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Applications of fuzzy optimization in operational research

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This paper deals with fuzzy optimization problems. A fuzzy mathematical programming problem is introduced and, as a particular case, a fuzzy linear mathematical programming problem is studied. Its basic formulations, as well as fuzzy versions of the transportation problem, vector-maximum problem and duality theory are discussed.

1. Introduction

In this paper we introduce basic concepts of Fuzzy Mathematical Programming (FMP) problems, and describe fundamental results of Fuzzy Linear Mathematical Programming (FLP). The main aim of this paper is to bring out applications related to the FLP problem. We consider applications to the Fuzzy Vectormaximum (FV) problem, Fuzzy Transportation (FT) problem and Fuzzy Duality Theory (FDT). Finally, references in these topics are given.

Fuzzy mathematical programming problems

Suppose there is a classical mathematical programming problem,

$$\begin{aligned} &\text{Max: } f(x) \\ &\text{s.t.: } g_i(x) \leqslant b_i, \ i \in I = \{1, \ldots, m\} \\ &\qquad x \geqslant 0 \end{aligned} \tag{1}$$

where

$$g_i, f \in \mathscr{R}(R^n) = \{h \mid h: R^n \to R\}, \ i \in I$$

and $x \in R^n$, $b \in R^m$.

Three possible fuzzy versions can be adopted to the problem (1)

$$\begin{aligned} &\underset{\sim}{\text{Max}}\colon f(x) \\ &\text{s.t.}\colon g_i(x) \leqslant b_i, \ i \in I \\ &\qquad x \geqslant 0 \end{aligned} \tag{2}$$

or

$$\begin{aligned} &\text{Max}\colon f(x) \\ &\text{s.t.}\colon g_i(x) \lesssim b_i, \ i \in I \\ &\qquad x \geqslant 0 \end{aligned} \tag{3}$$

or, finally,

$$\begin{aligned} &\underset{\sim}{\text{Max}}\colon f(x) \\ &\text{s.t.}\colon g_i(x) \lesssim b_i, \ i \in I \\ &\qquad x \geqslant 0 \end{aligned} \tag{4}$$

where the symbol "$\sim$" indicates the fuzziness in either the objective, or in the constraints or, both in the objective and in the constraint set.

Any of these three problems is called FMP problem and they can be interpreted in the following way.

a) Problem (2) is a FMP problem with a fuzzy objective, i.e., the decision-maker has a fuzzy set of objective functions such that

$$\mu_0\colon \mathscr{R}[R^n] \to [0, 1]$$

is a membership function (which be determines) giving the accomplishment degree of the decision-maker's aspirations for any $f \in \mathscr{R}[R^n]$.

b) Problem (3) is a FMP problem with a fuzzy constraint set. Thus, the existence of membership functions

$$\mu_i\colon R \to [0, 1], \ i \in I$$

is assumed, which is interpreted as if the decision-maker were prepared to tolerate certain violations in the accomplishment of the constraints, i.e., he permits $g_i(x) > b_i$ with a degree $u_i(g_i(x), b_i) \in [0, 1]$.

c) Finally, problem (4) is a mixture of (2) and (3) with fuzzification of both objective and constraint sets. Due to this fact, only FMP problems (2) and (3) shall be considered in further text.

Solving FMP problems

To show the solution method for FMP problems, the linear case is considered. In this case (2) becomes

$$\begin{aligned} &\underset{\sim}{\text{Max}}\colon cx \\ &\text{s.t.}\colon Ax \leqslant b \\ &\qquad x \geqslant 0 \end{aligned} \tag{5}$$

where c, b and A are $(1 \times n)$, $(m \times 1)$ and $(m \times n)$ matrices, respectively, and

$$\mu_0\colon R^n \to [0, 1]$$

is the membership function of the fuzzy objective. (Notice that in the linear case $\mathscr{R}[R^n]=R^n$) It can be shown, [31], that the existence of $\mu_0(\cdot)$ is equivalent to the existence of n membership functions,

$$\mu_{0j}: R \to [0, 1],\ j \in J=\{1, ..., n\}$$

defined for each component c_j, $j \in J$, of $c \in R^n$, respectively.

Thus by considering the $(1-\alpha)$-cuts, $\alpha \in [0, 1]$, of the fuzzy objective, and assuming the existence of inverse functions $\varphi_{0j}(\cdot)$, $j \in J$, for each μ_{0j}, respectively, it can be proved, [31], that a fuzzy solution to (5) can be found from the optimal solution of the parametric linear programming problem,

$$\begin{aligned} &\text{Max: } \varphi(1-\alpha)x \\ &\text{s.t.: } Ax \leqslant b \\ &\qquad x \geqslant 0,\ \alpha \in [0, 1] \end{aligned} \tag{6}$$

where $\varphi(\cdot)=[\varphi_{01}(\cdot), ..., \varphi_{0n}(\cdot)]$.

