Control and Cybernetics

VOL. 14 (1985) No. 4

Controllability of linear systems with nonuniquely determined resolvent operators

by

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In this paper the problem of controllability of systems is considered under assumptions of linearity of operators and spaces. Presented systems are described by right invertible operators and their initial operators. The author generalizes definitions of reachability of states and controllability of systems introduced by Nguyen Dinh Quyet (cf. [1]). in the cases of systems when resolvent operators need not be uniquely determined. It is shown that properties of reachability and controllability depend on resolvent operator. The problem is illustrated by some examples.

1. Preliminaries

Let X and Y be linear spaces over the space $\mathscr{F}$ of scalars. Denote by $L(X \to Y)$ the set of all linear operators A with domains $\operatorname{dom} A$ being linear subsets of X, and with values in Y. Write: $L_0(X \to Y) = \{A \in L(X \to Y)\colon \operatorname{dom} A = X\}$, $\ker A = \{x \in \operatorname{dom} A : Ax = 0\}$ and $\operatorname{im} A = A \operatorname{dom} A$ for $A \in L(X \to Y)$.

An operator $D \in L(X \to Y)$ is said to be right invertible if there exists an operator $R \in L_0(Y \to X)$ such that $RY \subset \operatorname{dom} D$ and $DR = I_Y$, where I_Y is the identity operator on the space Y. The operator R is called a right inverse of the operator D. Denote by R_D the set of all right inverses of the operator D. The set of all right invertible operators belonging to $L(X \to Y)$ will be denoted by $\mathbf{R}(X \to Y)$.

The set $\ker D$, where $D \in \mathbf{R}(X \to Y)$, is called the space of constants for D, so that every element $z \in \ker D$ is called a constant for D.

In [5] there is proved the following property of right invertible operators:

PROPERTY 1.1. $D \in \mathbf{R}(X \to Y)$ if and only if $\operatorname{codim} D = \dim Y/D \operatorname{dom} D = 0$.

It means that an operator $D \in L(X \to Y)$ is right invertible if and only if the operator D maps its domain onto space Y

An operator $D \in \mathbf{R}(X \to Y)$ such that $\ker D = \{0\}$ is invertible, i.e., there exists only one right inverse which is simultaneously a left inverse. The kind of invertibility of any linear operator depends on its considered domain.

An operator $F \in L(X \to X)$ is said to be an initial operator for D, if it satisfies: $\operatorname{dom} D \subset \operatorname{dom} F$, $F^2 = F$, $F \operatorname{dom} F = \ker D$. Denote by F_D the set of all initial operators for D. For any initial operator $F \in F_D$ there exists an unique right inverse $R \in R_D$ and these operators are connected by a formula: $F = I - RD$ on $\operatorname{dom} D$. Let us remark that $\ker F \cap \operatorname{dom} D = RY$.

Let Y_1 be a linear subspace of Y, i.e. $Y_1 \subset Y$. Let us consider the following system:

$$Dx = Ax + y \tag{1}$$

$$Fx = x_0 \tag{2}$$

where $D \in \mathbf{R}(X \to Y_1)$, $\ker D \neq \{0\}$, $A \in L(X \to Y)$, $\operatorname{dom} D \subset \operatorname{dom} A$, $F \in F_D$, $y \in (D - A) \operatorname{dom} D$, $x_0 \in \ker D$.

In [3] the following properties of solvability of the system (1)–(2) are proved.

THEOREM 1.1. *Let* $D \in \mathbf{R}(X \to Y_1)$, $A \in L(X \to Y)$, $\operatorname{dom} D \subset \operatorname{dom} A$. *Then for any operator* $R \in R_D$, $y \in (D - A)$ *dom* D *if and only if there exists a constant* $z \in \ker D$ *such that* $y + Az \in (I - AR) Y_1$.

THEOREM 1.2. *Let* $D \in \mathbf{R}(X \to Y_1)$, $A \in L(X \to Y)$, *dom* $D \subset$ *dom* A, $y \in (D - A)$ *dom* D. *The general solution of the equation* (1) *is given by*

$$\{x = R[R^A(y + Az_1 + Az) + s] + z_1 + z : s \in \ker(I - AR),$$
$$z \in \ker D,\ Az \in (I - AR) Y_1\},$$

where z_1 *is a constant determined in Theorem* 1.1. $R \in R_D$, $R^A \in R_{I-AR}$ *for* $I - AR \in L_0(Y_1 \to Y)$. *The general solution is independent of the choice of right inverses both* R *and* R^A.

