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Some aspects of bargaining in a firm

by

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A model of labor union - management bargaining about wages and production level is considered in order to illustrate the disadvantages of traditional solutions of the bargaining problem. A modified method for solving the problem is proposed. Its main feature is that the solution obtained is influenced by the whole shape of the set of feasible outcomes. This property enables to obtain reasonable solutions which are sensitive to the bargainers' characteristics in a way in which the other solutions are not.

1. Introduction

Bargaining plays an important role in many situations in economics, in which the partners must reach a consensus. An example of such a situation is labor union--management bargaining in a firm. Typically, these two groups of people, i.e. workers which are assumed to be represented by a labor union, and managers, have different interests which neither coincide nor are exactly opposite. In such a situation cooperation between them can be advantageous. By cooperation we mean here the correlation of decisions which, in turn, is bound up with an agreement determining these decisions. Such an agreement is usually reached as a result of a bargaining game, and in this paper same characteristic features of this particular game are discussed in order to illustrate the disadvantages of traditional solutions of the bargaining problem.

Then, a modified method for solving the bargaining problem is proposed. Its main feature is that the solution obtained is influenced by the whole shape of the set of feasible outcomes. This property enables to obtain solutions which are sensitive to the bargainers' characteristics in a way in which the other solutions are not. Features of the proposed solution are discussed and compared with the Nash's, Kalai-Smorodinsky's and Yu's solution concepts.

2. Labor union-management bargaining problem

We will consider a situation in an enterprise in which two decision makers, namely a labor union and management, are distinguished. The union, which represents workers, wants to maximize their income, while the management wants high net profit and a growing production. The goals are incorporated in the parties' objective functions which depend on the two decision variables we take into account in considerations. Namely, it is assumed that management determines the wage fund w, while employees work with a certain productivity and in this way they determine the production level q. The interests of the two parties embodied by their objectives I (union) and U (management) are not consistent, thus we have a non-constant sum game which has been formulated in [8]. The non-cooperative version of the game has been extensively discussed in [8] and [7]. All important tax regulations valid in Poland are incorporated in the model and they influence the behaviour of the parties very much.

The non-cooperative equilibrium obtained in a way described in [8], [7] appeared to be not Pareto optimal which means that cooperation between parties can be advantageous to them. We will treat this equilibrium as a status quo point $m=(u^*, i^*)$, which is the point of departure in negotiations and the disagreement result, i.e. the result of the game in the case when the parties do not reach an agreement. The pair of decisions resulting in $m=(u^*, i^*)$ is denoted by (u^*, q^*), i.e. $u^*=U(w^*, q^*)$ and $i^*=I(w^*, q^*)$.

The set of all feasible solutions of the game will be denoted by S,

$$S=\{(u, i): u=U(w, q), i=I(w, q) \quad \text{for all} \quad (w, q) \in A_\Omega\}, \tag{1}$$

where A_Ω is a set of admissible decisions [8]. Typically, to define a bargaining game it is sufficient to have the set S of feasible solutions and a status quo $m \in S$. Then the definition of the set $S^+ \subset S$ of solutions dominating m is straightforward. In our case however the set S^+ will be defined in a modified way which takes into account the union's incentive to negotiate.

The set of decisions which management prefers to (w^*, q^*) is

$$H_M=\{(w, q) \in A_\Omega: U(w, q)>u^*\}, \tag{2}$$

where the set A_Ω of admissible decisions is determined by management. The set of decisions which labor union prefers to (w^*, q^*) takes into account additional conditions:

$$H_U=\left\{(w, q): \frac{I(w, q)-i^*}{f(q)-f(q^*)} \geqslant \beta \quad \text{for} \quad f(q)>f(q^*) \quad \text{or} \quad I(w, q) \geqslant i^* \quad \text{for} \quad f(q) \leqslant f(q^*)\right\}, \tag{3}$$

