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Admissible systems for the exact model matching

by

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Considered is an orbit, with respect to the action of the control law: $u=Fy+Gv$ (with F, G constant), of a linear multivariable time invariant controllable and observable system described by the state space equations: $\dot{x}=Ax+Bu$, $y=Cx$. There has been established an explicit relation between parameters of the Luenberger observable companion forms of any element of the orbit and these of the system and matrices F and C. Topological characterisation of the orbit has been presented.

1. Introduction

Establishing an existence of solution and evaluating it — these are the main aspects of the exact model matching problem. Usually an existence question is answered when, following a given algorithm, we attempt to solve some resulting algebraic equations that describe relations between parameters of systems and regulators. However, such an approach makes it impossible to determine a whole class of systems which admit the use of an exact model matching scheme.

Thus, the task of establishing a simple characterization of class of systems for which the exact model matching problem can be solved, gains a special importance. For the exact model matching by a state feedback some results have already been published: Ackerman [1], Cramer [2], Wolovich, Falb [7], Wolovich [8]. Even in this simpler case, the connection between the problem stated above and sets of structural invariants of systems is unknown.

The paper presents a characterisation of a class of systems for which there exists a solution to the exact model matching by output feedback and input vector transformation. Characterization has been done in terms of structural invariants of systems and parameters of proportional regulators.

2. Problem statement

Consider a system

$$\dot{x}=Ax+Bu\,, \quad x(0)=0 \tag{2.1a}$$

$$y=Cx\,, \tag{2.1b}$$

where $x=x(t)\in R^n$, $y=y(t)\in R^p$, $u=u(t)\in R^q$ are state, output and input vectors respectively determined for $t\geqslant 0$. The system is assumed to be controllable and observable; A, B, C are constant real matrices of appropriate sizes, C is of a full rank. A transfer function matrix of (2.1) is

$$H(s)=C(s1_n-A)^{-1}B\,. \tag{2.2}$$

and is a strictly proper element of $R(s)^{p\times q}$ — set of $p\times q$ rational function matrices over reals. In the paper we consider a family of systems which can be obtained from (2.1) under an action of the control law

$$u=Fy+Gv \tag{2.3}$$

where $v=v(t)\in R^r$ is a vector of an external input and F and G are real constant matrices of appropriate sizes.

There exists such a nonsingular martix Q that a state space coordinate transformation

$$\bar{x}=Qx \tag{2.4}$$

yields following state space and output system equations for

$$\dot{\bar{x}}=\hat{A}\bar{x}+\hat{B}\bar{u}\,, \quad \bar{x}(0)=0 \tag{2.5a}$$

$$y=\hat{C}\bar{x} \tag{2.5b}$$

in so called Luenberger observable companion form

$$\hat{A}=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1p} \\ A_{21} & A_{22} & \cdots & A_{2p} \\ \cdot\cdot & \cdot\cdot & \cdots & \cdot\cdot \\ A_{p1} & A_{p2} & \cdots & A_{pp} \end{bmatrix}, \; A_{ij}\in R^{n_i\times n_j} \qquad i,j=1,\dots,p$$

$$A_{ii}=\begin{bmatrix} 0 & 0 & \cdots & 0 & * \\ 1 & 0 & \cdots & 0 & * \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & * \end{bmatrix},$$

$$A_{ij}=\begin{bmatrix} 0 & \cdots & 0 & * \\ \cdot & \cdots & \cdot & \cdot \\ 0 & \cdots & 0 & * \\ 0 & \cdots & 0 & 0 \\ \cdot & \cdots & \cdot & \cdot \\ 0 & \cdots & 0 & 0 \end{bmatrix} - \min(n_i, n_j)\,, \qquad i\neq j$$

$$C=[C_1\ C_2\ \cdots\ C_p],\ C_i\in R^{p\times n},\ i=1,\dots,p$$

$$C_i=\begin{array}{c}\begin{bmatrix}0 & \cdots & 0 & 0\\ \cdot & \cdots & \cdot & \cdot\\ 0 & \cdots & 0 & 0\\ 0 & \cdots & 0 & 1\\ 0 & \cdots & 0 & *\\ \cdot & \cdots & \cdot & \cdot\\ 0 & \cdots & 0 & *\end{bmatrix}\end{array}\begin{array}{l}\\ \\ \\ -i\\ \\ \\ \\ \end{array}$$

$(n_1, n_2, \dots, n_p)$ is a p-tuple of observability indices of (2.1), $*$ denotes possibly non zero elements of A_{ij} and C_i $i, j=1, \dots, p$. $\hat{B}$ has no special form (Luenberger [4]). Let $k_i=n_1+n_2+\dots+n_i$ for $i=1, \dots, p$.

We denote $\hat{A}_p$ and $\hat{C}_p$ matrices formed of columns numbered k_i $i=1, \dots, p$ of matrices $\hat{A}$ and $\hat{C}$ respectively. Since C has a full rank, matrix $\hat{C}_p$ is nonsingular. Thus matrices

$$K_{CA}=\hat{A}_p\,\hat{C}_p^{-1}\ ,\ M_{CA}=\hat{C}_p^{-1} \tag{2.6}$$

are well defined. They have the following form

$$K_{CA}=\begin{bmatrix}a_{11} & \cdots & a_{1p}\\ \cdot\cdot & \cdots & \cdot\cdot\\ a_{p1} & \cdots & a_{pp}\end{bmatrix},\ a_{ij}\in R^{n_i},$$

$$a_{ij}^T=[a_{ij0}, a_{ij1}, \dots, a_{ijn_i-1}],\ a_{ijk}=0 \text{ for } k\geqslant \min(n_i, n_j).$$

$$M_{CA}=\begin{bmatrix}1 & 0 & \cdots & 0 & 0\\ -b_{21} & 1 & \cdots & 0 & 0\\ \cdot & \cdot & \cdots & \cdot & \cdot\\ -b_{p1} & -b_{p2} & \cdots & -b_{p,\,p-1} & 1\end{bmatrix},\ b_{ij}\in R,$$

$b_{ij}=0$ for $n_i\geqslant n_j$. (Ackermann [1])

After Vardulakis ([9]) we call elements b_{ij} and these of vectors a_{ij} which by definition equal zero the "sacred zeros" of matrices M_{CA} and K_{CA}.