The operator $I - AR$ appearing during the decomposition of the system (1)–(2) need not have an uniquely determined resolvent.

THEOREM 1.3. *Let* $D \in \mathbf{R}(X \to Y_1)$, $A \in L(X \to Y)$, *dom* $D \subset$ *dom* A, $Y_1 \subset Y$, $F \in F_D$, *where* F *corresponds to* $R \in R_D$.

1. *For any pair* $(y, x_0) \in (D - A)$ *dom* $D \times \ker D$, *the system* (1)–(2) *possesses an unique solution if and only if* $A \ker D \subset (I - AR) Y_1$ *and* $\ker(I - AR) = \{0\}$.

This unique solution is given by $x = R(I - AR)^{-1}(y + Ax) + x_0$.

2. *If* $A \ker D \subset (I - AR) Y_1$ *and* $\ker(I - AR) \neq \{0\}$ *on* Y_1, *then for any pair* $(y, x_0) \in (D - A)$ *dom* $D \times \ker D$ *the system* (1)–(2) *possesses more than one solution. These solutions are given by* $x = R[R^A(y + Ax_0) + s] + x_0$, $s \in \ker(I - AR)$, *where* $R^A \in R_{I-AR}$, $I - AR \in L_0(Y_1 \to Y)$, R^A *is arbitrarily fixed.*

3. *If A ker $D \not\subset (I-AR)\, Y_1$, then there exist pairs $(y, x_0) \in (D-A)$ dom $D \times$ ker D for which the system (1)–(2) has no solutions.*

2. Controllability

Let U be a linear space over the field $\mathscr{F}$ of scalars. In this section we consider the problem of controllability of the following system:

$$Dx = Ax + Bu \tag{3}$$

$$Fx = x_0 \tag{4}$$

where $D \in \mathbf{R}(X \to Y_1)$, $\ker D \neq \{0\}$, $A \in L(X \to Y)$, $\mathrm{dom}\, D \subset \mathrm{dom}\, A$, $Y_1 \subset Y$, $B \in L_0(U \to Y)$, $BU \subset (D-A)\, \mathrm{dom}\, D$, $F \in F_D$, F corresponds to $R \in R_D$, $A \ker D \subset (I-AR)\, Y_1$, $u \in U$ and $x_0 \in \ker D$.

Denote by Φ the multi-valued mapping, $\Phi\colon \ker D \times U \to 2^X$ defined as follows:

$$\Phi(x_0, u) = \{R\,[R^A(Bu + Ax_0) + s] + x_0,\ s \in \ker(I-AR)\}$$

where $R^A \in R_{I-AR}$ is arbitraily fixed.

It is easy to show that the set $\Phi(x_0, u)$ is independent of the choice of R^A. If $\ker(I-AR) = \{0\}$, then the set $\Phi(x_0, u)$ contains only one element.

In the sequel X will be called the space of states, U will be called the space of controls and $\ker D$ will be called the space of initial states.

DEFINITION 2.1. The state $\bar{x} \in X$ is said to be reachable from the initial state $x_0 \in \ker D$, if there exists a control $u \in U$ such that $x \in \Phi(x_0, \bar{u})$.

If $\ker(I-AR) \neq \{0\}$, Definition 2.1, means that

$$\exists \bar{u} \in U\ \forall R^A \in R_{I-AR}\ \exists \bar{z} \in \ker(I-AR)$$
$$\bar{x} = R\,[R^A(B\bar{u} + Ax_0) + \bar{z}] + x_0$$

Denote by $\Phi(x_0) = \bigcup_{u \in U} \Phi(x_0, u)$. The set $\Phi(x_0)$ is a collection of all reachable solutions from the given initial state x_0 under a fixed space of controls. By properties of solvability of systems (3)–(4) we obtain.

PROPERTY 2.1. Let the system (3)–(4) be given. Then for any $x_0 \in \ker D$, $\Phi(x_0) \subset \mathrm{dom}\, D$.

Let D^k denote the k-th superposition of D.

