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Approximate controllability and input--output analysis*

by

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The interaction between interindustrial flows and final demand is studied in order to investigate the technological structure of an economic system. A "structural" method is designed, which allows basing on the analysis of the input-output structure of the system, the evaluation of its structural change. It is evidenced that systems analysis and control theory can be of great help in this analysis.

Introduction

In recent times a growing interest has been devoted by researchers, with different success, to the supply side of the economy. This new attitude that tends to privilege not only the process of formation of the demand components but to describe their impact on the technological structure reveals that it is no more sufficient to warrant predetermined demand levels. It is necessary to evaluate, how such demand is met by the existing technological structure and what changes a demand shock tends to generate on such a structure. Such problems that refer not only to economics but have relevant implications on the social field appear as not temporary and are linked to the availability of the primary resources, labour supply and shifts in the composition of internal and external demands. Transferring resources from one economic activity to another that appears to warrant a better performance may imply relevant economic and social costs so that such operation has to be justified in terms of the amelioration of the whole economic stucture.

In this context the economic problem can be viewed as a constant search of a structure that can fit the present technological and behaviour situation starting from a giving preexisting structure. But how to detect the point where a behaviour or a technology has become such that the structure has changed? Of course it is not

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sufficient to say that the economic structure has significantly changed when one parameter or a subset of the parametric structure has changed. Neither can we detect the peculiarities of a given structure investigating the paths described by the variables resulting from a simulation procedure. What we need is a "structural" method, that is the one that allows to go inside the economic variables, to the inner composition of the households consumption, foreign trade, capital stock and employment and evidences how the "reactivity" of the system to different input structures varies through time when the parametric structure of the model is kept constant. Any structural change can be then valued with reference to that situation.

The method is not intended to substitute for the simulation procedure and optimization experiments but it can provide a useful insight into designing the simulation scenarios and specifying the weights in the objective function.

1. The Input - Output Features

Input - Output approach provides two valuable features. The first one is connected with the consideration of the "supply side" of the economy. Even if the theory is not developed to specify explicitly a supply function that can limit the requirements coming from the demand side we have to take into account the fact that a great part of the economic activity takes place outside the final sectors of GDP, [8]. Intermediate consumption requirements directly follow from the production function [2] as well as labour, capital and energy demands according to more recent developments [5] that generalize such aggregate results to a multisectorial framework allowing for interesting applications [6]. Secondly the I–0 approach allows for the analysis of the "structure" i.e. the composition of the demand categories as household consumption, investement, inventories, import, export and public expenditure. Let us confine ourselves to the consideration of only two components of final demand: household expenditures and investement. The traditional dynamic model can be written as

$$x_t = Mx_t + Q\,(x_{t+1} - x_t) + f_t \tag{1}$$

where x_t is the vector of m outputs. M is an $(m \times m)$ intermediate coefficient matrix. Q is an $(m \times m)$ capital requirements matrix and f_t — the vector of consumption of order m.

System (1) has generated problems connected with the singularity of Q matrix that have been solved succesfully in terms of systems' theory. But the more substancial criticism is the one concerning the very meaning of q_{ij} coefficient and the possibility of using the model (1) for forecasting purposes.

If we want to model the behaviour of economic agents as investors and consumers we have to specify a set of consumption and a set of investment functions in a more suitable dominion and transform these demands to I–0 sectors. We can write then

$$x_t = Mx_t + Ri_t + Zc_t \tag{2}$$

where c_t is a vector of l elements representing the l items of the households budgets for which we can correctly specify a set of consumption functions. Z is the $(m \times l)$ consumption bridge matrix that transforms the l demands to the I–0 sectors. i_t is a vector of h elements representing a set of h investment functions according to the investing sectors while R is the investment bridge matrix that transforms the investment demands from the k investing sectors to n demands $(n<m)$ directed to the n sectors producing investement goods. Intermediate consumptions are defined at the I–0 level and explained through fixed coefficients.