REMARK. It is well known that elements of M_{CA}, below a diagonal, and of K_{CA} which are not the sacred zeros are so called β and α (respectively) parameters for an observable pair of matrices (C, A) and they are invariant to nonsingular change of coordinates in a state space. Conversely, given a set of α and β parameters one can easily write down matrices K_{CA} and M_{CA}. For notational reason we will use K_{CA} and M_{CA} instead of α and β parameters and even place them on a list of complete and independent invariants, always having in mind that they symbolize α and β parameters. According to this agreement, it will be shown in the proposition (3.9) that the following quadruple

$$(K_{CA}, M_{CA}, \hat{B}, (n_1, \dots, n_p)) \tag{2.7}$$

forms a complete set of independent invariants (shortly: structural invariant) of the observable system (2.1) (in fact, by assumption, it is controllable as well).

The task of the paper is to find a solution to the following problem: Given the structure invariant (2.7) of the controllable and observable system (2.1) and a pair of matrices (F, G) in (2.3), evaluate the structure invariant of the closed loop system (2.1 — 2.3) as a function of (2.7) and of the pair (F, G).

3. Minimal observability matrices, structural invariants

Before we solve the problem stated in the previous section, we recall some properties of minimal observability matrices, which will be our main tool.

DEFINITION (3.1). Given the controllable and observable system with a rational proper transfer function matrix $H \in R(s)^{p\times q}$, we call a matrix $S_H \in R[s]^{p\times(p+q)}$ (a $p\times(p+q)$ matrix whose elements are polynomials over reals) the minimal observability matrix for the system with the transfer function matrix H if

1° if $S_H^0=[P, R]$ and $P\in R[s]^{p\times p}$, then $H=P^{-1}R$

2° for each complex s, the matrix $S_H^0(s)$ has a full rank

3° a matrix $[P]_h \in R^{p\times p}$ consisting of coefficients of the highest degree s terms in each row of polynomial matrix P is nonsingular.

THEOREM (3.2). *Let $S_H^0 \in R[s]^{p\times(p+q)}$ be a minimal observability matrix for a system with transfer function matrix $H\in R(s)^{p\times q}$, then*

1° *Let m_i be the highest degree of terms in the i-th row of S_H^0 (the i-th row index) $i=1, 2, ..., p$. Then $\{m_1, m_2, ..., m_p\}$ is the set of observability indices for the system with transfer function matrix H. The sum $m_1+m_2+...+m_p$ is the dimension of its controllable and observable state space representation.*

2° *Given any two minimal observability matrices S_H^0 and Q_H^0 for the system with transfer function matrix H, there exists one and only one polynomial unimodular invertible matrix $D\in R[s]^{p\times p}$ such that $S_H^0=DQ_H^0$.*

3° *Let S_H^0 and Q_H^0 be any two minimal observability matrices for the system with transfer function matrix H. Assume that their row indexes $m_1, m_2, ..., m_p$ form nondecreasing sequence, then the unimodular polynomial matrix mentioned in* p. 2° *has a form*

$$D=\begin{bmatrix} D_1 & 0 & \cdots & 0 \\ U_{21} & D_2 & \cdots & 0 \\ \cdot\cdot & \cdot\cdot & \cdots & \cdot \\ U_{t1} & U_{t2} & \cdots & D_t \end{bmatrix} \tag{3.3}$$

A positive integer t can be evaluated as follows: cut the sequence $m_1, m_2, ..., m_p$ into constant subsequences, each of the maximal length, i.e. we have

$$m_1=...=m_{r_1}<m_{r_1+1}=...=m_{r_1+r_2}<...<m_{r_1+\dots+r_{t-1}+1}=...=m_p$$

Thus t is the smallest number of constant subsequences obtained from the sequence of the observability indices of the system with transfer function matrix H.

Let $k(i)=r_1+...+r_i$ for $i=1, 2, ..., t-1$. Let $k(t)=p$. Matrices $D_i \in R^{r_i \times r_i}$ are nonsingular for $i=1, 2, ..., t$. Matrices $U_{ij} \in R[s]^{r \times r}$ have entries with degree not exceeding the number $k(i)-k(j)$ for $i=2, 3, ..., t$, $j=1, 2, ..., i-1$.

4° *Let $S_H^0=[P, R]$ and $P \in R[s]^{p \times p}$. Then H is strictly proper (for any entry of H, degree of numerator is less than degree of denominator) if and only if for each $i=1, 2, ..., p$ $\deg r_{ij}<m_i$ $j=1, 2, ..., q$, where $R=[r_{ij}]$.*

Proof. The property 1° has been proved in [3]. The proof of 2° and 3° can be found in [6]. The property 4° is obvious. ■

PROPOSITION (3.4). Given any controllable and observable system (2.1),

1° the action of the control law (2.3) implies a linear transformation of a minimal observability matrix of the system

$$S_H^0 \mapsto S_H^0 \begin{bmatrix} 1_p & 0 \\ -F & G \end{bmatrix} \tag{3.5}$$

2° the action of output feedback F alone ($G=1_q$ in (2.3)) spoils neither its observability nor controllability.

3° the action of input vector transformation G alone ($F=0$ in (2.3)) always yields an observable system (in the case when G is square and nonsingular it is controllable as well).