On the other hand, for the linear case the problem (3) becomes

$$\begin{aligned} &\text{Max: } cx \\ &\text{s.t.: } Ax \lesssim b \\ &\qquad x \geqslant 0 \end{aligned} \tag{7}$$

Now,

$$\mu_i: R \to [0, 1],\ i \in I$$

are membership functions of the fuzzy constraints.

Taking into account that the α-cuts of the fuzzy constraint set are

$$\{x \in R^n \mid \mu_i(A_i x, b_i) \geqslant \alpha,\ i \in I\}$$

a fuzzy solution to (7) can be found from the optimal solution of the parametric linear programming problem,

$$\begin{aligned} &\text{Max: } cx \\ &\text{s.t.: } Ax \leqslant \psi(\alpha) \\ &\qquad x \geqslant 0,\ \alpha \in [0, 1] \end{aligned} \tag{8}$$

where $\psi(\cdot)=[\psi_1(\cdot), ..., \psi_n(\cdot)]$ is an n-vector defined by the inverse $(\cdot)$ *'s*.

Let us notice that both problems (6) and (8) are linear for any form of ψ_i, i.e., μ_i.

In the following, the presented parametric approach will be applied to solution of some "classical" fuzzy problems in O.R.

Fuzzy vectormaximum problem

In the first approach, a Fuzzy Vectormaximum (FV) problem can be defined as

$$\begin{aligned} &\underset{\sim}{\text{Max}}\text{: } z(x)=[z_1(x), ..., z_p(x)] \\ &\text{s.t.: } Ax \leqslant b \\ &\qquad x \geqslant 0 \end{aligned} \tag{9}$$

where "$\sim$" means "fuzzy optimization" in the sense that all the p objective linear functions are characterized by corresponding membership (or performance) functions $\mu_i\,[z_i\,(x)]$, $i \in P=\{1, ..., p\}$.

FV problem was first mentioned by Zimmermann in [36], other important contributions to this topic are [3] and [11].

In [29] a parametric approach was shown. This approach is valid for linear as well as non-linear functions $u_i\,(\cdot)$, $i \in P$. Moreover, it permits to find a fuzzy solution that produces, as a particular value, the solution to (9) proposed in [11].

Thus, with notation

$$\mu_0\,(x)=[\mu_1\,[z_1\,(x)], ..., \mu_p\,[z_p\,(x)]]$$

(9) becomes

$$\begin{aligned} &\text{Max: } \alpha \\ &\text{s.t.: } \mu_i\,[z_i\,(x)] \geqslant \alpha, \ i \in P \\ &\qquad Ax \leqslant b \\ &\qquad x \geqslant 0 \end{aligned} \tag{10}$$

Notice that if the constraints

$$\mu_i\,[z_i\,(x)] \geqslant \alpha, \ i \in P$$

are considered as the α-cuts of the objective functions, a fuzzy solution to (9) can be found from

$$\begin{aligned} &\text{Max: } \mu_k\,[z_k\,(x)] \\ &\text{s.t.: } \mu_i\,[z_i\,(x)] \geqslant \alpha, \ i \in P-\{k\} \\ &\qquad Ax \leqslant b \\ &\qquad x \geqslant 0, \ \alpha \in [0, 1] \end{aligned} \tag{11}$$

It can be proved that if $u_i\,(\cdot)$'s, $i \in P$, are continuous and strictly increasing functions, then (11) can be solved by means of

$$\begin{aligned} &\text{Max: } z_k\,(x) \\ &\text{s.t.: } z_i\,(x) \geqslant f_i\,(\alpha), \ i \in P-\{k\} \\ &\qquad Ax \leqslant b \\ &\qquad x \geqslant 0, \ \alpha \in [0, 1] \end{aligned} \tag{12}$$

where, as usual, $f_i\,(\cdot)$ are inverse functions of $\mu_i\,(\cdot)$, $i \in P$.