THEOREM 2.1. *Let the system (3)–(4) be given. Suppose that $Y = Y_1 = X$, $BU \subset (D-A)$ dom D^k and A ker $D \subset (I-AR)$ dom D^{k-1}, where $k \geqslant 1$ is a fixed integer. Then*

$$\Phi(x_0) \subset (I-F^A)\, \mathrm{dom}\, D^k \oplus F^A\, \mathrm{dom}\, D$$

where $F^A \in F_{I-AR}$, $I-AR \in L_0(Y_1 \to Y)$.

REMARK 2.1. If $\ker(I-AR)=\{0\}$, then $F^A \equiv 0$ on Y_1 and thesis of Theorem 2.1 takes form

$$\Phi(x_0) \subset \operatorname{dom} D^k$$

Next the symbol A^* will denote an algebraic adjoint of the operator $A \in L(\operatorname{dom} A \to Y)$, i.e., $A^* \in L_0(Y^* \to (\operatorname{dom} A)^*)$, $(A^*\varphi)(x) = \varphi(Ax)$ for any $x \in \operatorname{dom} A$ and $\varphi \in Y^*$.

THEOREM 2.2. *Let the system* (3)–(4) *be given.*

1. *Suppose that* $\ker(I-AR) \neq \{0\}$ *and* $R^A \in R_{I-AR}$. *If* $\ker(RR^A B)^* = \{0\}$ *then for any* $x_0 \in \ker D$, $\Phi(x_0) = RY_1 \oplus \{x_0\}$
2. *Suppose that* $\ker(I-AR) = \{0\}$, $\ker(B(I-AR)^{-1}B)^* = \{0\}$ *if and only if for any* $x_0 \in \ker D$, $\Phi(x_0) = RY_1 \oplus \{x_0\}$.

Proof. Let $x_0 \in \ker D$ be arbitrarily fixed. Since $\operatorname{dom} D = RY_1 \oplus \ker D$, the condition $BU \subset (D-A)\operatorname{dom} D$ implies $RR^A BU \subset \{x = RR^A[(I-AR)t - Az]: t \in Y_1, z \in \ker D\}$. Then $\ker(RR^A B)^* = \{0\}$ if and only if for any $t \in Y_1$ and $z \in \ker D$, there exists a control $u \in U$ such that $RR^A Bu = RR^A[(I-AR)t - Az]$. If we take $z = x_0$ and $s = F^A t$, where $F^A = I - R^A(I-AR)$ we obtain that for any $t \in Y_1$ there exist $u \in U$ and $s \in \ker(I-AR)$ such that $R[R^A(Bu + Ax_0) + s] + x_0 = Rt + x_0$. Finally, $\Phi(x_0) = RY_1 \oplus \{x_0\}$.
Now let $\ker(I-AR) = \{0\}$ and $\Phi(x_0) = RY_1 \oplus \{x_0\}$. Then for any $x_0 \in \ker D$ and $t \in Y_1$, there exists $u \in U$ such that $R(I-AR)^{-1}(Bu + Ax_0) + x_0 = Rt + x_0$ or $R(I-AR)^{-1}Bu = R(I-AR)^{-1}[(I-AR)t - Ax_0]$. Hence, $\ker(R(I-AR)^{-1}B)^* = \{0\}$ ∎

Let us remark that the set $RX \oplus \{x_0\}$ is the greatest set reachable from $x_0 \in \ker D$.

Now let F_1 be an initial operator for D such that $F_1 \neq F$.

DEFINITION 2.2. The state $x_1 \in \ker D$ is said to be F_1 — reachable from the initial state $x_0 \in \ker D$, if there exists a control $u \in U$ such that $x_1 \in F_1 \Phi(x_0, u)$. The state x_1 will be called a final state.

DEFINITION 2.3. The system (3)–(4) is said to be F_1 — controllable to 0 if for every initial state $x_0 \in \ker D$, $0 \in \Phi(x_0)$

DEFINITION 2.4. The system (3)–(4) is said to be F_1 — controllable if for every initial state $x_0 \in \ker D$, $F_1 \Phi(x_0) = \ker D$.

Obviously, F_1 — controllability implies F_1 — controllability to 0.

