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Compromise programming under fuzziness

by

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A fuzzy compromise programming approach is proposed to solve multiobjective optimization problems with fuzzy constraints. Conflicts among objectives are resolved through derivation of a compromise solution on the basis of an ideal solution. Since the constraints are fuzzy, the idea solution is inexact. With respect to a family of distance measures, a compromise solution set is found by minimizing the distance from a specific element of the fuzzy ideal. Depending on the element we are referring to, different compromise solution sets may be obtained. Though the ideal and the compromise solutions are inexact, the flexibility in resolving conflicts is substantially increased. A numerical example is provided to illustrate the basic ideals of fuzzy compromise programming.

1. Introduction

Except for special situations in which optimization of a single objective is sufficient, decisionmaking generally involves multiple and ordinarily conflicting objectives. To enable the selection of an alternative which best satisfies specified objectives, various methods have been proposed in recent years. Among existing techniques such as goal programming (see for example [1], [6], [5]), interactive methods (see for example [4], [2], [14]) and fuzzy sets based methods [13], the ideal solution methods (see for example [3], [9], [10]) appear to be versatile in resolving conflicts among objectives.

Under the maximization framework, an ideal solution, usually infeasible, is perceived as the solution which maximizes all individual objective functions simultaneously. A compromise solution is a solution which is closest, with respect to a distance measure, to the ideal solution.

Conventionally, an ideal solution is exactly defined. Specifically, it is point-valued. Inexact information or fuzzy cognitive and decisionmaking processes, however, may make it impossible to identify the ideal solution with exactitude. Zeleny [11] has suggested that an ideal may possibly be a region with fuzzy boundary. Employing concepts of a theory of possibility, Leung [8] provides a formal conceptualization of a fuzzy ideal. It is demonstrated that if the ideal solution is fuzzily designated, then the corresponding compromise solution set becomes imprecise. Depending on which element of the fuzzy ideal is selected as a reference, different compromise solution sets may be obtained.

Since the ideal solution method can be formulated as a compromise programming problem, to shed some more light on the concept of a fuzzy ideal, the present paper attempts to analyze conflict resolution through the compromise programming framework. Section 2 deals with the determination of a fuzzy ideal and compromis solutions in a multiobjective maximization problem with exact objectives and inexact constraints. The derivation of the most appropriate ideal and compromise solution set is also discussed. Section 3 employs a numerical example to illustrate the basic procedures of compromise programming under fuzziness. The paper then concludes with a discussion on the appropriateness of fuzzy compromise programming in resolving conflicts.

2. Compromise programming with fuzzy information

Let the following be a multiobjective maximization problem:

$$\begin{aligned} &\max F(x)=(f_1(x), \ldots, f_q(x)) \\ &\text{s.t. } a_i^T x \leqq b_i;\ b_i+d_i, \text{ for } i=1, \ldots, m \\ &\quad x \geqslant 0 \end{aligned} \tag{1}$$

where, $a_i \in R^n$ is the i-th vector of the coefficients of the decision variables $x \in R^n$, and b_i is a prescribed limit for the value of $a_i^T x$ which can be extended, if necessary, to b_i+d_i, $i=1, \ldots, m$. That is, d_i is the tolerance interval for the permissible violation of the constraint $a_i^T x \leqslant b_i$. The problem then is to maximize the set of q exact linear objective functions, $f_j(x)$, $j=1, \ldots, q$, with respect to a set of m fuzzy linear constraints.

Under the compromise programming framework, if the constraints are exact, a point-valued ideal solution can be determined as a reference point and the corresponding compromise solution set can be derived accordingly. Since the limits of the constraints in the vector maximization problem in eqn. (1) may be stretched from b_i to b_i+b_i, $i=1, \ldots, m$, then a natural consequence is that the ideal solution may be stretched respectively. That is, instead of a point-value, the ideal solution is a region with bounds derived from extending the limits of the constraints.