Proof. ad 1°. Perform a matrix multiplication in (3.5), and then use the property 1° of the definition (3.1). We obtain the closed loop system transfer function matrix of (2.1 — 2.3). ad 2°. The constant matrix on the right side of (3.5) is nonsingular and hence both properties 2° and 3° are always preserved, when (2.1) is subjected to (2.3). Thus, transformed as in (3.5), minimal observability matrix remains the minimal observability matrix for the closed loop system. Since its row degrees are the same as those of the matrix S_H^0 we conclude that the minimal state space representations of both systems have the same dimensions.

ad 3°. Under the action of (2.3) matrices A and C do not change, hence the closed loop system (2.1 — 2.3) remains observable. If G is square and nonsingular, by the same arguments as in p. 2°, we conclude that in (3.5), on the right side, there we have a minimal observability matrix. ■

Among all minimal observability matrices for the system (2.1) there exists especially nice one which we will call a Popov's canonical form (according to its connection with Popov's work). Let

$$[s^n]=\begin{bmatrix} s^{n_1} & 0 & \cdots & 0 \\ 0 & s^{n_2} & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & s^{n_p} \end{bmatrix} \tag{3.6a}$$

$$S=\begin{bmatrix} 1 & s & \cdots & s^{n_1-1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & s & \cdots & s^{n_2-1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 1 & s & \cdots & s^{n_p-1} \end{bmatrix} \tag{3.6b}$$

PROPOSITION (3.7). Given $(K_{CA}, M_{CA}, \hat{B}, (n_1, ..., n_p))$ defined in section 2, if

$$\hat{P}=[s^n]\, M_{CA}-SK_{CA} \tag{3.8a}$$

$$\hat{R}=S\hat{B} \tag{3.8b}$$

then $\hat{S}_H^0=[\hat{P}, \hat{R}]$ is a minimal observability matrix for the system (2.1) (we call it the minimal observability matrix in Popov's canonical form or shortly in Popov's form).

Proof. From [5] it follows that, given matrices $\hat{P}$ and $\hat{R}$ as in (3.8), condition 1° of the definition (3.1) is satisfied. We have to verify conditions 2° and 3°. But, $[\hat{P}]_h=$ $=M_{CA}$, so condition 3° is fulfilled. One can easily verify, that if there exists $s_0 \in C$ such that rank $[\hat{P}, \hat{R}]$ (s_0) is less than p, then there exists a polynomial matrix $Q \in R\,[s]^{p\times p}$ such that Q is left divisor of $[P, R]$ and determinant of Q is divisible by $s-s_0$, and hence m=deg det P exceeds the dimension of the minimal state space representation of the system, but this contradicts the assumption that $n_1, ..., n_p$ are the observability indces of controllable and observable system. ■

PROPOSITION (3.9). Given the observable system (2.1) with matrix C of a full rank, if $(n_1, ..., n_p)$ is a p-tuple of observability indices of the system, and K_{CA} and M_{CA} are defined in (2.6), $\hat{B}$ is defined in (2.5a), then $(K_{CA}, M_{CA}, \hat{B}, (n_1, ..., n_p))$ is a complete set of independent invariants for the system.

Proof. By dual version of result from [5], $(K_{CA}, M_{CA}, (n_1, ..., n_p))$ is a complete set of independent invariants for the system (2.1) with $B=Q^{-1}$ (see (2.4)). Given in (2.1) any matrix B, there exists uniquely determined matrix $\hat{B}$ such that (2.5) represents (2.1) in its observable companion form, and conversely, for any $n\times q$ matrix $\hat{B}$ there exists an observable system (2.1). Thus we conclude that $\hat{B}$ belongs to the set of structural invariants for (2.1). ■

4. Solution of the problem

In what follows we assume that G is square and nonsingular. Then, by the proposition (3.4) p. 3° the matrix on the right side of (3.5) is the minimal observability matrix. Suppose we have a minimal observability matrix S_H^0 of the system (2.1) in its Popov's form. Then

$$\hat{S}_H^0 \begin{bmatrix} 1_p & 0 \\ -F & G \end{bmatrix}=S_T^0 \tag{4.1}$$

is a minimal observability matrix for the closed loop system (2.1 — 2.3). By (3.2), there exists uniquely determined polynomial unimodular $p\times p$ matrix D such that

$$D\hat{S}_H^0 \begin{bmatrix} 1_p & 0 \\ -F & G \end{bmatrix}=\hat{S}_T^0 \tag{4.2}$$

and $\hat{S}_T^0$ is the Popov's form of a minimal observability matrix for the closed loop system. By (3.7), we may write

$$\hat{S}_T^0 = [\hat{U}, \hat{V}] \tag{4.3a}$$

and

$$\hat{U} = [s^{\bar{n}}]\, M_{C_1A_1} - SK_{C_1A_1} \tag{4.3b}$$

$$\hat{V} = S\hat{B}_1 \tag{4.3c}$$

where $(K_{C_1A_1}, M_{C_1A_1}, \hat{B}_1, (\bar{n}_1, ..., \bar{n}_p))$ is a structure invariant of the closed loop system (2.1 — 2.3) (unknown at the moment). Thus we have

PROPOSITION (4.4). Given a minimal observability matrix in the Popov's form for the system (2.1) and given a matrix of the control law

$$L = \begin{bmatrix} 1_p & 0 \\ -F & G \end{bmatrix} \tag{4.5}$$

where F and G are defined in (2.3), there exists one and only one unimodular poly nomial matrix D such that the minimal observability matrix $D\hat{S}_H^0 L$ of the closed loop system (2.1 — 2.3) is in Popov's form.

PROPOSITION (4.6). The matrix D in the proposition (4.4) has a form

$$D = 1_p - D_w^{-1}\, D_0 D_w \tag{4.7}$$

where D_w is a constant $p \times p$ matrix that brings rows of $\hat{S}_H^0$ in such an order that their indices form a nondecreasing sequence; D_0 has a block structure the same as (3.3) but now $D_i = 0$ for $i = 1, ..., t$; elements of polynomial matrices U_{ij} have degree less than $k(i) - k(j)$ for $i = 2, ..., t$ $j = 1, ..., i-1$.

