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Model matching adaptive controllers for minimumphase discrete—time plants

by

DARIUSZ JANECKI
ANATOL GOSIEWSKI

Institute of Automatic Control
Technical University of Warsaw
ul. Nowowiejska 15/19
00-665 Warszawa, Poland

Two indirect and two direct model matching adaptive controllers for minimumphase discrete time plants are presented in the unified approach. The stability analysis of indirect adaptive control system is given.

List of basic symbols

$y(t)$, $u(t)$	— plant output and input
$y_M(t)$, $u_M(t)$	— model output and input
$A(q^{-1})$, $B(q^{-1})$	— polynomials corresponding to the plant input and output, $A(q^{-1})$ monic
$\bar{A}(q^{-1})$, $\bar{B}(q^{-1})$	— as above but $\bar{B}(q^{-1})$ — monic
$A^M(q^{-1})$, $B^M(q^{-1})$	— polynomials corresponding to the model input and output
d	— time delay
$C(q^{-1})$, $P(q^{-1})$, $D(q^{-1})$	— Hurwitz polynomials
$x^P(t)$	— filtered scalar or vector variable: $P(q^{-1})\,x^P(t)=x(t)$
$T(q^{-1})$, $R(q^{-1})$, $\bar{T}(q^{-1})$, $\bar{R}(q^{-1})$	— regulator polynomials
$e_0(t)$	— output error: $e_0(t)=y(t)-y_M(t)$
$e_0^f(t)$	— filtered output error: $e_0^f(t)=C(q^{-1})\,e_0(t)$
θ, $\bar{\theta}$, ϑ, $\bar{\vartheta}$	— vectors of parameters
$\theta(t)$, $\bar{\theta}(t)$, $\vartheta(t)$, $\bar{\vartheta}(t)$	— vectors of estimated parameters
$\omega(t)$, $\bar{\omega}(t)$, $\varphi(t)$, $\bar{\varphi}(t)$	— vectors composed of inputs and outputs
$e_e(t)$	— estimation error: $e_e(t)=y^D(t)-\theta(t)^T\,\omega^D(t)$
$\bar{e}_e(t)$	— estimation error: $\bar{e}_e(t)=u^D(t)-\bar{\theta}(t)^T\,\bar{\omega}^D(t)$

β, α — lower and upper bound of b_0
$\|\cdot\|$ — Euclidean norm
$|\cdot|$ — absolute value

1. Introduction

The most frequent adaptive control problem statement for minimumphase plant with unknown parameters is to choose the plant input sequence in such a way that the plant output shall follow the output of the certain model (called sometimes the reference model) which represents the desired behaviour of the plant. Self Tuning Regulators (STR) is the most intuitive approach to this adaptive control problem. In this approach the unknown plant parameters are recursively estimated and, basing on the design procedure for known plant, the regulator parameters are computed at each discrete time instant using estimated parameters instead of the parameters of plant. For estimation purposes the plant model has to be parametrized. However, this may be done in various manners allowing various types of adaptive controllers to be applied. Two of them, called "direct" and "indirect", are of special importance. In the indirect adaptive controllers the plant model is parametrized in the standard minimal manner [2] (minimal with respect to the number of parameters). On the other hand in the direct ones a special nonminimal parametrization is applied which allows the direct estimation of regulator parameters [1]—[3]. Another interpretation of the direct adaptive controllers may be done on the basis of the Model Reference Adaptive Control (MRAC) approach [1], [5].

The most important problem in adaptive control theory is the global stability of adaptive schemes. This stability problem has been positively resolved recently for the direct adaptive controllers [1], [5], [6].

In this paper two types of indirect adaptive controllers and two types of direct ones are presented in the unified approach. One of them, called in the sequel the indirect adaptive controller II is a new one. Since up to now the direct controllers have been considered most frequently, more attention is paid in this paper to the indirect ones (there is some supposition that in the case of the plant with large time delay the indirect controllers may have better performance then the direct ones). Basing on Goodwin's, Ramadge's and Caines's idea [1] it is proved that if some assumptions about estimation algorithm are satisfied the indirect controllers presented in the paper guarantee the convergence of the error between the plant and model reference output to zero and that the proposed adaptive schemes are globally stable.

The paper is organized as follows. The problem statement and the design method for known plants are presented in section 2 and 3 respectively. Two indirect adaptive controllers are described in section 4 and 5. Next two direct adaptive controllers which correspond to two proposed indirect schemes are presented in section 6. Some stability results are given in section 7.

Notation

The plant and the controllers will be described in the discrete-time domain using a polynomial representation. The following notation will be used throughout the paper

$$F(q^{-1})=f_0+f_1 q^{-1}+\ldots+f_{n_f} q^{-n_f}$$

where q^{-1} is the backward — shift operator defined as $q^{-1} u(t)=u(t-1)$, f_i are constant coefficients, n_f is the degree of the polynomial $F(q^{-1})$. If $f_0=1$ then the polynomial $F(q^{-1})$ is said to be monic. If $F(z^{-1})$ has all zeros inside the closed unit disk then it is said to be Hurwitz polynomial. A polynomial with time — varying coefficients will be denoted respectively by

$$F_t(q^{-1})=f_0(t)+f_1(t) q^{-1}+\ldots+f_{n_f}(t) q^{-n_f}$$

Let $G_t(q^{-1})=\sum_{i=0}^{n_g} g_i(t) q^{-i}$. Let $F_t(q^{-1})\cdot G_t(q^{-1})$ and $F_t(q^{-1}) G_t(q^{-1})$ be two kinds of composition of operator polynomials $F_t(q^{-1})$, $G_t(q^{-1})$ defined respectively by

$$F_1(q^{-1})\cdot G_t(q^{-1})=\sum_{i=0}^{n_f} f_i(t) \sum_{j=0}^{n_g} g_i(t) q^{-(i+j)}$$

and

$$F_t(q^{-1}) G_t(q^{-1})=\sum_{i=0}^{n_f} f_i(t) \sum_{j=0}^{n_g} g_i(t-i) q^{-(i+j)}$$

2. Statement of the problem

Let us assume that the plant to be controlled is a single input — single output, discrete — time, linear, time — invariant system

$$A(q^{-1}) y(t)=q^{-d} B(q^{-1}) u(t), \quad d>0 \tag{1}$$

where $u(t)$, $y(t)$ are the plant input and output respectively, d denotes the time delay, $A(q^{-1})$, $B(q^{-1})$ are the polynomials of the form

$$A(q^{-1})=1+a_1 q^{-1}+\ldots+a_{n_a} q^{-n_a} \tag{2}$$

$$B(q^{-1})=b_0+b_1 q^{-1}+\ldots+b_{n_b} q^{-n_b}, \quad b_0\neq 0 \tag{3}$$

We assume that:
1. the time delay d is known
2. the degrees of the polynomials $A(q^{-1})$, $B(q^{-1})$ are known
3. the polynomial $B(q^{-1})$ is Hurwitz

Define the model whose output $y_M(t)$ represents the desired behaviour of the plant output. Let this model be described by

$$A^M(q^{-1}) y_M(t)=q^{-d} B^M(q^{-1}) u_M(t) \tag{4}$$

where $u_M(t)$ is a bounded command input, $A^M(q^{-1})$ is monic and Hurwitz polynomial.

