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A necessary condition for a problem of optimal control with equality and inequality constraints

by

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Some specification of the Dubovitskii-Milyutin method is applied to investigating a problem of optimal control with equality and inequality constraints on the phase coordinates and the control. A necessary condition in the form of a local extremum principle is obtained.

Introduction

A necessary condition for the problem of optimal control with equality and inequality constraints but without the condition $u(\cdot)\in U$, where U — some set, is proved in [1], [2], [3], [4], [6] and [12] by making use of the variational method.

The Dubovitskii-Milyutin method is useful to obtain a necessary condition for the extremal problems with only one equality constraint. A generalization of the Dubovitskii-Milyutin method in the case of n equality constraints in any form and under some assumptions on the cones is obtained in [14]. This generalization is applied to problems of optimal control with equality constraints on the phase coordinates in [15] and to a problem with no-operator equality constraint in [13]. In [10] some specification of the Dubovitskii--Milyutin method is given without any additional assumption about the cones but for the case of n equality constraints given in the operator form. This specification is applied in [11] to obtain a necessary condition for the problem of optimal control with equality constraints on the phase coordinates and the control.

In the paper the problem of optimal control with equality and inequality constraints is considered. This problem is more general than that in [11] because it contains inequality constraints, and more general than the problem from [6] because it contains the no-operator constraint $u(\cdot)\in U$ where U — some set. The local extremum principle for this problem is obtained in a more general and simpler form than in [11].

1. Fundamental definitions

DEFINITION 1. Denote by $W_{11}^n(0, 1)$ the space of absolutely continuous functions whose derivatives $\dot{x}\in L_1^n(0, 1)$. The norm in $W_{11}^n(0, 1)$ is defined by the formula

$$\|x\| = |x(0)| + \int_0^1 |\dot{x}(t)|\, dt.$$

Let $\bar{W}_{11}^n(0, 1)$ be the subspace of the space $W_{11}^n(0, 1)$ which contains functions satisfying the condition $x(0) = 0$.

DEFINITION 2. Let (Σ, μ) stand for a space with a Lebesgue measure defined on the Σ-field of subsets of the interval $[0, 1]$: Denote by *b.a.* $(0, 1)$ the family of additive functions $\varphi: \Sigma \to R$ satisfying the conditions:

a) if $A\in\Sigma$ and $\mu(A) = 0$, then $\varphi(A) = 0$,

b) the variation of the function φ is bounded, i.e. $|\varphi|_{(0,1)} < \infty$.

The space *b.a.* $(0, 1)$ is a Banach space with the norm $\|\varphi\| = |\varphi|_{(0,1)}$ (cf. [9], Part VI, §2).

Let $b^p a(0, 1)$ be the space of functions $\varphi: \Sigma \to R^p$ in the form

$$\varphi(A) = \big(\varphi_1(A), \varphi_2(A), \ldots, \varphi_p(A)\big)$$

with $A\in\Sigma$, $\varphi_i\in b.a(0, 1)$ for $i = 1, 2, \ldots, p$.
It is easy to show that the space $b^p a(0, 1)$ is a Banach space with the norm

$$\|\varphi\| = \sum_{i=1}^{n} |\varphi_i|_{(0,1)}.$$

DEFINITION 3. Let X be a Banach space of integrable functions defined on the interval $[0, 1]$. Each functional f defined on X of the form

$$f(x) = \int_0^1 \tilde{f}(t)\, x(t)\, dt \qquad \text{where } \tilde{f}\in L_1(0, 1),$$

will be called absolutely continuous.

LEMMA 1. *Any linear and continuous functional f can be written in the form*

$$f = f^a + f^s \tag{1}$$

where f^a — an absolutely continuous functional, f^s — a singular functional. This representation is univalent.

