

## **Interactive fuzzy goal programming for multiobjective nonlinear problems and its application to water quality management**

by

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This paper considers the use of fuzzy goal programming for the solution of multiobjective nonlinear programming problems by considering that the decision maker (DM) has fuzzy goals for each of the objective functions. The fuzzy goals of the DM are quantified by eliciting corresponding membership functions through the interaction with the DM. After determining the membership functions, if the DM specifies his goals in terms of membership values, the corresponding fuzzy goal programming problem is solved and the DM is supplied with Pareto optimal solutions. Then, by considering the current values of the membership functions together with the current values of negative deviation variables, the DM responds by updating his goals in terms of membership values. In this way a satisficing solution for the DM can be efficiently derived from among a Pareto optimal solutions. On the basis of the proposed method, a time-sharing interactive computer program is written and an application to water quality management is demonstrated together with some computer outputs.

### **1. Introduction**

In multiobjective nonlinear programming problems MONLP, there is no "optimal solution" in which all objectives are simultaneously optimized due to an inherent conflict between these objectives. Consequently, the aim in solving MONLP is to find a compromise or satisficing solution of the decision maker (DM) from a set of the Pareto optimal solutions.

Three most promising methodologies for dealing with MONLP problems which have been proposed are:

- (1) goal programming (e.g. [2—3], [8, 9], [11]);
- (2) interactive approaches (e.g. [1], [4—5], [14], [19], [21]);
- (3) fuzzy programming (e.g. [7], [10], [15—17], [20]).

The term goal programming, which assumes that the DM can specify his goals for the objective functions appeared first in 1961 in Charnes and Cooper [2] in order to deal with multiobjective linear programming (MOLP) problems. Subsequent works on goal programming methods have been numerous, e.g., [3], [8, 9], [11].

The term interactive approach, which assumes that the DM is able to give some preference information on a local level, was first initiated by Geoffrion et al. [4] and further developed by many researchers, e.g., [1], [5], [14], [19], [21].

The term fuzzy programming, which assumes that the fuzzy goals of the DM can be quantified by eliciting membership functions, was first introduced by Zimmermann [20] for solving MOLP problems and further extended by several authors, e.g., [7], [10], [15–17], [16].

Naturally, each of these approaches has its own advantages and disadvantages relative to the other approaches. Therefore, in this paper, assuming that the DM has fuzzy goals for each of the objective functions in MONLP problems, we present an interactive fuzzy goal programming algorithm by incorporating into goal programming attractive features from both the interactive approaches and fuzzy programming.

After determining the membership functions for each of the objective functions through an interaction with the DM, if the DM specifies his goals in terms of membership values, the corresponding fuzzy goal programming problem is solved and the DM is supplied with a Pareto optimal solution. Then, by considering the current values of membership functions together with the current values of the negative deviation variables, the DM responds by updating his goals in terms of membership values and a satisficing solution for the DM can be derived efficiently from among Pareto optimal solution. On the basis of the proposed method, a time-sharing interactive computer program is written in FORTRAN and its application to water quality management is demonstrated along with some computer outputs.

## 2. Interactive fuzzy goal programming

In general, a multiobjective nonlinear programming problem (MONLP) is represented as

$$\begin{array}{l} \min f(x) \triangleq (f_1(x), f_2(x), \dots, f_k(x))^T \\ \text{subject to } x \in X \subseteq E^n \end{array} \quad (1)$$

where  $f_1, \dots, f_k$  are  $k$  distinct objective functions of the decision vector  $x$  and  $X$  is a set of feasible constrained decisions. Here, it is assumed that all  $f_i$ ,  $i=1, \dots, k$  are convex and differentiable and the constraint set  $X$  is convex and compact.

Fundamental to the MONLP is the Pareto optimality concept, also known as noninferiority of solution. Qualitatively, a Pareto optimal solution of the MONLP is one in which any improvement of one objective function can be achieved only

at the expense of another. Mathematically, a formal definition of a Pareto optimal solution to the MONLP is given below:

**DEFINITION 1.** A decision  $x^*$  is said to be a Pareto optimal solution to the MONLP, iff there does not exist another  $x \in X$ , such that  $f_i(x) \leq f_i(x^*)$ ,  $j=1, \dots, k$ , with strict inequality holding for at least one  $i$ .

