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Two level model of the Polish economy, production and financial interrelations

by

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A two level model of controlling the national economy is presented. The model assumes financial regulations which are operative in the current reform of the Polish economy. It is designed as a decision aiding tool for the Central Planner and the Bank Director (the Center). The Center has institutional basis and executive power of influencing the activity of branches of the economy in such a way that the functioning of branches evolves in directions desired by the Center. This is done with the help of such instruments as taxes, credits for financing investments and subsidies.

A multiobjective, constrained problem is defined at each level of the model. Production, financial and material constraints are included.

1. Introduction

The presented model utilizes some previous definitions and results [1], [2], [3], [4], and extends the formulation due to the inclusion of financial flows and regulations. They are for instance investment credits and limits of the yearly increase of the branch wage fund determined by a special tax, called PFAZ. The two level structure is also newly introduced.

In the model, separately formulated problems are solved on two levels. However, they are interdependent; the solution of one of them parametrically depends on the other problem solution. The upper level problem (of the Center) expresses the goals and restrictions of the Central Planner and of the Bank. At this level financial parameters are determined which stimulate the financial activity of the lower level units (branches).

At the lower level branch models are formulated. Each branch is defined as a group of enterprises which produce a homogeneous good. Both, the upper level and the lower level problems are formulated as multiobjective ones.

Consistently with the present economic practice, selected investments are directly controlled by the Center. Means, bank credits and raw materials are assured by the Government and it is required that these investments be completed on time. They are to be used for changing the structure of the national economy. It is assumed that the Center's investments are independent of enterprises investments but they limit the total level of these investmens. The investmens of the Center, are determined exogeneously.

In section 2 the two level structure of controlling the national economy is described and interdependences between the upper and the lower levels are explained.

The description of the upper level problem is given in section 3 while a typical branch problem is discussed in the 4th section. The branch problems have identical structure and differ from one another in values of parameters.

2. The Two Level Structure of Controlling the National Economy

The model formulated in the paper reflects the structure of the Polish economy. The existing relations and limitations of values and variables which characterize the economic system and the mechanism of its functioning are described in some detail.

It is assumed that the Polish economy consists of a set of branches (each branch produces a homogeneous good), which function consistently with the valid regulations and aim at achievement of their objectives which are maximization of the profit and maximization of the revenue.

The Central Planner together with the Bank are trying to control the activity of branches with the help of various parameters. Given the parameters, the branches have the freedom of maximizing their own objectives.

Among the most important parameters are:
(exogeneously determined for a j-th branch and time t)

- — investments of the Center V^C_{tj} (new and continued ones);
- — financial subsidies Do_{tj};
- — the upper bounds on the amount of available raw materials $\bar{q}_{tj}$;
- — the upper bound on the foreign debt of the economy, US;
- — the rate of the turnover tax, σ_{tj}

(resulting from the solution of the Center problem)

- — investment credits Po_{tj}, granted to branches by the Bank;
- — the income tax rate r_{tj};
- — the share of the amortization fund in the development fund of an enterprise a_{Atj};
- — the level of a very progressive tax, which a branch has to pay on extra wage fund. These taxes are characterized by a parameter a_j and till the end of 1985 they were called PFAZ.

The objective of the Center is twofold.

The Center tries to achieve some growth paths which are designed to satisfy the needs of individual persons and to ensure given production and development programs. It expresses the implicit desire of maximization of individual and public consumption C_t, maximization of national income D_t and minimization of the foreign debt S_t.

In addition, its activity is motivated by a sound principle of profitability. The Bank selects only reliable creditors, for instance such branches which have credit worthiness, and the branches are allowed to maximize their profit.

It is assumed in the model that the branches make decisions independently one from another. Balacing of production, raw materials and labor force takes place at the upper level. The lower level problem of a branch is a multicriterion optimization problem. Each branch maximizes simultaneously its profit and its revenue given financial regulations parameters of the Center. The solution yields a vector of control variables u_{tj} which are: the investments V^P_{tj}, the wage fund Sa_{tj} and the employment level L_{tj} in the j-th branch at time instants $t=0, ..., T$. Additionally, the export E_{tj}, the production Q_{tj} and the fixed assets K_{tj} in the j-th branch are derived from equations defined in section 4.

The above values of E_{tj}, Sa_{tj}, V^P_{tj}, K_{tj}, L_{tj}, Q_{tj} and the credit repayments Dk_{tj}, are reported to the Center by the j-th branch $j=1, ..., m$ as a result of its activity. These values are functions of the parameters of the Center, which are used by the Bank and by the Central Planner in order to motivate the branches to act consistently with their desires. In Figure 1 the two-level structure of an interactive control of the economy is depicted.

