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The influence of some fuzzy implication operators on the steady-state and dynamical properties of a fuzzy logic controller

by

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In this paper, the influence of fuzzy implication operators on the performance of a fuzzy logic controller is considered. Some typical fuzzy implication operators are applied to the construction of a fuzzy controller. Settling time, overshoot, and steady-state value are used as the criteria to evaluate the performance of the controller.

Keywords: Fuzzy logic controller, fuzzy implication operators, performance criteria, fuzzy logics, fuzzy sets, expert systems.

1. Introduction

The design of fuzzy logic control of dynamic systems is based upon the knowledge of human experience in controlling a dynamic process. The human operator can take control decisions based on a qualitative information about states of the process. He can express his control strategy by means of linguistic description. The linguistic control algorithm consists of a set of linguistic implications linked together by connectives, e.g.:

$$\begin{aligned}&\text{IF } E=\mathit{big} \text{ THEN } X=\mathit{small}\\ &\text{ALSO}\\ &\text{IF } E=\mathit{medium} \text{ THEN } X=\mathit{medium}\\ &\text{ALSO}\\ &\text{IF } E=\mathit{null} \text{ THEN } X=\mathit{zero}\end{aligned} \tag{1.1}$$

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where:

E=a process state or error, and

X=a control action.

The state variable E and the control action X take the linguistic values ***big***, ***small***, ***medium***, etc. According to L. A. Zadeh (1975) the linguistic control algorithm (1.1) can be formalized by means of fuzzy sets, fuzzy implications, fuzzy relation, and compositional rule of inference. Taking fuzzy implication as the Cartesian product:

$$\text{IF } E \text{ THEN } X=\mu_{E\times X}(e,x)=\min\{\mu_E(e),\mu_X(x)\},\ \forall e,x\in E,X \tag{1.2}$$

and the connective ALSO as a fuzzy union, we obtain the fuzzy relation:

$$R:\{E\}\rightarrow\{X\} \tag{1.3}$$

where: $\mu_E(e)$, $\mu_X(x)$ — fuzzy sets; $\{E\}$, $\{X\}$ — families of the fuzzy sets.

Fuzzy relation (1.3) and compositional rule of inference yields a mathematical model of linguistic description (1.1) given by

$$X=E\circ R. \tag{1.4}$$

where:

$\circ$ — the max-min composition,

E — a fuzzy state, and

X — a fuzzy control action.

Taking into consideration a thinking process of human being it is worth to see that the inference processes have basic influence on the quality of linguistic description. More specifically, a mathematical formalism of the implication IF ... THEN has a fundamental influence on the quality of fuzzy control. The characteristics of the fuzzy implications have been studied by many investigators (Zadeh 1973, 1984), Sembi and Mamdani (1980), Willmot (1980) and Kiszka, Kochanska and Sliwinska (1984)).

It seems that operating properties of fuzzy implications should be interesting from the viewpoint of practical applications of a fuzzy logic controller. By an appropriate choice of the fuzzy implication definitions it is possible to obtain desirable properties of the fuzzy logic controller. The purpose of this paper is to investigate the effect of different definitions of fuzzy implications and the sentence connective ALSO on the operating properties of a fuzzy logic controller. Some of the operating properties considered are settling time, steadystate, overshoot and tolerance range.

An attempt will also be made to select a type of fuzzy implication which will ensure the best operating properties of the fuzzy logic controller.

