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A financial approach for operational investment

by

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In the paper we examine an operational investment problem under perfect market assumption. An application of a portfolio selection model to the description of the investment activity of a firm is presented. Allocation policy of a given sum of money among several production activities is given. Inventory products for stock evaluation are considered. Possible extensions are briefly discussed in which additional elements of uncertainty are included.

1. Introduction

The investment problem, with its financial and economic aspects plays a very important role in operational management. This paper presents a preliminary framework for investigation of an operational investment problem, which examines the portfolio model and looks at investments in both real and financial terms.

The investment problem can be of a long horizon, with several distinct sub-periods. Then, for instance, construction of a plant with long completion time, cannot bring money - profit in a short time. On the other hand many short-time investments made at a firm, e.g. purchasing of machines and tools, can be converted into money over a short period. The problem of supplying a firm with necessary raw materials belongs also to a class of short-time investment problems. Therefore, one can consider a firm acting on the market as similar to the individual investor.

Various conditions are examined in the paper which enable investigation of the operational investment problem under perfect market assumption.

2. General Conditions

We assume that the firm under consideration acts on a perfect market. Hence, the following conditions are imposed throughout the rest of the paper:

1) the firm is risk-averse and tends to maximize its profit at the end of the considered time period,
2) all firms act at a perfectly competitive market and have equal access to the same

homogeneous estimates of the return indices of goods: these return indices are assumed to be normally distributed,

3) a firm can invest, save or borrow, without any limit (at the market rate) for risk-free investments,
4) real goods can be bought or sold without limit and without price changes, but according to the following rules:
   a) the firm has to satisfy an internal minimal requirement for production,
   b) the firm may sell excess stock at the same price,
5) the global stock of goods available on the market is fixed, and goods are perfectly tradeable and divisible,
6) all firms share the same information that is freely available on the market,
7) the market is perfect and not constrained.

The above formulated assumptions are essential for the theory presented in this paper. Every firm should act accordingly to them. However, in the case of a particular firm they may differ. Therefore, if the considered models are applied to supporting the management decision of a firm they should be adjusted to satisfy the operational requirements.

We assume, as it is generally accepted in the financial theory, that the firm's objective is the maximization of its share price. Furthermore, the relevant structure of the firm is assumed, and hence the portfolio model can be applied for the description of its investment activity. The latter assumption implies that the maximization of the share price coincides with the minimization of the well-known mean--variance criterion.

3. Description of the model

In the paper we apply the approach of the classical financial theory. We assume the perfect market conditions, formulated in the previous section. Each firm is treated as a rational investor acting on the basis of its own utility function.

A firm is then an investor of available capital, in an amount W. It invests in raw material acquisitions.

Two additional assumptions are required for our presentation. The first one is that the profit rate of the capital does not depend upon the amount of the capital invested in the process. The second one is a linear dependence of the profits on the investments.

Then, the optium portfolio is the solution of the following quadratic programming problem (cf. Castellani [2]):

$$\max \left[\sum_{j=1}^{n} m_i x_i - \frac{1}{2a} \sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij} x_i x_j \right], \tag{3.1}$$

$$e^T x = 1, \tag{3.2}$$

$$x_i \geqslant 0 \quad (i=1, \ldots, n), \tag{3.3}$$

where:

a — positive constant; subjectively chosen according to the financial resources,

m_i — the expected value of the return rate of the i-th investment ($i=1, ..., n$),

σ_i — the variance of the return rate of the i-th investment ($i=1, ..., n$),

v_{ij} — (i,j) entry of the symmetric variance - covariance matrix $V(n \times n)$; $v_{ij}=\rho_{ij}\sigma_i\sigma_j$; ρ_{ij} represents the correlation coefficient,

x — a column vector with n components; x_i represents the proportion of the global quantity W, invested in the i-th material,

e^T — transposition of the column vector e with identical components equal 1.

The variance - covariance matrix V can be written as follows:

$$V=\begin{bmatrix} \sigma_1 & 0 & & \\ 0 & \sigma_2 & 0 & \\ & & \ddots & \\ & & 0 & \sigma_n \end{bmatrix} P \begin{bmatrix} \sigma_1 & 0 & & \\ 0 & \sigma_2 & 0 & \\ & & \ddots & \\ & & 0 & \sigma_n \end{bmatrix}, \tag{3.4}$$

where P is a symmetric matrix $(n \times n)$ with $p_{ij}=\rho_{ij}$. Let us assume that the det $V \neq 0$. Then V is positive definite and our problem has the unique optimal solution.