2. Problem formulation and basic assumptions

Let us consider the following problem:

$$I(x, u) = \int_0^1 f^0(x, u, t)\, dt \to \min \tag{2}$$

under the constraints

$$\dot{x} = f(x, u, t), \tag{3}$$

$$x(1) = x_1, \tag{4}$$

$$g(x, u, t) = 0, \tag{5}$$

$$h_i(x, u, t) \leqslant 0, \quad i = 1, 2, \ldots, l, \tag{6}$$

$$u(\cdot) \in U, \tag{7}$$

where $x(\cdot) \in \bar{W}_{11}^n(0,1)$, $u(\cdot) \in L_\infty^r(0,1)$; the set $U = \{u(\cdot) \in L_\infty^r(0,1): u(t) \in M\}$; the functions $f^0: R^n \times R^r \to R \to R$, $f: R^n \times R^r \times R \to R^n$, $g: R^n \times R^r \times R \to R^k$, $h_i: R^n \times R^r \times R \to R$ for $i = 1, 2, \ldots, l$; x_1 is a fixed point of R^n.

We assume that there exist derivatives

$$f_x^0, f_u^0, f_x, f_u, g_u, h_{ix}, h_{iu} \quad \text{for } i = 1, 2, \ldots, l \tag{8}$$

which are bounded for any $(x, u) \in W_{11}^n \times L_\infty^r$, the functions

$$f_0, f, g_x, f_x^0, f_u^0, f_x, f_u, h_i, h_{ix}, h_{iu} \quad \text{for } i = 1, 2, \ldots, l \tag{9}$$

are continuous with respect to (x, u) for any $t \in [0, 1]$ and measurable with respect to t, and the Frechet derivative $(g_x(x, u, t), g_u(x, u, t))$ is continuous with respect to (x, u) in the topology of the space

$$\mathrm{L}(\bar{W}_{11}^n \times L_\infty^r, L_\infty^k) \tag{10}$$

(cf. [8], §0.1), the set M is closed, convex and possesses a non-empty interior.

REMARK 1. Problem (2)–(7) under assumptions (8)–(10) will be called problem PI. We consider problem PI under the assumption $k \leqslant r$.

Let us put $X = \bar{W}_{11}^n \times L_\infty^r$.

Denote by $F_2: X \to \bar{W}_{11}^n$ the operator defined by the formula

$$F_2(x, u)(t) = x(t) - \int_0^t f(x(t), u(t), t)\, dt. \tag{11}$$

By $F_3: X \to L_\infty^k$ let us denote the operator of the form

$$F_3(x, u)(t) = g(x(t), u(t), t). \tag{12}$$

Denote by $F_4: X \to R^n$ the operator defined by the formula

$$F_4(x, u)(t) = x(1) - x_1. \tag{13}$$

3. The local extremum principle

Making use of the specification of the Dubovitskii-Milyutin method (cf. [10], th. 1) in the case of n equality constraints given in the operator form, we shall prove the following.

THEOREM 1 (*The local extremum principle*). *If*
1° (x^0, u^0) *is an optimal process in problem PI*,
2° *the operator* $A: \bar{W}_{11}^n \times L_\infty^r \to \bar{W}_{11}^n \times L_\infty^k \times R^n$ *in the form*

$$A(\bar{x}, \bar{u})(t) = \Big(\bar{x}(t) - \int_0^t \big(f_x(x^0, u^0, t)\,\bar{x}(t) + f_u(x^0, u^0, t)\,\bar{u}(t)\big)\,dt,$$
$$g_x(x^0, u^0, t)\,\bar{x}(t) + g_u(x^0, u^0, t)\,\bar{u}(t),\ \bar{x}(1)\Big)$$

is such that its image is closed in $\bar{W}_{11}^n \times L_\infty^k \times R^n$,

3° $$h_{ix}(x^0(t), u^0(t), t) \neq 0 \text{ or } h_{iu}(x^0(t), u^0(t), t) \neq 0$$

for any $t \in R_i$ *where*

$$R_i = \{t \in [0, 1]: h_i(x_0(t), u_0(t), t) = 0\} \tag{14}$$

for $i = 1, 2, \ldots, l$ *then there exist* $\lambda_0 \geq 0$ *and functions* $\psi \in L_\infty^n(0, 1)$, $\omega \in b^k.a(0, 1)$, $\gamma_i \in b.a(0, 1)$ *concentrated on the sets* R_i *for* $i = 1, 2, \ldots, l$, *such that*

1° $\|\omega\| + |\lambda_0| + \|\psi\| + |a| + \sum_{i=1}^{l} \|\gamma_i\| > 0$,