Usually, Pareto optimal solutions form sets of an infinite number of points, and some kinds of subjective judgement should be added to the quantitative analyses by the DM. The DM must select his compromise or satisficing solution from among Pareto optimal solutions.

In order to determine a compromise or satisficing solution of the DM, there are three major approaches, goal programming, interactive approaches and fuzzy programming. Each of these approaches has its own advantages and disadvantages relative to the other ones. Therefore, in this paper, we propose a new interactive fuzzy goal programming method by incorporating into goal programming desirable features of both the interactive approaches and fuzzy programming. In a minimization problem, a fuzzy goal stated by the DM may be to achieve the value of an objective function "substantially less" than some value  $A$ . This type of statement can be quantified by eliciting a corresponding membership function.

In order to elicit a membership function  $\mu_{f_i}(x)$  from the DM for each of the objective functions  $f_i(x)$ , we first calculate the individual minimum  $f_i^{\min}$  and maximum  $f_i^{\max}$  for each objective function  $f_i(x)$  under given constraints. By taking into account the calculated individual minimum and maximum of each objective function, the DM must determine his subjective membership function  $\mu_{f_i}(x)$  which is a strictly monotone decreasing function of  $f_i(x)$ . Here, it is assumed that  $\mu_{f_i}(x)=0$  if  $f_i(x) \geq f_i^0$  and  $\mu_{f_i}(x)=1$  if  $f_i(x) \leq f_i^1$ , where  $f_i^0$  is a worst acceptable level for  $f_i(x)$  and  $f_i^1$  is a totally desirable level for  $f_i(x)$  within  $f_i^{\min}$  and  $f_i^{\max}$ .

After determining the membership functions for each of the objective functions, the DM is asked to specify his aspiration levels of goal attainment in terms of membership values, called goal membership values, for all the membership functions. For the DM's goal membership values  $\hat{\mu}_{f_i}$ ,  $i=1, 2, \dots, k$ , the corresponding Pareto optimal solution, which is in a sense close to his requirement is obtained by solving the following fuzzy goal programming problem

$$\begin{array}{l} \min_{x \in X} \sum_{i=1}^k d_i^- \\ \text{subject to } \left. \begin{array}{l} \mu_{f_i}(x) + d_i^- - d_i^+ = \hat{\mu}_{f_i} \\ d_i^-, d_i^+ = 0 \\ d_i^-, d_i^+ \geq 0 \end{array} \right\} i=1, 2, \dots, k \end{array} \quad (2)$$

where  $d_i^-$  and  $d_i^+$  are respectively, the negative and positive deviation variables, which provide us with a way to measure non-attainment of the goal membership values. This particular formulation is called "one-sided" fuzzy goal programming.

The relationships between the optimal solutions of the fuzzy goal programming problem and the Pareto optimality concept of the MONLP can be characterized by the following theorem.

**THEOREM 1.** *If  $x^*$  is an optimal solution to the fuzzy goal programming problem with  $0 < \mu_{f_i}(x^*) < 1$  and  $d_i^+ = 0$  holding for all  $i$ , then  $x^*$  is a Pareto optimal solution to the MONLP.*

If  $x^*$  is an optimal solution of (2), and if in Theorem 1 none of the sufficiency conditions for the Pareto optimality are satisfied i.e.,  $\exists i: \mu_{f_i}(x^*) = \{0, 1\}$  or  $d_i^+ = 0$ , then we can test the Pareto optimality for  $x^*$  by solving the following problem:

$$\left. \begin{array}{l} \max_{x \in X} \sum_{i=1}^k s_i \\ \text{subject to } f_i(x) + s_i = f_i(x^*), s_i \geq 0 \quad (i=1, \dots, k) \end{array} \right\} \quad (3)$$

Let  $\bar{x}$  be an optimal solution to (3). If all  $s_i = 0$ , then  $x^*$  is a Pareto optimal solution. If at least one  $s_i > 0$ , it can easily be shown that  $\bar{x}$  is a Pareto optimal solution.