If $a_i^T x$ is restricted to the limit b_i, $i=1, \ldots, m$, then the ideal solution ${}_0x^* = ({}_0x_1^*, \ldots, {}_0x_n^*)$ which gives ${}_0f^* = ({}_0f_1^*, \ldots, {}_0f_q^*)$, with ${}_0f_j^* = f_j({}_0x^*)$, (throughout the text, the lower bar and upper bar indicate solutions obtained when b_i and b_i+d_i, $i=1, \ldots, m$, are employed, respectively), can be obtained by solving separately the following q linear programs (see, e.g., Zeleny [10, 11] for details):

$$\begin{aligned} &\max f_j(x) \\ &\text{s.t. } a_i^T x \leqslant b_i,\ i=1, \ldots, m \\ &\quad x \geqslant 0 \end{aligned} \tag{2}$$

Since $_0x^*$ is generally infeasible, a compromise solution is then sought so that the objective functions can be maximized as much as possible. To determine all possible compromise solutions, let the following

$$_0d_p=\left\{\sum_{j=1}^{q}\left[\frac{_0f_j^*-f_j(x)}{_0f_j^*}\right]^p\right\}^{1/p}, \quad p=1, 2, ..., \infty \tag{3}$$

be a family of distance measures depicting the distance between any alternative x to the ideal solution $_0x^*$ with respect to a weight p. A compromise solution is then the solution which minimizes $_0d_p$. Of course, $_0d_p$ can be defined by functions other than that in eqn. (3). By varying p from 1 to ∞, a set of compromise solutions can be determined.

Specifically, for $p=1$, the compromise solution $_0x^1$ (the superscript refers to the value of p) is obtained by solving

$$\begin{aligned} &\min {}_0d_1=\max \sum_{j=1}^{q}\frac{f_j(x)}{_0f_j^*} \\ &\text{s.t. } a_i^T x\leqslant b_i, \ i=1, ..., m \\ &\quad x\geqslant 0 \end{aligned} \tag{4}$$

For $p=\infty$, the compromise solution $_0x^\infty$ is found by solving

$$\begin{aligned} &\min {}_0d_\infty \\ &\text{s.t. } a_i^T x\leqslant b_i, \ i=1, ..., m \\ &\frac{_0f_j^*-f_j(x)}{_0f_j^*}\leqslant {}_0d_\infty, \ j=1, ..., q \\ &x\geqslant 0 \end{aligned} \tag{5}$$

These two compromise solutions serve as bounds of the compromise solution set for $1\leqslant p\leqslant\infty$. For any p within this interval, a compromise solution can likewise be obtained. In particular, when $p=2$, the compromise solution is determined by minimizing $_0d_2$ via the quadratic programming algorithm or by interpolation.

If the limit of $a_i^T x$ is extended to b_i+d_i, $i=1, ..., m$, then the ideal solution $^0x^*=(^0x_1^*, ..., {}^0x_n^*)$ which gives $^0f^*=(^0f_1^*, ..., {}^0f_q^*)$, with $^0f_j^*=f_j(^0x^*)$, can be determined by solving individually q linear programs as follows:

$$\begin{aligned} &\max f_j(x) \\ &\text{s.t. } a_i^T x\leqslant b_i+d_i, \ i=1, ..., m \\ &\quad x\geqslant 0 \end{aligned} \tag{6}$$

Replacing $_0f_j^*$ by $^0f_j^*$ in the distance measures in eqn. (3), the compromise solution set again can be obtained by minimizing ^0d_p, with $1\leqslant p\leqslant\infty$.