The concept of bridge matrices allows for the introduction in the I–0 framework of behaviour equations for final demand that can be econometrically estimated so that we realize within the same model the consistence of direct information. It usually regards the bridge matrices and intermediate coefficient matrix even if attempts have been made to estimate econometrically the technical coefficients [12]. The specification and estimation of the investement functions is crucial in the determination of the dynamics of the economic systems. Sectoral investment demand is given by the sum of expansion investment v and substitution investment s.

$$i=v+s$$

The substitution investment is given by the replacement rate r multiplied by the capital stock which is determined as the product of sectoral capital-output ratio k and smoothed output $\bar{q}$

$$s=rk\bar{q} \tag{3}$$

Expansion investment is equal to the capital-output ratio multiplied by a distributed lag on changes in output

$$v=k\sum_{i=0}^{n} w_i\, q_{t-i} \tag{4}$$

Thus, the sectoral investement for a second order distributed lag on changes in output is given by

$$i_t=rk\bar{q}+k\left(w_0\,(q_t-q_{t-1})+w_1\,(q_{t-1}-q_{t-2})+w_2\,(q_{t-2}-q_{t-3})\right), \tag{5}$$

weights w_i are constrained to sum to unity

$$w_0+w_1+w_2=1 \tag{6}$$

and smoothed output is given by

$$\bar{q}_t=a_1\,q_t+a_2\,q_{t-1}+a_3\,q_{t-2} \tag{7}$$

where

$$a_1+a_2+a_3=1 \tag{8}$$

Some deeper insight in the estimation of such functions is available in [1] and [2].

In matrix terms we can write

$$\bar{i}_t=D\,q^\circ{}_t+D\,q'{}_{t-1}+D\,q''{}_{t-2}+W^\circ\,(q_t-q_{t-1})+W'\,(q_{t-1}-q_{t-2})+ \\ +W''\,(q_{t-2}-q_{t-3}) \tag{9}$$

where i_t is the investment vector for the h investing sectors, D°, D' and D'' are $(h \times h)$ diagonal matrices whose elements are given by $a_1\, rk_i$, $a_2\, rk_i$ and $a_3\, rk_i$ with $i=1 \dots h$ and matrices $W^\circ\ W'$ and W'' are diagonal $(h \times h)$ matrices whose elements are given by $w_0\, k_i$, $w_1\, k_i$, $w_2\, k_i$ with $i=1 \dots h$. Output vector q_t is a vector of h elements consistent with the "sectorization" of investment and is tied to the I–0 output vector by the relation

$$q_t = Vx_t \tag{10}$$

where V is an $(h \times m)$ bridge matrix that transforms the output vector at I–0 level to the output vector by investing sectors. We now can rewrite equation (2) as

$$\begin{aligned} x_t = Mx_t + RD^\circ\, Vx_t + RD'\, Vx_{t-1} + RD''\, Vx_{t-2} + RW^\circ\, V(x_t - x_{t-1}) + \\ + RW'V(x_{t-1} - x_{t-2}) + RW''\, V(x_{t-2} - x_{t-3}) + Zc_t \end{aligned} \tag{11}$$

2. A state space realization

Equation (11) can be rewritten as

$$\begin{aligned} \big(I - (M + R(D^\circ + W^\circ)\, V)\big)\, x_t = R(D' - W^\circ + W')\, Vx_{t-1} + \\ R(D'' + W'' - W')\, Vx_{t-2} + R(-W'')\, Vx_{t-3} + Zc_t \end{aligned} \tag{12}$$

Given the R matrix with $m-n$ rows whose elements are zero, corresponding to the sectors that don't produce investment goods, we can rearrange the sectors so that first are the ones that produce investments goods.
If we pose

$$\begin{aligned} R(D' - W^\circ + W')\, V &= H' \\ R(D'' + W'' - W')\, V &= H'' \\ -W'' &= H''' \end{aligned} \tag{13}$$

and

$$\big(I - (M + R(D^\circ + W^\circ)\, V)\big) = L$$

we can define the following vectors

$$\begin{aligned} h'_t &= H'\, x_t \\ h''_t &= H''\, x_t \\ h'''_t &= H'''\, x_t \end{aligned} \tag{14}$$

and rewrite eq. (12) as

$$\begin{aligned} h'_t &= H'\, L^{-1}\, h'_{t-1} + H'\, L^{-1}\, h''_{t-2} + H'\, L^{-1}\, h'''_{t-3} + H'\, L^{-1}\, Zc_t \\ h''_t &= H''\, L^{-1}\, h'_{t-1} + H''\, L^{-1}\, h''_{t-2} + H''\, L^{-1}\, h''_{t-3} + H''\, L^{-1}\, Zc_t \\ h'''_t &= H'''\, L^{-1}\, h'_{t-1} + H'''\, L^{-1}\, h''_{t-2} + H'''\, L^{-1}\, h'''_{t-3} + H'''\, L^{-1}\, Zc_t \end{aligned} \tag{15}$$