The objective of the control is to determine an appropriate bounded control input $u(t)$ in such a way that the plant-model output error defined as

$$e_0(t)=y(t)-y_M(t) \tag{5}$$

shall tend to zero when $t\to\infty$

REMARKS

1. Note that we do not assume that $a_{n_a}\neq 0$ and $b_{n_b}\neq 0$. Therefore, in fact, only some upper bounds on the degrees of the polynomials $A(q^{-1})$, $B(q^{-1})$ are needed to be known.
2. The assumption 3 is necessary since the regulator applied cancels the zeros of polynomial $B(q^{-1})$.

3. Design of controller for a known plant

The design method for the plant with known parameters, as a basis for adaptive controller design, is derived below.

Let

$$C(q^{-1})=1+c_1 q^{-1}+\ldots+c_{n_c} q^{-nc} \tag{6}$$

$$P(q^{-1})=1+p_1 q^{-1}+\ldots+p_{n_p} q^{-np} \tag{7}$$

be arbitrary Hurwitz polynomials, and let the polynomials

$$S(q^{-1})=1+s_1 q^{-1}+\ldots+s_{d-1} q^{-d+1} \tag{8}$$

$$R(q^{-1})=r_0+r_1 q^{-1}+\ldots+r_{n_r} q^{-n_r}, \qquad n_r=\max(n_c-d, n_a-1) \tag{9}$$

be the solution of the following polynomial equation

$$C(q^{-1})=S(q^{-1})A(q^{-1})+q^{-d}R(q^{-1}) \tag{10}$$

(In appendix B it is shown that the solution of (10) exists and is unique).

Using (1), (5) and (10) we have

$$C(q^{-1})e_0(t+d)=C(q^{-1})y(t+d)-C(q^{-1})y_M(t+d)=$$

$$=S(q^{-1})A(q^{-1})y(t+d)+R(q^{-1})y(t)-C(q^{-1})y_M(t+d)=$$

$$=S(q^{-1})B(q^{-1})u(t)+R(q^{-1})y(t)-C(q^{-1})y_M(t+d)=$$

$$=P(q^{-1})\left[b_0\cdot u(t)+\frac{T(q^{-1})}{P(q^{-1})}u(t)+\frac{R(q^{-1})}{P(q^{-1})}y(t)-\frac{C(q^{-1})}{P(q^{-1})}y_M(t+d)\right] \tag{11}$$

where

$$T(q^{-1})=S(q^{-1})B(q^{-1})-b_0 P(q^{-1}) \tag{12}$$

$$\text{degree } T(q^{-1})=n_t=\max(n_b+d-1, n_p) \tag{13}$$

From equation (11) it follows that the control objective (5) is fulfilled by the control law

$$b_0 u(t)+\frac{T(q^{-1})}{P(q^{-1})} u(t)+\frac{R(q^{-1})}{P(q^{-1})} y(t)-\frac{C(q^{-1})}{P(q^{-1})} y_M(t+d)=0 \tag{14}$$

or equivalently

$$u(t)+\frac{\bar{T}(q^{-1})}{P(q^{-1})} u(t)+\frac{\bar{R}(q^{-1})}{P(q^{-1})} y(t)-\bar{a}_0 \frac{C(q^{-1})}{P(q^{-1})} y_M(t+d)=0 \tag{15}$$

where $\bar{a}_0=\frac{1}{b_0}$, $\bar{T}(q^{-1})=\frac{T(q^{-1})}{b_0}$, $\bar{R}(q^{-1})=\frac{R(q^{-1})}{b_0}$

Summarizing, the design method for known plant can be represented in the following four-step procedure

1. Choose monic Hurwitz polynomials $C(q^{-1})$, $P(q^{-1})$
2. Solve the polynomial equation (10) with respect to $S(q^{-1})$, $R(q^{-1})$
3. Compute the polynomial $T(q^{-1})$ from (12)
4. Use the control law (14) or (15).

REMARKS

1. Substituting the equation (14) into (11) we obtain

$$C(q^{-1})e_0(t+d)=0 \tag{16}$$

It follows from above that the choice of stable polynomial $C(q^{-1})$ affects the behaviour of plant only during the transient period i.e. the polynomial $C(q^{-1})$ characterizes the regulation property of the control scheme.

2. Let $B^M(q^{-1})=b_0^M+b_1^M q^{-1}+\ldots+b_{n_b}^M q^{-nb}$. Assume that $b_0^M\neq 0$ and that $B^M(q^{-1})$ is Hurwitz polynomial. Then choosing

$$C(q^{-1})=A^M(q^{-1})L(q^{-1}) \quad P(q^{-1})=\frac{1}{b_0^M}\cdot B^M(q^{-1})L(q^{-1}) \tag{17}$$

for an arbitrary monic Hurwitz polynomial $L(q^{-1})$, the control scheme without the input dynamics may be obtained.

4. Indirect adaptive controller I

Two kinds of indirect adaptive controllers are described in this and following sections. These controllers do not differ from each other very much and are identical when one assumes that the value of parameter b_0 is known.

As it has been pointed in the introduction the idea of indirect adaptive control is to estimate the unknown parameters of the plant equation (1) recursively and to compute the regulator parameters using estimated plant parameters at each step. The problem of estimation and regulator parameters computation are discussed in the following two subsection.

4.1. Plant parameters estimation

Assume that coefficients of $A(q^{-1})$, $B(q^{-1})$ are unknown and only time delay d and the degrees of $A(q^{-1})$, $B(q^{-1})$ are known. Let $D(q^{-1})$ be an arbitrary monic Hurwitz polynomial[1]) and define the filtered variables $u^D(t)$, $y^D(t)$ as

$$D(q^{-1})\,u^D(t)=u(t), \quad D(q^{-1})\,y^D(t)=y(t), \quad t=0,1,2,\ldots \tag{18}$$

where all initial condition, $u^D(-1)$, $u^D(-2)$, ..., $y^D(-1)$, $y^D(-2)$, ... are assumed to be identically zero[2]).

From the plant equation (1) it follows that the filtered variables satisfy the equation

$$A(q^{-1})\,y^D(t)=B(q^{-1})\,u^D(t-d) \tag{19}$$

Rewrite the equation (19) in the form

$$y^D(t)=\theta^T\omega^D(t) \tag{20}$$

where

$$\theta=[a_1, a_2, \ldots, a_{n_a}, b_0, \ldots, b_{n_b}]^T \tag{21}$$

$$\omega^D(t)=[-y^D(t-1), -y^D(t-2), \ldots, -y^D(t-n_a), u^D(t-d), \ldots, u^D(t-d-n_b)]^T \tag{22}$$

Since the vector θ is unknown it has to be recursively estimated. The stability analysis presented in section 6 requires that the sequence of estimates $\{\theta(t)\}$ generated by a recursive algorithm fulfils the following four requirements

I.1. The sequence $\{\|\theta(t)\|\}$ is bounded.

