\tilde{x}^j(\vartheta)=\int_{\vartheta_0}^{\vartheta}[X^{-1}(\vartheta)]'\,X'(t)\,Q'(t)\,[z(t)-Q(t)\,\lambda^j(t)]\,dt,\quad j=1,2,\dots \tag{3.10}$$

where

$$\lambda^j(t)\triangleq\lambda(t,x^{j-1}(.),u(.))=-\int_t^{\vartheta}X(t)\,X^{-1}(\xi)\,[B(\xi)\,u(\xi)+$$
$$+\mu f(x^{j-1}(\xi),u(\xi),\xi)]\,d\xi,\quad j=1,2,\dots \tag{3.11}$$

Further, j-th approximation for the current state $x(t)$, $\vartheta_0\leqslant t\leqslant\vartheta$, of the original system can be determined by

$$x^j(t)=X(t)\,X^{-1}(\vartheta)\,\tilde{x}^j(\vartheta)+\lambda^j(t),\quad j=1,2,\dots \tag{3.12}$$

Note that Eqs. (3.10) and (3.12) remain also valid for $j=0$, if in this case we set $\mu=0$.

From Eqs. (3.6), (3.7), (3.10) and (3.12) it is easy to see that a computer implementation of the above steps requires the construction of the inverse matrix $X^{-1}(t)$. In many cases, however, analytical evalution of the inverse matrix is difficult and, sometimes, even impossible. Moreover, in each iteration we must take definite integral in righthand side of the equation (3.10) and indefinite integral (3.11) that is not convenient and requires a lot of computer time. Therefore, it is of interest to find a modified formulation of the presented above iterative procedure which avoids these difficulties.

4. Modified formulation of the iterative procedure

First, we look at the problem of determining the Gramian matrix $G(\vartheta)$ which is needed in each iteration.

Differenting (3.5) with respect to t and noting that (see Ref. [15])

$$\frac{d\,[X^{-1}(t)]'}{dt}=-A'(t)\,[X^{-1}(t)]',\qquad \frac{dX^{-1}(t)}{dt}=-X^{-1}(t)\,A(t)$$

we obtain the following matrix differential equation

$$\frac{dG(t)}{dt}=-A'(t)\,G(t)-G(t)\,A(t)+Q'(t)\,Q(t) \tag{4.1}$$

subject to the initial conditon

$$G(\vartheta_0)=0 \tag{4.2}$$

Thus, the Gramian matrix $G(\vartheta)$ can be obtained by integrating Eq. (4.1) from ϑ_0 to ϑ subject to (4.2).

Further, it is easy to show that

$$\psi^j(\vartheta)=\int_{\vartheta_0}^{\vartheta}[X^{-1}(\vartheta)]'\,X'(t)\,Q'(t)\,[z(t)-Q(t)\,\lambda^j(t)]\,dt,\qquad j=0,1,2,\ldots \tag{4.3}$$

where $\psi^j(t)$ is the solution of the following system of differential equations

$$\frac{d\psi^j}{dt}=-A'(t)\,\psi^j+Q'(t)\,[z(t)-Q(t)\,\lambda^j(t)],\qquad j=0,1,2,\ldots \tag{4.4}$$

subject to

$$\psi^j(\vartheta_0)=0,\qquad j=0,1,2,\ldots \tag{4.5}$$

Note that the vector-valued function $\lambda^j(t)$ in the right-hand side of the Eq. (4.4) defined by (3.11) can be evaluated by solving the system of differential equations

$$\frac{d\lambda^j}{dt}=A(t)\,\lambda^j+B(t)\,u(t)+\mu_j f(x^{j-1}(t),u(t),t),\qquad j=0,1,2,\ldots \tag{4.6}$$

subject to

$$\lambda^j(\vartheta)=0,\qquad j=0,1,2,\ldots \tag{4.7}$$

where the numbers μ_j, $j=0,1,2,\ldots$ are defined by

$$\mu_j=\begin{cases}0 & \text{if } j=0\\ \mu & \text{if } j\neq 0\end{cases} \tag{4.8}$$

Clearly, it is very much easier to solve the Eqs. (4.6) subject to (4.7) than to evaluate the indefinite integral in the right-hand side of Eq. (3.11).

Thus, any j-th approximation for the unknown state $x(\vartheta)$ can be defined by solving the following algebraic equation

$$G(\vartheta)\,\tilde{x}^j(\vartheta)=\psi^j(\vartheta),\qquad j=0,1,2,\ldots \tag{4.9}$$

where $\psi^j(.)$ is the solution of the equation (4.4) subject to (4.5).

Now, any j-th approximation for the current state $x(t)$, $\vartheta_0\leqslant t\leqslant\vartheta$, of the considered system determined by (3.12) can be evaluated by formula

$$x^j(t)=y^j(t)+\lambda^j(t),\qquad j=0,1,2,\ldots \tag{4.10}$$

where $y^j(t)$ is the solution of the following differential equation

$$\frac{dy^j}{dt}=A(t)\,y^j,\qquad j=0,1,2,\ldots \tag{4.11}$$

subject to

$$y^j(\vartheta)=\tilde{x}^j(\vartheta),\qquad j=0,1,2,\ldots \tag{4.12}$$

Thus, it is not necessary to construct the inverse matrix $X^{-1}(t)$ in order to determine all the requisite values in each iteration.