2° $\dot{\varphi} = -\lambda_0 f_x^0(x^0, u^0, t) - f_x^*(x^0, u^0, t)\,\varphi + g_x^*(x^0, u^0, t)\tilde{f}_\omega - \sum_{i=1}^{l} h_{ix}^*(x^0, u^0, t)\tilde{f}_{\gamma_i}$

where $\varphi = -\psi$, $\tilde{f}_\omega, \tilde{f}_{\gamma_i}$ *are the integrable functions corresponding to the functionals* f_ω *and* f_{γ_i}, *respectively, in the sense of Lemma* 1 *and Definition* 3. *Functionals* f_ω *and* f_{γ_i}, $i = 1, 2, \ldots, l$ *are given in the form*

$$f_\omega(x(t)) = \int_0^1 x(t)\,d\omega,\quad f_{\gamma_i}(x(t)) = \int_0^1 x(t)\,d\gamma_i,\quad i = 1, 2, \ldots, l. \tag{15}$$

3° $$\lambda_0 \int_0^1 f_u^0(x^0, u^0, t)\,u^0(t)\,dt + \int_0^1 f_u((x^0, u^0, t)\,u^0(t)\,\varphi(t)\,dt -$$
$$-\int_0^1 g_u(x^0, u^0, t)\,u^0(t)\,d\omega + \sum_{i=1}^{l} \int_0^1 h_{iu}(x^0, u^0, t)\,u^0(t)\,d\gamma_i =$$
$$= \min_{u \in U} \Big(\lambda_0 \int_0^1 f_u^0(x^0, u^0, t)\,u(t)\,dt + \int_0^1 f_u(x^0, u^0, t)\,u(t)\,\varphi(t)\,dt -$$
$$-\int_0^1 g_u(x^0, u^0, t)\,u(t)\,d\omega + \sum_{i=1}^{l} \int_0^1 h_{iu}(x^0, u^0, t)\,u^0(t)\,d\gamma_i\Big).$$

Proof. Let us define the following sets:

$$Z_1 = \{(x, u) \in X: u \in U\},$$

$$Z_i = \{(x, u) \in X : F_i(x, u) = 0\}$$

for $i = 2, 3, 4$, where the operators F_2, F_3, F_4 are defined by formulae (11), (12) and (13), respectively.

$$Z_{4+j} = \{(x, u) \in X : h_j(x, u, t) \leqq 0\} \quad \text{for } j = 1, 2, \ldots, l.$$

Hence, problem PI may be represented in the form:

Determine the minimal value of the functional $I(x, u)$ defined on X, under the condition

$$(x, u) \in \bigcap_{i=1}^{4+l} Z_i.$$

In the proof we shall make use of a specification of the Dubovitskii-Milyutin method (cf. [10], th. 1 and 2).

Hence we shall find the following cones:

$C_0 = DC(I(x^0, u^0))$ — the cone of directions of decrease of the functional I at (x^0, u^0);

$C_i = FC(Z_i, (x^0, u^0))$ for $i = 1, 5, \ldots, 4+l$ — the cone of feasible directions of the set Z_i at (x^0, u^0);

$C_i = TC(Z_i, (x^0, u^0))$ for $i = 2, 3, 4$ — the cone of tangent directions of the set Z_i at (x^0, u^0);

as well as their dual cones C_i^*, $i = 0, 1, \ldots, 4+l$.

Proceeding identically as in [7], §7, 8, we can find the cones

$$C_0 = \{(\bar{x}, \bar{u}) \in X : \int_0^1 (f_x^0(x^0, u^0, t)\,\bar{x} + f_u^0(x^0, u^0, t)\,\bar{u})\,dt < 0\},$$

$$C_1 = \{(\bar{x}, \bar{u}) \in X : \bar{u} = \lambda(u - u^0) \text{ where } \lambda \geqq 0, u \in \operatorname{int} U\},$$

and we assume temporarily that $C_0 \neq \emptyset$.