For $p=1$, the compromise solution ${}^0x^1$ is determined by solving

$$\begin{aligned} &\min \ {}_0d_1=\max \sum_{j=1}^{q} \frac{f_j(x)}{{}^0f_j^*} \\ &\text{s.t. } a_i^T x \leqslant b_i+d_i, \ i=1, \ldots, m \\ &\quad x \geqslant 0 \end{aligned} \tag{7}$$

For $p=\infty$, the compromise solution ${}^0x^\infty$ is derived by solving

$$\begin{aligned} &\min \ {}^0d_\infty \\ &\text{s.t. } a_i^T x \leqslant b_i+d_i, \ i=1, \ldots, m \\ &\quad \frac{{}^0f_j^*-f_j(x)}{{}^0f_j^*} \leqslant {}^0d_\infty, \ j=1, \ldots, q \\ &\quad x \geqslant 0 \end{aligned} \tag{8}$$

Therefore, by stretching the limits of the constraints from b_i to b_i+d_i, $i=1, \ldots, m$, the maximum values of the individual objective functions $f_j(x)$, $j=1, \ldots, q$, change from ${}_0f_j^*$ (by solving the programs in eqn. (2)) to ${}^0f_j^*$ (by solving the programs in eqn. (6)). Consequently, the ideal solution moves from ${}_0f^*$ to ${}^0f^*$. Considering all variations within the intervals $[b_i, b_i+d_i]$, the ideal becomes fuzzy and can be identified as

$$\{(f_1, \ldots, f_q) \mid {}_0f_j^* \leqslant f_j \leqslant {}^0f_j^*, \ j=1, \ldots, q\}. \tag{9}$$

As previously discussed, by taking ${}^0f^*$ instead of ${}_0f^*$ in (9) as the ideal solution, a different compromise solution set is obtained. Thus, the compromise solution set with reference to the fuzzy ideal is similarly inexact.

So far, in determining an ideal solution, the fuzzy constraints in eqn. (1) have not been explicitly considered. Only the two critical points b_i and b_i+d_i have been employed to delimit the bounds of the fuzzy ideal. To determine the most appropriate ideal solution, the fuzziness of the objective functions and constraints should be explicitly considered in the multiobjective maximization problem.

Based on eqn. (9), the tolerance interval for the value of each objective function $f_j(x)$ is $[{}_0f_j^*, {}^0f_j^*]$. Thus, its fuzzy version $f_j(x)$ may be expressed as

$$f_j(x) \geqq {}^0f_j^*; {}_0f_j^*. \tag{10}$$

To derive the most appropriate ideal solution, we first need to solve separately q fuzzy linear programs as follows:

$$\begin{aligned} &f_j(x) \geqq {}^0f_j^*; \ {}_0f_j^* \\ &a_i^T x \leqq b_i; b_i+d_i, \quad \text{for } i=1, \ldots, m \\ &\quad x \geqslant 0 \end{aligned} \tag{11}$$

Let

$$\mu_j(f_j(x))=\begin{cases}1 & , \quad \text{if } f_j(x)\geqslant {}^0f_j^* \\ 1-\dfrac{{}^0f_j^*-f_j(x)}{{}^0f_j^*-{}_0f_j^*}, & \quad \text{if } {}_0f_j^*\leqslant f_j(x)<{}^0f_j^* \\ 0 & , \quad \text{if } f_j(x)<{}_0f_j^*\end{cases} \tag{12}$$

be the membership function of the satisfaction of the fuzzy objective $f_j(x), j=1, ..., q$. Let

$$\mu_i(a_i^T x)=\begin{cases}1 & , \quad \text{if } a_i^T x\leqslant b_i \\ 1-\dfrac{a_i^T x-b_i}{d_i}, & \quad \text{if } b_i<a_i^T x\leqslant b_i+d_i \\ 0 & , \quad \text{if } a_i^T x>b_i+d_i\end{cases} \tag{13}$$

be the membership function of the satisfaction of the fuzzy constraint $a_i^T x, i=1, ..., m$. Then, according to Zimmermann [12], the fuzzy linear program in eqn. (11) can be rewritten as

$$\max_x \min\,[\mu(f_j(x));\ \mu_i(a_i^T x), \quad \text{for } i=1, ..., m]\,. \tag{14}$$