I.2. $\lim\limits_{t\to\infty} \|\theta(t+k)-\theta(t)\|=0$ for every finite k

I.3. $\lim\limits_{t\to\infty} \dfrac{e_e(t)}{(1+\|\omega^D(t)\|^2)^{1/2}}=0$

where variable $e_e(t)$ is defined as

$$e_e(t)=y^D(t)-\theta(t)^T\omega^D(t) \tag{23}$$

and will be called an estimation error

I.4. The estimate $b_0(t)$ differs from zero for every t.

The requirements I.1—3 are satisfied by many well known estimation algorithms as recursive least squares algorithm and projection algorithm [1], stochastic approximation algorithm [6], some recursive algorithms derived via the stability theory [7], [8][3]).

1) The choice $D(q^{-1})=P(q^{-1})$ is of special interest.

2) If the initial conditions are nonzero an additional exponentially decaying term appears in the equation (19).

3) It is worth to note that algorithms mentioned (apart from the stochastic approximation algorithm) satisfy the stronger condition than required in I.3, namely

$$\lim_{t\to\infty} e_e(t)=0$$

However, in order to fulfil the requirement I.4, some modifications of the algorithm mentioned above are necessary [1]. An example of an appropriately modified projection algorithm is presented below.

Assume additionally that the sign of b_0 is known and let us assume $b_0>0$. Let β be a positive constant such that $0\leqslant\beta\leqslant b_0$. Then the following algorithm can be applied.

$$\theta'(t)=\theta(t-1)+\frac{\omega^D(t)\left(y^D(t)-\theta(t-1)^T\omega^D(t)\right)}{1+\|\omega^D(t)\|^2} \tag{24}$$

$$\theta(t)=\begin{cases}\theta'(t) \text{ if } \theta'_{n_a+1}(t)\geqslant\beta\\ [\theta'_1(t), \theta'_2(t), \ldots, \theta'_{n_a}(t), \beta, \theta'_{n_a+2}(t), \ldots, \theta'_{n_a+n_b+1}(t)]^T \text{ otherwise}\end{cases} \tag{25}$$

($\theta'_i(t)$ denotes i-th element of vector $\theta'(t)$)

In appendix A it is shown that the algorithm described above fulfils the requirements I.1—4.

4.2. Computation of regulator parameters

Define the polynomials

$$A_t(q^{-1})=1+a_1(t)q^{-1}+\ldots+a_{n_a}(t)q^{-na} \tag{26}$$

$$B_t(q^{-1})=b_0(t)+b_1(t)q^{-1}+\ldots+b_{n_b}(t)q^{-nb} \tag{27}$$

where $a_i(t)$, $b_i(t)$ are the current estimates of a_i, b_i at time t.

Let $S_t(q^{-1})$, $R_t(q^{-1})$ be the solution of the following polynomial equation

$$C(q^{-1})=S_t(q^{-1})\cdot A_t(q^{-1})+q^{-d}\cdot R_t(q^{-1}) \tag{28}$$

where according to (8) and (9) the polynomials $S_t(q^{-1})$, $R_t(q^{-1})$ have the form

$$S_t(q^{-1})=1+s_1(t)q^{-1}+\ldots+s_{d-1}(t)q^{-d+1} \tag{29}$$

$$R_t(q^{-1})=r_0(t)+r_1(t)q^{-1}+\ldots+r_{n_r}(t)q^{-nr} \tag{30}$$

Define the filtered variables $u^P(t)$, $y^P(t)$, $y_M^P(t)$ as

$$P(q^{-1})u^P(t)=u(t), \quad P(q^{-1})y^P(t)=y(t), \quad P(q^{-1})y_M^P(t)=y(t) \tag{31}$$

Therefore according to (14) the control input is determined by equation

$$b_0(t)u(t)+T_t(q^{-1})u^P(t)+R_t(q^{-1})y^P(t)-C(q^{-1})y_M^P(t+d)=0 \tag{32}$$

where

$$T_t(q^{-1})=S_t(q^{-1})\cdot B_t(q^{-1})-b_0(t)\cdot P(q^{-1}) \tag{33}$$

Since $b_0(t)\neq0$ (requirement I4) the solution of (32) with respect to $u(t)$ exists.

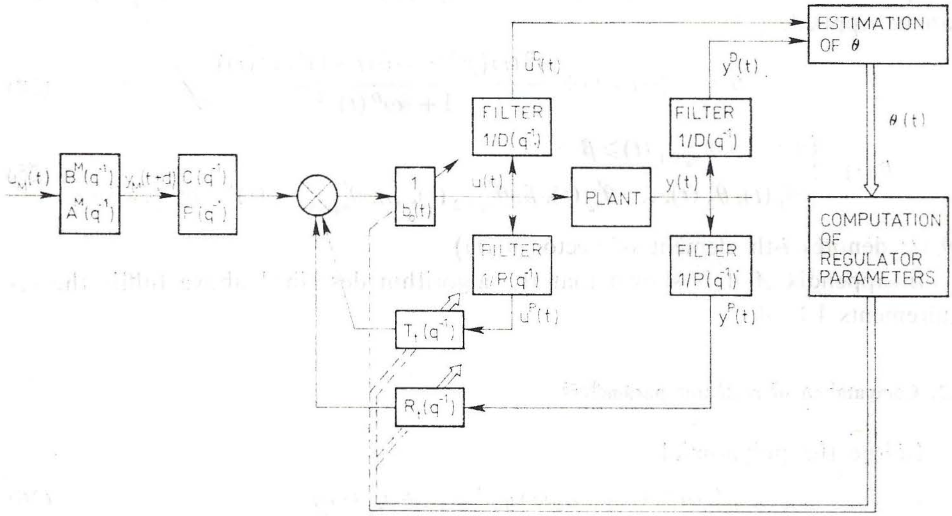
Summarizing, the adaptive controller derived in this section can be represented in the form of the following algorithm.

1. Estimate the parameters of the model (19) by recursive algorithm fulfiling I.1—4 (for instance by algorithm described by (24), (25)).

2. Using the current estimates of plant parameters compute the control input from (32).

The steps 1, 2 are repeated at each time t.

The block diagram of derived adaptive controller is presented in fig. 1.



Rys. 1

A complete analysis of the stability of adaptive system i.e. plant + adaptive controller I, is given in section 6 and appendix B. Below it is only shown that if the sequences of plant inputs and outputs are bounded (i.e. the adaptive system is stable) the output error e_0 converges to zero.

Actually, it follows from lemma B2 (see appendix B) that for special case $P(q^{-1})=D(q^{-1})$ the plant output generated by the adaptive system is such that the output error $e_0(t)$ satisfies the equation

$$C(q^{-1})\,e_0(t+d)=D(q^{-1})\,S_t(q^{-1})\,e_e(t+d)+$$

$$+D(q^{-1})\sum_{i=0}^{d-1} s_i(t)\big(\theta(t+d-i)-\theta(t)\big)^T\omega^D(t+d-i)\quad \big(s_0(t)=1\big) \tag{34}$$

The requirements I.1—4 imply that if the sequence $\{\|\omega^D(t)\|\}$ is bounded, all components of the right side of equation (34) converge to zero, and so does the output error e_0. Notice also that, in general, the convergence $e_e(t)\to 0$ does not imply that $\theta(t)\to\theta$.

5. Indirect adaptive controller II

The adaptive controller described in this section differs from that in section 4 by a slightly different way of the parametrization of plant.