where f is the effort function which associates production level q with the necessary employees' effort $f(q)$, and $\beta \geqslant 0$ is a threshold value. The first condition in (3) concerns the average income per effort unit. It means that the relative average income increase (in comparison with the status quo) ought to be above a specified threshold value β. Note, that for $\beta=0$ we obtain a standard domination rule, i.e. $I(w, q)\geqslant i^*$. It seems however, that the formula used in (3) better mirrors reality if $\beta>0$, which means that the unions are interested not only in the income increase (regardless of what the cost, i.e. employees' effort, is). They are interested also in keeping an average (per effort unit) income above a certain threshold. On the other hand, in the case of the effort decrease the union wants to maintain (at least) the income level. This is specified by the second condition in (3).

Combining (2) and (3) we obtain the set of decisions which are preferred to (w^*, q^*) both by the management and the union:

$$H=H_M \bigcup H_U, \tag{4}$$

which enables us to determine the set $S^{+m}\subset S$ of solutions dominating the status quo (u^*, i^*):

$$S^{+m}=\{(u, i): u=U(w, q), \quad i=I(w, q) \quad \text{for all} \quad (w, q)\in H\} \tag{5}$$

(the superscript m indicates that the definition of domination is modified).

Thus we have a bargaining game, with management and union as the parties, defined by the set of feasible solutions S, the status quo $m=(u^*, i^*)\in S$, and, defined in a modified way, the set $S^{+m}\subset S$ of solutions dominating m. The bargaining problem considered in the sequel consists in finding a point $(u, i)\in S$ which can determine a reasonable agreement between the parties.

3. Aspects of the bargaining game

The role of the threshold value β which occurs in (3) is twofold. First, it reflects the real union's preference concerning the relative average income increase, and second, it can be treated by the union as a decision variable and used during negotiations with management. The latter role is connected with the fact that β influences the shape of the set S^{+m} of feasible outcomes dominating the status quo and, in this way, it also influences the result of the bargaining game. In such a situation the question of a reasonable choice of the value of β arises. We will focus first on the dependence of the shape of S^{+m} on β.

Let us denote the boundary of the set H_U defined in the previous section by $\tilde{H}_U$:

$$\tilde{H}_U=\left\{(w, q): \frac{I(w, q)-i^*}{f(q)-f(q^*)}=\beta \quad \text{for} \quad f(q)>f(q^*) \quad \text{or} \right.$$
$$\left. I(w, q)=i^* \quad \text{for} \quad f(q)\leqslant f(q^*)\right\}, \tag{6}$$

and the associated set of outcomes by W:

$$W=\{(u, i): \quad u=U(w, q), \quad i=I(w, q) \quad \text{for all} \quad (w, q)\in \tilde{H}_U\}. \tag{7}$$

The set W plays an interesting role in the construction of the set S^{+m} of dominating solutions, which will be clarified a little bit later. The set W takes the form of a curve which is changing when the threshold β changes. The family of such curves W, for various values of $\beta \geqslant 0$, is shown in Fig. 1.

The sets S^{+m} for three different values of β are depicted in Figure 2. Looking at the Figures 2a, 2b, 2c it is not hard to grasp the association between W and S^{+m}. Let us introduce a subscript writing S_β^{+m}, in order to emphasize the dependence of S^{+m} on β. Note, that if $\beta \geqslant \gamma \geqslant 0$ then $S_\beta^{+m} \subset S_\gamma^{+m}$. For $\beta=0$ the formula (3) is equivalent to simple domination and S^{+m} is the greatest (Fig. 2a). But as β increases the set S^{+m} becomes smaller. In Figure 2b, which illustrates a typical example, the set S^{+m} has been divided into two subsets S_1^{+m} and S_2^{+m} such that (the frontier belongs to S_1^{+m}):

$$S^{+m}=S_1^{+m} \bigcup S_2^{+m} \quad \text{and} \quad S_1^{+m} \bigcap S_2^{+m}=\emptyset .$$

It is interesting that for each outcome $(u, i) \in S_1^{+m}$ there exist two different pairs of decisions (w, q) resulting in the same (u, i), while for an outcome from S_2^{+m} there exists only one such pair of decisions.