THEOREM 2.3. *Let the system* (3)–(4), $F_1 \in F_D$ *and* $R^A \in R_{I-AR}$ *be given. If the system* (3)–(4) *is* F_1 — *controllable to* 0 *and* $F_1(RR^A A + I)\ker D = \ker D$ *then the system* (3)–(4) *is* F_1 — *controllable.*

Proof. At first we prove that any $x_1 \in \ker D$ is F_1 — reachable from 0. Indeed, by F_1 — controllability to 0, for any $x_0 \in \ker D$ there exist $u_0 \in U$ and $s_0 \in \ker(I-AR)$ such that $0 = F_1[R(R^A(Bu_0+Ax_0)+s_0)+x_0]$. By second assumption there exists $\bar{x}_0$ such that $F_1(RR^A A+I)\bar{x}_0 = x_1$. Next if we take $x_0 = -\bar{x}_0$ we obtain $F_1[R(R^A(Bu_0+A0)+s_0)+0] = x_1$. But for any x_0 there exist $\hat{u}$ and $\hat{s}$ such that

$$F_1[R(R^A(B\hat{u}+Ax_0)+\hat{s})+x_0] = 0$$

Then, if we substitute $u = u_0+\hat{u}$ aand $s = s_0+\hat{s}$ we obtain thesis, i.e., for any $x_0, x_1 \in \ker D$ there exists $u \in U$ and $s \in \ker(I-AR)$ such that $F_1[R(R^A(Bu+ +Ax_0)+s)+x_0] = x_1$. Finally $x_1 \in F_1 \Phi(x_0)$ ■

In Section 3 there will be presented an example showing that the condition $F_1(RR^A A+I)\ker D = \ker D$ need not be necessary for F_1 — controllability.

Theorem 2.4. *Let the system* (3)–(4), $R^A \in R_{I-AR}$ *and* $F_1 \in F_D, F_1 \neq F$ *be given.*

1. *Suppose that* $\ker(I-AR) \neq \{0\}$. *Then, if* $\ker(F_1 RR^A B)^* = \{0\}$, *the system* (3)–(4) *is* F_1 — *controllable*
2. *Suppose that* $\ker(I-AR) = \{0\}$. *Then* $\ker(F_1 R(I-AR)^{-1} B)^* = \{0\}$ *if and only if the system* (3)–(4) *is* F_1 — *controllable.*

Proof. $F_1: RR^A BU \to \ker D$. The condition $\ker(F_1 RR^A B)^* = \{0\}$ is equivalent to $F_1 RR^A BU = \ker D$, provided $F_1 \neq F$. By the condition $BU \subset (D-A)\operatorname{dom} D$ we obtain $R^A BU \subset \operatorname{dom}(I-AR) = Y_1$. Then $F_1 RY_1 = \ker D$, i.e., for any $x_2 \in \ker D$, there exists $t \in Y_1$ such that $FRt = x_2$. Since $F_1 RR^A(D-A)\operatorname{dom} D = = \ker D$, then for any $t = Y_1$ and $z \in \ker D$ there exists $u \in U$ such that $F_1(Rt+z) = F_1(R(R^A(Bu+Az)+F^A t)+z)$, where $F^A \in F_{I-AR}$, $F^A = I-R^A(I- -AR)$. By the following substitution: $x_2 = x_1 - x_0$ and $z = x_0$, where $x_0, x_1 \in \in \ker D$ are arbitrary, we obtain $F_1(R(R^A(Bu+Ax_0)+s)+x_0) = x_1$, where $s = F^A t$ and $F_1 Rt = x_1 - x_0$. Hence, one part of the theorem is proved. If we now assume that $\ker(I-AR) = \{0\}$ and the system (3)–(4) is F_1 — controllable we have that for any $x_0, x_1 \in \ker D$ there exists $u \in U$ such that $F_1(R(I-AR)^{-1}(Bu+Ax_0)+x_0) = x_1$. In particular, for $x_0 = 0$, $F_1 R(I- -AR)^{-1} Bu = x_1$. Then $F_1 R(I-AR)^{-1} BU = \ker D$ and finally $\ker(F_1 R(I- -AR)^{-1} B)^* = \{0\}$ ■

3. Examples

Example 3.1. Let $X = C_n[0, T]$ over the complex field **C** of scalars, $0 < T < \infty$. The following system describes the linear differential stationary system by the right invertible operator

$$Dx = Ax+Bu \tag{5}$$

$$Fx = x_0 \tag{6}$$

where $x \in X$, $x(t) = [x_1(t), ..., x_n(t)]^T$, $u(t) = [u_1(t), ..., u_n(t)]^T$, $x_0 = [x_{01}, ...$ $..., x_{0n}]^T$, $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n}$, A and B are constant matrices, $D = \frac{d}{dt}$, $(Fx)(t) = x(t_0)$, $t_0 \in [0, T]$, $U = X$.