Proof. By (4.4), given a control law matrix L, there exists unimodular polynomial matrix D such that (4.2) holds. In the appendix we establish the following

LEMMA (4.8). *Let $\hat{S}_H^0 = [[s^n]\, M_{CA} - SK_{CA}, S\hat{B}]$ be the minimal observability matrix in Popov's form for the system* (2.1) *and $(n_1, ..., n_p)$ be its row indexes. Let $\hat{S}_T^0 = [[s^{\bar{n}}]\, M_{C_1A_1} - SK_{C_1A_1}, S\hat{B}_1]$ be the minimal observabiliy matrix in Popov's form of the closed loop system* (2.1 — 2.3) *and $(\bar{n}_1, ..., \bar{n}_p)$ be its row indexes. Then $\bar{n}_i = n_i$ for $i = 1, ..., p$ and $M_{C_1A_1} = M_{CA}$.*

Let D_w be a constant $p \times p$ matrix that brings rows of matrix $\hat{S}_H^0$ in suitable order (see the proposition (4.6)). By the preceding lemma, row indexes of $D_w S_T^0$ form a onndecreasing sequence. Multiplying from the left both sides of (4.1) by $D_w \tilde{D} D_w$, for some polynomial matrix $\tilde{D}$ we have

$$\tilde{D} D_w\, \hat{S}_H^0\, L = \hat{S}_T^0\,. \tag{4.9}$$

By the theorem (3.2) p. 3°, $D_w\bar{D}=\bar{D}$ has a form (3.3). Hence ,in (4.2) $D=D_w^{-1}\,\bar{D}D_w=$ $=\bar{D}D_w$. From (4.9) and (4.8) it follows that.

$$\bar{D}D_w\left([s^n]\,M_{CA}-S\,(K_{CA}+BF)\right)=D_w\left([s^n]\,M_{CA}-SK_{C_1A_1}\right) \tag{4.10}$$

Given the matrix $\bar{D}$, let $[\bar{D}]_0$ be a constant $p\times p$ matrix which has the following block structure

$$[\bar{D}]_0=\begin{bmatrix} D_1 & 0 & \cdots & 0 \\ U_{21}^0 & D_2 & \cdots & 0 \\ \cdot\cdot & \cdot\cdot & \cdots & \cdot \\ U_{t1}^0 & U_{t2}^0 & \cdots & D_t \end{bmatrix}$$

Blocks U_{ij}^0 have the same size as corresponding blocks U_{ij} in (3.3). Every term of U_{ij}^0 equals the coefficient of $(k\,(i)-k\,(j))$-th power of a variable s (according to the notation used in (3.2) p. 4°) of a polynomial standing in the same row and the same column of matrix U_{ij}. Nonsingular matrices D_i are the same as in (3.3). It is easy to see that

$$[\bar{D}D_w\,[s^n]]_h=[\bar{D}]_0\,[D_w\,[s^n]]_h=[\bar{D}]_0\,D_w\,.$$

On the other hand, from (4.10)

$$[\bar{D}D_w\,[s^n]]_h=D_w\,.$$

Hence we get $[\bar{D}]_0=1_p$, which completes the proof. ∎

Denote $\hat{D}$ the matrix $D_w^{-1}D_0D_w$ as in (4.7). By (4.6), it follows immediately that (4.2) is equivalent to

$$SK_{C_1A_1}=\hat{D}\,[s^n]\,M_{CA}+(1_p-\hat{D})\,S\,(K_{CA}+\hat{B}F) \tag{4.11a}$$

$$S\hat{B}_1=(1_p-\hat{D})\,S\hat{B}G \tag{4.11b}$$

What we shall do now is to compare coefficients of polynomial terms of both sides of (4.11). It seems to be reasonable to introduce two constant matrices which enable us to write down (4.11) as a parametrised system of linear equations.

Proposition (4.12). Let D_0 be any polynomial $p\times p$ matrix satisfying conditions specified in the proposition (4.6). Then given a matrix $\hat{D}=D_w^{-1}D_0D_w$ there always exist constant $n\times p$ and $n\times n$ matrices J_1 and J_2 such that

$$\hat{D}\,[s^n]=SJ_1 \tag{4.13a}$$

$$\hat{D}S=SJ_2 \tag{4.13b}$$

Proof. It is easy to see that polynomials in the j-th row of the matrix $D_w^{-1}\,D_0D_w\,[s^n]$ have degrees not exceeding n_j-1, which justifies (4.13a). Similarly, polynomial terms in the j-th row of $\hat{D}S$ have degrees less than n_j-1. ∎

Notice that $J_1=J_1\,(D_0)$ and $J_2=J_2\,(D_0)$, and terms of J_1 are uniquely determined by terms of J_2 and vice versa.

PROPOSITION (4.14). Let $(K_{CA}, M_{CA}, \hat{B}, (n_1, ..., n_p))$ be a structural invariant of the controllable and observable system (2.1). Let (F, G) be any pair of constant matrices $F\in R^{q\times p}$ and $G\in R^{q\times q}$ with G nonsingular. Let J_1 and J_2 be constant $n\times p$ and $n\times n$ matrices satisfying the following conditions:

1° there exists a polynomial matrix $\hat{D}=D_w^{-1} D_0 D_w$ with D_0 and D_w specified in the proposition (4.6), such that (4.13) holds,

2° a matrix

$$J_1 M_{CA}+(1_n-J_2)(K_{CA}+\hat{B}F) \tag{4.15}$$

has the same sacred zeros as the matrix K_{CA}.

Then

$$(J_1 M_{CA}+(1_n-J_2)(K_{CA}+\hat{B}F),\ M_{CA},\ (1_n-J_2)\hat{B}G,\ (n_1, ..., n_p)) \tag{4.16}$$

is a structural invariant of the closed loop system (2.1 — 2.3).

Proof. Let J_1 and J_2 be any constant matrices satisfying conditions 1° and 2°, then formally, we may write

$$K_{C_1A_1}=J_1 M_{CA}+(1_n-J_2)(K_{CA}+\hat{B}F).$$

By the lemma (4.8), we know that (4.16) is a structural invariant of some system, which we expect to be the closed loop system (2.1 — 2.3).

On the other hand, let $\hat{D}$ be a polynomial matrix satisfying condition 1°. Multiplying (4.15) from the left by the matrix S, we obtain the right side of (4.11a). Similarily, $S(1_n-J_2)\hat{B}G$ yields the right side of (4.11b). By equivalence of (4.2) and (4.10), we conclude that (4.16) is the structural invariant of the closed loop system (2.1 — 2.3). ∎

PROPOSITION (4.17). Given the structural invariant (2.7), there always exist constant matrices J_1 and J_2 satisfying conditions 1° and 2° of the proposition (4.14) and they are uniquely determined.