The cones C_0^* and C_1^* are given by the formulae (cf. [7], § 10):

$$C_0^* = \{f_0 \in X^* : f_0(\bar{x}, \bar{u}) = -\lambda_0 \int_0^1 (f_x^0(x^0, u^0, t)\,\bar{x} + f_u^0(x^0, u^0, t)\,\bar{u})\,dt,\ \lambda_0 \geqq 0\}, \tag{16}$$

$$C_1^* = \{f_1 \in X^* : f_1(\bar{x}, \bar{u}) = f_1'(\bar{u}) \text{ where } f_1' \text{ is a functional supporting the set } Z_1 \text{ at the point } (x^0, u^0)\}. \tag{17}$$

The set Z_2 is an equality constraint. Proceeding identically as in [11], we shall find the cones C_2 and C_2^* of the forms

$$C_2 = \{(\bar{x}, \bar{u}) = X : \dot{\bar{x}}(t) - f_x(x^0, u^0, t)\,\bar{x} + f_u(x^0, u^0, t)\,\bar{u} = 0\}, \tag{18}$$

$$C_2^* = \{f_2 \in X^* : f_2(\bar{x}, \bar{u}) = \int_0^1 (\dot{\bar{x}} - f_x(x^0, u^0, t)\,\bar{x} - f_u(x^0, u^0, t)\,\bar{u})\,\psi(t)\,dt \text{ where } \psi \in L_\infty^n(0, 1)\}. \tag{19}$$

The set Z_3 is an equality constraint, too. The operator F_3 given by formula (12) is strongly continuously differentiable (cf. assumption (9)) and its differential is given in the form

$$F_3'(x^0, u^0)(\bar{x}, \bar{u})(t) = g_x(x^0(t), u^0(t), t)\,\bar{x}(t) + \\ + g_u(x^0(t), u^0(t), t)\,\bar{u}(t). \tag{20}$$

Let us assume temporarily that the operator F_3 given in form (12) is regular at (x^0, u^0), i.e. the operator $F'_3(x^0, u^0): X \to L^k_\infty$ given by formula (20) is "onto". It is easy to show that the sufficient condition for the regularity of the operator F_3 is the following: there exist a minor rank k and $\alpha > 0$ such that

$$|m([g_u])| > \alpha \text{ for } t \in [0, 1] \text{ a.e.}$$

Hence the operator F_3 satisfies the assumption of the Lusternik theorem (cf. [8], § 0.2), and the cone C_3 is of the form:

$$C_3 = \{(\bar{x}, \bar{u}) \in X : g_x(x^0, u^0, t)\,\bar{x} + g_u(x^0, u^0, t)\,\bar{u} = 0\}.$$

Proceeding identically as in [11], we shall calculate the cone

$$C_3^* = \{f_3 \in X^* : f_3(\bar{x}, \bar{u}) = \int_0^1 (g_x(x^0, u^0, t)\,\bar{x} + \\ + g_u(x^0, u^0, t)\,\bar{u})\,d\omega \text{ where } \omega = b^k_\cdot a(0, 1)\}. \tag{21}$$

The set Z_4 is an equality constraint, too.

It is easily shown that cones C_4 and C_4^* are of the forms (cf. [7], §12)

$$\begin{aligned} C_4 &= \{(\bar{x}, \bar{u}) \in X : x(1) = 0\} \\ C_4^* &= \{f_4 \in X^* : f_4(\bar{x}, \bar{u}) = (\bar{x}(1), a) \text{ where } a \in R^n\}. \end{aligned} \tag{22}$$

Now, let us consider the inequality constraints Z_{4+i}, $i = 1, 2, \ldots, l$. Let us denote by H_i, $i = 1, \ldots, l$, the functionals

$$H_i(x, u) = \operatorname*{vrai\,max}_{t \in [0, 1]} h_i(x(t), u(t), t). \tag{23}$$

In view of example 6.6 from [7], the cone of directions of decrease of the functionals H_i at the point (x^0, u^0) is of the form

$$DC(H_i, (x^0, u^0)) = \{(\bar{x}, \bar{u}) \in X : H_i'(x^0, u^0)(\bar{x}, \bar{u}) < 0\} = \\ = \max_{t \in R_i} (h'_{ix}(x^0, u^0, t)\,\bar{x} + h_{iu}(x^0, u^0, t)\,\bar{u}) < 0\} \tag{24}$$

where R_i are given by the formula (14)} for $i = 1, 2, \ldots, l$.