We are interested in existence of a control u such that at the time $t_1 \in [0, T]$ solutions of (5)-(6) satisfy the condition $x(t_1) = x_1$, for a given $x_1 = [x_{11}, ..., x_{1n}]$. Let us define F_1 as follows: $(F_1 x)(t) = x(t_1)$. Then we obtain the problem of F_1 — controllability of (5)-(6). The operator $I - AR$ is invertible and by Theorem 2.4. the system (5)-(6) is F_1 — controllable if and only if $\ker (F_1 R(I-AR)^{-1} B)^* = \{0\}$ where the operator R is a right inverse of D corresponding to F. This takes place if and only if $F_1 \{x = R(I-AR)^{-1}(Bu + Ax_0) + x_0 : u \in U\} = \ker D$.

This last equality holds if and only if the rank of the matrix $[B, AB, ..., A^{n-1}B]$ equals n. This corresponds to Kalman Theorem of controllability for the differential linear stationary systems.

EXAMPLE 3.2. Let $X = C_2[0, T]$ over $\mathbf{C}$. We consider the system (5)-(6) with $A: a_{11} = a_{22} = 0, a_{12} = a_{21} = a$, $B: b_{11} = b_{22} = 0, b_{12} = b_{21} = b$, $a, b \in \mathbf{R}$. We are interested in satisfying, by solutions of (5)-(6), the condition $F_2 x = x_1$, where $(F_2 x)(t) = [x_1(t_1), x_2(t_2)]$ $t_1, t_2 \in [0, T]$. It is easy to show that the system is F_2 — controllable to 0 and $F_2(R(I-AR)^{-1}A + I)\ker D = \ker D$ holds. By Theorem 2.3 this system is F_2 — controllable.

EXAMPLE 3.3. Let $X = (s)$ be the space of all real sequences $x = \{x_n\}_{n=1}^{\infty}$ over the real field $\mathbf{R}$. We consider the system (3)-(4) where operators are defined as follows: $Dx = \{x_{n+1}\}_{n=1}^{\infty}$; $Fx = y$, $y = \{y_n\}_{n=1}^{\infty}$, $y_1 = x_1$, $y_n = 0$ for $n \geq 2$; $Ax = v$, $v = \{v_n\}_{n=1}^{\infty}$, $v_1 = x_2$, $v_n = x_{n-1}$ for $n \geq 2$; $B = I$ on $U = \{x = \{x_n\}_{n=1}^{\infty} : x_1 = 0\}$. $x_0 = \{x_{0n}\}_{n=1}^{\infty}$, $x_{01} = C$, $x_{0n} = 0$ for $n \geq 2$. Let us consider the initial operator for D defined as follows: $F_1 x = y$, $y = \{y_n\}_{n=1}^{\infty}$, $y_1 = x_2$, $y_n = 0$ for $n \geq 2$. $\ker (I-AR) = \{x = \{x_n\}_{n=1}^{\infty} : x_{2n+1} = C, x_{2n+2} = 0$ for $n \geq 0, C \in \mathbf{R}\}$, $A \ker D \subset (I-AR)X$. Let us take the following right inverse R^A defined on $(I-AR)X : R^A x = y$, $y = \{y_n\}_{n=1}^{\infty}$, $y_1 = 0$, $y_n = x_n + x_{n-2}$, $n \geq 2$, $x_0 = 0$. The following holds: $F_2(RR^A A + I)\ker D = \{0\}$ but the system (3)-(4) is F_2 — controllable. Indeed, for any x_0 it is sufficient to take $u = 0$ and $s = \{s_n\}_{n=1}^{\infty}$, $s_{2n+1} = C_1$, $S_{2n+2} = 0$ for $n \geq 0$, $s_1 = 0$ in order to fulfil the condition $F_2 x = x_1$, $x_1 = \{x_n^1\}_{n=1}^{\infty}$, $x_1^1 = C_1$, $x_n^1 = 0$ for $n \geq 2$, for any $x_1 \in \ker D$.