Proof. Suppose that given a matrix F, there is a contradiction in a set of equations obtained from 2°. Then there is no unimodular matrix D that brings the minimal observability matrix $\hat{S}_H^0 L$ to its Popov's form and this contradicts the proposition (4.4). Every arbitrariness in choice of J_1 contradicts the uniqueness of matrix D established in the proposition (4.4). ∎

Proposition (4.14) can be generalised. Actually, parameters of Popov's form of a minimal observability matrix via (2.6) are in one to one correspondence to those of the Luenberger observable companion form (2.5). For any $G\in R^{q\times r}$ and any $F\in R^{q\times p}$

$$L=\begin{bmatrix} 1_p & 0 \\ -F & G \end{bmatrix}=\begin{bmatrix} 1_p & 0 \\ -F & 1_q \end{bmatrix}\begin{bmatrix} 1_p & 0 \\ 0 & G \end{bmatrix}=L_F L_G \tag{4.19}$$

Given a minimal observability matrix S_H^0 of the system (2.1), $S_T^0=S_H^0 L$ may not be a minimal observability matrix, but $S_H^0 L_F$ is. If $S_{T_1}^0=S_H^0 L_F$, then there exists a polynomial unimodular matrix D such that $\hat{S}^0_{T_1}=DS^0_H L_F$ is the Popov's form of $S^0_{T_1}$. Moreover, since Popov's form is invariant to an action of G, we may write

$$\tilde{S}^0_T=DS^0_H L=(DS^0_H L_F) L_G=\hat{S}^0_{T_1} L_G$$

The matrix $\tilde{S}^0_T$ may not be minimal, but it can be uniquely written in the form (3.8). Thus we have

COROLLARY (4.20). *Let in the proposition* (4.14), C *be any constant* $q\times r$ *matrix. Then* (4.16) *is a structural invariant of the observable system* (2.1 — 2.3).

PROPOSITION (4.21). With assumptions of the corollary (4.20) no more than $(p+r)$ rank B elements of matrices K_{CA} and M_{CA} can be assigned arbitrarily by the control law (2.3).

Proof. Let J_1 and J_2 be any constant matrices satisfying condition 1° of the proposition (4.14). Then, condition 2° yields two systems of linear, with respect to elements of J_1, equations. The first one - that assigns zero value to sacred zeros in (4.15)--determines all values of the matrix J_1 and can be symbolically written in a form

$$E_1(\hat{B}F, J_1)=0 \tag{4.22}$$

By the theorem (3.2), there always (i.e. for any pair (F, G)) exists a solution J_1 to this equation and is unique. Since elements of J_1 are rational functions of elements of $\hat{B}F$, we conclude that the solution of (4.22)

$$J_1=Y(\hat{B}F) \tag{4.23}$$

is a continuous mapping. $Y\colon V\to R^{np}$, $V\subset R^{np}$ is a linear $p\cdot$rank B — dimensional subspace.

The second system assigns values to those elements of the matrix (4.15) which are not the sacred zeros. It can be written in a form

$$K_1^*=E_2(\hat{B}F, J_1) \tag{4.24}$$

E_2 defines a continuous, with respect to elements of $\hat{B}F$ and J_2, mapping. Substituting (4.23) into (4.24) we get

$$K_1^*=E_2(\hat{B}F, Y(\hat{B}F))=\tilde{E}_2(\hat{B}F) \tag{4.25}$$

$\tilde{E}_2\colon V\to W=\tilde{E}_2(V)\subset R^{np-m}$ is continuous. m is the number of the sacred zeros of K_{CA}. By (4.16),

$$B_1^*=E_3(\hat{B}G, J_1) \tag{4.26}$$

From (4.23) we get

$$B_1^*=E_3(\hat{B}G, Y(\hat{B}F)) \tag{4.27}$$

$E_3\colon V_1\times V\to W_1=E_3(V_1\times V)\subset R^{nr}$ is continuous in elements of $\hat{B}G$ and $\hat{B}F$. $V_1\subset R^{nr}$ is a linear $r\cdot$rank B-dimensional subspace.

We say that $(\hat{B}F_1, \hat{B}G_1) \sim (\hat{B}F_2, \hat{B}G_2)$ (the pairs are equivalent) if $\tilde{E}_2(\hat{B}F_1) = \tilde{E}_2(\hat{B}F_2)$ and $E_3(\hat{B}G_1, Y(\hat{B}F_1)) = E_3(\hat{B}G_2, Y(\hat{B}F_2))$. If $E = (\tilde{E}_2, E_3)$ and $\tilde{E}$ is a mapping

$$\tilde{E}: V_1 \times V/_\sim \to W_1 \times W.$$

induced by E, then $\tilde{E}$ is a continuous bijection, hence homeomorphsm and $W_1 \times W$ is a k-dimensional topological manifold with $k \leqslant (p+r)$ rank B.
Here we have two simple examples. ■

E1. Given a transfer function matrix of a system

$$H(s) = ((s-1)(s^2+s+3))^{-1} \begin{bmatrix} s^2+s+3 & 0 & s^2+s+3 \\ s^2+s+3 & s^2+s+3 & 0 \\ 3 & 3 & s-1 \end{bmatrix}$$

characterise a family of all observable systems obtained of it by means of (2.3) with constant F and G of appropriate size. The minimal observability matrix $\hat{S}_H^0$ in Popov's form is

$$\hat{S}_H^0 = \begin{bmatrix} s-1 & 0 & 0 & 1 & 0 & 1 \\ 0 & s-1 & 0 & 1 & 1 & 0 \\ 0 & -3 & s^2+s+3 & 0 & 0 & 1 \end{bmatrix} \quad n_1 = n_2 = 1, \; n_3 = 2.$$

$$[s^n] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s^2 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & s \end{bmatrix}$$

$$M_{CA} = \begin{bmatrix} 1 & 0 & 0 \\ \emptyset & 1 & 0 \\ \emptyset & \emptyset & 1 \end{bmatrix}, \quad K_{CA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & -3 \\ \emptyset & \emptyset & -1 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

where $\emptyset$ denotes the sacred zero.
We have now to determine the unimodular polynomial matrix D.

$$D_w = 1_3, \qquad \hat{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 & a_2 & 0 \end{bmatrix}, \text{ where } a_1, a_2 \in R.$$