By combining the above results with those of the previous section, the modified formulation of the iterative procedure is obtained. It entails the following steps.

Step 0. Determine Gramian matrix $G(\vartheta)$ by integrating Eq. (4.1) from ϑ_0 to ϑ subject to the initial condition (4.2).
Step 1. Set $j=0$, $\mu_j=0$ and integrate backward equation (4.6) from ϑ to ϑ_0 subject to (4.7). During the backward integration calculate vector-valued function $\lambda^j(t)$, $\vartheta_0 \leqslant t \leqslant \vartheta$.

Simultaneously integrate forward equation (4.4) from ϑ_0 to ϑ subject to (4.5) and at the end of the forward integration obtain vector $\psi^j(\vartheta)$.
Step 2. Solve algebraic equation (4.9) in order to find vector $\tilde{x}^j(\vartheta)$.
Step 3. Integrate backward equation (4.11) from ϑ to ϑ_0 subject to (4.12). During this backward integration calculate vector-valued function $y^j(t)$. Simultaneously evaluate vector-valued function $x^j(t)$ by (4.10). Store $x^j(t)$, $\vartheta_0 \leqslant t \leqslant \vartheta$.

Return to step 1 with j replaced by $j+1$ and set $\mu_j=\mu$. The process ends when $\tilde{x}^{j+1}(\vartheta)=\tilde{x}^j(\vartheta)$ with the requisite accuracy.

5. Some constructive sufficient conditions for the convergence of the iterative process

In the previous section we have employed the numerical technique developed by Nguyen in Refs. [1, 14] to solve the c-observation problem for nonlinear system and found the modified formulation of the iterative procedure which avoids some difficulties in computer implementation.

Now, it remains for us to establish some sufficient conditions under which the above iterative process is convergent and the considered nonlinear system (2.1) is c-observable at time $t=\vartheta$.

THEOREM. *Assume*

1) *The corresponding linear* ($\mu=0$) *system is c-observable at time* $t=\vartheta$.

2) *The vector-valued function* $f(x, u(t), t)$ *is continuous in all arguments in some closed domain of the space* (x, t) *determined by the expression*

$$D(\Delta, h)=\{(x, t): |x| \leqslant \Delta, \quad \vartheta_0 \leqslant t \leqslant \vartheta, \quad h=\vartheta-\vartheta_0>0\} \tag{5.1}$$

Here Δ *is some positive number which satisfies the condition*

$$\Delta > \sigma h(vbH + r\sigma q\gamma^-), \tag{5.2}$$

where

$$\sigma = \max_{\vartheta_0 < \xi \leqslant t \leqslant \vartheta} |X(t) X^{-1}(\xi)|, \; b = \max_{\vartheta_0 \leqslant t \leqslant \vartheta} |B(t)|, \; q = \max_{\vartheta_0 \leqslant t \leqslant \vartheta} |Q(t)|$$

$$\gamma^- = |G^{-1}(\vartheta)|, \; r = \max_{\vartheta_0 \leqslant t \leqslant \vartheta} |z(t)|, \; v = \max_{\vartheta_0 \leqslant t \leqslant \vartheta} |u(t)|, \; H = 1 + \sigma^2 q^2 h\gamma^-$$

The norm of the matrix x *is denoted by* $|x|$.

3) *In the domain* $D(\Delta, h)$ *the vector-valued function* $f(x, u(t), t)$ *is Lipschitzian with respect to* x *with Lipschitz's constant L. This means that for two arbitrary points*

(x^1, t), (x^2, t) of the domain $D(\Delta, h)$ and for any admissible system's input $u(t)$ the following condition is satisfied

$$|f(x^2, u(t), t) - f(x^1, u(t), t)| \leqslant L|x^2 - x^1| \tag{5.3}$$

4) *The parameter μ satisfies the condition*

$$0 < \mu < \mu^* = \frac{1}{\sigma hH} \min\left\{\frac{\Delta - \sigma h(vbH + r\sigma q\gamma^-)}{F}, \frac{1}{L}\right\} \tag{5.4}$$

where

$$F = \max |f(x, u(t), t)|, \ \forall (x, t) \in D(\Delta, h), \forall u(t)$$

Then, the proposed above iterative process for solving the c-observation problem is convergent and the considered nonlinear system (2.1) *is c-observable at time $t = \vartheta$.*

Proof. We show first that all the approximations for the current state $x(t)$ determined by (3.12) at any time t, $\vartheta_0 \leqslant t \leqslant \vartheta$, entirely belong to the domain $D(\Delta, h)$.

It is quite easy to show that, if the condition (5.2) is fulfilled, then $x^0(t)$ determined by (3.7) belongs to the domain $D(\Delta, h)$ .Suppose, therefore, that $x^j(.) \in D(\Delta, h)$, then we will prove that $x^{j+1}(.) \in D(\Delta, h)$.

