Control
and Cybernetics
VOL. 13 (1984) No. 3

Applications of fuzzy mathematics in seismological and meteorological research

by

FENG DE-YI
Tianjin Seismological Bureau,
Tianjin, People's Republic of China

LOU SHI-BO
Shanghai Institute of Railway Technology,
Shanghai, People's Republic of China

CHEN HUA-CHENG
Lenzhou Institute of Railway Technology,
Lanzhou, People's Republic of China

GU JIN-PING
Seismological Institute of Lanzhou,
Lanzhou, People's Republic of China

LIN MING-ZHOU
Shanghai Seismological Bureau,
Shanghai, People's Republic of China

By using a series of fuzzy mathematical methods, some important problems in seismology can be effectively investigated. Among these methods, a fuzzy description of earthquake precursors and a direct method for their fuzzy distinction have been studied; a method of fuzzy cluster analysis, a classification method for retrieving fuzzy information, and a fuzzy similarity method have been applied in earthquake prediction studies; fuzzy multifactorial evaluation has been applied to the evaluation of earthquake intensity; some fuzzy methods can also be suggested for studying many other seismological problems, such as examination of models for earthquake preparatory processes, investigation of earth interior structure, etc.

Fuzzy mathematical methods can also be successfully applied to meteorology, especially to weather forecasting. For example, the longer-period raining weather forecast in the Shanghai area, rainfall forecast in North China, and the influence of agricultural climate on planting natural rubber have been studied with the use of different methods based on the theory of fuzzy sets.

1. Introduction

Since 1965 when L. A. Zadeh proposed fuzzy set theory (Zadeh, 1965), fuzzy mathematics has developed rapidly. At present, it attracts much attention in various scientific domains.

For studying many problems in seismology and meteorology, in particular the difficult problems of earthquake prediction and weather forecasting, we often cannot avoid relying only on experience. Obviously, we must anyway deal with the problem of how to make earthquake prediction or weather forecast as correctly as possible with a required accuracy.

The treatment of empirical information and data by using methods of fuzzy mathematics has many advantages as, e.g. simplicity, convenience, easy operation. Moreover, human wisdom and experience can here easily be incorporated. Thus, the approach can take a good place in studying a series of seismological and meteorological problems on the basis of observational data.

This paper presents some important results obtained by the authors in applying methods of fuzzy mathematics to seismological and meteorological research. From these preliminary results we can see that these methods have promising perspectives in some domains of earth sciences including seismology and meteorology.

2. Applications of fuzzy mathematics in seismological research

2.1. A fuzzy description of earthquake precursors and a direct method for their fuzzy distinction

A direct method for fuzzy distinguishing of earthquake precursors consists in judging the earthquake risk and making prediction directly by the use of membership functions of different precursors. Its effectiveness depends on the technique of constructing these membership functions. Based on various precursor data, such as the radon content of underground water, apparent resistivity, seismic velocities etc., some methods and formulae for constructing the corresponding membership functions have been suggested by employing mainly the rate of precursor change with a correlation coefficient.

Quite a number of precursor data, as is well-known, indicate that the type of positive anomaly, i.e. the type of „Increase-Stationariness-Decrease-Earthquake Occurrence" and the type of negative anomaly, i.e. the type of "Decrease-Stationariness-Increase-Earthquake-Occurrence" of many kinds of precursors can be observed before a large earthquake. The smoothing mean precursor curve $y=A(t)$, varying over time, often reflects a main distinction between "Normality" and "Abnormality" in its "Flatness" and "Obliquity", i.e. in the size of the rate of precursor change. As the precursor change $y=A(x)$ can be either positive or negative, so for identifying the states "Normality" and "Abnormality" only the absolute value of the slope or the rate of precursor curve can be taken. Therefore, we have constructed the membership function of the single precursor in the form of (Feng De-yi, et al., 1978; 1981b):

$$\mu_i=\left(1+\frac{\alpha}{|K_1|\,|r_i|}\right)^{-1}, \tag{1}$$

where subscript i denotes the precursor number of a certain term, $|K_i|$ is the absolute value of the slope, r_i is the correlation coefficient, and α is an empirical constant. The value of μ is taken from [0, 1]. When μ is greater, the precursor is more obvious.

For describing the degree of obliquity of the precursor curve in fuzzy terms, we can define, for example:

$$\mu_{\text{slightly oblique}}=\mu^{0.25}$$

$$\mu_{\text{oblique}}=\mu^{1.0}$$

$$\mu_{\text{strongly oblique}}=\mu^{3.0}$$

Obviously, we can change the concrete indices for reducing the mistakes in judgements of the given samples.