2. Simulation studies and discussion

Suppose that the operator of a certain process has provided a hypothetical verbal description of his control strategy in the form given by:

$$\begin{array}{ll} \text{IF } E = \textit{negative big} & \text{THEN } X = \textit{negative medium} \\ \text{ALSO} & \\ \text{IF } E = \textit{negative medium} & \text{THEN } X = \textit{negative medium} \\ \text{ALSO} & \\ \text{IF } E = \textit{negative small} & \text{THEN } X = \textit{negative small} \\ \text{ALSO} & \\ \text{IF } E = \textit{zero} & \text{THEN } X = \textit{zero} \\ \text{ALSO} & \\ \text{IF } E = \textit{positive small} & \text{THEN } X = \textit{positive small} \\ \text{ALSO} & \\ \text{IF } E = \textit{positive medium} & \text{THEN } X = \textit{positive big} \\ \text{ALSO} & \\ \text{IF } E = \textit{positive big} & \text{THEN } X = \textit{positive medium} \end{array} \tag{2.0}$$

where: E is a systems error, and X is a control action. The membership functions of control error and control action are defined as follows:

$$\mu(y)=[1+(y-y_0)^2]^{-1} \tag{2.1}$$

where:

$$\text{if } y_0=\begin{cases} -6 \text{ then } \textit{negative big} \\ -4 \quad\quad\; \textit{negative medium} \\ -2 \quad\quad\; \textit{negative small} \\ \;\;0 \quad\quad\;\; \textit{zero} \\ +2 \text{ then } \textit{positive small} \\ +4 \quad\quad\; \textit{positive medium} \\ +6 \quad\quad\; \textit{positive big} \end{cases} \tag{2.2}$$

The support of the fuzzy sets is defined as follows:

$$s=\mathscr{X}=[-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6] \tag{2.3}$$

The dynamic characteristic of the fuzzy logic controller has been constructed based on the following compositional rule of inference:

$$X_{t+1}=X \circ R;\ t=0, 1, 2, ..., 10 \tag{2.4}$$

In (2.4), the following initial condition is taken:

$$X_0=\textit{positive big}.$$

It is to be noted that the dynamic characteristic of the fuzzy logic controller makes control actions $X=X(t)$ as a function of time. The static characteristic has been built based upon the expression:

$$X=E \circ R \tag{2.5}$$

where:

E = a fuzzy singleton of error,

X = a fuzzy singleton of control action.

Also, it should be noted that in the fuzzy controller the error and control action are statically related, $X=X(E)$.

In order to evaluate the usefulness of the fuzzy logic controller the following parameters have been used:

Dynamical Properties:

Steady state X_u

$$X_u=\lim_{t\to\infty} X_t;\; t=0, 1, 2, \dots \tag{2.6}$$

Setting time T_r was counted from the input moment to the steady-state moment.

Oscillation period T was counted using the following relation:

$$X(t+T)=X(t), \tag{2.7}$$

where: T=oscillation period.

Oscillation amplitude A is a maximum value of the dynamic response.

The maximum overshoot M is the maximum peak value of the response curve measured from the set value.

Steady-state Properties:

Forecasting ability α

$$\alpha=\frac{\Delta X}{\Delta E} \tag{2.8}$$

was defined as a ratio of incremental control action and incremental error. The coefficient shows the ability of the fuzzy controller to forecast the future states based upon the current states.

Tolerance range β

$$\beta=\{E \mid \Delta X/\Delta E=\text{constant}\} \tag{2.9}$$

This coefficient shows the error domain where a control action takes constant values.

It is an "insensitivity area" of the fuzzy controller.

As stated earlier, the definitions of fuzzy implication IF ... THEN and connective ALSO have strong influence on the fuzzy logic controller properties.

Eight typical definitions of these implementation parameters have been given in Appendix I. Linguistic description of equation (2.0) of the control actions has been simulated by applying the techniques described here. The corresponding dynamic and static characteristics are shown in Figures 2.1 to 2.8.

The characteristics of the fuzzy logic controller have been compiled in Table 2.1. Following interesting conclusions may be drawn from these studies.