3. The Upper Level Problem

The concern of the Center is the achievement of given development paths which in the view of the Central Planner and the Bank are desired by the society and at the same time ensure sufficient production program and development of the national economy.

The desired trajectories of consumption $\bar{C}^d_t$, national income $\bar{D}_t$ and of the foreign debt of the Polish economy $\bar{S}_t$ appear in the objective as follows

$$\min [\max (0, \bar{C}^d_t - C^d_t)], \tag{1}$$

$$\min [\max (0, \bar{D}_t - D_t)], \tag{2}$$

$$\min [\max (0, S_t - \bar{S}_t)], \quad t=0, ..., T \tag{3}$$

where T is the terminal time of the investigated period and C^d_t, D_t and S_t are defined further on by expressions (16), (15) and (14) respectively. In the upper level problem production and financial balances have to be maintained.

The production balance assumes the form

$$Q_t + G_t Q_t + \bar{B}_t V_t + M_t = A_t Q_t + B_t V_t + E_t + C_t + \Delta R_t, \quad t=0, ..., T \tag{4}$$

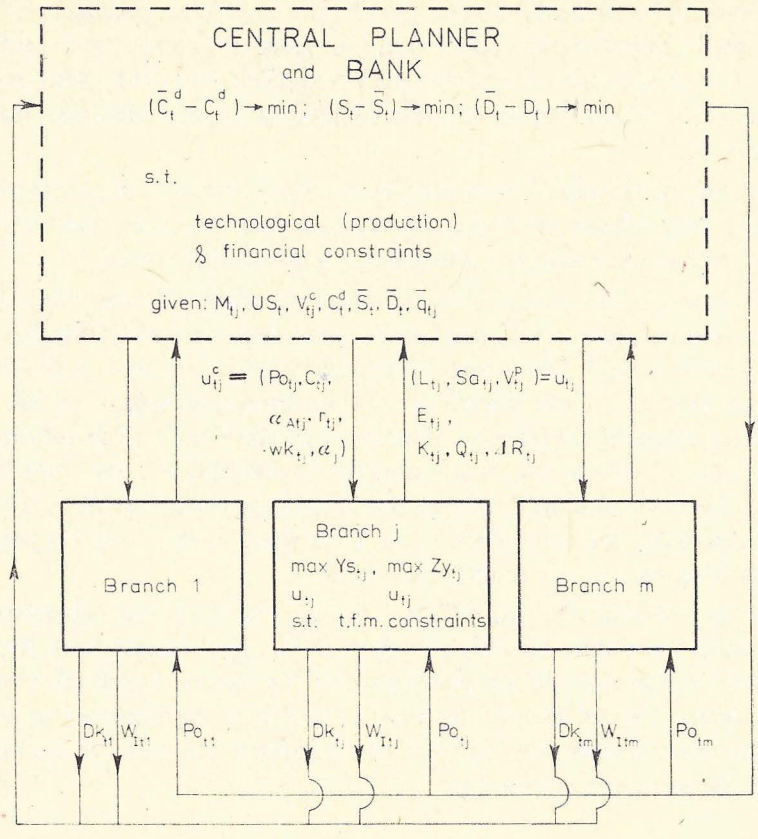

Fig. 1

where the variables Q_t, V_t, M_t, E_t, C_t, ΔR_t are vectors of m elements, and m is the number of branches in the economy.

The left hand side of the equation (4) is a sum of production Q_t and imports, for production $G_t Q_t$, for investment $\bar{B}_t V_t$, and for consumption purposes M_t which is to be divided among branches. The right hand side presents a division of the product between intermediate production $A_t Q_t$, investments $B_t V_t$, export E_t, total consumption C_t and the supplies and reserves ΔR_t.

It is assumed that the Leontief matrix A_t changes over time and encompasses an anticipated decrease of material coefficients. B_t is a changing over time total capital coefficient matrix, $\bar{B}_t$ is a matrix of imported capital coefficients and G_t is a matrix of imported material coefficients. The matrices $\bar{B}_t$ and G_t represent the propensity of the economy to import for investment and production purposes respectively.

The total investment V_{tj} is defined as a sum of the centrally financed investments V^C_{tj} and the branch investments V^P_{tj}

$$V_{tj}=V^C_{tj}+V^P_{tj}, \quad j=1, ..., m, \quad t=0, ..., T \tag{5}$$

Investments in the year t result from decisions made prior to time t. In order to account for earlier commitments it is assumed that the centrally financed investments in the $j-$th branch, at time t are not below the level planned at the year t.