So far we have considered a minimization problem and consequently assumed that the DM has a fuzzy goal such as " $f_i(x)$  should be substantially less than  $a_i$ ".

In the following, we consider a more general case where the DM has two types of fuzzy goals, namely those expressed as " $f_i(x)$  should be in the vicinity of  $b_i$ " (fuzzy equal) and " $f_i(x)$  should be substantially less than  $a_i$ " (fuzzy min). Therefore, the problem to be solved is now:

$$\left. \begin{array}{l} \text{fuzzy min } f_i(x) \quad (i \in I) \\ \text{fuzzy equal } f_i(x) \quad (i \in \bar{I}) \\ \text{subject to } x \in X \end{array} \right\} \quad (4)$$

where  $I \cup \bar{I} = \{1, 2, \dots, k\}$ .

In order to elicit from the DM a membership function for a fuzzy goal like " $f_i(x)$  should be in the vicinity of  $b_i$ ", it is obvious that we may use different functions for the left and right sides of  $b_i$ . After determining the membership functions for two types of fuzzy goals, if the DM specifies his goal membership values, the fuzzy goal programming problem (2) is solved.

Now, we introduce the concept of M-Pareto optimal solutions which are defined in terms of membership functions instead of objective functions.

**DEFINITION 2.** A decision  $x^*$  is said to be an M-Pareto optimal solution to (4), iff there does not exist another such  $x \in X$  that  $\mu_{f_i}(x) \geq \mu_{f_i}(x^*)$ ,  $i=1, \dots, k$ , with strict inequality holding for at least one  $i$ .

Note that the set of Pareto optimal solutions is a subset of that of M-Pareto optimal solutions.

Using the concept of M-Pareto optimality, the following M-Pareto version of Theorem 1 can be obtained under slightly different conditions.

**THEOREM 2.** *If  $x^*$  is an optimal solution to (2) with  $d_i^+ = 0$  holding for all  $i$ , then  $x^*$  is an M-Pareto optimal solution to (4).*

Similarly to the minimization case, a numerical test of M-Pareto optimality for  $x^*$  can be performed by solving the following problem:

$$\left. \begin{array}{l} \max_{x \in X} \sum_{i=1}^k s_i \\ \text{subject to} \\ \mu_{f_i}(x) - s_i = \mu_{f_i}(x^*), \quad s_i \geq 0 \quad (i=1, \dots, k) \end{array} \right\} \quad (5)$$

Let  $\bar{x}$  be an optimal solution to (5). If all  $s_i = 0$ , then  $x^*$  is an M-Pareto optimal solution. If at least one  $s_i > 0$ ,  $\bar{x}$  becomes an M-Pareto optimal solution.

Following the above discussion, we can now present an interactive fuzzy goal programming algorithm in order to derive a satisficing solution for the DM from among the (M-) Pareto optimal solution set. The steps marked with an asterisk involve an interaction with the DM.

**STEP 1.** Calculate the individual minimum and maximum for each objective function under given constraints.

**STEP 2\*.** Elicit a membership function from the DM for each of the objective functions.

**STEP 3.** Set all the initial goal membership values equal to 1, i.e.,  $\hat{\eta}_{f_i} = 1$  ( $i = 1, 2, \dots, k$ )

**STEP 4.** For the goal membership values specified by the DM, the corresponding fuzzy goal programming problem (2) is solved and the (M-) Pareto optimality test is performed, if necessary.

**STEP 5\*.** The DM is supplied with the corresponding (M-) Pareto optimal solution. If the DM is satisfied with the current membership values of the (M-) Pareto optimal solution, stop. Otherwise, the DM must update his goal membership values by considering the current values of the membership functions together with the current values of the negative deviation variables and return to Step 4. Here it should be stressed for the DM that any improvement of one membership function can be achieved only at the expense of at least one of the other membership functions.

