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# A formulation of fuzzy linear programming problem based on comparison of fuzzy numbers

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The paper aims to formulate Fuzzy Linear Programming (FLP) Problems with linear fuzzy constraints based upon the inequality relation between fuzzy numbers. The paper deals with two kinds of FLP problems, [A] and [B], which are written in the form of LP problems as follows. Problem [A] is to find an optimal solution $\mathbf{x}$ that maximizes $\mathbf{cx}$ subject to $a_1 x_1 + \ldots + a_n x_n \gtrsim b$, where $a$, $b$ and $c$ are fuzzy numbers. Problem [B] is to find an optimal fuzzy solution $x$ that maximizes $\mathbf{c}x$ subject to $a, x, + \ldots + a_n x_n \gtrsim b$. Problem [A] generates optimal crisp solution under consideration of possibility of coefficients. Problem [B] generates optimal fuzzy solution reflecting the possibility of coefficients in the right-hand-side of Problem [B]. These FLP problems can be reduced to conventional LP problems by defining the inequality and maximization with respect to the fuzzy numbers.

Keywords: fuzzy linear programming; fuzzy number.

## 1. Introduction

In fuzzy decision making problems the concept of a maximizing decision was proposed by Bellman & Zadeh [1]. By adopting this concept to a mathematical programming problem, we formulated the so-called "Fuzzy Mathematical Programming" [2] and showed that a compromise solution of the decision maker (DM) could be obtained through an iterative use of linear programming technique. Zimmermann [3] also presented a fuzzy approach to multi-objective linear programming problem.

Fuzzy Linear Programming (FLP) problem with fuzzy coefficients was formulated by Negoita [4] and called robust programming. Dubois & Prade [5] investigated linear fuzzy constraints. We also proposed a formulation of FLP with fuzzy coefficients [6].

In this paper two FLP Problems with fuzzy coefficients are formulated based upon the inequality relation between fuzzy numbers. Problem [A] is to find a crisp non-fuzzy solution **x** that maximizes $y=\mathbf{c}\mathbf{x}$ subject to $A\mathbf{x}<b$ and Problem [B] is to find a fuzzy solution $x$ that maximizes $y=cx$ subject to $Ax<b$. This fuzzy solution means a possibility distribution of solutions in Problem [B].

Since the FLP problem [A] takes the possibility distribution of coefficients into consideration, its solution is robust with regard to the uncertainty of the model, compared with the solution of the conventional LP problem. The FLP problem [B] provides us with possibilities of the solutions which reflects fuzziness of the coefficients.

Fuzzy sets are restricted to those with triangular membership functions. Owing to this simplification, the FLP problem can be transformed into a conventional LP problem with twice the number of constraints of the FLP problem. In modeling a real LP problem, coefficients in the LP problem are usually inexact, vague, ill-defined or in short fuzzy so that the FLP problems with fuzzy coefficients would be more adequate to describe reality. This approach seems tractable and applicable to real world decision problems where human estimation is used. Numerical examples are described to explain our FLP problems.

## 2. Possibility measure and its calculation

Fuzzy sets can be regared as a possibility distribution which is a fuzzy restriction [7]. Given a fuzzy set $F$ whose membership function $\mu_F(x)$ is normal, a possibility function $\pi_X(x)$ is defined as $\pi_X(x) \triangleq \mu_F(x)$.

DEFINITION 1. A possibility measure of a set $E$ is defined as

$$\pi_X(E)=\sup_{X \in E} \pi_X(x) \tag{1}$$

A possibility measure has the following properties:

$$\text{(i)} \quad \pi_X(\varphi)=0, \quad \pi_X(x)=1 \tag{2}$$

$$\text{(ii)} \quad \pi_X(E_1 \cup E_2)=\pi_X(E_1) \vee \pi_X(E_2) \tag{3}$$

A possibility space is represented by $(X, P(X), \pi_X(\cdot))$, where $P(X)$ is a set of all subsets of $X$. Let us consider two sets $X$ and $Y$, and a function $f: X \to Y$. A possibility space $(Y, P(Y), \pi_Y(\cdot))$ can be induced from a given possibility space as follows.