Investments planned to be committed in the year t are determined by the matrix $ZAA^C_{t-1,j}(n, 1)$. It describes the funds anticipated for continuation of investments which are to be committed in the year $t(t-1+1)$ and completion of which is anticipated for the year $t-1+n$.

$$V^C_{tj} > \sum_{n=1}^{N^C_{t-1,j-1}} ZAA^C_{t-1,j}(n, 1) \tag{6}$$

The value $N^C_{t-1,j}$ is the maximum length of continuation of the centrally financed investments in the j-th branch, measured at time $t-1$. The method of construction of the above investment matrix and examples of investment which are to be committed in the sections of the Polish economy over the period 1986–1990 are described in [5] and [10].

Additionally it is assumed that the flows of investment goods produced by the metal-machine and the construction sectors do not exceed production capacities n these sectors.

$$\sum_{j=1}^{m} b_{3j} V_{tj} \leqslant f(K_{t-1,3} L_{t,3}) \tag{7}$$

$$\sum_{j=1}^{m} b_{10j} V_{tj} \leqslant f(K_{t-1,10}, L_{t,10}), \quad t=0, ..., T \tag{8}$$

where indices 3 and 10 correspond to the investing sectors of the metal machine industry and the construction sector. In the expressions (1)–(8), the values $\bar{C}^d_t, \bar{D}_t, \bar{S}_t, G_t, \bar{B}_t, B_t, A_t, V^C_{tj}, M_t$ are determined exogeneously. The values C_{tj}, $D_t, S_t, Q_{tj}, V^P_{tj}, E_{tj}, \Delta R_t$ are computed, given parameter values of the Center, some of which are in turn calculated based on information from the lower level. The lower bound on consumption $\underline{C}_{tj}$, called the subsistence level is also exogeneous. The consumption computed from the model should not be below the subsistence level

$$C_{tj} \geqslant \underline{C}_{tj}, \quad j=1, ..., m, \quad t=0, ..., T \tag{9}$$

The second group of constraints concerns financial conditions. Some of them represent flows of money, the others describe reserves (the states) of monetary funds.

The state of the bank accounts in Polish currency at time $t+1$ is defined by $\overline{BANK}^1_{t+1}$:

$$\overline{BANK}^1_{t+1} = \overline{BANK}^1_t + BANK^1_t + \overline{FROZ}^1_t + \sum_{j=1}^{m} (Dk^1_{tj} + -Po^1_{tj} - V^{C1}_{tj}), \quad t=0, ..., T \tag{10a}$$

The state of the Bank accounts in foreign currencies

$$\overline{BANK}^2_{t+1}=\overline{BANK}^2_t+BANK^2_t+\sum_{j=1}^{m}(Dk^2_{tj}+$$
$$-Po^2_{tj}-V^{C2}_{tj})-Dz^t_t, \quad t=0,\ldots,T, \tag{10b}$$

where bank accounts are summed up in the following banks: The Polish National Bank (NBP), The Bank for Food Economy (BGŻ) and The Trading Bank SA (BH SA) in Warsaw.

$BANK^1_t$ represents sums of Polish money (flow at time t) that are either paid to the Bank (e.g. tax liabilities) or have to be paid by the bank to cover wages in branches financed from the budget, and other expenses in branches of the economy

$$BANK^1_t=\sum_{j=1}^{m}(Podo_{tj}+Podd_{tj}+F_{tj}+PFAZ_{tj}+(1-\alpha_{Atj})A_{tj}+$$
$$-Do^1_{tj}-Sa^1_{tj})+Os^1_t, \quad t=0,\ldots,T \tag{11a}$$

The money flow in foreign currencies is defined as:

$$BANK^2_t=Os^2_t-Z_t-\sum_{j=1}^{m}Do^2_{tj}, \quad t=0,\ldots,T \tag{11b}$$

The j-th branch pays to the Bank the turnover tax, $Podo_{tj}$, the income tax, $Podd_{tj}$, the wage fund tax F_{tj}, the progressive tax paid on the extra growth of the wage fund, $PFAZ$ and a part of a verified amortization fund $(1-\alpha_{Atj})A_{tj}$, $0<\alpha_{Atj}<1$. Do^1_{tj} denotes subsidies, in Polish zloty, granted by the Bank to the j-th branch, Sa_{tj} is the wage fund of the j-th branch which is financed from the central budget and Os^1_t is the balance of savings of individual citizens at time t. Os^2_t and Do^2_{tj} denote the balance of savings and the subsidies of the j-th branch, in foreign currency at time t. The trade balance with abroad Z_t is defined as a difference between import and export

$$Z_t=\sum_{j=1}^{m}[M_{tj}+(G_t\,Q_t)_j+(\bar{B}_t\,V_t)_j-E_{tj}], \quad t=0,\ldots,T. \tag{12}$$