EXAMPLE 3.4. Let $X = C([0, T] \times [0, T])$ over the complex field $\mathbf{C}$ of scalars, $0 < T < \infty$. Let us consider the following partial differential system

$$\frac{\partial^2}{\partial s\, \partial t} x(t, s) = u(t, s) \tag{7}$$

$$x(t, 0) = h(t), \; x(0, s) = g(s) \tag{8}$$

where $h(0) = g(0)$, $h, g \in C^1 [0, T]$

We are interested in the existence of such continuous functions $u(t, s)$ for which solutions of system (7)–(8) satisfy the conditions

$$x(t, s_0) = h_1(t), \; x(t_0, s) = g_1(s) \tag{9}$$

where $h_1(t_0) = g_1(s_0)$, $h_1, g_1 \in C^1 [0, T]$, $s_0, t_0 \in [0, T]$ under the assumption that functions h, g, h_1, g_1 are arbitrary.

Let us notice that the system (7)–(8) can be described by right invertible operator and the considered problem is the question about F_1 — reachability. Indeed, let us denote

$$D = \frac{\partial^2}{\partial s\, \partial t}, \text{dom } D = \left\{ x \in X: \frac{\partial^2}{\partial s\, \partial t} x(t, s) \text{ exists and is continuous} \right\},$$

$$(Fx)(t, s) = x(t, 0) + x(0, s) - x(0, 0), \; F \in F_D$$

$$D \in \mathbf{R}(X \to X), \ker D = \{h(t) + g(s); h, g \in C^1 [0, T]\} \neq \{0\}$$

$$(F_1 x)(t, s) = x(t, s_0) + x(t_0, x) - x(t_0, s_0), \; F_1 = F_D$$

Then the system (7)–(8) and the condition (9) take form:

$$Dx = u \tag{10}$$

$$Fx = x_0 \tag{11}$$

$$F_1 x = x_1 \tag{12}$$

where $x_0(t, s) = h(t) + g(s), x_1(t, s) = h_1(t) + g_1(s), U = X, A \equiv 0, B = I$ and additionaly $h(0) = g(0)$, $h_1(t_0) = g_1(s_0)$.

If there were not any additional conditions on functions h, g, h_1 and g_1, then the problem (12)–(14) would be a F_1 — controllability problem. For such defined systems the condition $\hat{F}_1 (RR^A A + I) \ker D = \ker D$ is fulfilled for any $\hat{F}_1 \in F_D$. Hence, by Theorem 2.3. for proving F_1 — controllability, it will do to verify F_1 — controllability to 0. It means that it will do to determine any $\hat{x}_0 \in \ker D$ and a control $\hat{u} \in U$ satisfying $F_1 (R\hat{u} + \hat{x}_0) = 0$. But it is easy to show that such controls do not exist, in general. Hence, the system (10)–(11) is not F_1 — controllable. On the other hand it is easy to show that for any $x_0(t, s) = h(t) + g(s)$, such that $h(0) = g(0)$ and for any $x_1(t, s) = h_1(t) + g_1(s)$ such that $h_1(t_0) = g_1(s_0)$, the state x_1 is F_1 — reachable from x_0.

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Received, June 1984

Sterowalność systemów liniowych z niejednoznacznie określonymi rezolwentami

W pracy rozważono sterowalność systemów przy założeniu liniowości operatorów i przestrzeni. Omawiane systemy są opisane przez operatory mające prawe odwrotności i przez ich operatory początkowe. Uogólniono definicję osiągalności stanów i sterowalności systemów zaproponowaną przez Nguyen Dinh Quyet na systemy, w których rezolwenty nie są jednoznacznie określone. Wykazano, że właściwości osiągalności i sterowalności zależą od rezolwent. Pracę zilustrowano przykładami.

Управляемость линейных систем с неоднозначно определеным разрешающим оператором

В работе рассмотрены системы описаны линейшыми операторами в линейных пространствах. Автор обобщает понятия достижимости состояний и управляемости систем на случай когда разрешающий оператор определен неоднозначно. Показано что управляемость и достижимость зависят от разрешающего оператора. Проблему проиллюсрировано примерами.