Denoting $E=\{x|y=f(x)\}$, a possibility distribution function of $y$ is induced from $\pi_X(\cdot)$ as

$$\pi_Y(y)=\pi_X(E) \tag{4}$$

This relation is analogous to that of probability $P_Y$ on $Y$ induced by the mapping $f: X \rightarrow Y$ and probability $P_X$ on $X$. Eq. (4) can be rewritten by the definition of possibility measure as

$$\pi_Y(y) = \sup_{\{x \mid y = f(x)\}} \pi_X(x) \tag{5}$$

Eq. (5) is similar to that of probability measure where the operator "Sup" is replaced by "$\Sigma$" (sum).

DEFINITION 2. An $N$-ary possibility distribution $\pi_{\mathbf{X}}(\mathbf{x})$ on $\mathbf{X} = X_1 \, x \ldots x \, X_n$ is defined as

$$\pi_{\mathbf{X}}(\mathbf{x}) = \min_j \, [\pi_{X_j}(x_j)] \tag{6}$$

where $\pi_X(\mathbf{x})$ is separable.

Eq. (6) is analogous to the joint probability distribution in which the "min" operator is replaced by "$\cdot$" (multiplication), when $X_i$ and $X_j$ $(i \neq j)$ are independent ($P_{X_i \cap X_j} = P_{X_i} \cdot P_{X_j}$).

A fuzzy function whose coefficients are fuzzy numbers is denoted by $f(\mathbf{x}, \boldsymbol{a})$. By Definition 1, the fuzzy number $y = f(\mathbf{x}, \boldsymbol{a})$ can be calculated as follows

$$\mu_Y(y) = \sup_{\{\boldsymbol{a} \mid y = f(\mathbf{x}, \boldsymbol{a})\}} \min \, [\mu_{a_j}(a_j)] \tag{7}$$

where $\mu_y(y) = 0$ for the case of $\{\boldsymbol{a} \mid y^f = f(\mathbf{x}, \boldsymbol{a}^f)\} = \varphi$.

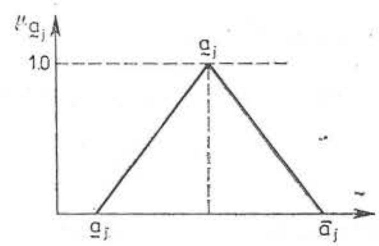


Fig. 1. Example of a fuzzy set $a$ with a triangular membership function.

It should be noted that the above gives an explanation of the extension principle [8], [9] from the viewpoint of possibility measures. For the sake of simplification fuzzy coefficients are restricted to those with triangular membership functions (see Fig. 1). The membership functions of these fuzzy coefficients are represented as

$$\mu_{a_j}(a_j) = \begin{cases} 1 - |2a_j - ({}^0a_j + {}_0a_j)| / ({}^0a_j - {}_0a_j); & {}_0a_j \leqslant a_j \leqslant {}^0a_j \\ 0 & ; \text{ otherwise} \end{cases} \tag{8}$$

where ${}^0a_j$, ${}_0a_j$ denote the upper limit and lower limit of the 0-level set of $a_j$, respectively.

Let $a_j \triangleq ({}_0a_j, {}^0a_j)$ denote a triangular fuzzy set.

THEOREM 1. *The membership function of a fuzzy linear function*

$$y = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \triangleq \boldsymbol{a}\mathbf{x} \tag{9}$$

is

$$\mu_y(y)=\begin{cases} 1-|2y-({}^0\mathbf{a}+{}_0\mathbf{a})\,\mathbf{x}|/({}^0\mathbf{a}-{}_0\mathbf{a})\,|\mathbf{x}|\,; & \mathbf{x}\neq 0 \\ 1 & ;\ \mathbf{x}=0,\ y=0 \\ 0 & ;\ \mathbf{x}=0,\ y\neq 0 \end{cases} \tag{10}$$

*where* ${}^0\mathbf{a}=[{}^0a_1, \ldots, {}^0a_n]$, ${}_0\mathbf{a}=[{}_0a_1, \ldots, {}_0a_n]$, $|\mathbf{x}|=[|x_1|, \ldots, |x_n|]^t$.