### 3. An interactive computer program

Interactive fuzzy goal programming processes for multiobjective nonlinear programming problems include eliciting a membership function for each of the objective functions and goal membership values from the DM. Thus, an interactive utilization of computer facilities is highly recommended. Based on the interactive algorithm described above, we have developed a new interactive computer program. Our program includes graphical representations by which the DM can portray

the shapes of his membership functions, and he can promptly find incorrect assessments or inconsistent evaluations revise them immediately and proceed to the next stage more easily.

Our program is composed of one main program and several subroutines. The main program calls in and runs the subprograms with commands indicated by the user (DM). Here we give a brief explanation of the commands prepared in our program.

- (1) MINMAX: Displays the calculated individual minimum and maximum of each of the objective functions under the given constraints.
- (2) MF: Elicits from the DM a membership function for each of the objective functions.
- (3) GRAPH: Depicts graphically the shape of membership function for each of the objective function.
- (4) GO: Derives a satisficing solution for the DM from among the (M-) Pareto optimal solutions by updating the goal membership values.
- (5) STOP: Exits from the program.

In our interactive computer program, the DM can select his membership function in a subjective manner from among the following five types of functions; linear, exponential, hyperbolic, hyperbolic inverse and piecewise linear functions. Then the parameter values are determined through an interaction with the DM. Here, except for the hyperbolic functions, it is assumed that  $\mu_{f_i}(x)=0$  if  $f_i(x) \leq f_i^0$  and  $\mu_{f_i}(x)=1$  if  $f_i(x) \geq f_i^1$ , where  $f_i^0$  is a worst acceptable level  $f_i(x)$  and  $f_i^1$  is a totally desirable level for  $f_i(x)$ .

For the particular functions we obtain:

- (1) Linear membership function:

$$\mu_{f_i}(x) = [f_i(x) - f_i^0] / [f_i^1 - f_i^0]. \quad (6)$$

The linear membership function can be determined by asking the DM to specify two points  $f_i^0$  and  $f_i^1$  within  $f_i^{\max}$  and  $f_i^{\min}$ .

- (2) Exponential membership function:

$$\mu_{f_i}(x) = a_i [1 - \exp \{ -b_i (f_i(x) - f_i^0) / (f_i^1 - f_i^0) \}] \quad (7)$$

The exponential membership function can be determined by asking the DM to specify three points  $f_i^0$ ,  $f_i^{0.5}$  and  $f_i^1$  within  $f_i^{\max}$  and  $f_i^{\min}$ , where  $f_i^0$  represents the values of  $f_i(x)$  such that the value of membership function  $\mu_{f_i}(x)$  is  $a$ .

- (3) Hyperbolic membership function:

$$\mu_{f_i}(x) = (1/2) \tanh ((f_i(x) - b_i) a_i) + (1/2). \quad (8)$$

The hyperbolic membership function can be determined by asking the DM to specify two points  $f_i^{0.25}$  and  $f_i^{0.5}$  within  $f_i^{\max}$  and  $f_i^{\min}$ .

- (4) Hyperbolic inverse membership function:

$$\mu_{f_i}(x) = a_i \tanh^{-1} ((f_i(x) - b_i) a_i) + (1/2) \quad (9)$$

The hyperbolic inverse membership function can be determined by asking the DM to specify three points  $f_i^0, f_i^{0.25}$  and  $f_i^{0.5}$  within  $f_i^{\max}$  and  $f_i^{\min}$ .

(5) Piecewise linear membership function:

$$\mu_{f_i}(x) = \sum_{j=1}^{N_i} \alpha_{ij} |f_i(x) - g_{ij}| + \beta_i f_i(x) + \gamma_i \quad (10)$$

Here, it is assumed that  $\mu_{f_i}(x) = t_{ir} f_i(x) + s_{ir}$  for each segment  $g_{ir-1} \leq f_i(x) \leq g_{ir}$ . The piecewise linear membership function can be determined by asking the DM to specify the grade of membership for each of several values of objective functions within  $f_i^{\max}$  and  $f_i^{\min}$ .