The fuzzy controller according to the definition (I.1) (see Appendix I) has very poor statical and dynamic properties (see Fig. 2.1 and Table 2.1). The required

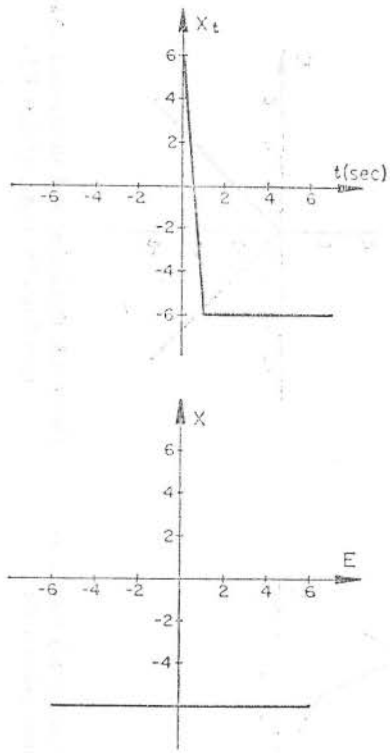

Figure 2.1. Dynamical and steady-state characteristics for fuzzy relation R1

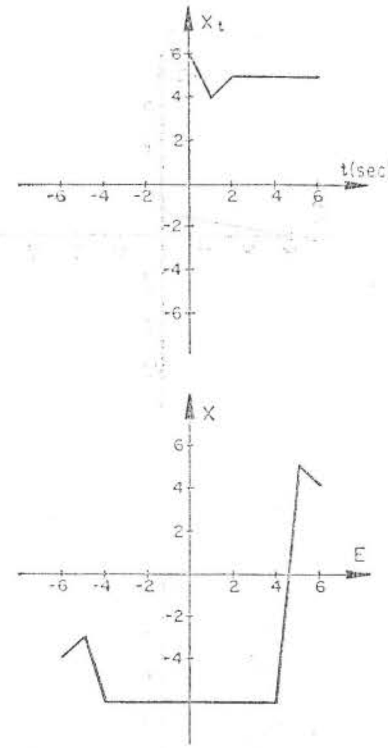

Figure 2.2. Dynamical and steady-state characteristics for fuzzy relation R2

value has not been reached. The forecasting ability $\alpha=\emptyset$, and the tolerance range $\beta=[-6, +6]$. We see that the fuzzy relation defined by formula (I.1) cannot be used as a mathematical formalism of the linguistic controller given by (2.$\emptyset$).

Using the definition given in (I.2), we see that the forecasting ability has heterogeneity property. The fuzzy controller (see Fig. 2.2) does not reach the required value. The tolerance range $\beta=[-4, 4]$ is very big. This means that a big change in control error does not cause any control action. This type of fuzzy controller also makes the overshoot $M=1$. It is easy to see that this type of controller cannot be used in practice.

The fuzzy controller defined by relation (I.3) (see Fig. 2.3) has a small forecasting ability, big tolerance range and steady-state value different from the required value. We can not accept these characteristics in a good regulator.

Implementation of the linguistic description by the relation (I.4) introduces instability into the fuzzy controller, (see Fig. 2.4). The oscillation period $T=1$, and the oscillation amplitude $X=2$, (See Table 2.1). The instability property precludes this type of controller as well.

The fuzzy controller implemented according to relation (I.5) (see Appendix I) yields the best properties, (see Fig. 2.5). This type of the fuzzy controller has a very good forecasting ability and a good tolerance range. In this controller the required