From Eq. (10) the $h$-level set of the fuzzy set $y$ is as follows:

$$R_h(y)=[\tfrac{1}{2}\{(1-h)({}^0\mathbf{a}-{}_0\mathbf{a})\,|\mathbf{x}|+({}^0\mathbf{a}+{}_0\mathbf{a})\,\mathbf{x}\},\ \tfrac{1}{2}\{(h-1)({}^0\mathbf{a}-{}_0\mathbf{a})\,|\mathbf{x}|+ +({}^0\mathbf{a}+{}_0\mathbf{a})\,\mathbf{x}\}] \tag{11}$$

Assuming that $\mathbf{x}\geqslant 0$, we obtain

$$R_h(y)=[\{(1-h/2){}^0\mathbf{a}+(h/2){}_0\mathbf{a}\}\,\mathbf{x},\ \{(h/2){}^0\mathbf{a}+(1-h/2){}_0\mathbf{a}\}\,\mathbf{x}] \tag{12}$$

Thus the possibility of $y$ is easily calculated in the case of a linear function.

## 3. Crisp solution of FLP problem [problem A]

The problem here is to obtain a crisp solution which reflects the ambiguity of fuzzy coefficients in the FLP problem. Consider the FLP problem whose coefficients are all fuzzy numbers. The FLP problem [A] can be written as

$$\begin{aligned} &\boldsymbol{a}_j\,\mathbf{x} \gtrsim^h b_i \quad (i=1, \ldots, m) \\ &\max\ y=\mathbf{c}\mathbf{x} \end{aligned} \tag{13}$$

where $\boldsymbol{a}_i=[a_{il}, \ldots, a_{in}]$, $\boldsymbol{c}=[c_1, \ldots, c_n]$, $\mathbf{x}=[x_1, \ldots, x_n]^t$ and level $h$ is specified by the decision maker a priori.

"Fuzzy max" $c=\max[a, b]$, is defined by the extension principle as [9]

$$\mu_C(c)=\max_{\{a,\, b\,|\, C=\max(a,\, b)\}} \min[\mu_a(a),\ \mu_b(b)]. \tag{14}$$

Definition 3. "$a$ is greater than $b$" is defined as

$$a\geqslant b \Leftrightarrow \max[a, b]=a \tag{15}$$

Let us depict an $h$-level set of $a$ as $R_h(a) \triangleq [({}^0a)_h, ({}_0a)_h]$, where $({}^0a)_h$ and $({}_0a)_h$ denote the upper and lower limit of the $h$-level set of $a$.

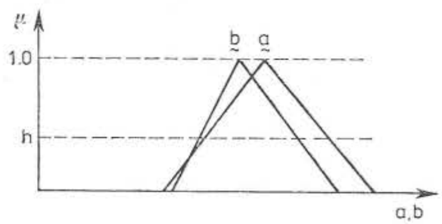


Fig. 2. Order relation between fuzzy sets $a \gtrsim^h b$, by definition 2.

From Definition 3 let us denote $a \gtrsim^h b$, if $({}^0a)_k > ({}^0b)_k$ and $({}_0a)_k > ({}_0b)_k$ hold for all $k \in [h, 1]$ as in Fig. 2. Level $h$ corresponds to the degree of optimism of the decision maker.

DEFINITION 4. We define the maximization of a fuzzy set $y$ as follows

$$\max y \Leftrightarrow \max (w_1\ {}^0y + w_2\ {}_0y) \tag{16}$$

where $w_1 + w_2 = 1$ and $w_1, w_2 \in [0, 1]$.

By Definitions 3 and 4, and Theorem 1, the FLP problem [A] can be reduced to the following conventional LP problem:

$$\begin{aligned} &\max (w_1\ {}^0\mathbf{c} + w_2\ {}_0\mathbf{c})\ \mathbf{x} \\ &\text{subject to} \\ &\quad \{(1-h/2)\ {}^0\mathbf{a}_i + (h/2)\ {}_0\mathbf{a}_i\}\ \mathbf{x} \leqslant (1-h/2)\ {}^0b_i + (h/2)\ {}_0b_i \\ &\quad \{(h/2)\ {}^0\mathbf{a}_i + (1-h/2)\ {}_0\mathbf{a}_i\}\ \mathbf{x} \leqslant (h/2)\ {}^0b_i + (1-h/2)\ {}_0b_i \qquad (i=1, \ldots, m) \end{aligned} \tag{17}$$

Example 1

Let us consider the following fuzzy linear programming problem.