Positive Z_t increases the foreign debt as new credits are then required. Assume, that T_R is the period by which credits granted by foreign banks must be repaid and that a coefficient γ^Z_t includes the interest on credits dependent on the repayment period T_R and the interest rate. Then, the repayment vector at time $t+1$ is of the form

$$Dz_{t+1}=[Dz^{t+1}_t+\gamma^Z_t(1)\,Z_t,\ldots,Dz^{t+T_R}+\gamma^Z_t(T_R)\,Z_t] \tag{13}$$

$$\gamma^Z_t=\begin{cases}0 & \text{if} \quad Z_t\leqslant 0\\ 0<\gamma^Z_t<1 & \text{if} \quad Z_t>0\end{cases}$$

if the credit repayment starts at time $t+n$, $n<T_R$, then $\gamma^Z_t(i)=0$ for $i=1,\ldots,n-1$. Thus, prolongation of credit repayment implies additional interest on granted credits. Eqn. (13) holds if the repayment time of new credits equals T_R.

$FROZ_t^1$ denotes the state of the development fund which is worked out at the branch by enterprises. The definition of $FROZ_t^1$ is given by (44) and (39), and the mechanism of its formation is illustrated in Figure 2.

Dk_{tj}^1 and Dk_{tj}^2 denote credit liabilities of the j-th branch in Polish and in foreign currencies respectively which are to be paid to the Bank at time t. Prolongation of the crediting period and postponement of the time of credit repayment induces additional interest, thus increasing Dk_{tj} which depends on the amount of credits and the length of its repayment.

Po_{tj}^1 and Po_{tj}^1 represent credits granted by the Bank to the j-th branch at time t.

The Bank collects interest on granted credits. It grants credits (investment, turnover, payment and other ones) to branches based on the solvency and worthiness of enterprises. For instance, it is required for investment credits that the percentage of the enterprise own fund amounts to at least 70% of the value of investment, the length of credit repayment is not longer than 5 years and a special measure of investment effectiveness is satisfied, [5].

It is assumed that the foreign debt of the economy, S_t, is equal to all repayments of foreign credits and does not exceed a number US_t, given by the Center

$$S_t = \sum_{k=1}^{T_R} \left(Dz_t^{t+k} + \gamma_t^Z (k)\, Z_t \right) \leqslant US_t, \qquad t=0, \ldots, T \tag{14}$$

Homogeneity of credits is assumed and (14) is purposefully simplified.

The national income is defined as

$$D_t = \sum_{j=1}^{m} (V_{tj} + C_{tj} + \Delta R_{tj}), \tag{15}$$

where $\Delta R_{tj} = R_{tj} - R_{t-1,\,j}$ is the increase of the reserves.

The global production of consumption goods, which appears in relation (1)

$$C_t^d = \sum_{j=1}^{m} C_{tj}, \qquad t=0, \ldots, T \tag{16}$$

The upper level problem of the Center can be formulated as follows:
Given the values of: M_{tj}, US_t, V_{tj}^C, $\bar{C}_t^d$, $\bar{D}_t$, $\bar{S}_t$, $\bar{q}_{tj}^1$, $\bar{q}_{tj}^2$, $\underline{C}_{tj}$, Os_t^1, Os_t^2, Do_{tj}^1, $Do\,(2)_{tj}$, Ul_{tj}, (where Os are savings, Do are subsidies, Ul are reliefs and $\bar{q}_{tj}$ upper bounds on raw materials), the structural matrices A_t, B_t, $\bar{B}_t$, G_t, the financial parameters: ξ_{tj}, Pr_{tj}, β_{tj}, $w_{\min}$, k_1, k_2, t_0, σ_{tj} defined in section 4 and $\gamma_z^t(k)$, and the values reported to the Center from the lower level E_{tj}, K_{tj}, L_{tj}, Q_{tj}, Sa_{tj}, V_{tj}^P, Dk_{tj}, ΔR_{tj}, find such levels of credits in the Polish, Po_{tj}^1, and in the foreign currencies, Po_{tj}^2 the production for consumption purposes C_{tj}, the value of the national income D_t and of the foreign debt S_t, and the values of financial parameters α_{Atj}, α_j, r_{tj}, wk_{tj} which yield the optimal values of expressions (1)–(3) and satisfy the constraints (4)–(16), (19), (20), for all $t=0, \ldots, T$ and for all sectors $j=1, \ldots, m$.

4. The Lower Level Problem of a Branch

The structure of all branch models is identical. The models vary in values of parameters of individual branch problems such as for instance initial financial resources, initial value of productive fixed assets, or parameters of functions which approximate production capacity of a branch.