#### 4. An application to a water quality management problem

An application of the proposed method to the Bow River Valley water quality management problem was considered, formulated as a six-objective optimization task (see for detail [6], pp. 187–209, [12, 13], [18]). It is concerned with pollution of an artificial river basin, Bow River Valley on a stretch of 100 miles. The Pierce-Hall Cannery, which is a source of industrial pollution is located near the head of the valley. Ten miles downstream the city of Bowville with 250 000 inhabitants, and seventy miles downstream from the cannery, the city of Plympton with 200 000 inhabitants are the sources of municipal waste pollution. In between, fifty miles downstream from the Pierce-Hall Cannery is Robin State Park, and the state boundary line is 100 miles downstream from the cannery.

Water quality is measured with respect to dissolved oxygen concentration (DO), and the industrial and municipal waste pollution is described by the number of pounds of biochemical oxygen demanding material (BOD), which is identified as the carbonaceous (BOD<sub>c</sub>) and nitrogenous (BOD<sub>n</sub>) material. The cannery and the two cities already have primary treatment facilities which reduce the BOD by 30% of their gross discharge. The cost of additional treatment facilities would increase the tax rate in Bowville and Plympton and decrease the investment return from the cannery.

The problem is formulated as follows:

The decision variables  $x_1, x_2, x_3$  represent the treatment levels of water discharge at the Pierce-Hall Cannery, at Bowville, and at Plympton, respectively. The treatment levels are specified in terms of the carbonaceous BOD<sub>c</sub> by the relation:

$$w_i = 0.39 / (1.39 - x_i^2)$$

where  $x_i$  is the proportion of the gross BOD<sub>c</sub> removed at location  $i$  and  $w_i$  is the corresponding value for BOD<sub>n</sub>.

The Bow River Valley Water Pollution Control Director (the DM in this problem) is concerned with the following six objective functions:

$$f_1(x) = 4.75 + 2.27(x_1 - 0.3)$$

$$f_2(x) = 2.0 + 0.524(x_1 - 0.3) + 2.79(x_2 - 0.3) + 0.882(w_1 - 0.3) + 2.65(w_2 - 0.3)$$

$$f_3(x) = 5.1 + 0.177(x_1 - 0.3) + 0.978(x_2 - 0.3) + 0.216(w_1 - 0.3) + 0.768(w_2 - 0.3)$$

$$f_4(x) = 7.5 - 0.012(59/(1.09 - x_1^2) - 59)$$

$$f_5(x) = 0.0018(532/(1.09 - x_2^2) - 532)$$

$$f_6(x) = 0.0025(450/(1.09 - x_3^2) - 450)$$

where  $f_1, f_2, f_3$  are the DO levels in mg/l at Bowville, Robin State Park, and Plympton, respectively,  $f_4$  is the percentage return on equity at the Pierce-Hall Cannery, and  $f_5$  and  $f_6$  are the additions to the tax rate at Bowville and Plympton, respectively. Therefore, the objective functions  $f_1, f_2, f_3$  and  $f_4$  should be maximized and the objective functions  $f_5$  and  $f_6$  should be minimized.

The DM must consider the constraint that the DO level at the state line must be no less than 3.5 mg/l, which is represented by:  $1.0 + 0.0332(x_1 - 0.3) + 0.0186(x_2 + 0.3) + 3.34(x_3 - 0.3) + 0.0204(w_1 - 0.3) + 0.778(w_2 - 0.3) + 2.62(w_3 - 0.3) \geq 3.5$ . In addition there are upper and lower bounds on the decision variables:  $0.3 \leq x_i \leq 1.0, i=1, 2, 3$ .