$$\begin{aligned} &\max y = 25x_1 + 18x_2 \\ &\text{subject to} \\ &\quad 15x_1 + 34x_2 \gtrsim^h 800 \\ &\quad 20x_1 + 10x_2 \gtrsim^h 430 \end{aligned} \tag{18}$$

where $h = 0.4$ is specified by the decision maker. The above inequalities and objective function can be written in the vector form as:

$$\begin{aligned} &\max y = \mathbf{c}\mathbf{x} \\ &\text{subject to} \\ &\quad \boldsymbol{a}_1\ \mathbf{x} \gtrsim^h b_1 \\ &\quad \boldsymbol{a}_2\ \mathbf{x} \gtrsim^h b_2 \end{aligned} \tag{19}$$

where the fuzzy sets $\boldsymbol{a}_i$, $b_i$ and $\boldsymbol{c}$ are assumed to be

$$\begin{aligned} &a_{11} = (12, 18), \ a_{12} = (32, 36), \ b_1 = (750, 850) \\ &a_{21} = (19, 21), \ a_{22} = (7, 13), \quad b_2 = (380, 480) \end{aligned}$$

$c_1 = (23, 27)$, $c_2 = (7, 9)$.

Assuming that $w_1 = 0.5$ and $w_2 = 0.5$, Eq. (15) becomes in terms of the conventional LP problem

$$\begin{aligned} &\max\ 15x_1 + 18x_2 \\ &\text{subject to} \\ &\quad 16.8x_1 + 35.2x_2 \leqslant 830 \\ &\quad 13.2x_1 + 32.8x_2 \leqslant 770 \\ &\quad 20.6x_1 + 11.8x_2 \leqslant 460 \\ &\quad 19.4x_1 + \ 8.2x_2 \leqslant 400 \end{aligned} \tag{20}$$

The optimal solution of (20) is $\mathbf{x}^* = (12.14, 17.78)^t$. To clarify the meaning of $\gtrsim^h$, the fuzzy sets $\boldsymbol{a}$, $\mathbf{x}$ corresponding to $\mathbf{x}^*$ are shown in Fig. 3. The fuzzy set $y = \boldsymbol{c}\mathbf{x}$

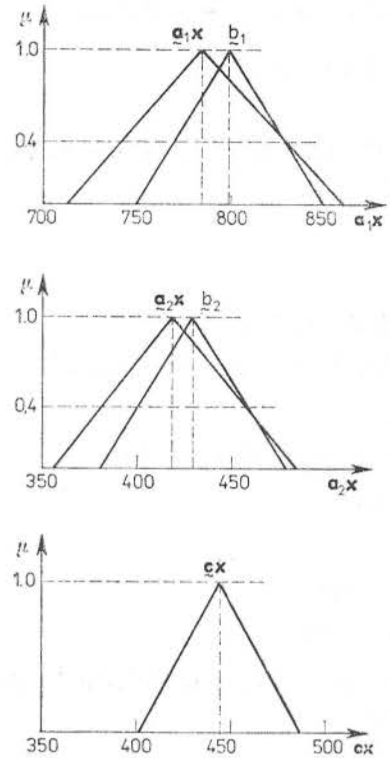


Fig. 3. Fuzzy sets $a_i$ x and cx in Example 1.

is also shown in Fig. 3. It is understood from Fig. 3 that the upper value of $b_1$ and $b_2$ in the right-hand-side of Eq. (19) provide restrictions, while there are leeways for the lower values of $b_1$ and $b_2$. $y=c\mathbf{x}$ attains approximately the value 444.

## 4. Fuzzy solution of FLP problem [problem B]

The problem presented here concerns obtaining a fuzzy solution reflecting ambiguity of fuzzy coefficients. First, we need to clarify what fuzzy solution will be meant optimal in the sense of the objective function.

Let $b_i$ $(i=1, ..., m)$ be fuzzy and $a_i$ be crisp, i.e. a non-fuzzy $a_i$ in (13). The problem [B] is reduced to obtaining a fuzzy set $x$ as an optimal solution in the sense that it maximizes $y$, The FLP problem [B] can be written as

$$\begin{aligned} &\max\ y=cx \\ &\text{subject to} \\ &\qquad a_i\, x \gtrsim^h b_i \quad (i=1, ..., m) \end{aligned} \tag{21}$$

where $a_i=[a_{i1}, ..., a_{in}]$, $c=[c_1, ..., c_n]$, and the level $h$ is given a priori.