Then, for a fuzzy set consisting of several kinds of precursors, the membership function can be written as:

$$\mu=\mu_1^{n_1} \vee \mu_2^{n_2} \vee \dots \vee \mu_m^{n_m}, \tag{2}$$

where: $\mu_1 \vee \mu_2=\max(\mu_1, \mu_2)$, and $n_1, n_2, \dots, n_m$ must be determined empirically.

Moreover, in some cases it is difficult to obtain a mathematical expression for a membership function, but the precursors can be represented by fuzzy descriptions according to experience. As an example, the inductive action of magnetic storm depends on its strength, so we can define the membership function as:

$$\mu=\begin{cases} 0.8, & \text{when there are two continuous magnetic storms with } K\geqslant 7; \\ 0.6, & \text{when there are one magnetic storm with } K\geqslant 7 \text{ and another with } K=6; \\ 0.4, & \text{when there is only one magnetic storm with } K\geqslant 7; \\ 0.2, & \text{when there is only one magnetic storm with } K=6; \\ 0, & \text{without any magnetic storm with } K\geqslant 6. \end{cases}$$

2.2. Application of fuzzy cluster analysis to earthquake prediction

The basic idea of applying cluster analysis to earthquake hazard assessment is that the earthquakes of different magnitudes are classified on the basis of fuzzy similarity relations according to different kinds of statistical indices or precursors. In particular, a method of cluster analysis based on the fuzzy equivalent relation has been applied. The method is described in detail in Lou Shi-bo and Chen Hua-cheng (1981) and includes mainly the following steps: obtaining the fuzzy compatibility relation according to the degrees of similarities between the samples; transforming the relation obtained into a fuzzy equivalent relation by the use of a combinational operation; finally, selecting a suitable value of a parameter λ, using the "Level cut-set method", and classifying the original samples.

Taking some statistical indices of seismicity or observational precursor data obtained in a given region a multi-approach earthquake prediction may be made by using the mentioned method of fuzzy cluster analysis. For illustrating this method we also choose the six basic statistical indices of seimicity in a given region namely: the earthquake frequency N, the maximum magnitude M, the average magnitude of earthquake $\bar{M}$ (or the b-value) and their derivatives versus time, $\dot{N}$, $\dot{M}$ and $\dot{\bar{M}}$ in the adjacent three time intervals hence we have 18 statistical indices for evaluating seismic risk in the forthcoming time interval (Feng De-yi at al. 1981a). By employing the method of fuzzy cluster analysis all the time intervals as samples taken for research can be classified into the samples "with earthquake" with magnitude $M \geqslant M_L$ and those "without earthquake" with $M \geqslant M_L$ according to the value of λ. Then the new samples "with earthquake" or those "without earthquake" may be examined. If the new samples belong to a certain type of "with earthquake" or "without earthquake" classified originally, then the prediction is successful; if the result is not the same as above, then this new type "with earthquake" or "without earthquake" should be added correspondingly to the original classification. The selection of the value of λ should ensure that the samples "with earthquake" and those "without earthquake" should be separated to the highest extent to be least confusing.

Now we give the following two real examples.

The first example is a study of earthquakes in the Yinchuan-Songpan section in the North-South Earthquake Zone of China. Using 12 indices of seismic activity i.e. M, N, b and $\dot{M}$, $\dot{N}$, $\dot{b}$ taken in the adjacent two months the "with $M \geqslant 4.5$ earthquake" or "without $M \geqslant 4.5$ earthquake" in the next month can be judged. The data from 1967 to 1979 were collected and hence 21 months were obtained as the samples. Taking $\lambda = 0.983$ for fuzzy cluster analysis, we can get eight types of samples belonging to "with $M \geqslant 4.5$ earthquake":

(1) October, 1967, Jin-Yuan, $M=5.3$; June, 1971, Wuzong, $M=5.5$. Both months have similar magnitudes and neighbouring epicenters of maximum earthquakes.
(2) September 1969, Wudu, $M=5.0$; May 1973, Nanping, $M=5.5$. Both months also have similar magnitudes and neighbouring epicenters of maximum earthquakes.
(3) October 1969, Abua, $M=5.3$; August 1976, Songpan, $M=7.2$. Both earthquakes were in the neighbouring regions but their magnitudes were obviously different.

The other five types of samples are classified similarly.

Then we took four "without earthquake" samples, and the results showed that all those samples could be classified into the type "without earthquake", in fact.

The second example is based on seismicity data taken from five provinces in West China, i.e. Sichuan, Yunnan, Gansu, Qinghai, and Ninxia Hui Nationality Autonomous Region. We had chosen 15 earthquake samples of those five provinces (regions). By using the indices N, $\dot{N}$, M, $\dot{M}$ in the adjacent three years, i.e. 12 indices, and employing the method of fuzzy cluster analysis, the earthquake risk in the next year is predicted. The results are as in Table 1.