Note, that for $\beta=0$ we have $S_2^{+m}=\emptyset$ and $S^{+m}=S_1^{+m}$. On the other hand for $\beta>\beta_1$, where β_1 is a characteristic value (see fig. 1), we have $S_1^{+m}=\emptyset$ and $S^{+m}=S_2^{+m}$. An important implication of that is, that for $\beta>\beta_1$ it is not possible to reach an agreement which is Pareto optimal in the set S of all feasible solutions (see Fig. 2c), i.e.

$$S_\beta^{+m} \bigcap P(S)=\emptyset \quad \text{for} \quad \beta>\beta_1, \tag{8}$$

where $P(S)$ is the set of Pareto optimal points in S.

In Figure 1 we have distinguished three characteristic values of β. The first is the above mentioned β_1 such that

$$W \bigcap P(S)=(u^*, i) \quad \text{for} \quad \beta=\beta_1, \tag{9}$$

where u^* is relevant to the status quo $m=(u^*, i^*)$. It is worth emphasizing here that the intersection $W \cap P(S)$ is a single point or, for $\beta>\beta_2$, this set is empty:

$$W \bigcap P(S)=\emptyset \quad \text{for} \quad \beta>\beta_2 . \tag{10}$$

The third characteristic value of β is β_3 such that

$$u^*=\max_{(u, i) \in W} u \quad \text{for} \quad \beta \geqslant \beta_3 . \tag{11}$$

The implication of (11) is that for $\beta \geqslant \beta_3$ the set S^{+m} contains only the status quo point m,

$$S^{+m}=\{m\} \quad \text{for} \quad \beta \geqslant \beta_3 . \tag{12}$$

In other words, if $\beta \geqslant \beta_3$ there is no use to negotiate since the set of solutions dominating m is empty. This means that the bargaining problems exists only for $0 \leqslant \beta<\beta_3$.

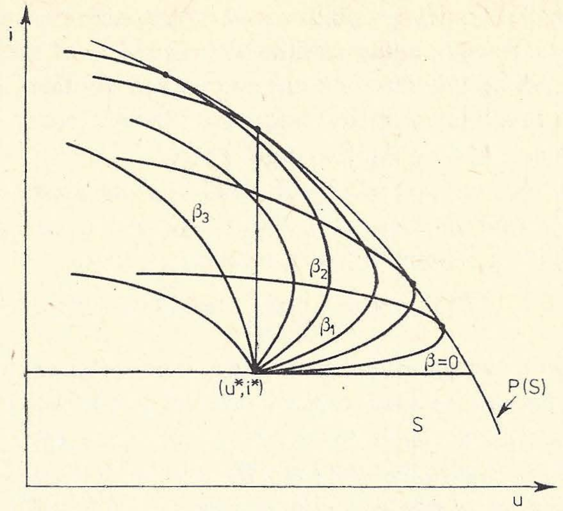

Fig. 1. The family of the curves W (for different values of β). $P(S)$ denotes the Pareto-border of S