As mentioned in section 2 a branch applies two objectives. It maximizes simultaneously at time t the branch profit:

$$\max_{u_{jt}} Zy_{tj}, \quad t=0, ..., T, \quad j=1, ..., m \tag{17}$$

and the net revenue (production sold by the branch)

$$\max_{u_{tj}} Ys'_{tj}, \quad t=0, ..., T, \quad j=1, ..., m \tag{18}$$

where $u_{tj}=(Sa_{tj}, V^P_{tj}, L_{tj})$, and Sa_{tj} is the wage fund and L_{tj} is an employment of the j-th branch.

Maximization of profit implies maximization of the development fund (39), which is the major source of financing the branch investment V^P_{tj} (the liabilities and the repayment of investment credits also burden the development fund). Thus, the Bank is interested in the profit maximization of the branches. Maximization of the branch net revenue directly maximizes the wage fund of the branch. At fixed prices, increased revenue contributes also to the growth of the national income. The controls u_{tj} have to satisfy a number of conditions which result from limited production capacities, scarcity of raw materials and financial funds, and from financial regulations.

The production capacities of the j-th branch Y_{tj} are approximated by a two factor production function of the *CES* type:

$$Y_{tj}=f_{CES}(K^B_{t-1,j}, L_{tj}), \quad t=0, ..., T \tag{19}$$

It depends on the gross value of the fixed assets in the preceding year $K^B_{t-1,j}$ and on the employment L_{tj}.

The utilization of the production capacities is described with the help of the expression

$$Q_{tj}=m_{tj}\, Y_{tj},$$
$$0<m_{tj}<1, \quad t=0, ..., T, \quad j=1, ..., m, \tag{20}$$

where m_{tj} is estimated exogeneously and Q_{tj} denotes production.

The maximum employment in the j-th branch $L_{tj\,\max}$ is defined by an increasing function g, which depends on the difference between the average wage in the branch w_{tj} and the average wage in the economy in the preceding year, W_{t-1}. Thus, we have

$$l_{tj}\, s\, L_{tj\,\max} - g(w_{tj}-W_{t-1}), \quad t=0, ..., T \tag{21}$$

where $w_{tj}=\dfrac{Sa_{tj}}{L_{tj}}$.

The fixed assests are calculated from the difference equations

$$K^B_{t+1,\,j}=(1-l_{tj})\,K^B_{tj}+\sum_{i=0}^{N_j-1}\psi^t_j(i)\,(V^C_{t-i,\,j}+V^P_{t-i,\,j}) \quad \text{for} \quad t\geqslant N_j \tag{22a}$$

$$K^B_{t+1,\,j}=(1-l_{tj})\,K^B_{tj}+\sum_{i=1}^{t}\psi^t_j(i)\,(V^C_{t-i,\,j}+V^P_{t-i,\,j})+\Delta_{tj} \quad \text{for} \quad t<N_j \tag{22b}$$

where

$$\Delta_{tj}=\sum_{i=t+1}^{N_j-1}\psi^t_j(i)\,\bar{V}_{t-i,\,j}, \qquad t=0,\ldots,T \tag{22c}$$

and

$$K_{0j}=\bar{K}_{0j} \tag{23}$$

For every sector $j=1,\ldots,m$:
N_j is the average investment delay, $\psi^t_j(i)$ denote coefficients of lagged investments and represent the effect of current ($i=0$) and lagged ($i=1,\ldots,N_j-1$) investments on fixed assets formation, $0\leqslant\psi_j(i)\leqslant 1$, l_{tj} is the liquidation rate of fixed assests, $\bar{V}_{t-i,\,j}$ denotes the central and branch investments together, committed prior to the investigated period $(0,T)$, i.e. for $t-i<0$, and K_{0j} is a given initial value of fixed assets.

Thus, the production in the model depends on the branch investments V^P_{tj} and on the branch employment L_{tj}.
It is assumed that the fixed assets do not decrease over time

$$K^B_{t+1,\,j}\geqslant K^B_{tj}, \qquad t=0,\ldots,T \tag{24}$$

Additionally, the funds for investments financed by the branch, V^P_{tj} should be at least sufficient for continuation of investment committed prior to time t. The funds required for continuation of these investments are determined by the matrix $ZAA^P_{t-1,\,j}(n,1)$

$$V^P_{tj}\geqslant\sum_{n=1}^{N^P_{t-1,\,j}-1}ZAA^P_{t-1,\,j}(n,1), \qquad t=0,\ldots,T \tag{25}$$

The value $ZAA_t(k,l)$ describes the funds for investments, which are planned to be committed in the year $t+l$, and whose completion is anticipated for the year $t+k$. The conditions (24) exemplify the tendency, articulated in the official economic plan 1986–1990, to restrain the decapitalization process of fixed assets. The conditions (24), (25) restrain the investment levels from below. The relation (25) can be violated in some situations. For instance in the case of investment suspension or postponement V^P_{tj} can be equal to or smaller than the value of the right hand side of (25).