In the following, results obtained employing our computer program run under TSS on ACOS-6 digital computer in the computer center of Kobe University are explained using some computer outputs.

```
COMMAND:
=MINMAX
```

INDIVIDUAL MINIMUM AND MAXIMUM					
	!	MIN	!	MAX	
F( 1)	!	0.475000000+01	!	0.633900000+01	
F( 2)	!	0.200000000+01	!	0.679220010+01	
F( 3)	!	0.510000000+01	!	0.659730010+01	
F( 4)	!	0.341336190+00	!	0.750000000+01	
F( 5)	!	-0.319633470-07	!	0.968239600+01	
F( 6)	!	0.957705220+00	!	0.113749950+02	

Illustration 1

Using the MINMAX command, the calculated individual minimum and maximum of each of the objective functions  $f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_6(x)$  are displayed.

```
COMMAND:
=MF
INPUT THE OBJECTIVE FUNCTION NUMBER:
=1
INPUT FUZZY GOAL:
(1) FUZZY MAX
(2) FUZZY MIN
(3) FUZZY EQUAL
=1
DO YOU WANT LIST OF MEMBERSHIP FUNCTION TYPE ?
=YES
LIST OF MEMBERSHIP FUNCTION TYPE
(1) LINEAR
(2) EXPONENTIAL
(3) HYPERBOLIC
(4) HYPERBOLIC INVERSE
(5) PIECEWISE LINEAR
INPUT MEMBERSHIP FUNCTION TYPE:
=2
INPUT THREE POINTS(F1,F2,F3) SUCH THAT
M(F1)=0.0 ( F1 : WORST ACCEPTABLE LEVEL )
M(F2)=0.5
M(F3)=1.0 ( F3 : TOTALLY DESIRABLE LEVEL )
=4.75 6.0 6.339
ANOTHER MSET ?
```

Illustration 2



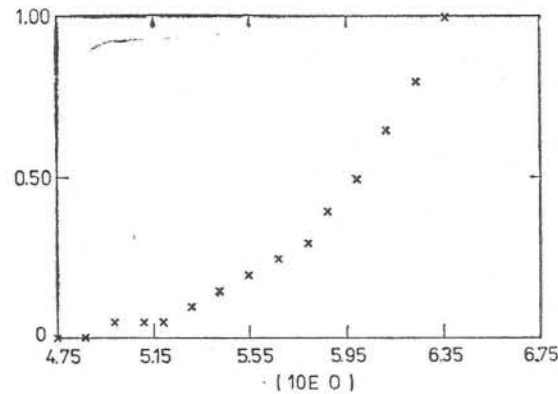


Illustration 3

```

COMMAND:
=GO

INPUT YOUR GOALS (BETWEEN 0 AND 1) FOR EACH OF THE MEMBERSHIP
FUNCTIONS
=1.0 1.0 1.0 1.0 1.0 1.0
( FRACTIONAL CHANGE IN OBJECTIVE )

----- < ITERATION 1 > -----
MINIMIZATION OF THE SUM OF THE NEGATIVE
DEVIATIONS FOR YOUR GOALS
-----
MEMBERSHIP          !      OBJECTIVE FUNCTION
-----
M(F 1) ! 0.4507046494D+00 ! F( 1) ! 0.5950796669D+01
M(F 2) ! 0.3862125013D+00 ! F( 2) ! 0.5078305304D+01
M(F 3) ! 0.6648070266D+00 ! F( 3) ! 0.6095216119D+01
M(F 4) ! 0.7289415986D+00 ! F( 4) ! 0.6450231317D+01
M(F 5) ! 0.5872914375D+00 ! F( 5) ! 0.2248719081D+01
M(F 6) ! 0.6452174847D+00 ! F( 6) ! 0.1300649757D+01
-----
NEGATIVE DEVIATION !      POSITIVE DEVIATION
-----
D( 1)- ! 0.5492955131D+00 ! D( 1)+ ! 0.
D( 2)- ! 0.6137956279D+00 ! D( 2)+ ! 0.
D( 3)- ! 0.3351973637D+00 ! D( 3)+ ! 0.
D( 4)- ! 0.2740574911D+00 ! D( 4)+ ! 0.
D( 5)- ! 0.4126174759D+00 ! D( 5)+ ! 0.
D( 6)- ! 0.3547833255D+00 ! D( 6)+ ! 0.
-----
SUM D- ! 0.2536746797D+01 ! SUM D+ ! 0.
-----
X( 1)= 0.8289853181D+00      X( 2)= 0.8895728198D+00
X( 3)= 0.7913322795D+00
-----
ARE YOU SATISFIED WITH THE CURRENT VALUES OF THE MEMBERSHIP
FUNCTIONS?
=NO
INPUT YOUR GOALS (BETWEEN 0 AND 1) FOR EACH OF THE MEMBERSHIP
FUNCTIONS
=0.8 0.8 0.8 0.8 0.8 0.8