By Definitions 3 and 4, and Theorem 1, the FLP problem [B] can be reduced to the following conventional LP problem:

$$\begin{aligned}&\max \boldsymbol{c}\,(w_1\,{}^0\mathbf{x}+w_2\,{}_0\mathbf{x})\\&\text{subject to}\\&\quad \mathbf{a}_i\,\{(1-h/2)\,{}^0\mathbf{x}+(h/2)\,{}_0\mathbf{x}\}\leqslant(1-h/2)\,{}^0b_i+(h/2)\,{}_0b_i\\&\quad \mathbf{a}_i\,\{(h/2)\,{}^0\mathbf{x}+(1-h/2)\,{}_0\mathbf{x}\}\leqslant(h/2)\,{}^0b_i+(1-h/2)\,{}_0b_i \qquad (i=1,\ldots,m)\end{aligned} \tag{22}$$

Example 2

Let us consider the following FLP problem.

$$\begin{aligned}&\max y=25x_1+18x_2\\&\text{subject to}\\&\quad 15x_1+34x_2 \gtrsim^h 800\\&\quad 20x_1+10x_2 \gtrsim^h 430\\&\quad 13x_1+37x_2 \gtrsim^h 800\end{aligned} \tag{23}$$

where $h=0.4$.

Eq. (23) can be written in the vector form as

$$\begin{aligned}&\max y=\boldsymbol{c}\boldsymbol{x}\\&\text{subject to}\\&\quad \boldsymbol{a}_1\,\boldsymbol{x} \gtrsim^h b_1\\&\quad \boldsymbol{a}_2\,\boldsymbol{x} \gtrsim^h b_2\\&\quad \boldsymbol{a}_3\,\boldsymbol{x} \gtrsim^h b_3\end{aligned} \tag{24}$$

where $\boldsymbol{a}_1=[15, 34]$, $\boldsymbol{a}_2=[20, 19]$, $\boldsymbol{a}_3=[13, 37]$, $\boldsymbol{c}=[25, 18]$, $b_1=(750, 850)$, $b_2=$ $=(380, 480)$, $b_3=(650, 950)$, $x_1=({}_0x_1, {}^0x_1)$ and $x_2=({}_0x_2, {}^0x_2)$.

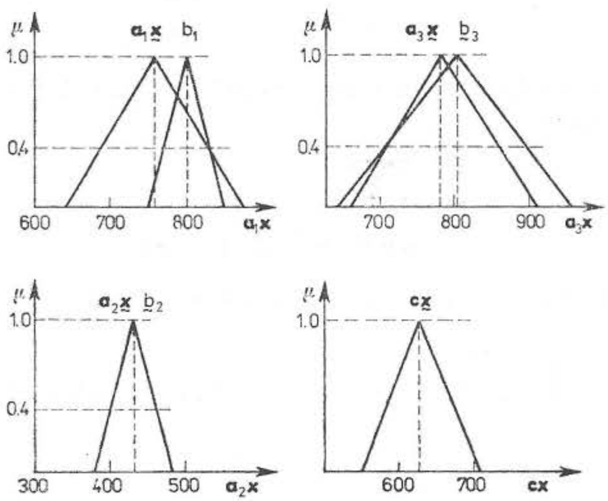


**Fig. 4. Fuzzy sets $a_i x$ and $cx$ in example 2.**

Assuming that $w_1=0.5$, $w_2=0.5$, (24) becomes in terms of the conventional LP problem

$$\begin{aligned}&\max\ 12.5\,{}^0x_1+9\,{}^0x_2+12.5\,{}_0x_1+9\,{}_0x_2\\&\text{subject to}\\&\quad 12\,{}^0x_1+27.2\,{}^0x_2+3\,{}_0x_1+6.8\,{}_0x_2\leqslant 830\\&\quad 3\,{}^0x_1+6.8\,{}^0x_2+12\,{}_0x_1+27.2\,{}_0x_2\leqslant 770\\&\quad 16\,{}^0x_1+8\,{}^0x_2+4\,{}_0x_1+2\,{}_0x_2\leqslant 460\\&\quad 4\,{}^0x_1+2\,{}^0x_2+16\,{}_0x_1+8\,{}_0x_2\leqslant 400\\&\quad 10.4\,{}^0x_1+29.6\,{}^0x_2+2.6\,{}_0x_1+7.4\,{}_0x_2\leqslant 890\\&\quad 2.6\,{}^0x_1+7.4\,{}^0x_2+10.4\,{}_0x_1+29.6\,{}_0x_2\leqslant 710\,.\end{aligned}\tag{25}$$