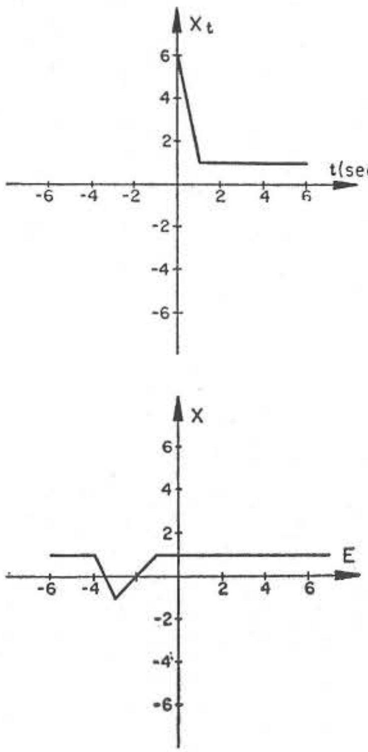

Figure 2.3. Dynamical and steady-state characteristics for fuzzy relation R3

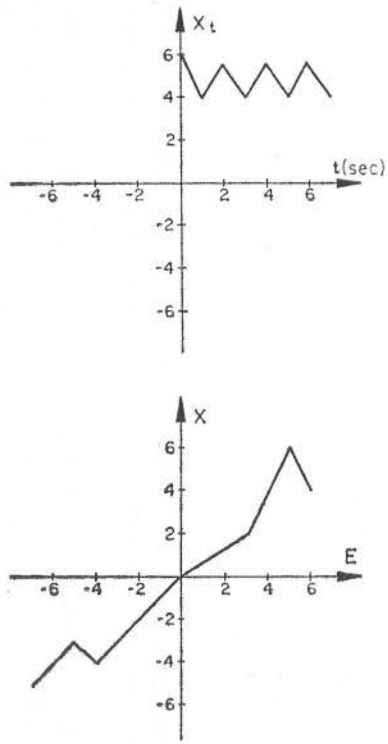

Figure 2.4. Dynamical and steady-state characteristics for fuzzy relation R4

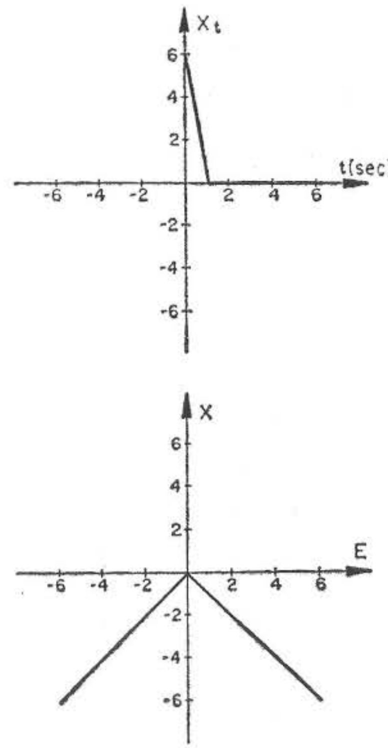

Figure 2.5. Dynamical and steady-state characteristics for fuzzy relation R5

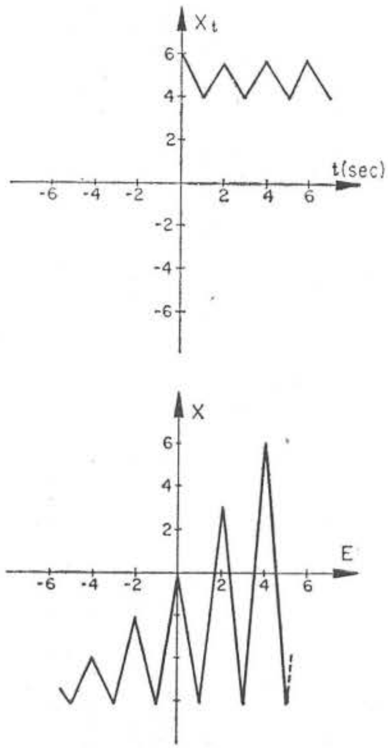

Figure 2.6. Dynamical and steady-state characteristics for fuzzy relation R6

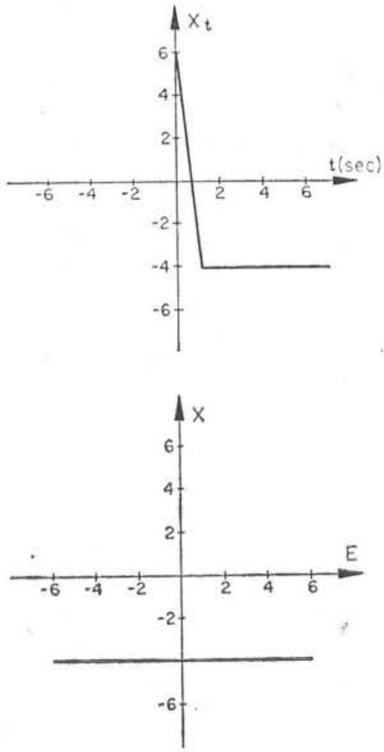

Figure 2.7. Dynamical and steady-state characteristics for fuzzy relation R7

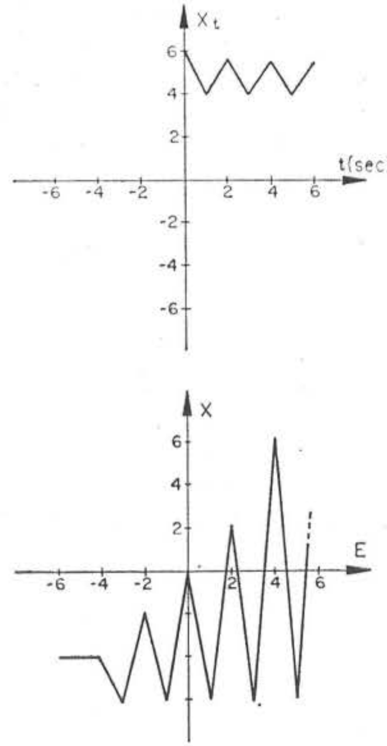

Figure 2.8. Dynamical and steady-state characteristics for fuzzy relation R8

Table 2.1

Steady-state and dynamic characteristics of the fuzzy logic controller

Fuzzy relation	Dynamic characteristics					Steady-state characteristics	
	Steady-state X_μ	Settling time T_r	Oscillation period T	Oscillation amplitude A	Overshoot M	Forecasting ability α	Tolerance range β
R_1	−6	0.5	0	0	0	0	[−6, 6]
R_2	5	2	1.5	1	1	11 −1, 0 −3, 1	[−4, 4]
R_3	1	1	0	0	0	−2 1	[−6, −4] [−1, 6]
R_4	0	0	1	2	0	0.7 1	∅
R_5	0	1	0	0	0	−1 1	[−6, 6]
R_6	0	0	1	2	0	3 6 1	—
R_7	−4	1	0	0	0	0	[−6, 6]
R_8	0	0	1	2	0	0 12	[−6, −4]

value has been reached. The setting time is very short, $T_r=1$. Here we do not observe any tendency for oscillations. This type of regulator is acceptable from an operating point of view.

It is very interesting to note how the fuzzy relations defined by relations (I.6) and (I.8) influence the controller properties. Both of them give the same dynamic unstable characteristics but the static characteristics are different (see Figs. 2.6 and 2.8 and Table 2.1). Performance results of the analysis of these two controllers show that both of these fuzzy controllers cannot be used in practice.

Implementation realized by relation (I.7) does not cause admittedly instability but the required value cannot be reached (see Fig. 2.7). This type of controller is also not good for practical use.

Based upon these studies we conclude that the fuzzy logic controller has a huge flexibility in its properties. By an appropriate choice of the fuzzy mathematical apparatus we may have very strong influence on statical and dynamical properties of the fuzzy logic controller. Let us formulate the following optimization problem:

"Choose the most appropriate operating properties of the fuzzy logic controller by varying the definition of the fuzzy implication operator."

Let

$$I:\ \mathcal{L}(E)\rightarrow\mathcal{L}(X) \tag{2.9}$$

be the fuzzy implication definitions where: $\mathcal{L}(X)$, $\mathcal{L}(E)$ are families of the fuzzy sets.

Let

$$W=\{X_u, T_r, T, A, M, \alpha, \beta\} \quad (2.10)$$

be the desired properties of the fuzzy logic controller.