We evaluate elements of matrices $J_1 \in R^{4\times 3}$ and $J_2 \in R^{4\times 4}$.

$$D[s^n] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 s & a_2 s & 0 \end{bmatrix} = S \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 & a_2 & 0 \end{bmatrix} = SJ_1,$$

$$DS = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 \end{bmatrix} = S \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = SJ_2.$$

We are able now to write down matrices $K_{C_1A_1}$ and $\hat{B}_1$ of the closed loop system (2.1 — 2.3).

$$K_{C_1A_1}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\a_1&a_2&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}1&0&0&0\\0&1&0&0\\-a_1&-a_2&1&0\\0&0&0&1\end{bmatrix}.$$

$$\cdot\left(\begin{bmatrix}1&0&1\\1&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}f_{11}&f_{12}&f_{13}\\f_{21}&f_{22}&f_{23}\\f_{31}&f_{32}&f_{33}\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&3&-3\\0&0&-1\end{bmatrix}\right)=$$

$$=\begin{bmatrix}1+f_{11}+f_{31} & f_{12}+f_{32} & f_{13}+f_{33}\\ f_{11}+f_{21} & 1+f_{12}+f_{22} & f_{13}+f_{23}\\ f_{31}-a_1(1+f_{11}+f_{31})-a_2(f_{11}+f_{21}) & 3+f_{32}-a_1(f_{12}+f_{32})+b & -3+f_{33}-a_1(f_{13}+f_{33})-a_2(f_{13}+f_{23}+3)\\ a_1 & a_2 & -1\end{bmatrix}$$

$b=-a_2(1+f_{12}+f_{22})$.

Since K_{CA} and $K_{C_1A_1}$ must have the same sacred zeros, we get $a_1=a_2=0$. Thus any feedback applied to the given system does not affect the canonical form of the minimal observability matrix $\hat{S}_H^0$ (we call such a feedback covariant — Vardulakis [9]). Matrix $K_{C_1A_1}$ has a form

$$K_{C_1A_1}=\begin{bmatrix}1+f_{11}+f_{31} & f_{12}+f_{32} & f_{13}+f_{33}\\ f_{11}+f_{21} & 1+f_{12}+f_{23} & f_{13}+f_{23}\\ f_{31} & 3+f_{32} & -3+f_{33}\\ \emptyset & \emptyset & -1\end{bmatrix}$$

Matrix $\hat{B}_1$ is

$$\hat{B}_1=\begin{bmatrix}1&0&1\\1&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}g_{11}&\cdots\\g_{21}&\cdots\\g_{31}&\cdots\end{bmatrix}=\begin{bmatrix}g_{11}+g_{31}&\cdots\\g_{11}+g_{21}&\cdots\\g_{31}&\cdots\\0&\cdots\end{bmatrix},$$

$$M_{C_1A_1}=\begin{bmatrix}1&0&0\\\emptyset&1&0\\\emptyset&\emptyset&1\end{bmatrix},\ (\bar{n}_1,\bar{n}_2,\bar{n}_3)=(1,1,2).$$

E2. Given in E1 the transfer function matrix H and given a matrix

$$T(s)=((s+1)(s^2+s+2))^{-1}\begin{bmatrix}s^2+s+2&0\\0&s^2+s+2\\1&s+3\end{bmatrix}$$

find such a pair of matrices (F, G), provided it exists, that a system represented by H under an action of control law (2.3) is controllable and observable and its transfer function matrix is T.
The minimal observability matrix in Popov's form for T is

$$\hat{S}_T^0=\begin{bmatrix} s+1 & 0 & 0 & 1 & 0 \\ 0 & s+1 & 0 & 0 & 1 \\ -1 & -2 & s^2+s+2 & 0 & 1 \end{bmatrix}$$

Thus $\bar{n}_1=\bar{n}_2=1$, $\bar{n}_3=2$.

$$M_{C_1A_1}=\begin{bmatrix} 1 & 0 & 0 \\ \emptyset & 1 & 0 \\ \emptyset & \emptyset & 1 \end{bmatrix}, \; K_{C_1A_1}=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & -2 \\ \emptyset & \emptyset & -1 \end{bmatrix}, \; \hat{B}_1=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

We compare invariants of the system described by the matrix T and those evaluated in E1. We obtain the following equations:

$$\begin{array}{lll} 1+f_{11}+f_{31}=-1 & f_{12}+f_{32}=0 & f_{13}+f_{33}=0 \\ f_{11}+f_{21}=0 & 1+f_{12}+f_{22}=-1 & f_{13}+f_{23}=0 \\ f_{31}=1 & 3+f_{32}=2 & f_{33}-3=-2 \end{array}$$

$$\begin{array}{ll} g_{11}+g_{31}=1 & g_{12}+g_{32}=0 \\ g_{11}+g_{21}=0 & g_{12}+g_{22}=1 \\ g_{31}=0 & g_{32}=1 \end{array}$$

Hence we get

$$F=\begin{bmatrix} -3 & 1 & -1 \\ 3 & -3 & 1 \\ 1 & -1 & 1 \end{bmatrix}, \; G=\begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}.$$

5. Appendix

First we formulate an algorithm of determining parameters (2.7) of the minimal observability matrix in Popov's form.

ALGORITHM (5.1). Let H be a strictly proper $p\times q$ transfer function matrix of the system (2.1).

STEP 1. Given a matrix H, multiply each row of a matrix

$$S_1=[1_p, H]$$

by the least common denominator of its elements. Let S_2 denote the resulting $p\times(p+q)$ polynomial matrix.

STEP 2. Performing a series of elementary operations on columns of the matrix S_2, we bring it to the form: $[T, 0]$, where T is an upper right triangular polynomial $p \times p$ matrix. Let

$$S_3 = T^{-1} S_2$$

Since T is a left divisor of S_2, S_3 is a polynomial $p \times (p+q)$ matrix).

STEP 3. Performing a series of elementary operations on rows of matrix S_3, we bring it to the following form

$$S_4 = [T_1, R]$$

where T_1 is a lower left triangular polynomial $p \times p$ matrix (diagonal elements of T_1 have degrees greater than any other elements of the corresponding column of S_4).