Dividing the equation (19) by b_0 (we recall that $b_0\neq 0$) we obtain

$$\bar{A}(q^{-1})\,y^D(t)=\bar{B}(q^{-1})\,u^D(t-d) \tag{35}$$

where

$$\bar{A}(q^{-1})=\frac{0}{b_0}\cdot A(q^{-1})=\bar{a}_0+\bar{a}_1 q^{-1}+\ldots+\bar{a}_{n_a}q^{-n_a} \tag{36}$$

$$\bar{B}(q^{-1})=\frac{1}{b_0}\cdot B(q^{-1})=1+b_1 q^{-1}+\ldots+\bar{b}_{n_b}q^{-n_b} \tag{37}$$

Using the notation

$$\bar{\theta}=[\bar{a}_0, \bar{a}_1, \ldots, \bar{a}_{n_a}, \bar{b}_1, \ldots, \bar{b}_{n_b}]^T \tag{38}$$

$$\bar{\omega}^D(t)=[y^D(t), y^D(t-1), \ldots, y^D(t-n_a), -u^D(t-d-1), \ldots, -u^D(t-d-n_b)]^T \tag{39}$$

the equation (35) can be rewritten in the form

$$u^D(t-d)=\bar{\theta}^T\bar{\omega}^D(t) \tag{40}$$

For estimating θ from (40) let us assume that a recursive algorithm is applied which generates the sequence of estimates $\theta(t)$ satisfing the following requirements (These requirements are important to ensure the global stability of adaptive system)

II.1. The sequence $\{\|\bar{\theta}(t)\|\}$ is bounded

II.2. $\lim\limits_{t\to\infty}\|\bar{\theta}(t+k)-\bar{\theta}(t)\|=0$ for every finite k

II.3. $\lim\limits_{t\to\infty}\dfrac{e_e(t)}{(1+\|\bar{\omega}^D(t)\|^2)^{1/2}}=0$

where the estimation error is defined now as

$$\bar{e}_e(t)=u^D(t-d)-\bar{\theta}(t)^T\bar{\omega}^D(t) \tag{41}$$

II.4. There exist a positive constant α such that for every $|\bar{a}_0(t)|\geqslant\alpha$.

If we assume that the sign of a_0 is known and positive and $\bar{a}_0\geqslant\alpha>0$ for a certain known constant α the following recursive algorithm corresponding to that described by (24), (25) can be applied

$$\bar{\theta}'(t)=\bar{\theta}(t-1)+\frac{\bar{\omega}^D(t)\left(u^D(t-d)-\bar{\theta}(t-1)^T\bar{\omega}^D(t)\right)}{1+\|\bar{\omega}^D(t)\|^2} \tag{42}$$

$$\theta(t)=\begin{cases}\bar{\theta}'(t) \text{ if } \bar{\theta}_1'(t)\geqslant\alpha\\ [\alpha, \bar{\theta}_2'(t), \ldots, \bar{\theta}_{n_a+n_b+1}'(t)]^T \text{ otherwise}\end{cases} \tag{43}$$

Let, according to (15), the control input be computed from

$$u(t)+\bar{T}_t(q^{-1})u^P(t)+\bar{R}_t(q^{-1})y^P(t)-\bar{a}_0(t)\cdot C(q^{-1})y_M^P(t+d)=0 \tag{44}$$

where

$$\bar{T}_t(q^{-1})=\bar{S}_t(q^{-1})\cdot\bar{B}_t(q^{-1})-P(q^{-1}) \tag{45}$$

and $\bar{S}_t(q^{-1})$, $\bar{R}_t(q^{-1})$ are the solution of the equation

$$\bar{a}_0(t)\cdot C(q^{-1})=\bar{S}_t(q^{-1})\cdot\bar{A}_t(q^{-1})+q^{-d}\cdot\bar{R}_t(q^{-1}) \tag{46}$$

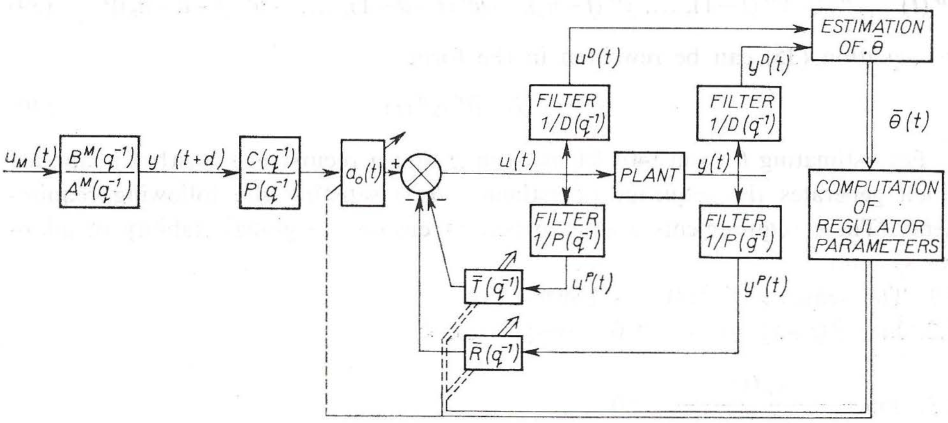
REMARKS

1) The solution of (46) always exists and is unique if $a_0(t) \neq 0$
2) The equations (45), (46) corespond to equations (10), (12) divided by b_0.

Summarizing, the following adaptive control algorithm has been obtained in this section

1. Estimate the parameters of the model (35) by recursive algorithm fulfiling II.1—4 (for instance by algorithm described by (42), (43))
2. Using the current estimates $\bar{\theta}(t)$ compute the control input from (44).

The steps 1, 2 are repeated at each time t.



Rys. 2

The block diagram of indirect adaptive controller II is presented in fig. 2. For special case $P(q^{-1}) = D(q^{-1})$ the plant output generated by the adaptive system (plant + adaptive controller II) is such that the output error $e_0(t)$ satisfies the equation

$$\bar{a}_0(t) \cdot C(q^{-1})\, e_0(t+d) = -D(q^{-1})\, \bar{S}_t(q^{-1})\, \bar{e}_e(t+d) +$$

$$+D(q^{-1}) \sum_{i=0}^{d-1} \bar{s}_i(t)\left(\bar{\theta}(t) - \bar{\theta}(t+d-i)\right)^T \bar{\omega}^D(t+d-i) \qquad (s_0(t)=1) \qquad (47)$$

Equation (47) and requirements II.1—4 ensure that if the sequence $\{\bar{\omega}^D(t)\}$ is bounded then the output error converges to zero.

REMARK Notice that in order to fulfil the requirement II.4 (for instance using the algorithm (42) (43)) an additional information about the parameter b_0 is necessary, namely a positive lower bound of $|a_0|$ i.e. an upper bound of $|b_0|$. In the case of indirect adaptive controller I the requirement I.4 is slightly less restrictive than II.4. However it is clear that in practice it is not admissible that the value $|b_0(t)|$ be too small. In this regard the requirement $|b_0(t)| > \beta$ should be fulfilled where β is a known, not too small, lower bound of b_0. (It can be done for instance by applying the algorithm (24) (25)). Now we see that to apply the adaptive controller I or II an additional information on the parameters b_0 is needed but different in either case.