A fuzzy solution to (9) is obtained from the optimal solution of the parametric linear programming problem (12). Moreover, [29], if $x\,(\alpha)$, $\alpha \in [0, 1]$, is an optimal solution to (12), Leberling's solution of (9) is $x\,(\hat{\alpha})$, where $\hat{\alpha} \in [0, 1]$ is the value that solves

$$\mu_k\,[z_k\,(x\,(\alpha)]=\alpha \tag{13}$$

For instance, consider the same example as in [11],

$$\underset{x \in X}{\widetilde{\text{Max}}}: \ z\,(x)=(-x_1+2x_2,\ 2x_1+x_2)' \tag{14}$$

where,

$$X=\{x\in R^2 \mid -x_1+3x_2\leqslant 21,\ x_1+3x_2\leqslant 27,\ 4x_1+3x_2\leqslant 45,\ 3x_1+x_2\leqslant 30,\ x_i\geqslant 0\}$$

and,

$$\mu_1(x)=(1/2)\tanh[(-x_1+2x_2-5.5)(6/17)]+(1/2)$$

$$\mu_2(x)=(1/2)\tanh[(2x_1+x_2-14)(6/14)]+(1/2)$$

are the membership functions of the fuzzy objectives.

Due to (12), we obtain

$$\begin{aligned}&\text{Max: } -x_1+2x_2\\ &\text{s.t.: } 2x_1+x_2\geqslant 14+(7/6)\ln[\alpha/(1-\alpha)]\\ &\qquad x\in X,\ \alpha\in[0,1]\end{aligned}$$

whose solution is

$$\left.\begin{aligned}x_1(\alpha)&=3+(7/10)\ln[\alpha/(1-\alpha)]\\ x_2(\alpha)&=8-(7/30)\ln[\alpha/(1-\alpha)]\end{aligned}\right.\quad \alpha\in[0.5,\ 0.986] \tag{15}$$

with

$$\hat{\alpha}=0.95$$

Thus, by substituting $\hat{\alpha}$ into (15), the solution proposed in [11] is obtained, i.e.

$$\hat{x}_1=5.03,\ \hat{x}_2=7.32 \tag{16}$$

2. Fuzzy transportation problem

The classical transportation problem occurs if a homogeneous product is to be shipped from m origins with supplies $a_1, \ldots, a_m$ to $n-1$ destinations with demands $b_1, \ldots, b_{n-1}$, so that the total transportation costs be minimized. It is assumed that the total supply is greater than or equal to the sum of the demands and that the costs c_{ij} of shipping one unit from the ith origin to the jth destination is independent of the amount of goods shipped from origin i to destination j, i.e., the transportation problem can be formulated as follows,

$$\begin{aligned}&\text{Min: } cx\\ &\text{s.t.: } \sum_{i\in I} x_{ij}\geqslant b_j,\ j\in J'=\{1,\ldots,n-1\}\\ &\qquad \sum_{j\in J} x_{ij}\leqslant a_i,\ i\in I=\{1,\ldots,m\}\\ &\qquad\quad x_{ij}\geqslant 0,\ (ij)\in I\times J',\end{aligned} \tag{17}$$

with

$$\sum_{i\in I} a_i \geqslant \sum_{j\in J'} b_j \tag{18}$$

as a feasibility condition.

A possible fuzzy version of (17), the one proposed in [20], is

$$\begin{aligned}&\text{Min: } cx\\ &\text{s.t.: } \sum_{i\in I} x_{ij} \gtrsim b_j,\ j\in J'\\ &\qquad \sum_{j\in J'} x_{ij} \lesssim a_i,\ i\in I\\ &\qquad\quad x_{ij}\geqslant 0\ (ij)\in I\times J'\end{aligned} \tag{19}$$

where $\mu_j(\cdot)$, $j\in J'$, and $\mu_i(\cdot)$, $i\in I$, are the respective membership functions of the constraints.

If (18) holds and the membership functions are continuous and strictly monotonic (decreasing for $i\in I$ and increasing for $j\in J'$) it can be proved, [30], that

$$\sum_{i\in I} \varphi_i(\alpha) \geqslant \sum_{j\in J'} \psi_j(\alpha),\quad \alpha\in[0,1] \tag{20}$$

where $\varphi_i(\cdot)$, $i\in I$, and $\psi_j(\cdot)$, $j\in J'$, are the respective inverse functions of $\mu_i(\cdot)$, $i\in I$, and $\mu_j(\cdot)$, $j\in J'$.