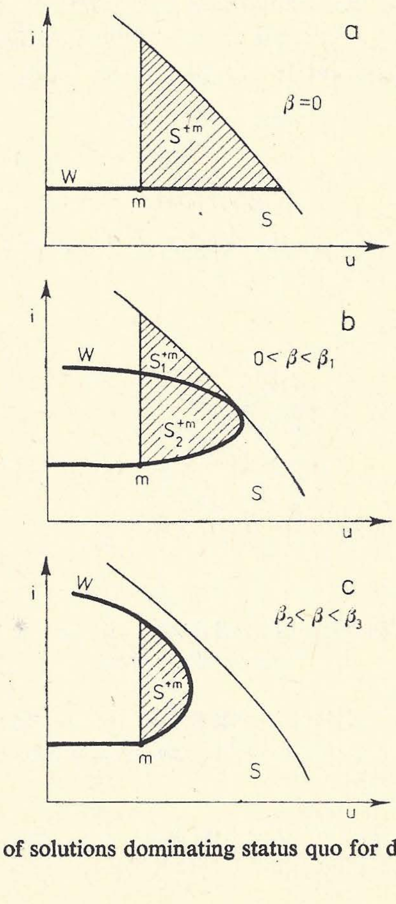

Fig. 2. Sets S^{+m} of solutions dominating status quo for different values of β

Note, that in the bargaining game for $0<\beta<\beta_3$ there is no possibility of increasing the management's utility u without the increase of the union's objective i (see Fig. 2b,c). Taking this fact into account we can say that, except for the case when $\beta=0$, the situation is disadvantageous to management, whose position in negotiations is weakened by the threshold $\beta>0$.

We will focus now on the methods of solution of the above outlined bargaining problem. The classical solution of a bargaining game, i.e. the Nash solution $N(S^{+m}, m)=(u^N, i^N)$ is determined in the following way:

$$(u^N, i^N)=\arg \max_{(u,\, i)\in S^{+m}} (u-u^*)(i-i^*), \tag{13}$$

where $(u^*, i^*)=m$ is the status quo point. One of the most controversial properties of this solution (as well as some others — see [6], [5]) is that it is independent of irrelevant alternatives. In other words, if we take into account two bargaining games (S, m) and (T, m) such that $S\subset T$ and the solution $N(T, m)\in S$, then $N(S, m)= =N(T, m)$. This property has been criticised for a long time [3], [6]. This was connected with the feeling that the changes of the set of feasible solutions should not be ignored.

In 1975 Kalai and Smorodinsky [2] proposed a modified solution concept which partly takes into account the influence of the shape of S^{+m}. This is the solution designated by K in Figure 3. It is the point at which the line between the status quo (u^*, i^*) and the so called ideal (or utopia) point $(\bar{u}, \bar{i})$, where

$$\begin{aligned} \bar{u}&= \max_{(u,\, i)\in S^{+m}} u, \\ \bar{i}&= \max_{(u,\, i)\in S^{+m}} i, \end{aligned} \tag{14}$$

intesects $P(S^{+m})$, i.e. the Pareto frontier of S^{+m} (see Fig. 3).

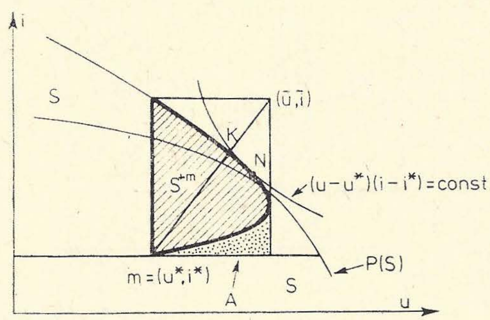

Fig. 3. Illustration of Kalai-Smorodinski's and Nash's solutions, and their independence on irrelevant alternatives

There exist also other solutions depending on the ideal point [9], [1]. Those solutions concepts select a point in S^{+m}, which is "closest" to the ideal point $\bar{x}$ in terms of some metric D:

$$Y(S^{+m}, m)=\arg \min_{x\in S^{+m}} D(x, \bar{x}). \tag{15}$$

However, the above solutions, i.e. K and $\underline{Y}$, depend only on the status quo and the ideal point. In our particular example concerning the union - management negotiations they do not take into account the fact that, for some values of β, there are no solutions in S^{+m} which are much more advantageous to management then to the union. See Fig. 3 — for the above mentioned solution concepts it does not matter if the area designated by A belongs to S^{+m} or not (in our game it does not, which weakens the management's bargaining power, and it seems that this fact should be reflected by the final compromise solution).