The revenue (the value of the sold production) maximized in the j-th branch, is proportional to the production given by expression (20)

$$Ys_{tj}=\kappa_{tj}\,Q_{tj}, \tag{26}$$

where κ_{tj} is an exogeneously determined parameter which symbolizes an aggregate branch price.

The profit, which is also maximized is defined consistently with the operating regulations

$$Zy_{tj}=Ys_{tj}-Ko_{tj}+Do_{tj}-Podo_{tj}+Sal_{tj}, \tag{27}$$

where Ko_{tj} represents the costs of production of the j-th branch

$$Ko_{tj}=A_{tj}+Ma_{tj}+Sa_{tj}+F_{tj}, \tag{28}$$

The subsidies from the Bank, Do_{tj}, are determined at the upper level.

The turnover tax $Podo_{tj}$ is defined as a part of the revenue

$$Podo_{tj}=\sigma_{tj}\cdot Ys_{tj}, \tag{29}$$

where parameters σ_{tj}, $t=0, ..., T$ are elements of the control vector of the upper level problem, with the help of which the Center stimulates financial policy of branches, Sal_{tj} denotes the balance of extraordinary benefits and losses in the j-th branch.

The costs of production Ko_{tj} and the turnover tax $Podo_{tj}$, are calculated based on economic and financial gains of the branch and on the parameters with the help of which Bank controls the branch activity. The value of the amortization fund A_{tj} is approximated as a proportion of fixed assets

$$A_{tj}=\xi_{tj}K_{tj}, \qquad t=0, ..., T \tag{30}$$

where ξ_{tj} is given by the Center. Thus, A_{tj} implicitly depend on the branch investments.

The material costs of production Ma_{tj} are computed from the equation

$$Ma_{tj}=\sum_{i=1}^{m} a_{tij}Q_{tj}+(G_tQ_t)_j, \tag{31}$$

where the coefficients a_{tij} are elements of the material coefficient Leontief matrix A_{tj}.

The wage fund Sa_{tj} is the control variable, which satisfies the following relations.

$$Sa^W_{tj}=Sa_{t-1,j}+wk_{tj}\frac{\Delta Ys''_{tj}}{Ys''_{t-1,j}} \tag{32}$$

where Sa^W_{tj} is the wage fund, which is not subject to a progressive tax on the extra increase of the wage fund at time t, $PFAZ_{tj}$. It depends on the relative increase of the net revenue Ys''_{tj}, multiplied by a correction coefficient wk_{tj}. Additional wages can be assured, but a high tax has to be paid. The tax, called $PFAZ$, is defined below

$$PFAZ_{tj}=\begin{cases}0 & \text{if} \quad Sa_{tj}>Sa^W_{tj}\\ (Sa_{tj}-Sa^W_{tj})^{\alpha_j} & \text{if} \quad Sa_{tj}\leqslant Sa^W_{tj}\end{cases} \tag{33}$$

where α_j denotes the growth rate of the $PFAZ$-tax. Both parameters, wk_{tj} and α_j are elements of the control vector of the Center.

The net revenue is calculated as follows

$$Ys''_{tj}=Ys^1_{tj}-Ma_{tj} \tag{34}$$

$$Ys^1_{tj} = Ys_{tj} - Podo_{tj} - Do_{tj} \tag{35}$$

The income tax $Podo_{tj}$ assumes the form

$$Podo_{tj} = r_{tj}\,(Zy_{tj} + Kn_{tj}) \tag{36}$$

where r_{tj} is a parameter given exogeneously by the Center. The profit Zy_{tj} is defined by (27). Kn_{tj} denotes unjustified costs and losses and is exogeneous in the branch model. The value of F_{tj} denotes the wage tax, which chages over time.

The profit Zy_{tj} can be uniquely determined by the: exogeneous parameters of the Center, its control variables and parameters, and by the branch controls: the investments V^P_{tj}, the wage fund Sa_{tj}, and the employment L_{tj}. The value of investment funds V^P_{tj} is limited from above by the financial resources of the j-th branch W_{Itj} and the investment credits, granted by the Bank.

$$V^P_{tj} \leqslant W_{Itj} + Po_{tj} \tag{37}$$

$$W_{Itj} = \overline{FROZ}_{t-1,\,j} + (1+Pr)\,FROZ_{tj} - Dk_{tj} - \Delta R_{tj} \tag{38}$$

The development fund is defined as

$$FROZ_{tj} = Zyp_{tj} - PFAZ_{tj} - Fs_{tj} - Fm_{tj} + \alpha_{tj} \cdot A_{tj} - Fz_{tj} \tag{39}$$

where the verified profit

$$Zyp_{tj} = Zy_{tj} - Podd_{tj} - Ul_{tj} \tag{40}$$

Ul_{tj} represent special reliefs granted by the Center. The social benefits fund $Fsoc_{tj}$ and the housing fund Fm_{tj} depend on the employment L_{tj}.