a state space realization for eq. (12) can be given by

$$\begin{bmatrix} h'_t \\ h''_t \\ h'''_t \\ h''_{t-1} \\ h'''_{t-1} \\ h'''_{t-2} \end{bmatrix}_t = \begin{bmatrix} H'L^{-1} & 0 & 0 & H'L^{-1} & 0 & H'L^{-1} \\ H''L^{-1} & 0 & 0 & H''L^{-1} & 0 & H''L^{-1} \\ H'''L^{-1} & 0 & 0 & H'''L^{-1} & 0 & H'''L^{-1} \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} h'_t \\ h''_t \\ h'''_t \\ h''_{t-1} \\ h'''_{t-1} \\ h'''_{t-2} \end{bmatrix}_{t-1} + \begin{bmatrix} H'L^{-1}Z \\ H''L^{-1}Z \\ H'''L^{-1}Z \\ 0 \\ 0 \\ 0 \end{bmatrix} \hat{c}_t \tag{16}$$

the dimension of the state space is $6\times m$ but can be reduced to $p=6\times n$ considering that $(m-n)$ elements in each h subvector are equal to zero.

We can define the following two operators

$$J' = \begin{bmatrix} \boxed{I'_n \; 0} & & & \\ & \boxed{I''_n \; 0} & & \\ & & \ddots & \\ & & & \boxed{I_n \; 0} \end{bmatrix} \qquad J'' = \begin{bmatrix} \boxed{\begin{matrix} I_n \\ 0 \end{matrix}} & & \\ & \boxed{\begin{matrix} I_n \\ 0 \end{matrix}} & \\ & & \boxed{\begin{matrix} I_n \\ 0 \end{matrix}} \end{bmatrix} \tag{17}$$

where J' is a diagonal block matrix with diagonal blocks formed by a unit matrix I_n of rank n and a zero matrix $(n\times (m-n))$ and J'' is a diagonal block matrix with diagonal block formed by a unit matrix I_n of rank k and a zero matrix of order $((m-n)\times n)$ so that J' can transform the left hand side vector in eq. (16) excluding the $(m-n)$ zero elements in each h_t subvector and J'' can transform this new vector into the previous one. We can write (16) as

$$J'h_t \text{—} J'AJ''J'h_{t-1}+J'Bc_t \tag{18}$$

where A and B are the right hand side matrices in eq. (16) If we define

$$\begin{aligned} J'h_t &= \hat{h}_t \\ J'AJ'' &= \hat{A} \\ J'B &= \hat{B} \end{aligned} \tag{19}$$

we can write the state equation

$$\hat{h}_t = \hat{A}\hat{h}_{t-1} + \hat{B}c_t \tag{20}$$

where the state vector has now $6\times n$ elements; n is the number of the I–0 sectors producing investment goods that normally is less than half the number of I–0 sectors and number 6 indicates the fact that in the state equation a higher order lag structure has to be expressed in terms of a first order difference equation.

The search of the output transformation E has to start from the definition of a matrix that transforms the total output vector at least into one component of the state vector h'_t. In order to avoid singularities the remaining $(m-n)$ elements in the transformed vector can be constituted by sectoral availabilities for final demand.

$$\begin{bmatrix} \hat{h}. \\ \cdot \\ \cdot \\ \hat{h}'_n \\ \hline f_{n+1} \\ f_m \end{bmatrix} = \begin{bmatrix} \\ \\ E \\ \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix} \tag{21}$$

Let us define the following transformation

$$J''' = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & I_{m-n} \end{array}\right] \tag{22}$$

where I_{m-n} is a unit matrix of rank $(m-n)$. We can define E matrix as

$$E = H_1 + J''' (I - M) \tag{23}$$

which in general is nonsingular. We can partition the inverse of E so that

$$E^{-1} = [E_1 | E_2] \tag{24}$$

where E_1 is an $m \times n$ block and E_2 is an $(m \times (m-n))$ block of the inverse of E.