STEP 4. Perform a series of operations on rows of matrix S_4, following a diagram presented below. Let

$$W = [X, Y]$$

be a polynomial matrix with two $p \times p$ and $p \times q$ blocks X and Y. W_i denotes the i-th row of the matrix W. $a_{ij} = \deg x_{ij}$ — degree of an element of block X, standing in the i-th row and j-th column. $'$denotes the modified element.
Transformation type 1: make

$$W_i' = W_i - f s^{a_{ij} - a_{jj}} W_j \tag{5.2a}$$

$$W_j' = W_j \tag{5.2b}$$

with $f \in \boldsymbol{R}$ such that $a_{ij}' < a_{ij}$.
Transformation type 2: make

$$W_i' = W_j + f s^{a_{jj} - a_{ij}} W_i \tag{5.3a}$$

$$W_j' = W_i \tag{5.3b}$$

with $f \in \boldsymbol{R}$ such that $a_{ij}' < a_{jj}$.
Transformation type 3: make

$$W_m' = W_m + f s^{a_{mk} - a_{kk}} W_k \tag{5.4a}$$

$$W_k' = W_k \tag{5.4b}$$

with $f \in \boldsymbol{R}$ such that $a_{mk}' < a_{mk}$.
Transformation type 4: make

$$W_m' = W_k + f s^{a_{kk} - a_{mk}} W_m \tag{5.5a}$$

$$W_k' = W_m \tag{5.5b}$$

with $f \in \boldsymbol{R}$ such that $a_{mk}' < a_{kk}$.

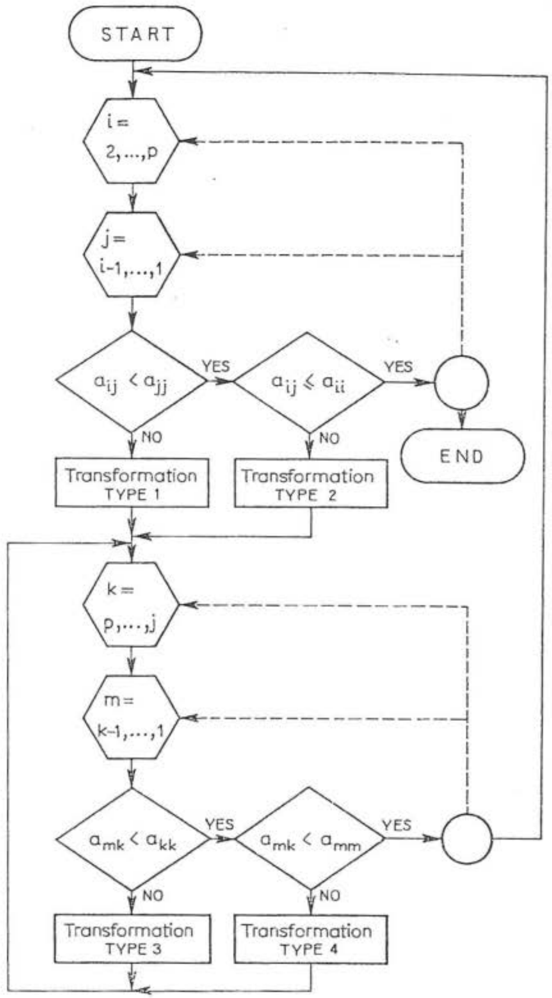

Fig. 1.

LEMMA (5.6). *Let $\hat{S}^0_H=[\hat{P}, \hat{R}]$ be the minimal observability matrix in Popov's form for the system with the strictly proper $p\times q$ transfer function matrix H. $\hat{P}$ is a polynomial $p\times p$ matrix, $\hat{R}$ is a polynomial $p\times q$ matrix. Let $p_{ij}, r_{ij}\in \boldsymbol{R}\,[s]$ be elements of $\hat{P}$ and $\hat{R}$ respectively. Let n_i for $i=1, ..., p$ be row indexes of $\hat{S}^0_H$. Then*

$$p_{ij}=\begin{cases}\text{monic polynomial of degree } n_i, \text{ for } i=j\\ \text{polynomial of degree } \min(n_i, n_j)-1, \text{ for } i<j\\ \text{polynomial of degree } \min(n_i, n_j-1), \text{ for } i>j\end{cases} \tag{5.7a}$$

$$r_{ij}=\text{polynomial of degree } n_i-1, \text{ for } i=1, ..., p \tag{5.7b}$$

Proof. By (3.8),

$$\hat{P}=[s^n]\,M_{CA}-SK_{CA}$$
$$\hat{R}=S\hat{B}$$

The straightforward consequence of properties of matrices K_{CA} and M_{CA} is that:

$$\deg p_{ii}=n_i \text{ for } i=1,\dots,p$$

$$\text{for } i>j \ \deg p_{ij}=\begin{cases} n_i, & \text{if } n_i<n_j \\ n_j-1, & \text{if } n_i\geqslant n_j\,, \end{cases}$$

$$\text{for } i<j \ \deg p_{ij}=\begin{cases} n_i-1, & \text{if } n_i<n_j \\ n_j-1, & \text{if } n_i\geqslant n_j\,, \end{cases}$$

$$\deg r_{ij}<n_i \text{ for } i=1,\dots,p\,.$$

∎

PROPOSITION (5.8). Given a matrix S_4 in the step 3, a finite number of transformations specified in step 4 yields Popov's form of the minimal observability matrix of the system (2.1).

Proof. If $W=[X, Y]$ is a final polynomial matrix obtained in the step 4, then it is easy to verify that $X^{-1}\,Y=H$. Matrix T (obtained in the step 2) was the greatest left divisor of matrix S_2. Hence matrices S_2, S_3 as well as S_4 satisfy condition 2° of definition (3.1).

Transformations 1—4 are, in fact, elementary operations on rows of matrix W which have to be performed in order to bring X to desired form. In each loop condition (5.7a) is checked. In the case when any entry fails to satisfy it, appropriate rows are modified.