In the next section we propose a new solution of a bargaining game, which is influenced by the shape of the set S^{+m} in a much more extensive way.

4. The concept of a new solution

We will consider a two-player bargaining game (S, m), where S is the set of feasible outcomes, and $m \in S$ is the status quo point (which determines the solution of the game in the case in which players will not reach an agreement). We will assume that S is a compact and connected subset of R^2, and that there exists $x \in S$ such that $x > m$. Let $P(S)$ denote the set of strongly Pareto optimal points in S, i.e.

$$P(S)=\{x \in S\colon (y \geqslant x \quad \text{and} \quad y \neq x) \Rightarrow y \notin S\}. \tag{16}$$

We will assume that $P(S)$ is connected.

In this section we define the new solution J which we propose, and we shortly discuss some of its properties.

First, we define a mapping φ such that $\varphi(S, x) \in R^2$ and $x \in E(S) \subset R^2$, where

$$E(S)=\{x\colon x \geqslant \underline{x}(S) \quad \text{and} \quad x \leqslant s,\ s \in S\}, \tag{17}$$

where, in turn, $\underline{x}(S)=(\underline{x}_1, \underline{x}_2)$ and

$$\underline{x}_i=\min_{x \in S} x_i. \tag{18}$$

Note that $S \subset E(S)$ (see Figure 4). Let $y=(y_1, y_2)$ be such a point that

$$\begin{aligned} y_1&=\max\{x_1, \max_{(v, x_2) \in S} v\} \\ y_2&=\max\{x_2, \max_{(x_1, v) \in S} v\} \end{aligned} \tag{18}$$

The point $\varphi(S, x)=(\varphi_1, \varphi_2)$ is defined in the following way

$$\varphi_i=\begin{cases} y_i & \text{if} \quad y \neq x \\ \max\limits_{z \in C(S, \delta, x)} z_i & \text{if} \quad y=x, \end{cases} \tag{19}$$

where

$$C(S, \delta, x)=\{z \in S\colon z \geqslant x \quad \text{and} \quad d(x, z) \leqslant \delta\}, \tag{20}$$

$\delta \geqslant 0$, and d is a distance in R^2, $d(x, y)=\sqrt{(y_1-x_1)^2+(y_2-x_2)^2}$. The idea of φ is illustrated in figure 5. Taking into account x as a point of departure, y_i is the maximal utility, player i-th can attain without the change of the partner's utility level. In situations in which $x \in E(S) \setminus S$ it may happen that e.g. $x_1 < \max_{(v, x_2) \in S} v$ — in such a case $y_1 = x_1$. In a very particular case when $y=x$, the point (φ_1, φ_2) is obtained in a slightly modified way which is not hard to grasp when looking at (19) and (20).

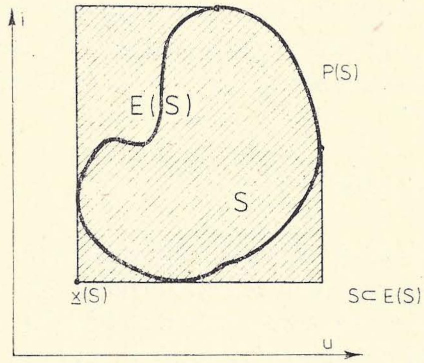

Fig. 4. Illustration of the set $E(S)$

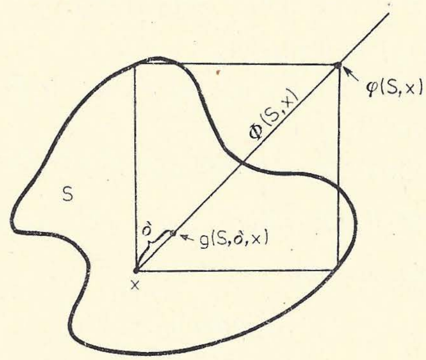

Fig. 5. Illustration of the definitions of the mappings φ and g, and the set $\Phi(S, x)$