We can now write the output as

$$x_t = [E_1 | 0] J'' \hat{h}_t + E^{-1} J''' Zc_t \tag{25}$$

where the block 0 in the first term of the right hand side of eq. (24) is a zero block of order $m \times (6 \times (m-n))$.

Redefining (24) we have the classical output transformation equation:

$$x_t = C\hat{h}_t + Dc_t \tag{26}$$

Equations (20), (25) represent a state space realization for a simplified I–0 model expressed in eq. (2) with sectoral investment functions of the type of eq. (9) and consumption demands expressed in terms of the l items from the households budgets. The investment theory chosen has defined the form of the state equation where the values of the state variables depend on values of such variables in the previous periods and on the simultaneous values of inputs.

3. Forced evolution and approximate controllability

To the couple of equations (20), (25) that represent the implicit form of the system correspond two equations that describe explicitely the evolution of the state and output variables:

$$h_t = A^t h_0 + \sum_{\tau=1}^{t} A^{t-\tau} Bc_\tau \tag{27}$$

$$x_t = CA^t h_0 + \sum_{\tau=1}^{t} CA^{t-\tau} Bc_\tau + Dc_t, \qquad t_0 = 0 \tag{28}$$

The first term of the right hand side of the eq. (26) represents the effect of initial conditions on the free evolution of the state variables, the second term the effect of an input sequence $c_1 \ldots, c_t$, while the first term of the right hand side of eq. (27) represents the free evolution of the output variables, and the remaining terms represent the "forced" evolution of the system.

Each one of these components can provide interesting information on the structural properties of the system but for the purposes of the present work let us concentrate on the forced evolution of the output variables.

This shall be done, with no loss of generality, by assuming initial condition equal to zero. Eq. (27) then becomes

$$x_t = \sum_{\tau=1}^{t} CA^{t-\tau} Bc_\tau + Dc_t \tag{29}$$

The possibility of controlling the outputs is then tied to the properties of the matrix sequence

$$[CB + DCABCA^2 B \ldots CA^{t-1} B] = G \tag{30}$$

i.e. the output is controllable if and only if G contains at least m independent columns [10].

Since every matrix satisfies its own characterictic equation, powers of A higher than $p-1$ can be expressed as linear combination of powers of order less than p. Then the inspection of $t-1$ periods is limited to $p-1$ periods. The condition for complete controllability is then given by:

$$\text{rank}\,[CB + DCAB \ldots CA^{p-1} B] = m \tag{31}$$

so that the system's output is controllable in $p-1$ periods or not at all.

The concept of complete controllability applied to economic models has been criticized under many aspects. The possibility of forcing the output vector to assume predetermineted values with a sequence of limited inputs in a limited number of time periods doesn't give a valuable information when applied to short term models which by their own design appear usually controllable.

When applied to I–0 models the dimension of the state space is usually much higher than the number of time horizon periods. Thus, the condition (30) could in general be fulfilled outside the relevant economic time horizon. For this reason we need to refer to the matrix sequence (29).

If we write the input vector sequence c_t $t = t_0, \ldots, T$ in the form

$$[c'_T\, c'_{T-1}, \ldots, c'_0]' = c \tag{32}$$

where the sign $'$ denotes transposition, we can express the value of the output vector at the final time period T in terms of the matrix sequence (29)

$$x_T = Gc \tag{33}$$

where matrix G is of order $(m\times(T\times 1))$. We can apply the singular value decomposition so that

$$x = USV'\,c \tag{34}$$

where U is a real $m\times m$ orthogonal matrix, V is a real $(T\times 1)\times(T\times 1)$ orthogonal matrix and S is a diagonal matrix whose elements can be arranged as

$$\bar{s}_1 > \bar{s}_2 > \bar{s}_3 > \ldots s_r > 0$$

where r is the rank of G.

Applying the transformation U on output and the V transformation on input

$$\bar{x} = U'\,x$$

$$\bar{c} = V'\,c$$

eq. (33) can be written as

$$\bar{x}_i = s_i\,\bar{c}_i \qquad i = 1 \ldots, r \tag{35}$$

In the new coordinates the linear transformation G multiplies the units of the ith coordinate by a factor s_i. In particular if $\bar{c}_i$ describes a sphere, $\bar{x}_i$ shall describe an ellipsoid with semiaxes of length $s_1, s_2, \ldots, s_r$. We can then look at $\bar{x}_t = U'\,x_{tm}$ as at a set of orthogonal planes that define the inner composition i.e. the inner structure of total output and similarly $\bar{c} = V'\,c$ as a set of consumption structures that can have different impact on output structure according to the s_i.