Its solution is equivalently obtained by solving

$$\begin{aligned}&\max\ \lambda\\ &\text{s.t. } \frac{f_j(x)}{{}^0f_j^*-{}_0f_j^*}-\frac{{}^0f_j^*}{{}^0f_j^*-{}_0f_j^*}\geqslant\lambda\\ &\qquad \frac{b_i}{d_i}-\frac{a_i^T x}{d_i}\geqslant\lambda, \quad \text{for } i=1, ..., m\\ &\qquad x\geqslant 0\end{aligned} \tag{15}$$

Let x and λ^* be the optimal solution of the above program, then taking the optimal solutions of the q fuzzy linear programs into consideration, the most appropriate ideal solution becomes $x^*=(x_1^*, ..., x_n^*)$ with $f^*=(f_1^*, ..., f_q^*)$.

Based on the conditions

$$1-\frac{a_i^T x-b_i}{d_i}, \quad \text{if } b_i<a_i^T x\leqslant b_i+d_i, \quad i=1, ..., m \tag{16}$$

imposed on the fuzzy linear program in eqn. (14), with reference to λ^*, the decision space on which a compromise solution is obtained should then be bounded by the following constraints

$$\begin{aligned}&a_i^T x\leqslant b_i+(1-\lambda^*)\,d_i, \quad \text{for } i=1, ..., m.\\ &x\geqslant 0\end{aligned} \tag{17}$$

Taking x^* as the ideal solution, the most appropriate compromise solution set can be found minimizing d_p, replacing ${}_0f_j^*$ by f_j^* in eqn. (3), with $1\leqslant p\leqslant\infty$.

For $p=1$, the most appropriate compromise solution is obtained by solving

$$\begin{aligned}&\min\ d_1=\max\sum_{j=1}^{q}\frac{f_j(x)}{f_j^*}\\&\text{s.t.}\ a_i^T x\leqslant b_i+(1-\lambda^*)\,d_i,\quad i=1,\ldots,m\\&\qquad x\geqslant 0\end{aligned}\tag{18}$$

For $p=\infty$, the most appropriate compromise solution is again determined by solving

$$\begin{aligned}&\min\ d_\infty\\&\text{s.t.}\ a_i^T x\leqslant b_i+(1-\lambda^*)\,d_i,\quad i=1,\ldots,m\\&\qquad \frac{f_j^*-f_j(x)}{f_j^*}\leqslant d_\infty,\quad j=1,\ldots,q\\&\qquad x\geqslant 0\end{aligned}\tag{19}$$

Since λ indicates the degree of satisfaction of the fuzzy objective functions and constraints in eqn. (15), $1-\lambda$ can be interpreted as the degree of dissatisfaction or violation. Therefore, by varying the value of λ we are varying our reference point in the fuzzy ideal, and the compromise solution set changes accordingly. When $1-\lambda=0$, programs (18) and (19) become programs (4) and (5), respectively. By the same token, when $1-\lambda=1$, programs (18) (19) become programs (7) and (8), respectively. For $0<1-\lambda<1$, programs (18) and (19) give compromise solutions when the constraints are violated by $1-\lambda$. If we treat $\theta=1-\lambda$ as a parameter, we can generate the entire compromise solution set with respect to a distance measure p.

Such a formulation is versatile in the sense that the fuzziness of constraints allows us to generate alternatives, within permitted limits of tolerance, to resolve conflicts which may not be resolved or dissolved otherwise. That is, decisionmakers do not have to restrict themselves to a single point-valued ideal in conflict resolution. Moreover, the process of determining varying ideal and compromise solutions in tractable.

To illustrate the basic concepts discussed, a simple multiobjective maximization problem is provided as a numerical example.