The optimal solution of (25) is $\boldsymbol{x}^*=(x_1^*, x_2^*)^t=[(12.21, 14.26), (13.57, 19.48)]^t$. The fuzzy sets $\boldsymbol{a}_i\,\boldsymbol{x}$ and $y=\boldsymbol{cx}$ are illustrated in Fig. 4. It is understood from Fig. 4 that there are leeways for the lower value of $b_1$ and the upper value of $b_3$. $y=\boldsymbol{cx}$ attains approximately the value 625.

## 5. Extention to normal and convex fuzzy coefficients

In the above discussion, fuzzy coefficients are restricted to triangular fuzzy sets. In this section let us discuss the extension to normal and convex fuzzy coefficients.

From the decomposition theorem of fuzzy sets [10], a fuzzy set $a$ may be represented as

$$a=\bigcup_{h\in[0,\,1]} h\cdot R_h(a)\tag{26}$$

where the fuzzy set $h\cdot R_h(a)$ is characterized by the membership function

$$\mu_{h\cdot R_h(a)}(a)=\begin{cases}h;\ a\in R_h(a)\\0;\ a\notin R_h(a)\end{cases}\tag{27}$$

According to this decomposition theorem and Definition 2, the crisp substitute of the FLP problem

$$\begin{aligned}&\max\ y=\mathbf{c}\mathbf{x}\\&\text{subject to}\\&\quad \boldsymbol{a}_i\,\boldsymbol{x}\gtrsim^h b_i\qquad (i=1, \ldots, m)\end{aligned}\tag{28}$$

in the multiobjective linear programming problem

$$\begin{aligned}&\max\begin{cases}({}^0y)_k=({}^0\mathbf{c})_k\,\mathbf{x}\\({}_0y)_k=({}_0\mathbf{c})_k\,\mathbf{x}\end{cases}\quad (\text{for all } \mathrm{k}\in[h, 1],\quad i=1, \ldots, m)\\&\text{subject to}\\&\quad ({}^0\mathbf{a}_i)_k\,\mathbf{x}\leqslant({}^0b_i)_k\\&\quad ({}_0\mathbf{a}_i)_k\,\mathbf{x}\leqslant({}_0b_i)_k\end{aligned}\tag{29}$$

where

$$({}^0\mathbf{a}_i)=[({}^0a_{i1})_k, \ldots, ({}^0a_{in})_k], \quad ({}_0\mathbf{a}_i)_k=[({}_0a_{i1})_k, \ldots, ({}_0a_{in})_k].$$

Consider the normal and convex fuzzy sets $a_{ji}$, $b_i$, $c_j$ whose membership functions assume only a limited number of values:

$$\mu_{a_{ji}}, \mu_{b_i}, \mu_{c_i} \in \{k_1, \ldots, k_n\} \tag{30}$$

with $0 \leqslant k_1 < k_2 < \ldots < k_p \leqslant 1$. Then, since we have finite level sets the objective functions and constraints can be rewritten as

$$\begin{aligned} &\max \begin{cases} ({}^0y)_{k_q} = ({}^0\mathbf{c})_{k_q}\,\mathbf{x} \\ ({}_0y)_{k_q} = ({}_0\mathbf{c})_{k_q}\,\mathbf{x} \end{cases} \\ &\text{subject to} \\ &\quad ({}^0\mathbf{a}_i)_{k_q}\,\mathbf{x} \leqslant ({}^0b_i)_{k_q} \\ &\quad ({}_0\mathbf{a}_i)_{k_q}\,\mathbf{x} \leqslant ({}_0b_i)_{k_q} \qquad (j=1, \ldots, m) \end{aligned} \tag{31}$$

for all $k_q \in \{k_j \mid q=1, \ldots, p,\ k_q \in [0, 1]\}$.