We can then explore the changes in the properties of an I–0 model as the planning period changes by successively applying the singular value decomposition to matrix G. In order to clarify these points let us refer to a two-dimensional example taken from [4].

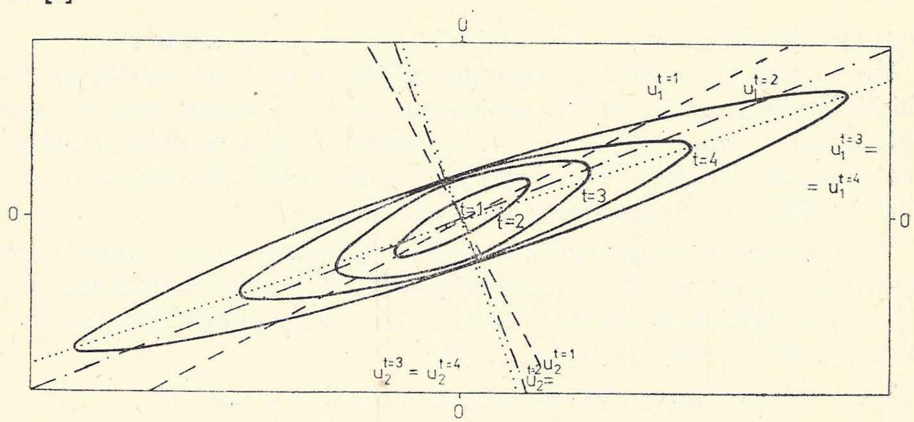

Fig. 1. Implicit state structures and their unit input sensitivity

Fig. 1. shows output structures identified by V matrix. The directions identified by vectors u_1^t show the evolution of the output structure which completely converges after two periods to the limit structure (dotted direction) identified by vector $u_1^{t=3}=u_1^{t=4}$. Such structure is the most easy to perturb with shocks on the final demand vector. While vectors u_2 describe the sectoral output structure along which growth is more difficult to be stimulated.

4. A structural analysis of the impact of consumption on total output. Difficulties and preliminary results

In this paragraph we shall show the preliminary results obtained in an attempt of application of the analysis to the Italian economy. The results are to be considered as preliminary since while implementing the analysis to the available data some difficulties has arisen which result from inconsistences with regard to the characteristics of data sources (either the disaggregation criterion and (or) the coherence of I–0 data sources and final demand data sources). If we take the 44 sector I–0 table for Italy in 1975 we find that 16 sectors out of the 44 produce also investment goods. If we rearrange the sectors so that the first 16 produce investment goods we obtain the ordering shown in Table 1. The number preceeding the sectoral

Table 1. I-0 sectors titles

1 agriculture	32 sea transport	2 coal
8 non ferrous min. and prod.	17 milk and dairy prod.	16 meat
10 metal products	18 other foods	33 other transp. serv.
11 agric. and industr. mach.	19 alcoh. and non alc. bev.	34 communications
12 office, opt. prec. instr.	20 tobacco	35 banking and insurance
13 electrical apparel	21 textiles and clothing	36 services to firms
14 motor and vehicles	9 chemical products	37 house rents
15 other transp. equip.	3 coke	38 education (priv.)
22 leather and footwear	24 paper and printing	39 health (priv.)
23 wood and furniture	4 petroleum, oil, nat. gas	40 recreation serv.
25 rubber and plastic prod.	5 electricity gas and water	41 administration
26 other manif. prod.	6 nuclear fuels	42 education (publ.)
27 constructions	28 recover and repairs	43 health (publ.)
29 trade	7 ferrous min. and min. prod.	44 household serv.
31 inland transport	30 hotels and restaurants	

title indicates the original NACE/GLIO ordering. Consumption items in the family budgets amount to 40 and are shown in Table 2. The investing sectors form the final demand accounts and sum up to 23, as shown in Table 3. For each sector an investment function has to be estimated. For such a purpose the results obtained in a previous work [1] were used but they generated an unsatisfactory dynamical pattern probably due to the compensating effect of the replacement investment with the expansion investment terms and to the depressing effect of condition (6). Thus, a simple new investment scheme was chosen by imposing $r=0$ in (3) and