3. Conflict resolution of a two-objective maximization problem

Let the following be a multiobjective maximization problem

$$\begin{aligned}\max\ & f_1=2x_1+x_2\\ & f_2=-3x_1+2x_2\\ \text{s.t.}\ & x_1\leqslant 5;\ 6\\ & x_2\leqslant 6;\ 8\\ & x_1+x_2\leqslant 9;\ 10\\ & 2x_1+3x_2\geqslant 8;\ 6\\ & x_1\geqslant 0,\ x_2\geqslant 0\end{aligned}\tag{20}$$

in which two objectives functions are to be maximized with respect to four fuzzy constraints.

By eqn. (2), the separate maximization of f_1 and f_2 with respect to the following exact constraints

$$\begin{aligned} x_1 \quad &\leqslant 5 \\ x_2 &\leqslant 6 \\ x_1+x_2 &\leqslant 9 \\ 2x_1+3x_2 &\geqslant 8 \\ x_1\geqslant 0,\ x_2 &\geqslant 0 \end{aligned} \tag{21}$$

gives the ideal solution ${}_0x^*=(2.288, 9.438)$ with ${}_0f^*=(14, 12)$.

Employing the distance measure in eqn. (3), the compromise solution, ${}_0x^1=$ $=(0, 6)$ with ${}_0f^1=(6, 12)$, for the distance measure ${}_0d_1$ is found by solving

$$\begin{aligned} \max\ & -.107x_1+.238x_2 \\ \text{s.t.}\ & x_1 \leqslant 5 \\ & x_2\leqslant 6 \\ & x_1+x_2\leqslant 9 \\ & 2x_1+3x_2\geqslant 8 \\ & x_1\geqslant 0,\ x_2\geqslant 0 \end{aligned} \tag{22}$$

Likewise, solving

$$\begin{aligned} \min\ & d_\infty \\ \text{s.t.}\ & x_1 \leqslant 5 \\ & x_2\leqslant 6 \\ & x_1+x_2\leqslant 9 \\ & 2x_1+3x_2\geqslant 8 \\ & .143x_1+.071x_2+{}_0d_\infty\geqslant 1 \\ & -.25x_1+.167x_2+{}_0d_\infty\geqslant 1 \\ & x_1\geqslant 0,\ x_2\geqslant 0 \end{aligned} \tag{23}$$

gives the compromise solution ${}_0x^\infty=(1.466, 6)$ with ${}_0f^\infty=(0.932, 7.602)$.

Thus, for $1\leqslant p\leqslant\infty$, the minimization of ${}_0d_p$ enables us to find a set of compromise solutions with reference to the ideal solution ${}_0x^*$. Figures 1 and 2 depict, respectively, the problem and its solutions in the decision and objective spaces.

By stretching the constraints in eqn. (20) to the other ends of the tolerance intervals, the separate maximization of f_1 and f_2, eqn. (6), with respect to the following set of constraints

$$\begin{aligned} x_1 \quad &\leqslant 6 \\ x_2 &\leqslant 8 \\ x_1+x_2 &\leqslant 10 \\ 2x_1+3x_2 &\geqslant 6 \\ x_1\geqslant 0,\ x_2 &\geqslant 0 \end{aligned} \tag{24}$$

gives the ideal solution ${}^0x^*=(2.288, 11.44)$ with ${}^0f^*=(16, 16)$.

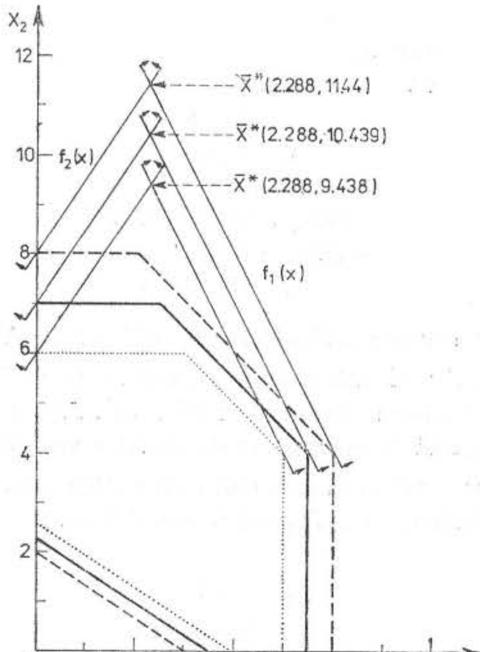
Again, the compromise solution, ${}^0x^1=(0, 8)$ with ${}^0f^1=(8, 16)$, with respect to the measure ${}^0d_1$ is obtained by solving