The workers fund, Fz_{tj} is the source of the prizes, which are granted to the workers of the j-th branch and its value depends on the employment and the wage fund

$$Fs_{tj} = 0.5 W_{mtj} \cdot L_{tj} \tag{41}$$

$$Fm_{tj} = 0.25 W_{mtj} \cdot L_{tj} \tag{42}$$

$$Fz_{tj} = 0.085 w_{tj} L_{tj}, \qquad t = 0, \ldots, T \tag{43}$$

where w_m is the minimum wage, and w_{tj} the average wage in the j-th branch. The state of the accumulated development fund is formulated as

$$\overline{FROZ}_{t+1,\,j} = \overline{FROZ}_{tj} + (1+Pr_{tj})\,FROZ_{tj} + Po^1_{tj} - Dk^1_{tj} - V^P_{tj}, \tag{44}$$

where Pr_{tj} denotes interest rate on $FROZ_{tj}$.

Export in the j-th branch depends on the branch production and on the financial situation of the branch. The maximum share of the production in export is determined by the parameter k which represents the growth rate of export. The minimum production $Q_{j\min}$ is specified, below which no export results from production. The financial situation is characterized by the weighted relative total import and export in the preceding year:

$$E_{tj} = [\max\,(Q_{tj} - Q_{tj\min})]^{1/k} + \beta_1\, Jm_{tj} + \beta_2\, E_{t-1,\,j} \tag{45}$$