Table 2. Consumption sectors

1 bread and cereals	15 shoes and repairs	28 means of transport.
2 meat	16 house rents	29 user cost of trans.
3 fish	17 energy and fuels	30 transport services
4 dairy prod.	18 furniture	31 communications
4 butter, margarine	19 textiles	32 radio and tv sets
6 fruit and vegetables	20 household applian.	33 newspapers and books
7 potatoes	21 glasswork and pottery	34 education
8 sugar	22 non durables and serv.	35 entertainment
9 tea, coffee, cocoa	23 domestic services	36 personal care artic.
10 other foods	24 medicine	37 hotels, cafe, rest.
11 non alcoh. bever.	25 medical apparel	38 other goods
12 alcoholic bever.	26 medical care	39 financ. serv. and ins.
13 tobacco	27 hospital care	40 other services
14 clothing and repairs		

Table 3. Investing sectors

1 agriculture, for., fishery	9 electrical goods	16 constructions
2 energy	10 transport equipment	17 trade
3 ferrous and non ferrous ores	11 food, beverages and tobacco	18 hotels and restaurants
4 non metal min. and prod.	12 textiles, clothing and shoes	19 transports
5 chemical products	13 paper and printing	20 communications
6 non machinery metal prod.	14 rubber and plastics	21 banking and insurance
7 agric. and ind. machinery	15 wood and furniture	22 other services
8 office, precision, opt. prod.		23 non market services

Table 4. Coefficient of the investment functions

k1	1.11500	k13	0.27900
k2	0.87700	k14	0.37100
k3	0.95800	k15	0.10400
k4	0.61100	k16	0.14100
k5	0.78200	k17	0.58600
k6	0.31700	k18	0.41000
k7	0.30900	k19	1.46000
k8	0.34200	k20	1.05600
k9	0.34300	k21	0.39500
k10	0.62500	k22	1.17700
k11	0.16800	k23	1.39500
k12	0.20100		

$w_2=1$, $w_0=w_1=0$ in (5) generating a second order difference equation easily detectable in final results. A further problem has arisen connected, with the "quasi singularity" of the matrices to be inverted, caused essentially by the structure of the investment bridge matrix. In the 1975 I–0 table 16 sectors produce investment goods, but at least for five of them (I–0 sectors 1, 22, 26, 19, 16) the amount is negligibile so that doubts can arise whether or not the sector has to be considered as producing investment goods. In this preliminary experiment it was decided to consider all the 16 sectors as producing investment goods but some elements of the bridge matrix has to be modified.

The singular values obtained are shown in Table 5.

Table 5. Magnitudes of the scale effects of consumption on total output

S1	2.41	3.87	5.01
S2	2.26	3.03	3.11
S3	1.86	2.11	2.59
S4	1.71	2.05	2.56
S5	1.63	1.77	2.04
S6	1.57	1.73	1.88
S7	1.51	1.68	1.81
S8	1.40	1.64	1.78
S9	1.33	1.57	1.76
S10	1.32	1.49	1.67
S11	1.29	1.41	1.63
S12	1.22	1.34	1.52
S13	1.15	1.30	1.44
S14	1.14	1.24	1.38
S15	1.10	1.20	1.31
S16	.99	1.13	1.28
S17	.98	1.10	1.25
S18	.94	1.05	1.20
S19	.88	1.01	1.10
S20	.84	.97	1.07
S21	.82	.91	.96
S22	.73	.86	.94
S23	.66	.82	.89
S24	.57	.81	.89
S25	.52	.77	.85
S26	.47	.71	.81
S27	.45	.67	.78
S28	.33	.66	.73
S29	.29	.61	.70
S30	.25	.54	.60
S31	.19	.50	.58
S32	.18	.48	.57
S33	.14	.43	.51
S34	.13	.40	.48
S35	.10	.38	.43
S36	.08	.34	.39
S37	.04	.32	.35
S38	.02	.28	.34
S39	.01	.25	.27
S40	.01	.22	.25
S41	.00	.16	.19
S42	.00	.12	.16
S43	.00	.10	.12
S44	.00	.01	.01