```

Illustration 4a. Iteration 1.

The MF command is utilized to subsequently determine the membership functions for each of the objective functions  $f_1(x)$ ,  $f_2(x)$ ,  $f_3(x)$ ,  $f_4(x)$ ,  $f_5(x)$ ,  $f_6(x)$ . Here the interaction with a hypothetical DM establishes the following membership functions and corresponding assessment values.

```

----- < ITERATION 5 > -----
      MINIMIZATION OF THE SUM OF THE NEGATIVE
      DEVIATIONS FOR YOUR GOALS
-----
      MEMBERSHIP      !      OBJECTIVE FUNCTION
M(F 1) ! 0.5668000922D+00 ! F( 1) ! 0.6060073218D+01
M(F 2) ! 0.3703644308D+00 ! F( 2) ! 0.5034057491D+01
M(F 3) ! 0.6531120149D+00 ! F( 3) ! 0.6077708686D+01
M(F 4) ! 0.5000000000D+00 ! F( 4) ! 0.6000000000D+01
M(F 5) ! 0.6770714625D+00 ! F( 5) ! 0.1946894362D+01
M(F 6) ! 0.6000000000D+00 ! F( 6) ! 0.1357459691D+01
-----
      NEGATIVE DEVIATION      !      POSITIVE DEVIATION
D( 1)- ! 0.3319990784D-01 ! D( 1)+ ! 0.
D( 2)- ! 0.1296502556D+00 ! D( 2)+ ! 0.
D( 3)- ! 0.4689592339D-01 ! D( 3)+ ! 0.
D( 4)- ! 0. ! D( 4)+ ! 0.
D( 5)- ! 0.2282865687D-01 ! D( 5)+ ! 0.
D( 6)- ! 0. ! D( 6)+ ! 0.
-----
      SUM D- ! 0.2325747437D+00 ! SUM D+ ! 0.
-----
      X( 1)= 0.8771247674D+00      X( 2)= 0.8719541807D+00
      X( 3)= 0.7980103191D+00
-----
ARE YOU SATISFIED WITH THE CURRENT VALUES OF THE MEMBERSHIP
FUNCTIONS?
=YES
THE FOLLOWING VALUES ARE YOUR SATISFICING SOLUTION:
-----
      MEMBERSHIP      !      OBJECTIVE FUNCTION
M(F 1) = 0.5668000922D+00 ! F( 1) = 0.6060073218D+01
M(F 2) = 0.3703644308D+00 ! F( 2) = 0.5034057491D+01
M(F 3) = 0.6531120149D+00 ! F( 3) = 0.6077708686D+01
M(F 4) = 0.5000000000D+00 ! F( 4) = 0.6000000000D+01
M(F 5) = 0.6770714625D+00 ! F( 5) = 0.1946894362D+01
M(F 6) = 0.6000000000D+00 ! F( 6) = 0.1357459691D+01
-----
      X( 1)= 0.8771247674D+00      X( 2)= 0.8719541807D+00
      X( 3)= 0.7980103191D+00
-----

```

Illustration 4b. Continuation of illustration 4a. Iteration 5.