From (3.12) it follows that

$$\|x^{j+1}(t)\| \leqslant \sigma|\tilde{x}^{j+1}(\vartheta)| + \sigma vbh + \mu\sigma hF \tag{5.5}$$

where

$$\|x^{j+1}(t)\| = \max_{\vartheta - h \leqslant t \leqslant \vartheta} |x^{j+1}(t)|$$

From (3.10) we obtain

$$|\tilde{x}^{j+1}(\vartheta)| \leqslant \sigma q\gamma^- h(r + \sigma vqbh + \mu\sigma qhF) \tag{5.6}$$

With this inequality for $|\tilde{x}^{j+1}(\vartheta)|$ substituted into (5.5) we have

$$\|x^{j+1}(t)\| \leqslant \sigma h(vbH + r\sigma q\gamma^-) + \mu\sigma hHF \leqslant \Delta$$

Thus, by induction we conclude that all the approximations for the current state $x(t)$ at any t, $\vartheta_0 \leqslant t \leqslant \vartheta$, belong to the domain $D(\Delta, h)$.

Now, we pass to the problem of the convergence.

From the equations (3.10), (3.11) and (3.12) it is easy to derive the following estimations

$$\|x^{j+1}(t) - x^j(t)\| \leqslant \sigma|\tilde{x}^{j+1}(\vartheta) - \tilde{x}^j(\vartheta)| + \mu\sigma hL\|x^j(t) - x^{j-1}(t)\| \tag{5.7}$$

$$|\tilde{x}^{j+1}(\vartheta) - \tilde{x}^j(\vartheta)| \leqslant \mu\sigma^2 q^2 h^2 \gamma^- L\|x^j(t) - x^{j-1}(t)\| \tag{5.8}$$

Substituting (5.8) into (5.7) gives

$$\|x^{j+1}(t) - x^j(t)\| \leqslant \mu\sigma hHL\|x^j(t) - x^{j-1}(t)\|$$

or

$$\frac{\|x^{j+1}(t) - x^j(t)\|}{\|x^j(t) - x^{j-1}(t)\|} \leqslant \mu\sigma hHL < 1 \tag{5.9}$$

The inequality (5.9) shows that majorant series converges (due to d'Alambert criterion). Hence, the sequence of the approximations $\{x^{j+1}(t)\}$ must converge uniformly to a certain continuous vector-valued function $x(t) \in D(\Delta, h)$, and due to (5.8), the sequence $\{\tilde{x}^{j+1}(\vartheta)\}$ converges to the state $x(\vartheta)$.

It is easy to see that the limit function $x(t)$ satisfies the original integral equation (3.1) and $x(\vartheta)$ — Eq. (3.4), i.e. they are a solution of the c-observation problem for nonlinear system (2.1).

Finally, we show that the mentioned above limits are unique. Let $x^1(t)$ and $x^2(t)$, $\vartheta_0 \leqslant t \leqslant \vartheta$, be the limit functions simultaneously satisfying equation (3.1) and let $x^1(\vartheta)$ and $x^2(\vartheta)$ be the limit states simultaneously satisfying Eq. (3.4), then we have

$$\|x^2(t)-x^1(t)\| \leqslant \sigma |x^2(\vartheta)-x^1(\vartheta)| + \mu\sigma hL \|x^2(t)-x^1(t)\| \tag{5.10}$$

$$|x^2(\vartheta)-x^1(\vartheta)| \leqslant \mu\sigma^2 q^2 h^2 \gamma^- L \|x^2(t)-x^1(t)\| \tag{5.11}$$

Substituting (5.11) into (5.10) yields

$$\|x^2(t)-x^1(t)\| \leqslant \mu\sigma hHL \|x^2(t)-x^1(t)\| < \|x^2(t)-x^1(t)\|$$

Latter inequality is possible if and only if $x^2(t) \equiv x^1(t)$, $\vartheta_0 \leqslant t \leqslant \vartheta$, and due to (5.11), $x^2(\vartheta) = x^1(\vartheta)$.

Thus, if all assumptions 1(−4) are fulfilled, then the nonlinear system (2.1) will be c-observable at time $t=\vartheta$.

The proof of theorem is complete. ■

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Rozwiązania numeryczne problemu *c*-obserwacji dla układów nieliniowych

W artykule przedstawiono praktyczną i efektywną metodę wyznaczania bieżącego stanu układów nieliniowych na podstawie kompletnej znajomości wejść i wyjść układu w pewnym przedziale czasowym. Metoda ta wymaga rozwiązywania, w każdej iteracji, jedynie pomocniczych układów równań różniczkowych o znanych warunkach początkowych lub końcowych oraz układu równań algebraicznych, co daje się łatwo zrealizować przy pomocy maszyn cyfrowych lub hybrydowych.

Численные решения задачи *c*-наблюдений для нелинейных систем

В статье представлен практический и эффективный метод определения текущего состояния нелинейных систем на основе полного знания входных и выходных сигналов системы в некотором диапазоне времени. Этот метод требует решения, в каждой итерации только лишь вспомогательных систем дифференциальных уравнений с известными начальными или конечными условиями, а также системы алгебраических уравнений, что несложно реализовать с помощью цифровой или гибридной вычислительной машины.