From Table 1 it can be seen that the predicted maximum magnitude M_{max} is in good agreement with the magnitude of the actual event, and the predicted average magnitude of large earthquakes $\bar{M}$ is by $M=0.5 \div 1.0$ smaller than that of the latter (see Feng D-yi et al., 1982a).

Table 1

Time (Year)	Predicted Maximum Magnitude		Actual Largest Event	
	M_{max}	$\overline{M}$	M	Region
1973	7.7	6.9	7.9	Luhuo, Sichuan
1974	7.0	6.6	7.1	Yongshan, Yunnan
1976	7.25	6.2	7.6	Longling, Yunnan
			7.2	Songpan, Sichuan

Excluding statistical indices of seismicity, such as N, M, b, etc., the precursor data (earth deformation, tilt, radon content, apparent resistivity, etc.) can also be chosen for earthquake prediction by the method of fuzzy cluster analysis. Some preliminary results were obtained for earthquakes with $M \geqslant 5.0$ which occurred in Sichuan Province (see Feng De-yi et al., 1983b).

2.3. Application of a fuzzy information retrieval method in earthquake prediction research

As an aspect of earthquake prediction research, a classification method for retrieving information based on the theory of fuzzy sets, has been studied as well (Feng De-yi et al., 1982b; Lou Shi-bo et al., 1983). By using this method and taking some basic premonitory indices in a given region as descriptors, the time intervals can be classified into different kinds with different ranges of the maximum magnitude of earthquakes. Such a classification may be used to predict the time of large earthquake occurrence in the given region. The basic premonitory indices used as descriptors may be some statistical indices of seismicity, such as N, M, b, $\dot{N}$, $\dot{M}$, $\dot{b}$ and other earthquake precursors.

Table 2

Type of Seismic Risk	Retrieval and Predicted Period	Period of Actual Earthquake Occurrence
Occurrence of earthquake with $M=4.0 \div 4.9$	February, May and November, 1976	February, June and November, 1976
Occurrence of earthquake with $M=5.0 \div 5.9$	September and December, 1976	September and December, 1976
Occurrence of earthquake with $M=6.0 \div 6.9$	No	No
Occurrence of earthquake with $M>7.0$	August, 1976	August, 1976
No occurrence of earthquake with $M>4.0$	January, March, June, July, October, 1976 and January, 1981	January, March, April, May, July, October, 1976 and January, 1981

Adopting this method and the seismic activity data for statistical indices N, M, $\dot{N}$, $\dot{M}$, obtained at the section from Nanping, Songpan of Sichuan Province to Yinohuan of Ninxia Region, as an example, we have studied the earthquakes of $M_L \geqslant 2.6$ from 1966 to December, 1980, and made an earthquake prediction examination. The results are shown in Table 2. From this table we can see that the predicted earthquake risk results are also in good agreement with the actual cases.

2.4. Applications of fuzzy mathematics in engineering seismology

We have been studying applications of fuzzy mathematics in engineering seismology including seismic zoning and a quantitative evaluation of earthquake intensity.

In seismic zoning and evaluation of earthquake intensity, a series of qualitative and quantitative indices may be used simultaneously as is well-known. Among them, many indices, particularly the qualitative indices are fuzzy. For example, in the evaluation of earthquake risk and hazard based on seismic zoning the used indices have often to do with geological structure, intensities of historical earthquakes, recent seismic activity, etc. In the evaluation of earthquake intensity some qualitative indices, such as building damage, human sensitivity, rupture (crack) of earth surface and others are mainly used. These fuzzy indices or standards are difficult for a quantitative evaluation and so is the earthquake itself. Fortunately, the concepts and methods of fuzzy mathematics developed rapidly in recent years seem to be an efficient tool for such quantitative evaluations. Here we describe a method of fuzzy mathematics for evaluating earthquake intensity (Feng De-yi et al., 1982c).

Based on the concept of the degree of approaching for normal fuzzy sets and by means of fuzzy synthetic judgement, the abundant macroscopic observational data relevant to the evaluation of earthquake intensity by the Chinese seismic scale are investigated and treated quantitatively. These macroscopic data have been collected from seismological literature and reports on the intensities of Luhuo (M=7.9), Tangshan (M=7.8), Longling (M=7.6), Haicheng (M=7.3), Sandan (M=7.25), Xintai (M=7.2), Songpan (M=7.2), Yongshan (M=7.1), Yangkiang (M=6.4), Ushi (M=6.1), Liyang (M=6.0) and other earthquakes with $M \geqslant 6.0$ which occurred in China. Setting the degrees of damage for buildings of different types as a macroscopic standard, quantitative comparison table for degrees VI to XI for an empirical judging of earthquake intensity has been preliminarily compiled. From this comparison table we can obtain membership functions for such a macroscopic standard. Then, we can apply a simple method for quantitatively evaluating earthquake intensity by the use of this fuzzy macroscopic standard of seismic scale. This method consists of the following main steps:

(1) Collection of macroscopic observational data as samples of the assumed standard after occurrence of an earthquake;

(2) Determination of the membership functions for different cases of the assumed macroscopic standard;

(3) Calculation of all elements of the degree matrix of approaching;
(4) Averaging and normalization of the above-mentioned matrix;
(5) Consideration of certain weight distributions of different cases of the assumed standard and evaluation of the intensity of the given earthquake by means of a fuzzy synthetic judgement.