Now, we define a set $\Phi(S, x)$ which consists of the points on the half-line which starts from $x \in E(s)$ and goes through $\varphi(S, x)$ (see Fig. 5):

$$\Phi(S, x)=\left\{z: z_i=x_i+t\frac{y_i-x_i}{d(x, y)}, \quad y=\varphi(S, x), \quad t \geqslant 0\right\}. \tag{21}$$

Next, we introduce a mapping g, where $g(S, \delta, x) \in R^2$, $\delta \geqslant 0$ (the same as in (19), (20)), which associates with each point $x \in E(S)$ such a point from $\Phi(S, x)$ that the distance between it and x is δ (see Fig. 5):

$$g(S, \delta, x)=\{z \in \Phi(S, x): d(x, z)=\delta\}. \tag{22}$$

Because $\Phi(S, x)$ has the form of a half-line beginning at x, $g(S, \delta, x)$ is always a single point.

The bargaining solution $x^J = J(S, m)$ is reached in an iterative process which consists of the following sequence of steps:

1. Set $k=0$, $x^k = m$.
2. Compute $\varphi(S, x^k)$. If $\varphi(S, x^k) = x^k$, then $x^J = x^k$ and Stop.
3. Compute $g(X, \delta, x^k)$. If $g(S, \delta, x^k) \in E(S)$ then $x^{k+1} = g(S, \delta, x^k)$, otherwise $x^{k+1} = \arg\max_{z \in G} d(x^k, z)$, where $G = E(S) \cap \Phi(S, x^k)$.
4. $k = k+1$, go to step 2.

The algorithm stops at the step 2 if $x^k \in P(S)$, because from the definition of the mapping φ and the assumption about connectivity of S follows that

$$\varphi(S, x) = x \Rightarrow x \in P(S). \tag{23}$$

The situation relevant to the second case at step 3, i.e. when $g(S, \delta, x^k) \notin E(S)$, is shown in figure 6. The whole process of reaching a solution is illustrated in figure 7.

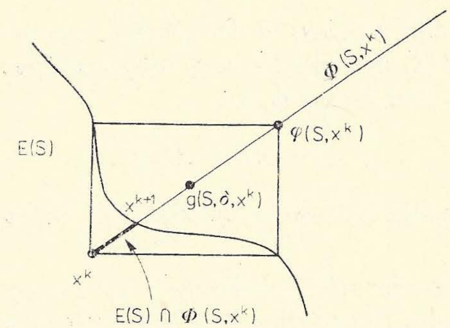

Fig. 6. Illustration of the step 3 of the algorithm, when $g(S, \delta, x^k) \notin S$

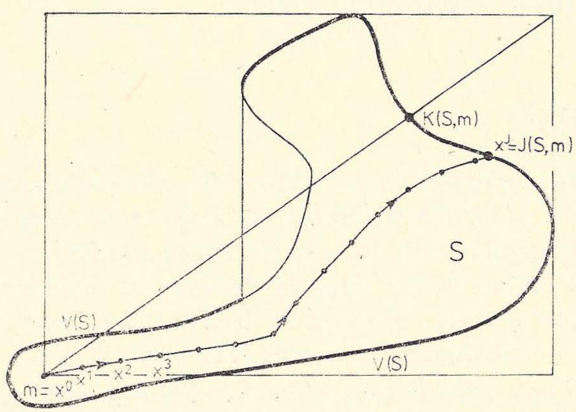

Fig. 7. Illustration of the process of reaching $J(S, m)$ (and comparison with the Kalai-Smorodinsky's solution $K(S, m)$)

It is easy to see that the solution $J(S, m) \in S$ exists and is unique. $J(S, m)$ has the following properties:

1. Strong Pareto optimality: $J(S, m) \in P(S)$.
2. Strict individual rationality: $J(S, m) > m$.
3. Symmetry: if the game is symmetric (i.e. $(x_1, x_2) \in S \Leftrightarrow (x_2, x_1) \in S$ and $m_1 = m_2$), then $J(S, m) = (v, v)$.