Consequently, we can formulate the FLP problem [A] in the case of normal and convex fuzzy coefficients. By defining the fuzzy max in (28) as described in Definition 3, we can obtain a conventional LP problem from (31). Similarly, the FLP problem [B] with a fuzzy solution may also be formulated likewise and be reduced to a conventional LP problem.

## 6. Concluding remarks

The FLP problems with fuzzy coefficients were formulated in the paper, with introduction of the concepts of a crisp solution and a fuzzy solution in a fuzzy environment. A FLP problem is a generalization of the conventional one because the conventional LP problem requires precise coefficients. Our solutions of both the problem [A] and [B] are robust with regard to fuzziness of coefficients in the model. Hence, our approach seems more tractable and applicable to real decision problems than the conventional one.

## Reference

[1] Bellman A. E., Zadeh L. A. Decision Making in a Fuzzy Environment. *Management Science*, **17** (1970), 141—164.

[2] Tanaka H., Okuda T., Asai K. On fuzzy mathematical programming. *J. Cybernet.*, **3** (1974) 4, 37—46.

[3] Zimmerman H. J. Description and Optimization of Fuzzy Systems. *Int. J. General Systems*, 2 (1976), 209—215.

[4] Negoita C. V. Fuzziness in Management. ORSA/TIMS, Miami, (1970).

[5] Dubois D., Prade H. System of Linear Fuzzy Constraints. *Fuzzy Sets and Systems*, **3** (1980) 1, 37—48.

[6] TANAKA H., ASAI K. Fuzzy Linear Programming Problem with Fuzzy Numbers. *Fuzzy Sets and Systems*, **12** (1984) 3.

[7] ZADEH L. A. Fuzzy Sets as a Basis for a Theory of Possibility *Fuzzy Sets and Systems*, **1** (1979) 1, 3—28.

[8] ZADEH L. A. The Concepts of Linguistic Variable and its Application to Approximate Reasoning. Part 1. *Information Science*, **8** (1975), 199—249.

[9] DUBOIS D., PRADE H. Operations on Fuzzy Number. *Int. J. Systems Science*, **9** (1978) 6, 613—626.

[10] ZADEH L. A. The Concepts of Linguistic variable and its Application to Approximate Reasoning. Part 1. *Information Science*, **8** (1975), 199—249.

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## Sformułowanie zadania rozmytego programowania liniowego z wykorzystaniem liczb rozmytych

Sformułowano zadanie rozmytego programowania liniowego z rozmytymi ograniczeniami, oparte na relacji nierówności liczb rozmytych. Zadanie to rozważane jest w dwóch postaciach: a) znajdowania x maksymalizującego $\boldsymbol{c}$x przy ograniczeniach $\boldsymbol{a}_1 x_1+...+\boldsymbol{a}_n x_n \lesssim \boldsymbol{b}$, gdzie $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ są liczbami rozmytymi, oraz b) znajdowania rozmytego $\boldsymbol{x}$ maksymalizującego $c\,\boldsymbol{x}$ przy ograniczeniach $a_1 \boldsymbol{x}_1+...+a_n x_n<b$. Te dwie wersje zadania rozmytego programowania liniowego mogą zostać sprowadzone do standardowych postaci programowania liniowego przez odpowiednie określenie relacji nierówności i operacji maksymalizacji dla liczb rozmytych.

## Формулировка задачи нечеткого линейного программирования с использованием нечетких чисел

Формулируется задача нечеткого линейного программирования с нечеткими ограничениями, основанного на отношении неравенства нечетких чисел. Эта задача рассматривается в двух видах: а) нахождение х, максимизирующего $\boldsymbol{c}$x при ограничениях $\boldsymbol{a}_1 \mathrm{x}_1+...+\boldsymbol{a}_n \mathrm{x}_n<\boldsymbol{b}$, где $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ являются нечеткими числами; б) нахождение нечеткого $x$ максимизирующего $Cx$ при ограничениях $a_1 \mathrm{x}_1+...+a_n \mathrm{x}_n<b$. Эти два вида задачи нечеткого линейного программирования могут быть сведены к стандартному виду линейного программирования путем соответствующего определения отношения неравенства и операции максимизации для нечетких чисел.