Taking account of assumptions (8)–(9) concerning functions h_i and assumption 3° of this theorem, we shall use the corollary from theorem 8.1 [7]. This corollary implies that the cones C_{4+i}, $i = 1, 2, \ldots, l$ have the form

$$C_{4+i} = \{(\bar{x}, \bar{u}) \in X : h_{ix}(x^0, u^0, t)\,\bar{x} + h_{iu}(x^0, u^0, t)\,\bar{u} < 0 \quad \text{for } t \in R_i\}, \quad i = 1, 2, \ldots, l. \tag{25}$$

Now, we shall obtain the formula for cones C^*_{4+i}, $i = 1, 2, \ldots, l$.

Let us consider the operators $A_i : \bar{W}^n_{11} \times L^r_\infty \to L^r_\infty$ given below

$$A_i(\bar{x}, \bar{u})(t) = h_{ix}(x^0, u^0, t)\,\bar{x} + h_{iu}(x^0, u^0, t)\,\bar{u} \tag{26}$$

for $i = 1, 2, \ldots, l$ and define the cones

$$\bar{C}_{4+i} = \{\bar{y} \in L^r_\infty : \bar{y}(t) < 0 \text{ for } t \in R_i\} \quad \text{for } i = 1, 2, \ldots, l. \tag{27}$$

Hence, from (25), (26) and (27) we obtain that

$$\bar{C}_{4+i} = AC_{4+i} \quad \text{for } i = 1, 2, \ldots, l.$$

It is easy to check that the cones $\bar{C}_{4+i}$ satisfy the assumptions of the Minkowski-Farkasz theorem (cf. [7], § 10), hence

$$C^*_{4+i} = A^*\,\bar{C}^*_{4+i} \quad \text{for } i = 1, 2, \ldots, l.$$

Let f_{4+i} be an arbitrary element of the cone C^*_{4+i}. Hence, for any $(\bar{x}, \bar{u}) \in X$,

$$\begin{aligned}(f_{4+i}, (\bar{x}, \bar{u})) &= (A^*\bar{f}_{4+i}, (\bar{x}, \bar{u})) = (\bar{f}_{4+i}, A(\bar{x}, \bar{u})) = \\ &= (\bar{f}_{4+i}, h_{ix}(x^0, u^0, t)\,\bar{x} + h_{iu}(x^0, u^0, t)\,\bar{u})\end{aligned} \tag{28}$$

where $\bar{f}_{4+i} \in \bar{C}^*_{4+i}$ for $i = 1, 2, \ldots, l$.

Making use of the formula for a linear and continuous functional defined on L^r_∞ (cf. [9], Part VI, §2), from (28) we obtain that

$$\bar{C}^*_{4+i} = \{\bar{f}_{4+i} \in (L^r_\infty)^* : \bar{f}_{4+i}(\bar{y}) = -\int_0^1 y(t)\,d\gamma_i \tag{29}$$

where the function $\gamma_i \in b.a\,(0, 1)$ is concentrated on the set R_i given in form (14) for $i = 1, 2, \ldots, l$,

Combining (28) and (29), we get the formula for C^*_{4+i}:

$$\begin{aligned}C^*_{4+i} = \{f_{4+1} \in X^* : f_{4+i}(\bar{x}, \bar{u}) &= \\ &= -\int_0^1 (h_{ix}(x^0, u^0, t)\,\bar{x} + h_{iu}(x^0, u^0, t)\,\bar{u})\,d\gamma_i\end{aligned} \tag{30}$$

where the function $\gamma_i \in b.a\,(0, 1)$ is concentrated on the set $R_i\}$, $i = 1, 2, \ldots, l$.

Let us introduce the operator $F : X \to W^n_{11} \times L^k_\infty \times R^n$ by the formula

$$F(x, u) = (F_2(x, u), F_3(x, u), F_4(x, u))$$

where the operators F_2, F_3, F_4 are given by equalities (11), (12) and (13), respectively.

It is easy to demonstrate that the operator F is strongly differentiable at point (x^0, u^0) and its differential at this point is the operator A from assumption 2° of the theorem. Then, in view of assumption 2° the differential of the operator F has the closed image in $W_{11}^n \times L_\infty^k \times R^n$.