Let us give an example. For judging the intensity of Songpan (M=7.2) earthquake at Beima and Wanbachu points together, we can use data about building damage and destruction at those points, as shown in Table 3, where a is the mean percentage value of all the assumed indices and b is its mean standard deviation. Then, we can get the degree matrix of approaching as:

$$S=\begin{bmatrix} 0.5 & 0.547 & 0.678 & 0.893 & 0.826 \\ 0.5 & 0.577 & 0.589 & 0.959 & 0.837 \end{bmatrix}.$$

Its normalized form is:

$$Q=\begin{bmatrix} 0.15 & 0.16 & 0.20 & 0.26 & 0.24 \\ 0.15 & 0.17 & 0.17 & 0.28 & 0.24 \end{bmatrix}.$$

Table 3

Type of Building	Slight Damage	Damage	Destruction	Severe Destruction
II-type buildings	a=0.412 b=0.059	a=0.47 b=0.028	a=0.018 b=0.025	a=0 b=0
III-type buildings	a=0.529 b=0.426	a=0.08 b=0.042	a=0.0025 b=0.0035	a=0 b=0

Supposing that both the II-type and III-type buildings have equal weight for judging earthquake intensity, i.e. taking W=(0.5, 0.5), we can obtain the synthesized indices for a synthetic judgement of the intensity as a vector $P=W \circ Q$, where "$\circ$" is the max-min composition, and its normalized form as:

$$H=[0.14,\ 0.16,\ 0.18,\ 0.27,\ 0.24].$$

The components of H correspond to the X, IX, VIII, VII, VI degrees, respectively. From these components we can see that H_4=0.27 corresponding to the VII degree is the largest. Therefore, the intensity at Baima and Wangbachu together must be judged as the VII degree, but it also may be approaching the VI degree. This result agrees well with the result of a macroscopic intensity investigation obtained by Sichuan Seismological Bureau.

2.5. Methods of fuzzy mathematics for the examination of an earthquake preparatory model

The concept of the degree of approaching and the method of fuzzy similarity choice can be used to examine a different theoretical earthquake preparatory source model.

As is well-known, two main preparatory source models for earthquake research have been proposed by the American and Soviet scientists, i.e. the DD and IPE models, respectively (Brace, 1975). Taking the Liyang (M=6.0) earthquake in 1979 as an example, we tried to apply these concepts and methods on the basis of some precursor data. Among them, we took the seismic velocity ratio V_P/V_s, the radon contents in underground water R_n and the levelling data Δh. The precursory time can be approximately assumed from February, 1978, and the precursors may be divided into several steps.

In the data used the durations and maximum anomalous amplitudes of different precursors must be normalized to be comparable.

The normalized degree matrix of approaching is hence obtained, and it can be represented quantitatively as:

$$A=\begin{bmatrix}0.45 & 0.45\\ 0.43 & 0.49\\ 0.60 & 0.54\end{bmatrix}\begin{matrix}V_p/V_s\\ Rn\\ \Delta h\end{matrix}$$
$$\quad\ \ \text{DD}\ \ \text{IPE}$$

where the rows include the degrees of approaching of precursors V_p/V_s, Rn, Δh, respectively, and the columns include the type of the source model, i.e. DD and IPE.

Considering the weight vector $W=[W_1, W_2, W_3]$ with equal values $W=W_2= =W_3=1/3$, we can obtain:

$$H=[0.33,\ 0.33].$$

From vector H we can conclude that the degrees of approaching for the observational precursor data of Liyang earthquake for the DD and IPE models are equal, and both these degrees of approaching are also small enough which indicates that the new earthquake preparatory model must be further developed.

2.6. A study on crust structure model employing a fuzzy similarity choice

For studying earth crust structure, different theoretical models may be assumed. The actual observational data must be compared with them, and one would be selected according to the best agreement. Obviously, problems of selection of a theoretical model of its testing can also be solved by applying methods of fuzzy mathematics.