Therefore, in accordance with (7) and (8), the fuzzy transportation problem (19) can be solved by means of the classical parametric transportation problem,

$$\begin{aligned}&\text{Min: } cx\\ &\text{s.t.: } \sum_{j\in J'} x_{ij} \leqslant \varphi_i(\alpha),\ i\in I\\ &\qquad \sum_{i\in I} x_{ij} \geqslant \psi_j(\alpha),\ j\in J'\\ &\qquad\quad x_{ij}\geqslant 0,\ (ij)\in I\times J',\ \alpha\in[0,1]\end{aligned} \tag{21}$$

If, in order to get $\Sigma\varphi_i(\alpha)=\Sigma\psi_j(\alpha)$, an n-th dummy destination is introduced, (21) becomes,

$$\begin{aligned}&\text{Min: } cx\\ &\text{s.t.: } \sum_{j\in J} x_{ij} = \varphi_i(\alpha),\ i\in I\\ &\qquad \sum_{i\in I} x_{ij} = \psi_j(\alpha),\ j\in J=\{1, \ldots, n\}\\ &\qquad\quad x_{ij}\geqslant 0,\ (ij)\in I\times J,\ \alpha\in[0,1]\end{aligned} \tag{22}$$

Now (22) can be solved as a classical parametric transportation problem giving a fuzzy solution to (19). This is illustrated with the following example,

4	5	2	1	$\Delta(8, \infty, 5)$
6	2	4	3	$\Delta(6, \infty, 3)$
3	1	1	1	$\Delta(5, \infty, 1)$
$\Delta(4, 1, \infty)$	$\Delta(5, 2, \infty)$	$\Delta(6, 3, \infty)$	$\Delta(4, 3, \infty)$	

(23)

where $\Delta(x, \underset{\sim}{x}, \infty)$ and $\Delta(x, \infty, \bar{x})$ are triangular shaped membership function as it is shown in Figure 1.

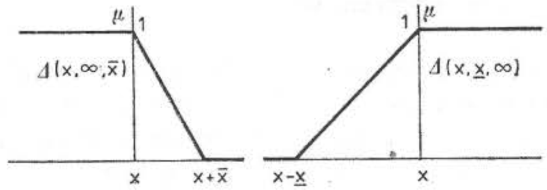

Fig. 1. Shapes of membership functions of the example

By introducing the 5-th dummy destination with its membership function $\Delta(0, \infty, 18)$ and applying (22), one obtains

4	5	2	1	0	$13-5\alpha$
6	2	4	3	0	$9-3\alpha$
3	1	1	1	0	$6-\alpha$
$3+\alpha$	$3+2\alpha$	$3+3\alpha$	$1+3\alpha$	$18-18\alpha$	

where

$$\sum_i \varphi_i(\alpha) = \sum_j \psi_j(\alpha), \quad \alpha \in [0, 1]$$

The optimal solution that provides a fuzzy solution to (23) is,

$3+\alpha$		6α	$1+3\alpha$	$9-15\alpha$
				$9-3\alpha$
	$3+2\alpha$	$3-3\alpha$		

$\alpha \in [0, 3/5]$

$3+\alpha$		$9-9\alpha$	$1+3\alpha$	
	$-9+15\alpha$			$18-18\alpha$
	$12-13\alpha$	$-6+2\alpha$		

(24)

$\alpha \in [3/5, 12/13]$

$3+\alpha$		$-3+4\alpha$	$13-10\alpha$	
	$3+2\alpha$		$-12+13\alpha$	$18-18\alpha$
		$6-\alpha$		

$\alpha \in [12/13, 1]$

By solving (13) with the performance function used in [20]:

$$\mu_0(cx) = (45 - cx)/10 \qquad 35 \leqslant cx \leqslant 45$$

the point-wise solution to (23) obtained in [20] can be found.

Notice that from these results potential applications to other problems can be attempted, as, for instance to the assignment problem. Moreover, a fuzzy transportation problem with a fuzzy objective can be studied, too.

3. Duality in fuzzy linear programming

It is well known that a fuzzy mathematical programming problem is a particular case of the general fuzzy decision problem. Taking this into account it becomes clear that the objectives and constraints can be treated identically in the formulation of a decision, i.e., in the resolution of a FMP problem. This point of view can be understood in the sense that the fuzzy objective and fuzzy constraint are dual concepts.

More precisely, it can be shown, [31], that the dual of a FMP problem with fuzzy constraints is a FMP problem with fuzzy objective, both having the same fuzzy solution, and conversely.

To clarify these considerations about fuzzy duality, the following easy example is developed.