$$\begin{aligned} \max\quad & .063x_1+.188x_2 \\ \text{s.t.}\quad & x_1 \leqslant 6 \\ & x_2\leqslant 8 \\ & x_1+x_2\leqslant 10 \\ & 2x_1+3x_2\geqslant 6 \\ & x_1\geqslant 0,\ x_2\geqslant 0 \end{aligned} \tag{25}$$

And, the compromise solution, ${}^0x^\infty=(1.585, 8)$ with ${}^0f^\infty=(11.17, 11.245)$, for the distance measure ${}^0d_\infty$ is found by solving

$$\begin{aligned} & \min\ {}^0d_\infty \\ \text{s.t.}\quad & x_1 \leqslant 6 \\ & x_2\leqslant 8 \\ & x_1+x_2\leqslant 10 \\ & 2x_1+3x_2\geqslant 6 \\ & .125x_1+.063x_2+{}^0d_\infty\geqslant 1 \\ & -.188x_1+.125x_2+{}^0d_\infty\geqslant 1 \\ & x_1\geqslant 0,\ x_2\geqslant 0 \end{aligned} \tag{26}$$

The graphical solutions of the problem are again depicted in Figures 1 and 2.



········ feasible region when b_i are effective [for eqn. (21)]

——— feasible region when $b_i+(1-\lambda^*)\,d_i$ are effective [for eqn. (33)]

– – – – feasible region when b_i+d_i are effective [for eqn. (24)]

Fig. 1. The ideals in the fuzzy decision space for the two-objective problem

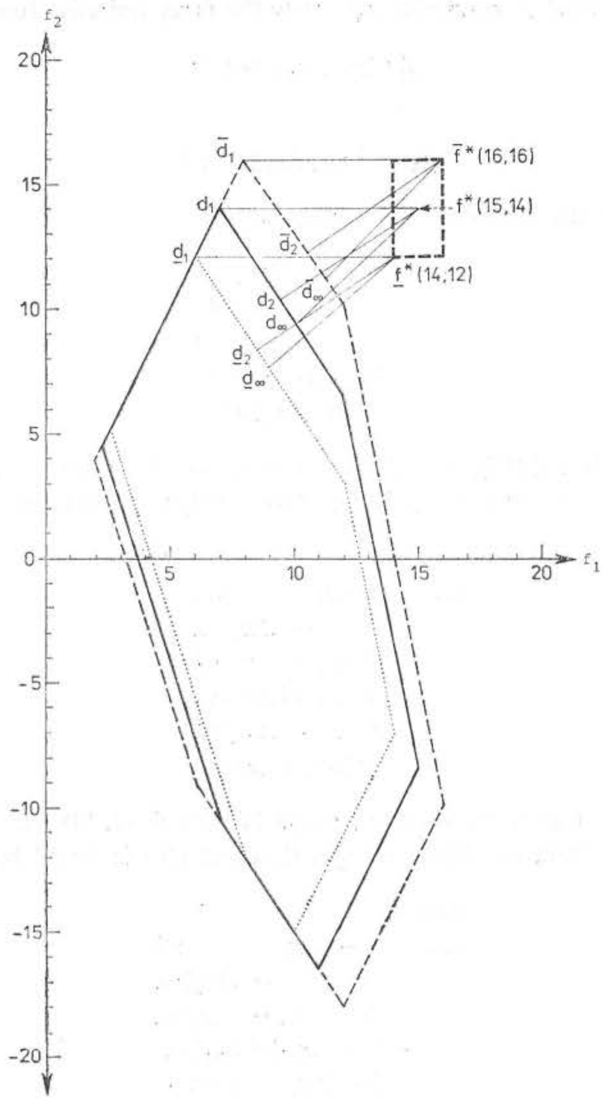


Fig. 2. The ideals and the compromise solutions in the fuzzy objective space for the two-objective problem (keys: same as for Fig. 1)