- $f_1$ : exponential,  $(f_1^0, f_1^{0.5}, f_1^1) = (4.75, 6.0, 6.339)$ ,  
 $f_2$ : linear,  $(f_2^0, f_2^1) = (0.4, 6.792)$ ,  
 $f_3$ : linear,  $(f_3^0, f_3^1) = (5.1, 6.597)$ ,  
 $f_4$ : hyperbolic,  $(f_4^{0.25}, f_4^{0.5}) = (5.5, 6.0)$ ,  
 $f_5$ : exponential,  $(f_5^0, f_5^{0.5}, f_5^1) = (3.5, 2.0, 0.0)$ ,  
 $f_6$ : exponential,  $(f_6^0, f_6^{0.5}, f_6^1) = (7.0, 1.5, 0.958)$ .

For each type of membership functions, the corresponding assessment values are input in a subjective manner by considering the calculated individual minimum and maximum of each of the objective functions.

The shape of the exponential membership function for  $f_1(x)$  is shown graphically with the GRAPH command. Thus the DM can visually check properties of his membership functions.

Using the GO command, the fuzzy goal programming problem is solved for the initial goal membership values and the DM is supplied with the corresponding Pareto optimal solution. Since the DM is not satisfied with the current membership values, the DM updates his goal membership values.

The same procedure continues in this manner, until the DM is satisfied with the current values of the membership functions. In this example, in the fifth iteration, a satisficing solution for the DM is obtained.

## 5. Conclusion

In this paper, we have proposed an interactive fuzzy goal programming algorithm in order to deal with fuzzy goals of the DM in multiobjective nonlinear programming problems. In our interactive scheme, after determining the membership functions for each of the objective functions, a satisficing solution for the DM can be derived by updating his goal membership values based on the current values of the membership functions together with the current values of the negative deviation variables. Furthermore, (M-) Pareto optimality of the generated solution in each iteration is guaranteed. Based on the proposed method, a time-sharing computer program has been written to facilitate the interaction processes.

An application to the Bow River Valley water quality management problem has demonstrated the feasibility and efficiency of both the proposed algorithm and its corresponding interactive computer program under a hypothetical DM. Although the actual DM for the water quality management problem would of course select other values of the six objectives than the ones which were selected by the hypothetical DM used in this paper, the way of iterating and calculating is essentially the same. However, application to real-world problems must be carried out in cooperation with a person actually involved in decision making. Such experiences may lead to revisions of the proposed algorithm and its computer program.

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### **Interaktywne programowanie celowe w wielokryterialnych zadaniach nieliniowych i jego zastosowanie do sterowania jakością wód**

Rozpatruje się zastosowanie rozmytego programowania celowego do rozwiązania wielokryterialnych zadań programowania nieliniowego przy założeniu rozmytości poszczególnych celów. Cele te są specyfikowane interaktywnie przez podanie ich funkcji przynależności. Następnie rozwiązywane jest odpowiednie zadanie rozmytego programowania celowego, którego wynikiem jest rozwiązanie optymalne w sensie Pareto. Kolejnym krokiem procedury interaktywnej jest uaktualnienie funkcji przynależności celów na podstawie otrzymanego rozwiązania i otrzymanie następnego rozwiązania. W ten sposób można efektywnie wyznaczać rozwiązania zadawalające ze zbioru optymalnego w sensie Pareto. Program komputerowy realizujący przedstawioną metodę został zastosowany do zadania sterowania jakością wód.

### **Интерактивное целевое программирование во многокритериальных нелинейных задачах и его применение к управлению качеством воды**

Рассматривается применение нечеткого целевого программирования к решению многокритериальных задач нелинейного программирования при предпосылке нечеткости отдельных целей. Эти цели определяются интерактивно посредством их функций инцидентности. Затем решается соответствующая задача нечеткого целевого программирования, в результате чего получаем решение, оптимальное в смысле Парето. Следующим шагом интерактивной процедуры является актуализация функции инцидентности целей на основе полученного решения и получение следующего решения. Таким образом можно эффективно определять удовлетворительные решения из множества, оптимального в смысле Парето. Вычислительная программа, реализующая представленный метод, была использована в задаче управления качеством воды.