Given a matrix S_4, in a finite number of elementary operations performed on its rows, we can make it a minimal observality matrix (Forney [3]). Since any minimal observability matrix can be brought to its Popov's form by multiplicating it from the left by a polynomial unimodular matrix (which simply represents elementary operation on rows), we conclude that after a finite number of transformations, to be performed in step 5, we obtain a minimal observability matrix in Popov's form. Since it is unique, it is the minimal observability matrix in Popov's form of the system (32.1).

∎

We can now prove the lemma (4.8).

Proof of the lemma (4.8). We subject the matrix $\hat{S}^0_H\,L$ to the step 4 of the algorithm (5.1). We shall show that any transformation performed on rows of $\hat{S}^0_H\,L$ preserves the matrix M_{CA} and the order of row indexes.

Suppose that for some $i>j$ we have $a_{ij}\geqslant a_{jj}$. On the other hand we have $a_{ij}\leqslant a_{ii}$, so $a_{jj}\leqslant a_{ij}\leqslant a_{ii}$. Transformation 1 is to be done. We claim that

$$a_{jk}+a_{ij}-a_{jj}<n_i \text{ for } k=1,\dots,j\,.$$

It is equivalent to $a_{jk}-n_j<n_i-a_{ij}$. But $n_i-a_{ij}\geqslant 0$ and $a_{jk}-n_j\leqslant 0$. The inequality stated above is not valid when and only when simultaneously $a_{jk}=n_j$ and $a_{ij}=n_i$.

Recall that for $n_i \geqslant n_j$ (as a property of the matrix M_{CA}) we have $a_{ij} \leqslant n_i - 1$, that contradicts the foregoing statement. For $k = j+1, \ldots, p$ $a_{jk} + a_{ij} - a_{jj} \leqslant n_i + a_{jk} - a_{jj} \leqslant \leqslant n_i - 1$.
Thus we see, that the coefficient of s^{n_i} in term x'_{ik} is that of x_{ik}, for $k = 1, \ldots, p$.
Similarly, if transformation 3 is to be accomplished (in the case when for $i<j$ we have $a_{ij} \geqslant a_{jj}$), one can easily verify that for $k=1, \ldots, i$ $a_{ij} - a_{jj} + a_{jk} \leqslant n_i - 1$, that shows that the coefficient of s^{n_i} in term x'_{ik} equals that of x_{ik}. Moreover, for $k = i+1, \ldots$ $\ldots, p$ $k \neq j$ $a'_{ik} \leqslant n_i - 1$ and $a_{ij} \leqslant a'_{ij} - 1$.
Hence, we conclude that transformations 2 and 4 (which cause a change of the order of rows) will not be used and that the matrix M_{CA} is invariant to transformations 1 and 3, which completes the proof. ■

As an example we compute the minimal observability matrix in Popov's form of the system with a transfer function matrix as below

$$H(s) = ((s-1)(s^2+s+3))^{-1} \begin{bmatrix} s^2+s+3 & 0 & s^2+s+3 \\ s^2+s+3 & s^2+s+3 & 0 \\ 3 & 3 & s-1 \end{bmatrix}$$

The matrix S_2 in step 1 is

$$S_2 = \begin{bmatrix} s-1 & 0 & 0 & 1 & 0 & 1 \\ 0 & s-1 & 0 & 1 & 1 & 0 \\ 0 & 0 & (s-1)(s^2+s+3) & 3 & 3 & s-1 \end{bmatrix}$$

The matrix T is

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & s-1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

The matrix S_3 in step 2 is

$$S_3 = \begin{bmatrix} s-1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1/3\,(s^2+s+3) & 0 & 0 & -1/3 \\ 0 & 0 & 1/3\,(s^2+s+3)\,(s-1) & 1 & 1 & 1/3\,(s-1) \end{bmatrix}$$

The matrix S_4 in step 3 is

$$S_4 = \begin{bmatrix} s-1 & 0 & 0 & 1 & 0 & 1 \\ 0 & s-1 & 0 & 1 & 1 & 0 \\ 0 & -3 & s^2+s+3 & 0 & 0 & 1 \end{bmatrix}$$

It is already the minimal observability matrix in Popov's canonical form.

6. Conclusions

As we have shown, it is possible to characterize strictly proper systems, obtainable from a given system by means of the control law (2.3), by its structural invariant and elements of matrices F and G. Though we have not presented any new invariant with respect to the application of the control law (2.3) (the invariance of observability indicies and of the matrix M_{CA} is the fact well known), the corollary (4.20) shows how the structural invariants of the system transform while the system is subjected to the output feedback. The proposition (4.20) establishes the topological property of the set of all systems that can be obtained from the given controllable and observable system by use of the control law (2.3) — it is a manifold of dimension not greater than $(p+r)$ rank B. Thus the probability that, given two linear systems (2.1) there exists a control law (2.3) that transforms one system into the another (even in the case when they have the same observability indices) is zero, unless rank $B=n$.

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Dopuszczalne układy dla zadania układu o zadanej macierzy transmitancji operatorowych

W pracy badana jest orbita wielowymiarowego, stacjonarnego, liniowego, sterowalnego i obserwowalnego układu opisywanego równaniami

$$\dot{x}=Ax+Bu,\ y=Cx \qquad (*)$$

względem działania proporcjonalnego sprzężenia zwrotnego

$$u=Fy+Gv$$

Wprowadzone zostały zależności wiążące parametry obserwowalnej postaci kanoniczej Luenbergera równań (*) układu zamkniętego z parametrami układu otwartego oraz elementami macierzy F i G. Przedstawiona została topologiczna charakteryzacja rozpatrywanej orbity.

Допустимые системы для задачи синтеза системы с заданной матрицей операторных передаточных функций

В работе исследуется орбита многомерной, стационарной, линейной, управляемой и наблюдаемой системы, описываемой уравнениями

$$\dot{x}=Ax+Bu,\ y=Cx \qquad (\ast)$$

по отношению к воздействию пропорциональной обратной связи

$$u=Fy+Gv$$

Представлены полученные зависимости, связывающие параметры в наблюдаемом каноническом виде Люенбергера уравнений ($\ast$) замкнутой системы с параметрами разомкнутой системы, а также элементами матрицы F и G. Представлена топологическая характеристика рассматриваемой орбиты.