Therefore, the fuzzy ideal can be expressed as

$$\{(f_1, f_2) \mid 14 \leqslant f_1 \leqslant 16,\ 12 \leqslant f_2 \leqslant 16\}. \tag{27}$$

Taking into consideration the intervals within which f_1 and f_2 may take their values, the most appropriate ideal solution may be obtained by incorporating the fuzzy specifications of the objective functions (eqn. (12)), and constraints (eqn. (13)) into the programming framework in eqn. (11).

That is, we need to maximize separately the fuzzy objective functions

$$f_1: 2x_1+x_2 \geqslant 16;\ 14, \tag{28}$$

and

$$f_2: -3x_1+2x_2 \geqslant 16;\ 12 \tag{29}$$

with respect to the following set of constraints

$$\begin{aligned} x_1 \quad &\leqslant 5;\ 6 \\ x_2 &\leqslant 6;\ 8 \\ x_1+x_2 &\leqslant 9;\ 10 \\ 2x_1+3x_2 &\geqslant 8;\ 6 \\ x_1 \geqslant 0,\ x_2 &\geqslant 0 \end{aligned} \tag{30}$$

By eqns. (14) and (15), the solution, $x=(5.5, 4)$ with $f=(15, -8.5)$ and $\lambda^*=.5$, for the fuzzy linear program consisting of eqns. (28) and (30) is obtained by solving

$$\begin{aligned} &\max \lambda \\ &\text{s.t.} \quad 5-x_1 \geqslant \lambda \\ &\qquad 3 - .5x_2 \geqslant \lambda \\ &\qquad 9-x_1-x_2 \geqslant \lambda \\ &\qquad -4+x_1+1.5x_2 \geqslant \lambda \\ &\qquad -8+x_1+.5x_2 \geqslant \lambda \\ &\qquad x_1 \geqslant 0,\ x_2 \geqslant 0 \end{aligned} \tag{31}$$

By the same token, the solution, $x=(0, 7)$ with $f=(7, 14)$ and $\lambda^*=.5$, for the fuzzy linear program consisting of eqn. (29) and (30) is found by solving

$$\begin{aligned} &\max \lambda \\ &\text{s.t.} \quad 5-x_1 \geqslant \lambda \\ &\qquad 3 - .5x_2 \geqslant \lambda \\ &\qquad 9-x_1-x_2 \geqslant \lambda \\ &\qquad -4+x_1+1.5x_2 \geqslant \lambda \\ &\qquad -4-.75x_1+.5x_2 \geqslant \lambda \\ &\qquad x_1 \geqslant 0,\ x_2 \geqslant 0 \end{aligned} \tag{32}$$

Thus, the most appropriate ideal solution of the program in eqn. (20) is $x^*= =(2.288, 10.439)$ with $f^*=(15, 14)$.

According to eqns. (16) and (17), with reference to $\lambda^*=.5$, the decision space on which the corresponding compromise solutions may be determined is

$$\begin{aligned} x_1 \quad &\leqslant 5.5 \\ x_2 &\leqslant 7 \\ x_1+x_2 &\leqslant 9.5 \\ x_1+1.5x_2 &\geqslant 3.5 \\ x_1 \geqslant 0,\ x_2 &\geqslant 0 \end{aligned} \tag{33}$$

With reference to the most appropriate ideal solution $f^*=(15, 14)$, the most likely compromise solution set can be found by minimizing d_p, with $1 \leqslant p \leqslant \infty$.