6. Adaptive controllers with direct estimation of regulator parameters — direct adaptive control

In the indirect adaptive control approach the plant is parametrized in the simplest natural way. However another possibilities can also be utilized. One of them is to rewrite the plant equation in such a way that the parameters of this rewritten equation would be coincident with the regulator parameters. Such parametrization enables the ditect estimation of regulator parameters and leads to the approach which is called "direct". In the following two subsections two direct adaptive controllers corresponding to the indirect ones described previously are presented.

6.1. Direct adaptive controller I

Using (10), (19) and (12) consecutively we obtain

$$C(q^{-1})y^D(t)=S(q^{-1})A(q^{-1})y^D(t)+R(q^{-1})y^D(t-d)=$$
$$=S(q^{-1})B(q^{-1})u^D(t-d)+R(q^{-1})y^D(t-d)=b_0\cdot P(q^{-1})u^D(t-d)+$$
$$+T(q^{-1})u^D(t-d)+R(q^{-1})y^D(t-d) \tag{48}$$

Introducing notations

$$\vartheta=[b_0, t_1, t_2, \ldots, t_{n_t}, r_0, \ldots, r_{n_r}]^T \tag{49}$$

$$\varphi(t)=[P(q^{-1})u(t),\ u(t-1), \ldots, u(t-n_t),\ y(t), \ldots, y(t-n_r)]^T \tag{50}$$

we obtain

$$C(q^{-1})y^D(t+d)=\vartheta^T\varphi^D(t) \tag{51}$$

where

$$D(q^{-1})\varphi^D(t)=\varphi(t) \tag{52}$$

Notice that the elements of vector ϑ can be considered to be the parameters of the plant equation rewritten in a special way. On the other hand the elements of ϑ are also the regulator parameters. Hence by estimating the vector ϑ we estimate the regulator parameters directly.

The direct adaptive controller I can be represented in the form of the following algorithm.

1. Estimate recursively the parameters of the model (51) (i.e. the elements of ϑ)
2. Using the current estimate $\vartheta(t)$ compute the control input from (32) or equivalently from

$$\vartheta(t)^T\varphi^P(t)-C(q^{-1})y_M^P(t+d)=0 \tag{53}$$

where $P(q^{-1})\varphi^P(t)=\varphi(t)$.

The adaptive controllers similar to that described above for special case $P(q^{-1})=D(q^{-1})$ have been considered by many authors [1], [4].

6.2. Direct adaptive controller II

Dividing (48) by b_0 we obtain

$$P(q^{-1})\,u^D(t-d)=\frac{1}{b_0}\,C(q^{-1})\,y^D(t)-\frac{1}{b_0}\,T(q^{-1})\,u^D(t-d)-$$
$$-\frac{1}{b_0}R(q^{-1})\,y^D(t-d)=\bar{a}_0\,C(q^{-1})\,y^D(t)-\bar{T}(q^{-1})\,u^D(t-d)-$$
$$-\bar{R}(q^{-1})\,y^D(t-d)=\bar{\vartheta}^T\,\bar{\varphi}^D(t) \tag{54}$$

where

$$\bar{\vartheta}=[a_0,\bar{t}_1,\bar{t}_2,\ldots,\bar{t}_{n_t},\bar{r}_0,\ldots,\bar{r}_{n_r}]^T \tag{55}$$

$$\bar{\varphi}^D(t)=[C(q^{-1})\,y^D(t)\;-u^D(t-d-1),\;-u^D(t-d-2),\ldots,\;-u^D(t-d-n_t),$$
$$y^D(t-d),\ldots,y^D(t-d-n_r)]^T \tag{56}$$

Direct adaptive controller we obtain by estimating the vector $\bar{\vartheta}$ from (54) and by using $\bar{\vartheta}(t)$ to compute the control input from (44).

Similar adaptive controller (but in continuous time domain) is considered in [9].

REMARK. Notice, that $\dim\theta=\dim\bar{\theta}=n_a+n_b+1$ and $\dim\vartheta=\dim\bar{\vartheta}=n_t+n_r+$ $+2\geqslant n_a+n_b+d$. Therefore for $d>1$ the number of estimated parameters in the indirect adaptive controllers is greater than in the direct ones. This implies that among elements of vector ϑ (and $\bar{\vartheta}$) at least $d-1$ elements are not independent. However this fact is not utilized in estimation i.e. the elements of ϑ (and $\bar{\vartheta}$) are estimated as if they were all independent. So one can say that if the plant model is nonminimally parametrized (with respect to the number of parameters), some information about plant is lost. This fact may negatively affect the efficiency of estimation and also the convergence rate of the output error to zero. Therefore for large d it may be preferable to use the indirect controller rather than the direct one.

7. Some stability and convergence results

In this section some stability and convergence results are given. It is shown that the proposed adaptive control methods assure the boundedness of the sequences $\{u(t)\}$, $\{y(t)\}$ and the convergence of output error to zero. This features mean that the control objectives stated in section 2 are accomplished.

Analysis presented is based on Goodwin's, Ramadge's and Caines's concept [1]. First, two lemmas are given

Appendix A

Define

$$\varepsilon(t)=y^D(t)-\theta(t-1)^T\omega^D(t) \tag{A1}$$

and

$$V(t)=\|\theta-\theta(t)\|^2 \tag{A2}$$

From (24), (25) and (A1) we obtain

$$V(t)-V(t-1)=\|\theta-\theta(t)\|^2-\|\theta-\theta(t-1)\|^2\leqslant$$

$$\leqslant\|\theta-\theta'(t)\|^2-\|\theta-\theta(t-1)\|^2=$$

$$=\left\|\theta-\theta(t-1)-\frac{\omega^D(t)\,\varepsilon(t)}{1+\|\omega^D(t)\|^2}\right\|^2-\|\theta-\theta(t-1)\|^2=$$

$$=\frac{\|\omega^D(t)\|^2\,\varepsilon^2(t)}{(1+\|\omega^D(t)\|^2)^2}-\frac{2\varepsilon(t)(\theta-\theta(t-1))^T\omega^D(t)}{1+\|\omega^D(t)\|^2}=$$

$$=\frac{\|\omega^D(t)\|^2\,\varepsilon^2(t)}{(1+\|\omega^D(t)\|^2)^2}-\frac{2\varepsilon^2(t)}{1+\|\omega^D(t)\|^2}\leqslant-\frac{\varepsilon^2(t)}{1+\|\omega^D(t)\|^2}\leqslant 0 \tag{A3}$$

Hence $V(t)\leqslant V(0)$ and consequently the sequence $\{\theta(t)\}$ is bounded (the requirement I.1).