The associated ouput structures for the first three singular values are shown in Table 6. The magnitude of the scale effect ranges from 5.01 to 0. In particular

Table 6. Structural effect on the output vector associated with the three dominat scale effects for $t=0, 2$

Sectors	S1			S2			S3		
	$t=0$	$t=1$	$t=2$	$t=0$	$t=1$	$t=2$	$t=0$	$t=1$	$t=2$
1 Agriculture, For., Fish.	−.07	−.15	−.15	.04	−.01	.02	−.09	−.15	.05
8 Non Ferrous Min. & Prod.	−.15	−.10	−.11	−.10	−.04	−.01	.00	−.06	.01
10 Metal Products	−.11	−.09	−.09	−.07	−.03	.00	−.16	−.08	−.01
11 Agric. & Indus. Machines	−.17	−.13	−.13	−.08	−.02	.02	.06	−.12	−.01
12 Office, Prec., Opt. Instr.	−.24	−.14	−.14	.26	−.03	.01	.45	−.08	.00
13 Electrical Goods	−.16	−.17	−.20	.25	−.06	−.01	.14	−.06	.00
14 Motor Vehicles	−.05	−.11	−.11	.03	−.03	−.01	.00	−.04	−.01
15 Other Transport Equipment	−.05	−.23	−.24	−.04	−.13	−.15	.07	.50	.32
22 Leather & Shoe	−.17	−.25	−.26	.15	−.15	−.16	−.05	.54	.43
23 Wood & Furniture	−.13	−.21	−.24	.05	−.10	−.07	−.06	−.05	.12
25 Rubber & Plastic	−.09	−.16	−.19	.00	−.07	−.07	−.03	.00	.16
26 Other Manufact.	−.12	−.11	−.10	−.03	−.05	−.03	.05	.02	.03
27 Construction	.03	−.17	−.19	.32	−.05	.00	−.05	−.06	−.03
29 Trade	−.09	−.08	−.08	.05	−.02	.00	−.12	−.07	.00
31 Inland Transports	−.18	−.06	−.06	−.04	−.02	.00	.13	−.05	−.01
32 Sea & Air Transports	−.13	−.07	−.07	−.03	−.02	.00	.01	−.05	−.01
17 Mill	−.14	−.12	−.13	.02	−.05	−.03	−.08	−.04	.03
18 Other Foods	−.05	−.11	−.10	.18	.00	.02	−.21	−.05	.03
19 Non Alcohol, Alcoh. Bev.	−.07	−.08	−.07	.06	.00	.01	−.01	.01	.02
20 Tobacco	−.10	−.14	−.12	.05	−.06	−.04	−.05	.06	.10
21 Textiles & Clothing	−.18	−.16	−.14	−.01	−.07	−.04	−.03	−.03	.01
9 Chemical Products	−.16	−.16	−.16	.06	−.05	−.01	−.05	−.10	.03
3 Coke	−.13	−.12	−.12	.04	−.04	−.01	.01	−.10	.01
24 Paper & Printing	−.35	−.10	−.11	.03	−.02	−.01	−.44	.00	.06
4 Petroleum, Gas, Refineries	−.25	−.14	−.14	−.04	−.05	−.02	−.15	.02	.04
5 Electricity, Gas, Water	−.18	−.11	−.11	.00	−.05	−.02	−.01	−.04	.02
6 Nuclear Fuels	−.04	−.15	−.14	.01	−.07	−.02	.02	−.32	−.08
28 Recovery & Repair	−.06	−.11	−.12	.00	−.01	.02	−.04	−.08	−.02
7 Ferrous, Non Ferrous	−.30	−.36	−.20	−.04	.91	.95	−.17	.07	.08
30 Hotels & Restaurants	−.09	−.15	−.13	.03	.10	.13	−.01	.01	−.07
2 Coal	−.16	−.13	−.13	.03	−.05	−.04	−.09	−.05	−.22
16 Meat	−.21	−.20	−.22	−.09	−.09	−.07	−.10	−.06	−.54
33 Transport Services	−.02	−.09	−.10	.01	−.04	−.03	.00	−.04	−.08
34 Communication	−.10	−.17	−.16	.01	−.07	−.04	.00	−.22	−.40
35 Banking & Insurance	−.10	−.13	−.15	.01	−.04	−.01	−.03	−.08	−.05
36 Other Private Services	−.08	−.11	−.12	−.01	−.05	−.03	.01	−.01	.03
37 Real Estate	−.04	−.15	−.18	.09	−.07	−.03	−.06	−.13	−.01
38 Private Education	−.06	−.11	−.12	.01	−.03	.00	.00	−.10	−.06
39 Private Health Services	−.24	−.23	−.26	.27	−.08	−.01	.42	−.21	−.16
40 Recreation & Cultur	−.16	−.10	−.12	.06	−.04	−.01	.08	−.07	−.03
41 Government Services	.20	−.14	−.13	−.35	−.07	−.06	.12	.18	−.25
42 Public Education	.31	−.13	−.14	.63	−.07	−.07	−.34	.17	−.03
43 Public Health	.18	−.17	−.18	.24	−.09	−.07	.13	.21	.11
44 Domestic Servants	−.13	−.10	−.11	.10	−.05	−.04	.25	.01	,06