For $p=1$, the most appropriate compromise solution $x^1=(0, 7)$ with $f^1=(7, 14)$ is found by solving

$$\begin{aligned} \max\ & -.081x_1+.21x_2 \\ \text{s.t.}\ & x_1 \leqslant 5.5 \\ & x_2 \leqslant 7 \\ & x_1+x_2 \leqslant 9.5 \\ & x_1+1.5x_2 \geqslant 3.5 \\ & x_1 \geqslant 0,\ x_2 \geqslant 0 \end{aligned} \tag{34}$$

The solution $x^\infty=(1.533, 7)$ with $f^\infty=(10.066, 9.401)$ for $p=\infty$ is likewise obtained by solving

$$\begin{aligned} \min\ & d_\infty \\ \text{s.t.}\ & x_1 \leqslant 5.5 \\ & x_2 \leqslant 7 \\ & x_1+x_2 \leqslant 9.5 \\ & x_1+1.5x_2 \geqslant 3.5 \\ & .133x_1+.067x_2+d_\infty \geqslant 1 \\ & -.214x_1+.143x_2+d_\infty \geqslant 1 \\ & x_1 \geqslant 0,\ x_2 \geqslant 0 \end{aligned} \tag{35}$$

(see Figures 1 and 2).

Thus, with λ varying from 0 to 1, we are taking different elements of the fuzzy ideal in eqn. (27) as the reference point for obtaining a compromise solution set.

4. Conclusion

Concepts and procedures of compromise programming under fuzziness have been examined in this paper. Due to the inexactness of the constraints in a multiobjective optimization problem, the ideal solution becomes a region bounded by the tolerance intervals of the objective functions. The corresponding compromise solution set is again fuzzy. With reference to different elements of the fuzzy ideal, different compromise solution sets may be derived.

Though each compromise solution set gives nondominating solutions over the respective decision and objective spaces, among the compromise solution sets themselves, depending on how much a violation of the constraints the decisionmakers are permitting themselves to make, some may be more preferable than others. The fuzziness of the ideal and the compromise solution may appear to be undesirable, but in fact it increases the flexibility in conflict resolution. Should an ideal solution fail to give an acceptable compromise solution, it can always be replaced by another element in the fuzzy ideal. Moreover, if the information in a multiple objective problem is fuzzy, it is natural to expect that its solution can only be fuzzily identified.

Since conflict resolution processes are usually interactive and recursive, displacement of the fuzzy ideal [7] may then be necessary. The current frawework can also be extended to approximate such dynamic conflict resolution processes.

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Programowanie kompromisowe w warunkach rozmytości

Zastosowano programowanie kompromisowe jako metodę rozwiązywania zadań optymalizacji wielokryterialnej z rozmytymi ograniczeniami. Rozwiązanie kompromisowe jest otrzymywane na podstawie rozwiązania „idealnego". Ponieważ ograniczenia są rozmyte, więc i rozwiązanie idealne jest rozmyte. Dla pewnej klasy miar odległości zbiór rozwiązań kompromisowych może być znaleziony poprzez minimalizację odległości od określonego elementu rozmytego „ideału". W zależności od wyboru tego elementu otrzymuje się różne zbiory kompromisowe. Przedstawiona metoda pozwala na znaczne zwiększenie elastyczności w rozwiązaniu konfliktów. Jest ona zilustrowana przykładem numerycznym.

Компромиссное программирование в условиях нечеткости

Используется компромиссное программирование как метод решения задач многокритериальной оптимизации с нечеткими ограничениями. Компромисное решение достигается на основе „идеального" решения. Поскольку ограничения являются нечеткими, поэтому и идеальное решение является нечетким. Для некоторого класса меры расстояния множество компромиссных решений может быть найдено посредством минимизации расстояния от определенного элемента нечеткого „идеала". В зависимости от выбора этого элемента получаем разные компромиссные множества. Представленный метод позволяет значительно расширить эластичность при решении конфликтов. Метод иллюстрируется численным примером.