Since the function $V(t)$ is bounded from below it follows from (A3) that

$$\lim_{t\to\infty}\frac{\varepsilon(t)}{(1+\|\omega^D(t)\|^2)^{1/2}}=0 \tag{A4}$$

Now, using (24), (25), we obtain

$$\|\theta(t+1)-\theta(t)\|\leqslant\|\theta'(t+1)-\theta(t)\|=$$

$$=\frac{|\varepsilon(t+1)|}{(1+\|\omega^D(t+1)\|^2)^{1/2}}\cdot\frac{\|\omega^D(t+1)\|}{(1+\|\omega^D(t+1)\|^2)^{1/2}}\leqslant\frac{|\varepsilon(t+1)|}{(1+\|\omega^D(t+1)\|^2)^{1/2}} \tag{A5}$$

Hence, using (A4)

$$\lim_{t\to\infty}\|\theta(t+k)-\theta(t)\|\leqslant\lim_{t\to\infty}\sum_{i=1}^{k}\|\theta(t+i)-\theta(t+i-1)\|\leqslant$$

$$\lim_{t\to\infty}\sum_{i=1}^{k}\frac{|\varepsilon(t+i)|}{(1+\|\omega^D(t+i)\|^2)^{1/2}}=0 \tag{A6}$$

This implies that the requirement I.2 holds.
Furthermore, from definitions (A1) and (23) we have

$$e_e(t)=\varepsilon(t)-(\theta(t)-\theta(t-1))^T\omega^D(t) \tag{A7}$$

Hence, using (A4) and (A6)

$$\lim_{t\to\infty}\frac{e_e(t)}{(1+\|\omega^D(t)\|^2)^{1/2}}=\lim_{t\to\infty}\frac{\varepsilon(t)}{(1+\|\omega^D(t)\|^2)^{1/2}}+$$
$$+\lim_{t\to\infty}\left[(\theta(t)-\theta(t-1))^T\cdot\frac{\omega^D(t)}{(1+\|\omega^D(t)\|^2)^{1/2}}\right]=0 \qquad \text{(A8)}$$

and so the requirement I.3 holds.

Fulfilment of I.4 follows directly from (25).

Appendix B

First, two lemmas will be proved.

LEMMA B1. *The solution* $S(q^{-1})$, $R(q^{-1})$ *of polynomial equation* (10) *always exists, is unique and the function* Ω *defined as* $\Omega:(a_1, a_2, \ldots, a_{n_a})\,(s_1, \ldots, s_{d-1}, r_0, \ldots, r_{n_r})$ *is continuous.*

P r o o f. Rewrite the equation (10) in the matrix form

$$\begin{bmatrix}
1 \\
a_1 & 1 \\
a_2 & a_1 & 1 \\
\vdots & \vdots & \vdots & \ddots \\
a_{d-1} & a_{d-2} & \cdots & a_1 & 1 \\
a_d & a_{d-1} & & & a_1 & 1 \\
a_{d+1} & \vdots & & & a_2 & 0 & 1 \\
\vdots & & & & a_3 & 0 & 0 & 1 \\
 & & & & \vdots & \vdots & & & 1 \\
 & & & & & 0 & 0 & & 0 & 1
\end{bmatrix}
\begin{bmatrix} 1 \\ s_1 \\ s_2 \\ \vdots \\ s_{d-1} \\ r_0 \\ r_1 \\ \vdots \\ r_{n_r} \end{bmatrix}
=
\begin{bmatrix} 1 \\ c_1 \\ c_2 \\ \vdots \\ c_{d-1} \\ \vdots \end{bmatrix} \qquad \text{(A9)}$$

Since the determinant of triangular matrix in (A9) is always equal to 1 so the solution of equation (9) exists and is unique. The continuity of Ω is obvious. ■

COROLLARY *Let* $\{R_t(q^{-1})\}$, $\{S_t(q^{-1})\}$ *be the sequences of solutions of* (28) *and let* $\{T_t(q^{-1})\}$ *be determined by* (33). *If the requirements* I.1—2 *are fulfilled then for* $i=1, \ldots, d-1$, $j=0, 1, \ldots, n_r$, $k=1, \ldots, n_t$

1. *the time sequences* $\{s_i(t)\}$, $\{r_j(t)\}$, $\{t_k(t)\}$ *are bounded,*
2. $\lim\limits_{t\to\infty}(s_i(t+m)-s_i(t))=0$, $\lim\limits_{t\to\infty}(r_j(t+m)-r(t))=0$,
   $\lim\limits_{t\to\infty}(t_k(t+m)-t(t))=0$ *for every finite* m

LEMMA B2. *Let the vectors* ϑ *and* $\varphi(t)$ *be defined by* (49), (50) *respectively, and let* $\vartheta(t)$ *be a vector of regulator parameters corresponding to* ϑ *and computed* *at time t. The plant output generated by the adaptive system (plant + adaptive controller* I) *is such that the output error* $e_0(t)$ *satisfies the equation.*

$$C(q^{-1})\, e_0(t+d) = D(q^{-1})\, S_t(q^{-1})\, e_e(t+d) +$$

$$+ \sum_{j=0}^{n_d} d_i \sum_{i=0}^{d-1} s_i(t-j)\,(\theta(t+d-i-j)-\theta(t-j))^T \omega^D(t+d-i-j) +$$

$$+ \sum_{i=1}^{n_d} d_i\,(\vartheta(t-i)-\vartheta(t))^T \varphi^D(t-i) - \sum_{i=1}^{n_p} p_i\,(\vartheta(t-i)-\vartheta(t))\, \varphi^P(t-i)$$

$$(d_0=1,\ s_0(t)=1) \quad \text{(A10)}$$

Proof: Using definitions of vectors $\vartheta(t)$, $\varphi(t)$ the equation (32) can be rewritten in the form

$$C(q^{-1})\, y_M^P(t+d) = b_0(t)\, u(t) + T_t(q^{-1})\, u^P(t) + R_t(q^{-1})\, y^P(t) = \vartheta(t)^T \varphi^P(t) \quad \text{(A11)}$$

We have

$$P(q^{-1})(\vartheta(t)^T \varphi^P(t)) = \vartheta(t)^T (P(q^{-1})\, \varphi^P(t)) +$$

$$+ (P(q^{-1})\, \vartheta(t) - \vartheta(t)\, P(q^{-1}))^T \varphi^P(t) = \vartheta(t)^T \varphi(t) +$$

$$+ \sum_{i=1}^{n_p} p_i\,(\vartheta(t-i)-\vartheta(t))^T \varphi^P(t-i) \quad \text{(A12)}$$

Similarly

$$D(q^{-1})(\vartheta(t)^T \varphi^D(t)) = \vartheta(t)^T \varphi(t) + \sum_{i=1}^{n_d} d_i(\vartheta(t-i)-\vartheta(t))^T \varphi^D(t-i) \quad \text{(A13)}$$

From (A11), (A12), (A13) we obtain

$$D(q^{-1})\,[\vartheta(t)^T \varphi^D(t) - C(q^{-1})\, y_M^D(t+d)] =$$

$$= \sum_{i=1}^{n_d} d_i\,(\vartheta(t-i)-\vartheta(t))^T \varphi^D(t-i) - \sum_{i=1}^{n_p} p_i\,(\vartheta(t-i)-\vartheta(t))\, \varphi^P(t-i) \quad \text{(A14)}$$