For example, crust structure in the Middle Asian Region of the USSR was explained by applying a fuzzy similarity choice model. According to the results obtained by Deep Seismic Sounding, three crust structure models may correspond to observational hodographs of basic body waves. They are: model I as a one-layer homogeneous medium, model II as a two-layer homogeneous medium and model III as a one-layer heterogeneous medium (Alekseyev, et al., 1963).

For judging and selecting the best crust model among the three above mentioned theoretical models, the dynamic characteristics of seismic waves must be considered. We have taken observational and theoretical amplitude curves of the boundary P-wave in that region reflected from Moho. From the three theoretical models we can choose the best one according to the nine points (distants) of those reflected P-wave amplitude curves taken as the similarity factors.

Based on these data nine fuzzy relation matrices have been obtained by applying a fuzzy dominant choice method. Then, using the level cut-set method, the model being most similar to the observational data can be chosen. Table 4 shows the dominant similarity choice rank order for the three models.

Table 4

Model \ Factor	1	2	3	4	5	6	7	8	9	Product	Rank Order
I	2	1	2	2	2	2	2	1	1	64	2
II	3	3	3	3	3	3	3	3	2	13122	3
III	1	2	1	1	1	1	1	2	3	12	1

From Table 4 we can see that model III, i.e. the one-layer heterogeneous crust model is the best of them and model II, i.e. the two-layer homogeneous model is the worst for the given region.

3. Applications of fuzzy mathematics in meteorological research

3.1. The lasting spring rainy weather forecast in the Shanghai Area

This research was conducted by Lou Shi-bo and Cheng Hua-cheng (1981). It is known that the lasting spring raining in the Shaghai area depends on ten meteorological factors, hence each lasting raining sample could be considered as a 10-dimension vector, denoted X_i. A method of fuzzy cluster analysis can be used. Its consecutive steps are:

1. Establishing a fuzzy consistency relation R' between the samples using the following formula:

$$\mu_{R'}(X_i, X_j)=\frac{(X_i, X_j)}{\|X_i\|\cdot\|X_j\|}, \tag{3}$$

where:

$$(X_i, X_j)=\sum_{k=1}^{10} X_{ik}\cdot X_{jk},$$

$$\|X_i\|=\left(\sum_{k=1}^{10} X_{ik}^2\right)^{1/2}.$$

2. Changing R' into a fuzzy equivalence relation R, taking $\lambda=0.94$ and using the level cut-set method, the samples are classified into thirteen groups.
3. Each group is a general subset of the lasting raining sample set

$$X^{(i)}=\{X_{i_1}, X_{i_2}, \ldots, X_{i_p}\}.$$

where:

$$X^{(i)} \text{ represents the } i\text{-th group.}$$

4. Constructing a complex pattern of lasting raining as follows:

$$X = \bigvee_i X_i^{(i)}$$

where "$\vee$" is maximum. Obviously, X is a fuzzy set. Its membership function is:

$$\mu_X(X) = \bigvee_i \mu_{X^{(i)}}(X) \tag{4}$$

5. Having a threshold value λ, the forecasting criterion becomes:
   If $\mu_X(X) \geqslant \lambda_0 = 0.94$, then the lasting raining will occur in the day after the next;
   If $\mu_X(X) < \lambda_0$, then the lasting raining will not occur in the day after the next.
6. Using soft partitioning (ISODATA), the lasting raining samples were classified into thirteen groups. The cluster centre is denoted by:

$$V_i;\ i=1, 2, ..., 13.$$

The forecasting criterion is:

$$\mu_{V_i}^{(1)}(X) = \begin{cases} 1-0.5\left(\dfrac{\|X-V_i\|}{0.352}\right)^2, & \text{if } \left(\dfrac{\|X-V_j\|}{0.352}\right)^2 \leqslant 2; \\ & i=1, 2, 3, 4, 8, 11 \\ 0 & , \text{ otherwise.} \end{cases}$$

$$\mu_{V_i}^{(2)}(X) = \begin{cases} 1-0.5\left(\dfrac{\|X-V_i\|}{0.607}\right)^2, & \text{if } \left(\dfrac{\|X-V_j\|}{0.607}\right)^2 \leqslant 2; \\ & i=9 \\ 0 & , \text{ otherwise.} \end{cases}$$

$$\mu_{V_i}^{(3)}(X) = \begin{cases} 1-0.5\left(\dfrac{\|X-V_j\|}{0.087}\right)^2, & \text{if } \left(\dfrac{\|X-V_i\|}{0.087}\right)^2 \leqslant 2; \\ & i=5, 6, 7, 10, 12, 13 \\ 0 & , \text{ otherwise.} \end{cases}$$

If each $\mu_{V_i}(X) < 0.5$, $i=1, 2, ..., 13$, then the lasting raining will not occur in the day after the next; otherwise, it will occur.