Consider the following FLP problem,

$$\begin{aligned} &\text{Max: } 8x_1+6x_2 \\ &\text{s.t.: } 5x_1+3x_2 \lesssim 30 \\ &\qquad x_1+3x_2 \lesssim 18 \\ &\qquad x_i \geqslant 0 \end{aligned} \tag{25}$$

with the membership functions,

$$\begin{aligned} &\mu_1(5x_1+3x_2, 30)=(36-5x_1-3x_2)^2/36 \quad \text{if } 30 \leqslant 5x_1+3x_2 \leqslant 36 \\ &\mu_2(x_1+3x_2, 18)=(23-x_1-3x_2)^2/25 \quad \text{if } 18 \leqslant x_1+3x_2 \leqslant 23 \end{aligned} \tag{26}$$

taking on the values 0 and 1 outside the above intervals, as usually.

Due to (7) and (8), one obtains,

$$\begin{aligned} &\text{Max: } 8x_1+6x_2 \\ &\text{s.t.: } 5x_1+3x_2 \leqslant 36-6\sqrt{\alpha} \\ &\qquad x_1+3x_2 \leqslant 23-5\sqrt{\alpha} \\ &\qquad x_i \geqslant 0,\ \alpha \in [0, 1] \end{aligned} \tag{27}$$

that is, a classical parametric linear programming problem whose dual is

$$\begin{aligned} &\text{Min: } (36-6\sqrt{\alpha})\, u_1+(23-5\sqrt{\alpha})\, u_2 \\ &\text{s.t.: } 5u_1+u_2 \geqslant 8 \\ &3u_1+3u_2 \geqslant 6 \\ &\alpha \in [0, 1],\ u_i \geqslant 0 \end{aligned} \tag{28}$$

Introducing notation

$$\begin{aligned} &\psi_1(\beta)=(36-6\sqrt{1-\beta}) \\ &\psi_2(\beta)=(23-5\sqrt{1-\beta}),\ \beta=1-\alpha,\ \alpha \in [0, 1] \end{aligned}$$

one can interprete these functions as the inverses of the membership functions of a fuzzy objective in (28). Thus, this problem can be rewritten as

$$\begin{aligned} &\text{Min: } cu \\ &\text{s.t.: } 5u_1+u_2 \geqslant 8 \\ &\quad\quad 3u_1+3u_2 \geqslant 6 \\ &\quad\quad \mu_i(c_i) \geqslant 1-\beta,\ i=1,2 \\ &\quad\quad \alpha \in [0,1],\ x_i \geqslant 0,\ c_i \in R,\ i=1,2 \end{aligned} \tag{29}$$

or, equivalently,

$$\begin{aligned} &\underset{\sim}{\text{Min}}\text{: } cu \\ &\text{s.t.: } 5u_1+u_2 \geqslant 8 \\ &\quad\quad 3u_1+3u_2 \geqslant 6 \\ &\quad\quad u_i \geqslant 0 \end{aligned} \tag{30}$$

with (26) being membership functions of the objective.

Notice that (25) is an FLP with fuzzy constraints while (30) is an FLP with a fuzzy objective characterized by the membership functions of (25), and vice versa.

On the other hand, if (27) is solved,

$$x_1(\alpha)=(65-5\sqrt{\alpha})/20,\ x_2(\alpha)=(79-19\sqrt{\alpha})/12,\ \alpha \in [0,1]$$

is obtained, and therefore,

$$z[x(\alpha)]=(131-23\sqrt{\alpha})/2,\ \alpha \in [0,1]$$

Hence, the fuzzy solution to (27) is,

$$\Omega=\{x;\ (131-2x)^2/529:\ x \in [54,\ 131/2]\} \tag{31}$$

If (30) is solved by means of (28), one obtains

$$u_1=3/2,\ u_2=1/2,\ w(u)=(131-23\sqrt{\alpha})/2,\ \alpha \in [0,1]$$

which produces, exactly, the fuzzy solution (31).

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Zastosowania optymalizacji rozmytej w badaniach operacyjnych

Artykuł dotyczy zadań optymalizacji rozmytej. Sformułowano w nim zadanie rozmytego programowania matematycznego, przy czym analizowany jest jego szczególny przypadek, tj. rozmyte programowanie liniowe. Rozważa się zarówno ogólne sformułowanie zadania rozmytego, programowania liniowego, jak i w wersji rozmytej, takie jego przypadki jak: zadanie transportowe, zadanie maksymalizacji wektorowej, a także teorię dualności.

Использование нечеткой оптимизации в операционных исследованиях

Статья касается задач нечеткой оптимизации. Формулируется задача нечеткого математического программирования, причем анализируется особый случай, т.е. нечеткое линейное программирование. Рассматривается как общая формулировка задачи нечеткого линейного программирования, так и в нечетком варианте случай транспортной задачи и задачи векторной максимизации, а также теории дуальности.