From (23) we have

$$y^D(t) = \theta(t)^T \omega^D(t) + e_e(t) \quad \text{(A15)}$$

or equivalently

$$A_t(q^{-1})\, y^D(t) = B_t(q^{-1})\, u^D(t-d) + e_e(t) \quad \text{(A16)}$$

Now, using (28), (A16), (32) consecutively we obtain

$$C(q^{-1})\, e_0^D(t+d) = C(q^{-1})\, y^D(t+d) - C(q^{-1})\, y_M^D(t+d) =$$

$$= S_t(q^{-1}) \cdot A_t(q^{-1})\, y^D(t+d) + R_t(q^{-1})\, y^D(t) - C(q^{-1})\, y_M^D(t+d) =$$

$$= S_t(q^{-1})\, A_{t+d}(q^{-1})\, y^D(t+d) + R_t(q^{-1})\, y^D(t) - C(q^{-1})\, y_M^D(t+d) + \ldots$$

$$+(S_t(q^{-1})\cdot A_t(q^{-1})-S_t(q^{-1})A_{t+d}(q^{-1}))\,y^D(t+d)=$$
$$=S_t(q^{-1})B_{t+d}(q^{-1})\,u^D(t)+R_t(q^{-1})\,y^D(t)-C(q^{-1})\,y_M^D(t+d)+$$
$$+S_t(q^{-1})\,e_e(t+d)+(S_t(q^{-1})\cdot A_t(q^{-1})-S_t(q^{-1})A_{t+d}(q^{-1}))\,y^D(t+d)=$$
$$=S_t(q^{-1})\cdot B_t(q^{-1})\,u^D(t)+R_t(q^{-1})\,y^D(t)-C(q^{-1})\,y_M^D(t+d)+$$
$$+S_t(q^{-1})\,e_e(t+d)+(S_t(q^{-1})\cdot A_t(q^{-1})-S_t(q^{-1})A_{t+d}(q^{-1}))\,y^D(t+d)-$$
$$-(S_t(q^{-1})\cdot B_t(q^{-1})-S_t(q^{-1})B_{t+d}(q^{-1}))\,u^D(t)=$$
$$=b_0(t)P(q^{-1})\,u^D(t)+T_t(q^{-1})\,u^D(t)+R_t(q^{-1})\,y^D(t)-C(q^{-1})\,y_M^D(t+d)+$$
$$+S_t(q^{-1})\,e_e(t+d)+\sum_{i=0}^{d-1}s_i(t)(\theta(t+d-i)-\theta(t))^T\omega^D(t+d-i)=$$
$$=\vartheta(t)^T\varphi^D(t)-C(q^{-1})\,y_M^D(t+d)+S_t(q^{-1})\,e_e(t+d)+$$
$$+\sum_{i=0}^{d-1}s_i(t)(\theta(t+d-i)-\theta(t))^T\omega^D(t+d-i) \qquad \text{(A17)}$$

Multiplying equation (A17) by $D(q^{-1})$ and using (A14) we obtain equation (A10). ■

Proof of theorem 1. As it was pointed earlier, the fulfilment of the three assumptions of lemma 2 is only to be shown. This is done in the following three points respectively.

1. Define a new time-varying vector $\psi(t+d)$ as a vector constructed from all different elements of the following vectors: $\omega^D(t+d-i)$ for $i=0,1,\ldots,d+n_a-1$, $\varphi^D(t-i)$ for $i=1,\ldots,n_a$ and $\varphi^P(t-i)$ for $i=1,\ldots,n_p$. Using this vector, the equation (A10) can be presented in the form

$$C(q^{-1})\,e_0(t+d)=D(q^{-1})\,S_t(q^{-1})\,e_e(t+d)+\sigma(t+d)^T\psi(t+d) \qquad \text{(A18)}$$

where $\sigma(t+d)$ is the appropriate time-varying vector of the same dimension as $\psi(t+d)$. From the corollary it follows that

$$\lim_{t\to\infty}\|\sigma(t)\|=0 \qquad \text{(A19)}$$

Let $v_i(t)\,(i=0,1,\ldots,n_a+d-1)$ be the coefficients of polynomial $D(q^{-1})\,S_t(q^{-1})$ and denote $K'=\sup_t \max_{0\leq i\leq n_a+d-1}|v_i(t)|$. From (A18) we obtain

$$e_0(t+d)|=|C(q^{-1})\,e_0(t+d)|\leq|D(q^{-1})\,S_t(q^{-1})\,e_e(t+d)|+$$
$$+|\sigma(t+d)^T\psi(t+d)|\leq(n_a+d-1)\cdot K'\max_{0\leq i\leq n_a+d-1}|e_e(t+d-i)|+$$
$$+\|\sigma(t+d)\|\cdot\|\psi(t+d)\|\leq K\cdot\max_{0\leq i\leq N}|e_e(t+d-i)|+\eta(t+d)\cdot\|\psi(t+d)\| \qquad \text{(A20)}$$

where

$$K=K'\cdot(n_a+d-1),\qquad \eta(t)=\|\sigma(t)\|,\qquad N=n_a+d-1 \qquad \text{(A22)}$$

Hence, the assumption 1 of lemma 2 holds.

LEMMA 1. [1]. *Let $\{e_0^f(t)\}$, $\{\psi(t)\}$ be some real scalar and vector sequences, respectively, and assume that*

1. $$\lim_{t\to\infty}\frac{e_0^f(t)}{(1+\|\psi(t)\|^2)^{1/2}}=0 \tag{57}$$

2. *for some positive constants K_1, K_2*
$$\|\psi(t)\|\leqslant K_1+K_2\cdot\max_{0\leqslant\tau\leqslant n}|e_0^f(\tau)| \text{ for every } t\geqslant 0 \tag{58}$$

Then $\lim_{t\to\infty} e_0^f(t)=0$ and the sequence $\{\psi(t)\}$ is bounded.

The next lemma is essential for stability proofs of adaptive schemes presented in this paper.

LEMMA 2. *Let the output error be determined by (1)—(5). Assume that for a positive constant K, a positive integer N, a scalar real sequence $\{\eta(t)\}$ subject to $\lim_{t\to\infty}\eta(t)=0$ and a vector sequence $\{\psi(t)\}$ the following conditions are fulfilled*

1. $$|e_0^f(t)|\leqslant K\cdot\max_{0\leqslant i\leqslant N}|e_e(t-i)|+\eta(t)\cdot\|\psi(t)\| \tag{59}$$
where $e_0^f(t)=C(q^{-1})e_0(t)$ and $e_e(t)$ is the estimation error defined by (23) or (41)

2. $$\lim_{t\to\infty}\frac{e_e(t-i)}{(1+\|\psi(t)\|^2)^{1/2}}=0 \text{ for } 0\leqslant i\leqslant N \tag{60}$$

3. *For some positive constants K_1, K_2 the linear boundedness condition (58) is satisfied.*

Then the sequences $\{u(t)\}$, $\{y(t)\}$ are bounded and $\lim_{t\to\infty} e_0(t)=0$

Proof. We conclude from (60) that

$$\lim_{t\to\infty}\frac{\max_{0\leqslant i\leqslant N}|e_e(t-i)|}{(1+\|\psi(t)\|^2)^{1/2}}=0 \tag{61}$$