3.2. Rainfall forecast in North China

This research was conducted by Zong Rong-xiang (1982). According to experience in meteorological studies, the factors influencing the rainfall of North China are:

X_1: the average temperature in April in the Shanghai area.
X_2: the total rainfall in March in Beijin.
X_3: the geometric index C_L in May.
X_4: the duration of 500 mm bar W circulation in April.

The forecasted object y is the total rainfall in July to August at five weather stations: Beijin, Tianjin, Yingkou, Taiyuan and Shijiazhuang. The data for seven years from 1961 to 1967 are shown in Table 5.

Tahle 5

Factor / Year	X_1	X_2	X_3	X_4	Y	U
1961	14.8	20.1	0.64	13	410	0.80
1962	12.5	2.3	0.36	4	255	0.28
1963	14.5	12.4	0.69	12	527	1
1964	16.4	10.6	0.58	26	510	1
1965	12.2	0.3	0.35	4	226	0
1966	13.8	12.3	0.42	23	456	1
1967	13.6	7.7	0.82	35	389	0.70
1968	13.7	0.6	0.68	13	179	0
1969	14.2	16.5	0.65	15	439	0.90

First, the original data must be processed:

$$X=\frac{X'-\bar{X}'}{S} \tag{5}$$

where X' is the original data $\bar{X}'$ and S are the mean value and variance of the original data, respectively.

The similarity coefficients of the samples are calculated using the following formula:

$$\gamma_{ij}=\frac{\sum_{k=1}^{4} X_{ik}\cdot X_{jk}}{\sqrt{\sum_{k=1}^{4} X_{ik}^2\cdot \sum_{k=1}^{4} X_{jk}^2}} \tag{6}$$

where X_{ik} denotes the value of the k-th factor of the i-th sample due to (5). Using (6), the following matrix is obtained:

$$R_s=\begin{bmatrix} 1 & -0.68 & 0.84 & 0.28 & -0.74 & -0.05 & 0.06 \\ & 1 & -0.57 & -0.76 & -0.99 & -0.19 & -0.60 \\ & & 1 & 0.20 & -0.61 & -0.56 & -0.26 \\ & & & 1 & -0.76 & -0.29 & 0.23 \\ & & & & 1 & -0.19 & -0.49 \\ & & & & & 1 & -0.14 \\ & & & & & & 1 \end{bmatrix}.$$

The calculated similarity coefficients are between $[-1, 1]$. Transformation of each element is performed by the formula:

$$\gamma'_{ij}=0.5+\tfrac{1}{2}\gamma_{ij}.$$

Thus, the R_s matrix is transformed into:

$$R=\begin{bmatrix} 1 & 0.16 & 0.92 & 0.64 & 0.13 & 0.48 & 0.53 \\ & 1 & 0.22 & 0.12 & 0.99 & 0.42 & 0.20 \\ & & 1 & 0.60 & 0.20 & 0.22 & 0.63 \\ & & & 1 & 0.12 & 0.65 & 0.62 \\ & & & & 1 & 0.40 & 0.26 \\ & & & & & 1 & 0.43 \\ & & & & & & 1 \end{bmatrix}.$$

Obviously, the matrix R corresponds to a fuzzy consistency relation between the seven weather samples. In order to perform cluster analysis, it is necessary to transform the fuzzy consistency relation into a fuzzy equivalence relation:

$$R=\begin{bmatrix} 1 & 0.42 & 0.92 & 0.64 & 0.42 & 0.64 & 0.63 \\ & 1 & 0.42 & 0.42 & 0.99 & 0.42 & 0.42 \\ & & 1 & 0.64 & 0.42 & 0.64 & 0.63 \\ & & & 1 & 0.42 & 0.65 & 0.63 \\ & & & & 1 & 0.42 & 0.42 \\ & & & & & 1 & 0.63 \\ & & & & & & 1 \end{bmatrix}.$$

Analysing aggregation of the samples and according to the requirement of the forecast, we assume the threshold $\lambda=0.64$ and the following matrix is obtained:

$$R_{0.64}=\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

As a result, all samples are divided into the following three types:

Type I: 1961, 1963, 1964, 1966;

Type II: 1962, 1965;

Type III: 1967.

We have used the mean value of various types to denote the given sample and mark them as sample I, sample II and sample III, respectively. Among them, sample III has only one element. The factor value of its own is used. We take the factor value of 1968 as sample IV and repeat the above described procedure to make fuzzy clustering of samples I, II, III, IV. The fuzzy equivalence relation matrix below is deduced:

$$R=\begin{bmatrix} 1 & 0.45 & 0.62 & 0.45 \\ & 1 & 0.45 & 0.59 \\ & & 1 & 0.45 \\ & & & 1 \end{bmatrix}.$$

For $\lambda=0.59$, we get:

$$R_{0.59}=\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}.$$

Both sample IV and sample II belong to the same type, that is, the rainfalls in July and August of 1968 are similar to those in 1962 and 1965.