We have shown that the properties of the fuzzy controller depend upon the definition of the fuzzy implication. We can rewrite relation (2.10) as a function of formula (2.3):

$$W(I)=\{X_u(I), T_r(I), T(I), A(I), M(I), \alpha(I), \beta(I)\}$$

The optimization problem may now be formulated as follows:

$$W=\operatorname*{ext.}_{I}\{X_u(I), T_r(I), T(I), A(I), M(I), \alpha(I), \beta(I)\} \quad (2.12)$$

By virtue of the techniques described in this paper we may have

$$W_{\mathrm{opt}}=\{X_u^*, T_r^*, T^*, A^*, M^*, \alpha^*, \beta^*\} \quad (2.12)$$

where

"*" denotes optimal values of static and dynamic characteristics given, in this case, by fuzzy implication defined by the relation (I.5).

3. Conclusions

Eight different definitions of fuzzy implication operators are examined, and their operational properties are studied.

This study shows that the fuzzy logic controller has very flexible operating properties and it is possible to carry out an optimization of these properties in order to give the best possible operating characteristics.

APPENDIX I

Definitions of fuzzy relations

$$\mu_{R_1}(e, x_j)=\bigvee_{k=1}^{7}\begin{cases}1, & \mu_E(e_.)\neq 1,\ \mu_X(x_j)=1\\ 0, & \text{otherwise}\end{cases} \quad (I.1)$$

$$\mu_{R_2}(e_i, x_j)=\bigwedge_{k=1}^{7}\begin{cases}1, & \mu_E(e_i)\neq 1,\ \mu_X(x_j)=1\\ 0, & \text{otherwise}\end{cases} \quad (I.2)$$

$$\mu_{R_3}(e_i, x_j)=\bigwedge_{k=1}^{7}\begin{cases}1, & \mu_E(e_i)=\mu_X(x_j)\\ \mu_X(x_j), & \text{otherwise}\end{cases} \quad (I.3)$$

$$\mu_{R_4}(e_i, x_j)=\bigwedge_{k=1}^{7}\{\min[1, 1-\mu_E(e_i)+\mu_X(x_j)]\} \quad (I.4)$$

$$\mu_{R_5}(e_i, x_j)=\bigwedge_{k=1}^{7}\{\min[\mu_E(e_i), \mu_X(x_j)]\} \quad (I.5)$$

$$\mu_{R_6}(e_i, x_j) = \bigvee_{k=1}^{7} \{[\mu_E(e_i) \Lambda \mu_X(x_j)] \, V [1 - \mu_X(x_j)]\} \tag{I.6}$$

$$\mu_{R_7}(e_i, x_j) = \bigwedge_{k=1}^{7} \{\max [1 - \mu_E(e_i), \ \mu_X(x_j)]\} \tag{I.7}$$

$$\mu_{R_8}(e_i, x_j) = \bigwedge_{k=1}^{7} \{\max [1 - \mu_E(e_i), \ \mu_X(x_j)]\} \tag{I.8}$$

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Wpływ wyboru operatora implikacji rozmytej na własności dynamiczne i w stanie ustalonym regulatora opartego na rozmytej logice

Rozważono wpływ wyboru postaci operatora rozmytej implikacji na jakość działania regulatora opartego na rozmytej logice. Pewne typowe postacie operatora implikacji zostały zastosowane do skonstruowania tego regulatora. Takie własności czasowej charakterystyki układu jak czas ustalania, przeregulowanie i wartość w stanie ustalonym są używane jako kryteria działania regulatora. Na podstawie tych kryteriów dokonano oceny wpływu operatora implikacji na jakość regulatora.

Влияние выбора оператора нечеткой импликации на динамические и установившиеся свойства регулятора, основанного на нечеткой логике

Рассмотрено влияние выбора вида оператора нечеткой импликации на качество действия регулятора, основанного на нечеткой логике. Некоторые типовые виды оператора импликации были использованы для разработки этого регулятора. Такие свойства временной характеристики системы, как время успокоения, перерегулирование и значение в установившемся состоянии используются в качестве критериев действия регулятора. На основе этих критериев проводится оценка влияния оператора импликации на качество регулятора.