The main characteristic feature of the proposed solution J is that it is essentially sensitive to changes if the set S of feasible outcomes (while the solutions mentioned in the previous section depend only on the status quo point or the ideal point). The natural question concerns the points in S which influence $J(S, m)$. When determing the set of such points the following definition is useful

$$V(S) = \{x \in S: (v, x_2) \geqslant x \Rightarrow (v, x_2) \notin S \quad \text{or} \quad (x_1, v) \geqslant x \Rightarrow (x_1, v) \notin S\}. \tag{24}$$

We will call $V(S)$ the set of active points in S or the active frontier of S. The idea of $V(S)$ is illustrated in Figure 7. Note, that the Pareto frontier of S is a subset of $V(S)$,

$$P(S) \subset V(S). \tag{25}$$

An important role this set plays in our solution concept is bound up with the fact that the solution $J(S, m)$ is influenced by the points from $V(S^+)$, where $S^+ \subset S$ is the set of solutions dominating m, i.e.

$$S^+ = \{x \in S: x \geqslant m\}. \tag{26}$$

It is worth emphasising that for $\delta \to 0$ the solution $J(S, m)$ depends on all the points in $V(S^+)$.

Taking into account the union-management bargaining problem described in the two previous sections the solution $J(S^{+m}, m)$, where S^{+m} is given by (5), is influenced by the whole frontier $V(S^{+m})$. In other words, the specific shape of S^{+m}, bound up with the characteristic features of the game in a firm, is reflected in the final bargaining solution $J(S^{+m}, m)$.

5. Concluding remarks

We have presented a model which describes labor union - management bargaining in a firm. The goal of the bargaining process is to reach an agreement which is advantageous, in comparison to the noncooperative equilibrium, to both parties. The characteristic feature of this bargaining problem is the non-symmetric shape of the set of feasible outcomes of the game. It seems that the traditional solutions do not reflect this non-symmetry in a sufficient way.

In the paper a new solution concept for bargaining games has been proposed, which is much more sensitive to the changes of the shape of the set of feasible outcomes. The final compromise solution is obtained in an iterative process which yields a path from a status quo point to this solution $J(S, m)$. The mentioned path (and, in consequence, the solution) is influenced by the so called active frontier $V(S)$ which mirrors the substantial properties of the shape of S.

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Pewne aspekty zagadnienia targu w przedsiębiorstwie

W pracy sformułowano problem targu pomiędzy dyrekcją i załogą w przedsiębiorstwie dotyczący ustalenia funduszu płac oraz poziomu produkcji. Problem ten ilustruje pewne wady znanych rozwiązań zagadnienia targu. W pracy zaproponowano nową metodę rozwiązania zadania targu. Główną cechą tej metody jest zależność otrzymanego rozwiązania od kształtu całego zbioru rozwiązań dopuszczalnych. Właściwość ta umożliwia otrzymywanie racjonalnych rozwiązań kooperacyjnych, które są w istotny sposób wrażliwe na cechy charakterystyczne opisujące uczestników negocjacji.

Некоторые аспекты проблемы спора на предприятии

В работе формулируется проблема спора между правлением и коллективом на предприятии, касающегося определения фонда зарплаты и уровня производства. Эта проблема отображает некоторые недостатки известных решений задачи спора. В работе представлен новый метод решения задачи спора. Основной чертой этого метода является зависимость получаемого решения от вида всего множества допускаемых решений. Это свойство позволяет получать рациональные кооперативные решения, которые существенным образом чувствительных к характерным признакам, описывающим участников переговоров.