In order to distinguish the amount of rainfall, we stipulate the membership function of a fuzzy subset "more rainfall" as follows:

$$\mu(y)=\begin{cases}1 , & \text{if } y\geqslant 450 \text{ mm};\\ (y-250)/200, & \text{if } 450 \text{ mm}>y>250 \text{ mm};\\ 0 , & \text{if } 250 \text{ mm}\geqslant y.\end{cases}$$

where y denotes the total rainfall from July to August each year. It can be seen from Table 5 that 1961, 1963, 1964, 1966 were years of more rainfall, and 1962, 1965 were years of less rainfall. The rainfall in June and July of 1968 was less. It was 179 mm. This is in good agreement with the forecast.

We have once more taken 1969 as sample V, and it has been shown by cluster analysis that samples V and I belong to the same type. It should be a year of more rainfall. In fact, the rainfall in July and August of 1969 was 439 mm., indicating a year of "more rainfall". The actual situation is in conformity with the forecasted one.

3.3. Assessment of agricultural climate for planting natural rubber

This research was conducted by Gao Su-hua (1982). In order to perform the assessment, many meteorological factors for planting rubber must be considered They are, for instance:

T — the mean temperature over a year;

T_n — the lowest temperature in a year;

F — the mean wind speed over a year.

It is supposed that a single factor judgement matrix R and the distribution of weights of each meteorological factor are well known. Then, we can apply the method of fuzzy synthetic decision.

Thus:

$$B=A\circ R,$$

where B is a fuzzy set of the judgement set V which is, for instance:

$$V=\{\text{very good, better, good, bad}\},$$

and

$$A=[0.80,\ 0.19,\ 0.01]$$

$$R_{\text{Nanning}}=\begin{bmatrix}0.42 & 0.58 & 0 & 0\\ 0 & 0 & 0.26 & 0.74\\ 0 & 0.11 & 0.26 & 0.63\end{bmatrix},$$

$$R_{\text{Wanning}}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0.95 & 0.05 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

and so on.

In this example, we would judge the agricultural climate for planting rubber in six areas, i.e., Nanning, Guongzhou, Yunjinghong, Haoko, Wanning and Langzhou.

Defining the membership function as follows:

$$\mu(T)=\begin{cases}1 & \text{if } T\geqslant 23°\text{C},\\ \dfrac{1}{1+a_T(T-23)^2} & \text{if } T<23°\text{C};\end{cases}\quad (a_T=0.0625)$$

$$\mu(T_n)=\begin{cases}1 & \text{if } T_n\geqslant 8°\text{C},\\ \dfrac{1}{1+a_{T_n}(8-T_n)^2} & \text{if } -4°\text{C}\leqslant T_n<8°\text{C}\\ 0 & \text{if } T_n<-4°\text{C};\end{cases}\quad (a_{T_n}=0.0833)$$

$$\mu(F)=\begin{cases}1 & \text{if } F\leqslant 1\text{M/S},\\ \dfrac{1}{1+a_F(F-1)^2} & \text{if } F>1\text{M/S};\end{cases}\quad (a_F=0.8182)$$

and taking

$\mu\geqslant 0.90$, for "very good";
$0.90>\mu\geqslant 0.80$, for "better";
$0.80>\mu\geqslant 0.70$, for "good";
$\mu<0.70$, for "bad";

the result obtained for fuzzy synthetical decision is:

$B_{\text{Nanning}}=[0.30,\ 0.42,\ 0.14,\ 0.14]$;
$B_{\text{Wanning}}=[0.80,\ 0.05,\ 0.0]$;
$B_{\text{Yunjinghong}}=[0.53,\ 0.31,\ 0.16,\ 0]$;
$B_{\text{Goungzhou}}=[0.47,\ 0.27,\ 0.10,\ 0.14]$;
$B_{\text{Haiko}}=[0.80,\ 0.19,\ 0.01,\ 0]$;
$B_{\text{Longzhou}}=[0.62,\ 0.09,\ 0.15,\ 0.14]$;

which shows that Wanning and Haiko are very good for planting rubber and Guongzhou and Nanning are not good for planting rubber.

4. Concluding remarks

On the basis of the preliminary positive results reported in the paper, it may be viewed that prospects for application of the methods of fuzzy mathematics in seismology and meteorology are optimistic.

References

[1] ALEKSEYEV A. S. et al. On the problem of the physical nature of waves recorded in the deep seismic sounding. *Izvestiya AN SSSR*, Physics of the Solid Earth, **1** (1963).