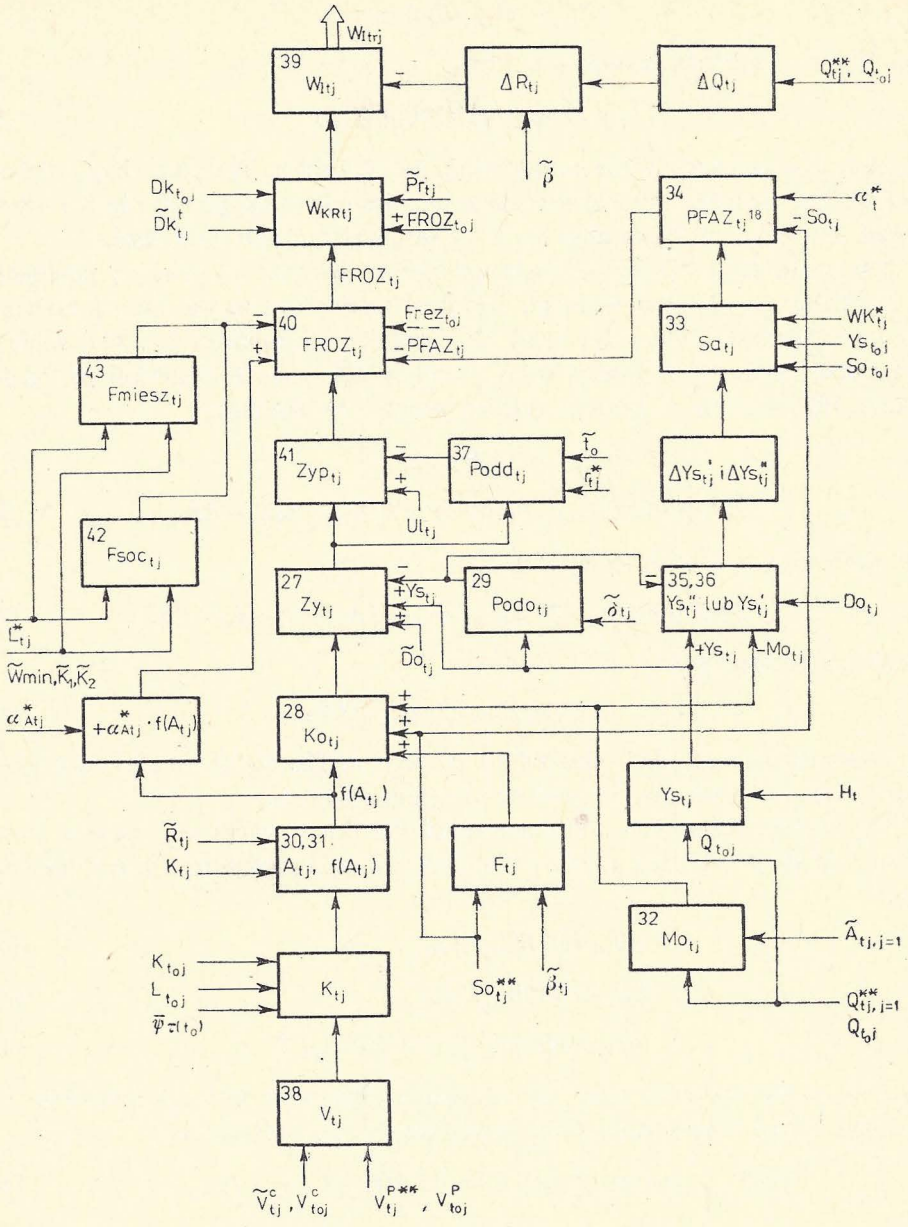

Fig. 2

where β_1, β_2 are estimated constants and

$$Jm_{tj}=M_{tj}+(G_t Q_t)_j+(\bar{B}_t V_t)_j$$

The simplicity of this equation results from the fact that export and import prices are not defined in the model. Therefore, the world market is not considered either.

The scarcity of raw materials is assumed

$$\sum_{k=1}^{m} a_{tjk} Q_k \leqslant \bar{q}_{tj}(1) \tag{46}$$

and

$$\sum_{k=1}^{m} g_{tjk} Q_k \leqslant \bar{q}_{tj}(2), \tag{47}$$

$j=1, \ldots, m$,

where $\bar{q}_{tj}(2)$ are the upper levels of the imported materials and components and $\bar{q}_{tj}(1)$ are the upper levels of home produced materials, both given exogeneously.

5. Summary and Conclusions

The presented model includes many relations, which describe the major objectives and regulation functions of the current Polish economy. However, some essensial simplifications have been applied. To the major ones belongs the assumption of an exogeneous quasi-price for aggregated goods produced by branches. This assumption implies that the branch (a unit of the lower level problem) decides for itself only about the level of its own investments and about its wage fund and employment. The homogeneity of branches, to which some regulations are applied as if they were enterprises is also a simplification. However, this difficulty can be hardly overcome at this stage of research.

The complexity of the model results from considering multiple objectives of the joint coordinator, represented by the union of Polish banks and by the Central Planning Commission, and from selecting the objectives by branches in such a way that they satisfy an individual employee (by applying an implicit maximization of the wages) and also indirectly contribute to the satisfaction of the objectives of the Center. The financial regulations and limitations with respect to raw materials, production capacities and home and foreign finances are adequatly described, thus creating a complex picture of the Polish economy. The simplifications mentioned above can be of benefit at the first stages of investigations. They can allow for thorough investigations of a mechanism of promotion of the branch (enterprise) investments.

It is anticipated in the further research to introduce several categories of prices for production, investment, consumption, export and import goods. This should enable to investigate the dynamic relations between wages, employement, investment and prices.

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Dwupoziomowy model gospodarki narodowej Polski: relacja pomiędzy sferą finansową i produkcyjną

W pracy przedstawiono dwupoziomowy model sterowania gospodarką narodową. W modelu założono występowanie regulatorów finansowych, które wprowadzono w gospodarce polskiej w ramach obecnej reformy. Jest on przygotowywany jako narzędzie wspomagania decyzji dla Centralnego Planisty i Dyrektora NBP (Centrum). Centrum posiada środki i bazę instytucjonalną pozwalające oddziaływać na branże gospodarki w taki sposób, by branża funkcjonowała w sposób pożądany przez Centrum. Uzyskuje się to za pomocą takich instrumentów jak: podatki, kredyty na finansowanie inwestycji i dotacje.

Na każdym poziomie definiuje się zadanie wielokryterialne z ograniczeniami. Uwzględniono ograniczenia produkcyjne, finansowe i materiałowe.

Двухуровневая модель народного хозяйства Польши: связь между финансовой и производственной сферами

В работе представлена двухуровневая модель управления народного хозяйства. В модели предполагается существование финансовых регуляторов, которые введены в польскую экономику в результате текущей реформы. Модель подготовлена в качестве вспомогательного инструмента для принятия решений Центральным Органом Планирования, а также Директором Польского Народного Банка (Центром). Центр обладает средствами и базой, позволяющими воздействовать на отрасли народного хозяйства таким образом, чтобы отрасли функционировали согласно постановлениям Центра. Это достигается с помощью таких инструментов, как: налоги, кредиты по капиталовложениям и дотации.

На каждом уровне формулируется многокритериальная задача с ограничениями. Учитываются производственные, финансовые и материальные ограничения.