the magnitudes of order 5 and 4 (5.01 and 3.87) appear only once; twice the order 3 (3.11 and 3.03), 10 times occurs the magnitude 2 (from 2.59 to 1.81). The remaining is almost equally divided among magnitude 1 (57 times from 1.71 to .81) and less than 1 (61 times from .78 to 0.0). The dominant singular value has associated a depressing effect. From Table 5. we can see that the dynamic pattern of the composition of total output is prevalently negative with reference to actual initial conditions, in each of the three periods considered (with exception of Government Services, Health and Education). The scale effect of order 4 (3.87) has an expansive effect on sectors 12, 13, 22, 27, 18, 39, 42, 43 and 44 and a depressing one in sector 41 in the first period, while in the remaining two periods a rather surprisingly high structural effect is detected. The structural effect associated with the highest singular value of order 2 (2.59) has rather interesting restructuring effects from period 1 to period 2: a relevant depressing effect on sectors 16 and 34 is almost compensated by expansion in sectors 15 and 22 and other depressing effects on sectors 41, 39, 34 and 2 seems almost compensated by expansion in sectors 23, 25, 20 and 43.

Conclusions

The analysis developed allows for a substantial insight of the interaction between interindustrial flows and final demand components as it allows for a synthetic representation of the output structures that can be more easily influenced by demand structure revealing the strong and weak points of the system. We did not go into detailed comments of the example presented because of the simplifying assumptions we had to pose for overcoming some difficulties connected with the "sectorization" and the investment functions. A more general analysis can endogenize consumption functions which can also imply lagged variables so that the dynamics of the system becomes more complex and foreign demand impact on the technological and behavioural patterns of the system can be introduced and studied.

The method can also be used for evaluating the impact of changes in the parametric structure of the system. As we have seen the "reactivity" of the economic systems changes according to predetermined time paths when the parametric structure is kept constant. If the system experiences a shock on a subset of its parametric structure we are able, through this analysis, to evaluate the order relevance of such structural changes. If input-output analysis provides a framework for coherent modelling of the technological features of the production system (and linking it with the theory and the pratice of final demand components, at a level where the components of each traditional macrovariable can be specified) systems analysis and control theory can greatly help in synthesizing its behaviour. They are complementary to simulation procedures and serve for detecting and evaluating structural change by means of a "structural" method.

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Przybliżona sterowalność i analiza modelu Input-Output

W pracy badano relacje pomiędzy przepływami międzygałęziowymi i składowymi popytu finalnego w celu określenia technologicznej struktury systemu ekonomicznego. Zaproponowano specjalizowaną „strukturalną" metodę badawczą, która umożliwia, na podstawie badań przepływów międzygałęziowych, analizę zmian strukturalnych systemu. Wykazano, że analiza systemowa i teoria sterowania są bardzo pomocne w powyższej analizie.

Приближенная управляемость и анализ модели Вход-Выход

В работе исследуются зависимости между межотраслевыми потоками и составляющими конечного спроса для определения технологической структуры экономической системы. Предложен специализированный „структурный" исследовательский метод, который позволяет, на основе исследования межотраслевых потоков, проводить анализ структурных изменений системы. Показано, что системный анализ и теория управления весьма полезны для проведения выше указанного анализа.