Now using (59) and (61) we obtain

$$\lim_{t\to\infty}\frac{|e_0^f(t)|}{(1+\|\psi(t)\|^2)^{1/2}}\leqslant K\cdot\lim_{t\to\infty}\frac{\max_{0\leqslant i\leqslant N}|e_e(t-i)|}{(1+\|\psi(t)\|^2)^{1/2}}+\lim_{t\to\infty}\eta(t)\cdot\frac{\|\psi(t)\|}{(1+\|\psi(t)\|^2)^{1/2}}=0 \tag{62}$$

Thus the sequences $\{e_0^f(t)\}$ and $\{\psi(t)\}$ fulfil the assumption of lemma 1. Therefore $\lim_{t\to\infty} e_0^f(t)=0$. Since $e_0^f(t)=C(q^{-1})e_0(t)$ and $C(q^{-1})$ is Hurwitz polynomial then also

$$\lim_{t\to\infty} e_0(t)=0 \tag{63}$$

In section 2 it was assumed that the command input $u_M(t)$ is bounded. Since $y_M(t)=\frac{q^{-d}(B^M(q^{-1}))}{A^M(q^{-1})}u_M(t)$ and $A^M(q^{-1})$ is Hurwitz polynomial, the output of the model $y_M(t)$ is also bounded, and then from (5) and (63), the plant output, too. Therefore the plant input is also bounded because $u(t)=\frac{A(q^{-1})}{B(q^{-1})}y(t+d)$ and $B(q^{-1})$ is Hurwitz polynomial. ∎

In order to prove the stability of particular adaptive schemes the fulfilment of assumptions of lemma 2 is only to be shown. In this way the following two theorems can be proved.

THEOREM 1. *If the requirements* I.1—4 *are fulfilled, the indirect adaptive controller* I *described in section 4 assures both the uniform boundedness of plant input and output and convergence*

$$\lim_{t\to\infty} e_0(t)=0$$

For proof see appendix B.

THEOREM 2. *If the requirements* II.1—4 *are fulfilled, the indirect adaptive controller* II *described in section 5 assures both the boundedness of plant input and output and convergence*

$$\lim_{t\to\infty} e_0(t)=0$$

The proof of theorem 2 is very similar to that of theorem 1 and will not be given in this paper.

REMARK. Although the indirect adaptive controller has been analysed only the same method as in appendix B can be applied to the stability analysis of adaptive controllers based on any other way of parametrization.

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2. For $0 \leqslant i \leqslant N$ we have

$$\frac{|e_e(t-i)|}{(1+\|\psi(t)\|^2)^{1/2}} = \frac{|e_e(t-i)|}{(1+\|\omega^D(t-i)\|^2)^{1/2}} \cdot \left(\frac{1+\|\omega^D(t-i)\|^2}{1+\|\psi(t)\|^2}\right)^{1/2} \leqslant$$

$$\leqslant \frac{|e_e(t-i)|}{(1+\|\omega^D(t-i)\|^2)^{1/2}} \quad \text{(A23)}$$

because $\|\omega^D(t-i)\| \leqslant \|\psi(t)\|$.

Now, using requirement I.3, the assumption 2 of lemma 2 follows.

3. Denote $y^f(t) = C(q^{-1})y(t)$ and $y_M^f(t) = C(q^{-1})y(t)$. Let $Q_i(z^{-1})$ be a transfer function specified by equation $\psi_i(t) = Q_i(q^{-1})y^f(t)$ ($\psi_i(t)$ denotes the i-th element of $\psi(t)$). From the definition of vector ψ and from the fact that the plant is minimumphase it follows that for all elements $\psi_i(t)$ the transfer function $Q_i(z^{-1})$ is proper and asymptoticaly stable. Therefore from the known property of dynamic systems [1], for each element $\psi_i(t)$ there exist some positive constants L_{2i}, L_{2i+1} that

$$|\psi_i(t)| \leqslant L_{2i} + L_{2i+1} \max_{0 \leqslant \tau \leqslant t} |y^f(\tau)| \text{ for every } t \geqslant 0 \quad \text{(A24)}$$

Hence, for some positive constants K_1, K_2 some

$$\|\psi(t)\| \leqslant \bar{K}_1 + \bar{K}_2 \max_{0 \leqslant \tau \leqslant t} |y^f(t)| \text{ for every } t \geqslant 0 \quad \text{(A25)}$$

Since the command input is bounded so is the filtered output of the model $y_M^f(t)$, i.e. $|y_M^f(t)| \leqslant \bar{K}_3$ for a certain positive $\bar{K}_3$. Hence

$$|y^f(t)| = |e_0(t) + y_M^f(t)| \leqslant |e_0^f(t)| + |y_M^f(t)| \leqslant |e_0^f(t)| + \bar{K}_3 \quad \text{(A26)}$$

Now, substituting (A26) into (A25) the linear boundedness condition (58) can be obtained. Hence the assumption 3 of lemma 2 holds, too. ■

Regulatory adaptacyjne dostrajania do modelu dla dyskretnych w czasie obiektów minimalnofazowych

W literaturze rozróżnia się obecnie dwa podstawowe podejścia do problemu adaptacyjnego sterowania obiektów o nieznanych parametrach. Podejścia te prowadzą do dwóch różnych metod projektowania układów adaptacyjnych: metody pośredniej i metody bezpośredniej. W metodzie pośredniej parametry regulatora są wyznaczone w każdej dyskretnej chwili czasu na podstawie bieżącej estymaty parametrów obiektu. W metodzie bezpośredniej natomiast, estymowane są bezpośrednio parametry regulatora.

W niniejszej pracy przedstawiono w zunifikowanym ujęciu cztery typy regulatorów adaptacyjnych dla obiektów minimalnofazowych, z których dwa są pośrednie, a pozostałe dwa — bezpośrednie. Jeden z przedstawionych typów, nazywany w pracy pośrednim regulatorem adaptacyjnym II, stanowi pewną nową propozycję. Ponieważ w literaturze częściej są rozważane regulatory bezpośrednie w niniejszej pracy większy nacisk położono na regulatory pośrednie. Opierając się o koncepcje zawarte w pracy [1] pokazano, że przedstawione regulatory pośrednie zapewniają stabilność układu i zbieżność błędu wyjściowego (różnicy między wyjściem obiektu a wyjściem modelu) do zera.

Адаптивные регуляторы настройки по модели для минимальнофазовых дискретных по времени объектов

В настоящее время различаются в литературе два основных подхода к проблеме адаптивного управления объектов с неизвестными параметрами. Эти подходы ведут к двум различным методам проектирования адаптивных систем: косвенному и прямому. В косвенном методе параметры регулятора определяются для каждого дискретного момента времени на основании текущей оценки параметров объекта. Вместо этого в прямом методе параметры регулятора оцениваются непосредственно.

В статье представлены в единой постановке четыре типа адаптивных регуляторов, два из которых косвенные, а остальные два — прямые. Один из рассмотренных типов, называемый в статье косвенным адаптивным регулятором II, представляет собой новое решение. Поскольку в литературе рассматриваются главным образом прямые регуляторы, в настоящей статье большее внимание уделяется косвенным регуляторам. Основываясь на идеях представленных в работе [1] показано, что рассмотренные косвенные регуляторы обеспечивают устойчивость системы и сходимость выходной ошибки (разницы между выходом объекта и выходом модели) к нулю.