[2] BRACE W. F. The physical basis for earthquake prediction. *Technology Review*, **77** (1975) 5.

[3] FENG De-yi Anomalous variations of seismic velocity ratio before the Youngshan-Daguan earthquake (M=7.1) on May 11, 1974. *Acta Geophysica Sinica*, **18** (1975) 4; *Chinese Geophysics*, **1** (1978).

[4] FENG De-yi, GU Jing-ping. et al. The statistical indexes of seismicity and the probability prediction for the time of earthquake occurrence. *Journal of Seismological Research*, **4** (1981).

[5] FENG De-yi, LIN Ming-zhou et al. Applications of fuzzy mathematics to earthquake prediction and investigation of the preparatory model of earthquake source. *Northwestern Seismological Journal*, **3** (1981) 4.

[6] FENG De-yi, ZHONG Ting-jiao et al. An application of fuzzy sets method to earthquake prediction based on seismic activity, *Di Zhen*, **4** (1982).

[7] FENG De-yi, GU Jin-ping et al. The application of retrieval method of fuzzy information in earthquake prediction based on seismic activity. *Di Zhen*, 5 (1982).

[8] FENG De-yi, LOU Shi-bo et al. Application of fuzzy mathematics in evaluating earthquake intensity, *Earthquake Engineering and Engineering Vibration*, **2** (1982) 3.

[9] FENG De-yi, GU Jin-ping, LIN Ming-zhou The Applications of choice method based on fuzzy dominant ratio to earthquake studies, *Di Zhen*, **3** (1983).

[10] FENG De-yi, LOU Shi-bo et al. The Methods of Fuzzy Mathematics and their Applications. Publishing House of State Seismological Bureau, 1983.

[11] GAO Su-hua Fuzzy comprehensive assessment in agroclimatic analysis. *Mathematics in Practice and Theory*, (1982) 1.

[12] LOU Shi-bo, CHEN Hua-cheng Fuzzy clustering and weather forecast. *Fuzzy Mathematics*, **1** (1981) 1.

[13] LOU Shi-bo, FENG De-yi et al. The application of retrieval method of fuzzy information in earthquake prediction researches. *Fuzzy Mathematics*, **3** (1983) 1.

[14] ZADEH L. A. Fuzzy sets. *Information and Control*, **8** (1965) 3.

[15] ZONG Rong-xiang An application of fuzzy mathematics to long-range weather forecasts. *Mathematics in Practice and Theory*, 1982 1.

Received, May 1984.

Zastosowania metod teorii zbiorów rozmytych w badaniach sejsmologicznych i meteorologicznych

W artykule pokazano, jak stosując metody teorii zbiorów rozmytych można w sposób efektywny badać pewne istotne zagadnienia sejsmologiczne i meteorologiczne. Zastosowane metody stanowią procedurę badawczą, na którą składają się, w odniesieniu do zagadnień sejsmologicznych, m.in.: rozmyty opis prekursorów trzęsień ziemi, bezpośrednia metoda ich rozmytego rozróżnienia, metoda analizy skupień rozmytych, metoda klasyfikacji służąca do uzyskiwania rozmytej informacji, czy metoda rozmytego podobieństwa. Procedura ta została zilustrowana konkretnymi przykładami numerycznymi. Niezależnie od tego przedstawiono zastosowanie poszczególnych metod opartych na teorii zbiorów rozmytych do rozwiązywania szczegółowych zadań zarówno z zakresu sejsmologii (ocena intensywności trzęsień ziemi, badanie wewnętrznej struktury ziemi itp.), jak i z zakresu meteorologii (prognozy wiosennych długotrwałych deszczów w okolicach Szanghaju, prognozy opadów w Płn. Chinach, podejmowanie decyzji co do uprawy kauczuku naturalnego).

Применение методов теории нечетких множеств в сейсмологических и метеорологических исследованиях

В статье показано, как используя методы теории нечетких множеств можно эффективным способом исследовать некоторые существенные сейсмологические и метеорологические вопросы. Используемые методы являются исследовательской процедурой, которая состоит, в случае сейсмологических вопросов, в том числе из нечеткого описания предвестников землетрясений, непосредственного метода их нечеткого различения, метода нечеткого кластер--анализа, метода классификации для получения нечеткой информации, а также метода нечеткого подобия. Эта процедура иллюстрируется конкретными численными примерами. Независимо от этого представлено применение отдельных методов, основанных на теории нечетких множеств, для решения конкретных задач, как в области сейсмологии (оценка интенсивности землетрясений, исследование внутренней структуры земли и т.п.), так и в области метеорологии (прогноз весенних длительных дождей в окрестностях Шанхая, прогноз града в Северном Китае, принятие решений по выращиванию натурального каучука).