We can now apply theorem 1 from [10]. Making use of the formulae for the cones C_i^*, $i = 0, 1, \ldots, 4+l$ ((16), (17), (19), (21), (22) and (30), respectively), we obtain

$$-\lambda_0 \int_0^1 (f_x^0 \bar{x} + f_u^0 \bar{u})\, dt + f_1'(\bar{u}) + \int_0^1 (\dot{\bar{x}} - f_x \bar{x} - f_x \bar{x} - f_u \bar{u})\, \psi(t)\, dt +$$

$$+ \int_0^1 (g_x \bar{x} + g_u \bar{u})\, d\omega + (a, \bar{x}(1)) - \sum_{i=1}^{l} \int_0^1 (h_{ix} \bar{x} + h_{iu} \bar{u})\, d\gamma_i = 0 \tag{31}$$

where $\lambda_0 \geqq 0$, $\psi \in L_\infty^n$, $\omega \in b^k a\,(0, 1)$, $a \in R^n$, $\gamma_i \in b.a\,(0, 1)$ are concentrated on the sets R_i of he form (14) for $i = 1, 2, \ldots, l$.

Let us apply formula (15) and put $(\bar{x}, \bar{u}) = (\bar{x}, 0) \in X$ in (31). We get the following equation

$$-\lambda_0 \int_0^1 f_x^0 \bar{x}\, dt + \int_0^1 (\dot{\bar{x}} - f_x \bar{x})\, \psi(t)\, dt + f_\omega(g_x \bar{x}) +$$

$$+ (a, \bar{x}(1)) - \sum_{i=1}^{l} f_{\gamma_i}(h_{ix} \bar{x}) = 0 \qquad \text{for any } x \in W_{11}^n. \tag{32}$$

Now, we shall apply Lemma 1 and Definition 3 of this paper to the functionals f_ω and f_{γ_i}. From (32) we obtain

$$-\lambda_0 \int_0^1 f_x^0 \bar{x}\, dt + \int_0^1 (\dot{\bar{x}} - f_x \bar{x})\, \psi(t)\, dt + (a, x(1)) +$$

$$+ \int_0^1 (g_x \bar{x})\, \tilde{f}_\omega\, dt - \sum_{i=1}^{l} \int_0^1 (h_{ix} \bar{x})\, \tilde{f}_{\gamma_i}\, dt =$$

$$= -f_\omega^s(g_x \bar{x}) + \sum_{i=1}^{l} f_{\gamma_i}^s(h_{ix} \bar{x}) \qquad \text{for any } \bar{x} \in W_{11}^n. \tag{33}$$

The left-hand side of the last equation is a functional absolutely continuous with respect to $\bar{x}$, the right-hand side is a functional singular with respect to $\bar{x}$. We apply the fact that the functional which is simultaneously singular and absolutely continuous is equal to zero (cf. [4]), i.e.

$$-f_\omega^s(g_x \bar{x}) + \sum_{i=1}^{l} f_{\omega_i}^s(h_{ix} \bar{x}) = 0 \qquad \text{for any } \bar{x} \in W_{11}^n.$$

Taking account of the last equation, and (33), after simple calculations we obtain

$$\int_0^1 \Big(\int_0^t (\lambda_0 f_x^0 - f_x^* \varphi - g_x^* \tilde{f}_\omega + \sum_{i=1}^{l} h_{ix}^* \tilde{f}_{\gamma_i}\, d\tau - \dot{\varphi}\Big)\, \dot{\bar{x}}(t)\, dt +$$

$$+ \Big(a - \int_0^1 (\lambda_0 f_x^0 - f_x^* \varphi - g_x^* \tilde{f}_\omega + \sum_{i=1}^{l} h_{ix}^* \tilde{f}_{\gamma_i})\, dt,\ \bar{x}(1)\Big) = 0$$

for any $\bar{x} \in \bar{W}^n_{11}$, where $\varphi = -\psi \in L^n_1$, $\tilde{f}_\omega \in L^k_1$, $\tilde{f}_{\gamma_i} \in L_1$ for $i = 1, 2, \ldots, l$. From the last equation, in view of the Dubois-Reymond lemma (cf. [8]), we obtain equation 2° of the proposition.

Let us consider equation (31) for $(\bar{x}, \bar{u}) = (0, \bar{u}) \in X$. We obtain

$$-\lambda_0 \int_0^1 f_u^0 \bar{u}\, dt - \int_0^1 f_u \bar{u}\psi(t)\, dt + \int_0^1 g_u \bar{u}\, d\omega - \\ - \sum_{i=1}^{l} \int_0^1 h_{iu} \bar{u}\, d\gamma_i + f'(\bar{u}) = 0 \tag{34}$$

for any $\bar{u} = L^r_\infty$, where $\lambda_0 \geqq 0$, $\psi \in L^n_\infty$, $\omega \in b^k\!.a$, $\gamma_i \in b.a\,(0, 1)$ are concentrated on the sets R_i for $i = 1, 2, \ldots, l$.

By the use of the definition of the functional supporting a set (cf. [7], §4), the last equation implies condition 3° of the proposition.

Now, let us assume that operator F_3 given by formula (12) is not regular at point $(x^0, u^0) \in X$. Then, in view of theorem 2 from [10], proceeding analogously as in theorem 1 from [11], we obtain conditions 2° and 3° of the proposition.

We shall now show that $|\lambda_0| + \|\psi\| + \|\omega\| + \sum_{i=1}^{l} \|\gamma_i\| + |a| > 0$. This condition follows from equality (31). If $\lambda_0 = 0$, $\psi = 0$, $\omega = 0$, $a = 0$, $\gamma_i = 0$ for $i = 1, 2, \ldots$ $\ldots, l$, then $f_i = 0$ for $i = 0, 2, \ldots, 4+l$ and, by equality (31), $f_1 = 0$, which contradicts theorem 1 from [10].

We have thus proved this theorem under the assumption that $C_0 \neq \emptyset$. In the opposite case, proceeding analogously as in theorem 1 from [11], we obtain the proposition.

REMARK 2. Let us consider problem PI without the condition $u \in U$, as in [1], [2], [6] and others. Let us denote this problem by $P\,\mathrm{II}$. It is easy to show that, for problem $P\,\mathrm{II}$, the local extremum principle is the following:

If assumptions 1°–3° of theorem 1 are satisfied, then there exist some $\lambda_0 \geqq 0$, an absolutely continuous function $\varphi\colon [0, 1] \to R^n$, functions $v \in L^k_1$, $p_i \in L_1$ for $i = 1, 2, \ldots, l$, not vanishing simultaneously and satisfying the equation

$$\dot{\varphi} = -\lambda_0 f_x^0(x^0, u^0, t) - f_x^*(x^0, u^0, t)\,\varphi + \\ + g_x^*(x^0, u^0, t)\cdot v - \sum_{i=1}^{n} h_{ix}^*(x^0, u^0, t)\cdot p_i$$

and such that

$$\lambda_0 f_u^0(x^0, u^0, t) + f_u^*(x^0, u^0, t)\,\varphi - g_u^*(x^0, u^0, t)\cdot v + \\ + \sum_{i=1}^{l} h_{iu}^*(x^0, u^0, t)\cdot p_i = 0,$$

$$p_i\, h_i(x^0, u^0, t) = 0 \quad \text{for } t \in [0, 1] \text{ a.e.}$$

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Warunek konieczny dla zadania sterowania optymalnego z ograniczeniami równościowymi i nierównościowymi

Pewna specyfikacja metody Dubowickiego-Milutina stosowana jest do badania zadań sterowania optymalnego z ograniczeniami typu nierówności i nierówności na współrzędne stanu i sterowanie. Otrzymany jest warunek konieczny w postaci lokalnej zasady ekstremum.

Необходимое условие для задачи оптимального управления с ограничениями типа равенств и неравенств

В работе рассмотрена задача оптимального управления с ограничениями наложенными на управления и состояния. Эти ограничения типа равенств и неравенств. Для решения задачи применено частный случай метода Дубовицкого-Милутина. Получено необходимое условие для оптимума. Условие это сформулировано в форме локального принципа экстремума.