If the expected return on portfolio — π, is known, then the linear term in the utility function is negligible and our problem becomes (in matrix notation):

$$\min x^T Vx \tag{3.5}$$

s.t.

$$r^T x=\pi, \tag{3.6}$$

$$e^T x=1, \tag{3.7}$$

$$x \geqslant 0. \tag{3.8}$$

If the optimal solution satisfies nonnegativity constraints (3.8) as strict inequalities then the optimal solution can be found as the stationary point of the Lagrangean function:

$$L(x, \lambda_1, \lambda_2)=x^T Vx-\lambda_1 (r^T x-\pi)-\lambda_2 (e^T x-1), \tag{3.9}$$

i.e. as the solution of the following system of equations:

$$\frac{\partial L}{\partial x}=2Vx-\lambda_1 r-\lambda_2 e=0, \tag{3.10}$$

$$\frac{\partial L}{\partial \lambda_1}=r^T x-\pi=0, \tag{3.11}$$

$$\frac{\partial L}{\partial \lambda_2}e^T x-1=0. \tag{3.12}$$

It is a linear system of equations with $n+2$ unknown variables $x_1, x_2, ..., x_n, \lambda_1, \lambda_2$.

The model presented here is known in literature. First, it was developed by Markovitz. It was also studied by many other authors as Lintner, Sharp, Merton, Castellani and Szegö.

4. Model adjustment

In this section the model under consideration is slightly changed in order to adjust it to the operational framework of a firm. There are some differences in comparison with the ordinary financial market. The firm investing in raw materials buys first of all goods used in its own production. There exist some particular technical constraints: for example, the proportion of each material for each productis fixed.

However, goods purchases can be treated as financial investments because they have changing prices, similarly to the financial market assets. Furthermore, raw materials are bought or sold easily. It justifies the above introduced assumption of a perfect market. Goods represent for a firm both an instrumental good and a financial investment.

The portfolio model has to be redefined to take into account these two aspects: new assumptions and constraints are to be introduced. It makes it useful for a broader framework and transforms it into a decision-aiding tool for operational applications.

First of all, a new vector $z=(z_1, ..., z_i, ..., z_n)$ is defined. Each element z_i is the ratio of the difference between the quantity of the i-th raw material, necessary for production, and the quantity actually available in stock, to the global invested quantity W.

Each z_i can be either positive or negative, or be equal to zero. In the first case the firm cannot produce without new purchases.

Considering the problem of the maintenance of the stock for production we suppose that a firm choose safe strategy to avoid the risk of exhaustion.

It implies that $z_i \leqslant 0$, in order to satisfy production requirements.

Now we introduce the first change to the model (3.5–8). We substitute z for the null vector in the constraint (3.8).

A first important problem is the evaluation of the value of the vector z. It is a technical problem. For example Miller-Orr model can be applied for this purpose. It is a control theory model, based on a stochastic approach to the problem.

For some reasons it may be also useful to introduce an upper bound h on the vector of proportions x. Hence, finally the nonnegativity constraint (3.8) is replaced by:

$$z \leqslant x \leqslant h, \tag{4.1}$$

where:

h — upper bound on x_i resulting from stock capacity,

z — explained above; if $z<0$, the firm can sell as much as $|z| \cdot W$.

If the constraints (4.1) are satisfied as strict inequalities, the optimal solution x^* can be found from the system of equations (3.10–12). Then:

$$x^* = \frac{\pi\gamma-\beta}{\alpha\gamma-\beta^2} V^{-1} r + \frac{\alpha-\pi\beta}{\alpha\gamma-\beta^2} V^{-1} e, \tag{4.2}$$

and the optimal value of the functional 3.5:

$$v^*=(x^*)^T V(x^*)=\frac{\pi^2 \gamma-2\pi\beta+\alpha}{\alpha\gamma-\beta^2}. \tag{4.3}$$

However, x^* is generally not the optimal solution of the problem (3.5–7, 4.1). Then it is necessary to use a library subroutine realizing a quadratic programming algorithm and solve the problem numerically on a computer.

5. Concluding remarks

In this paper an application of a portfolio selection model to the description of the investment activity of a firm is presented. The problem is to find an allocation policy of a given sum W among several production activities. A very restrictive assumption of the perfect market is assumed.

As already mentioned, our model has been constructed to examine also inventory elements for stock evaluation: in the case of an uncertain framework this problem is particularly difficult to solve.

Furthermore, transaction and opportunity costs are relevant for a firm, and more realistic model should include them as the other uncertain elements.

However, this kind of model disturbances is not introduced into our considerations, because of the perfect market assumptions. It indicates the directions of the further research in the field of operational investments.

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Podejście finansowe do inwestycji operacyjnych

W niniejszej pracy analizuje się zagadnienie inwestycji operacyjnych przy założeniu działania w warunkach idealnego rynku. Przedstawiono zastosowanie modelu wyboru portfela dla opisu działalności inwestycyjnej firmy. Określono politykę rozdziału danej sumy pieniędzy pomiędzy wiele rodzajów działalności produkcyjnej. Rozważono problem zakupu do magazynu produktów zapasowych niezbędnych dla utrzymania produkcji. Omówiono w skrócie możliwe rozszerzenie zakresu badań, co pozwoliłoby na uwzględnienie dodatkowych elementów niepewności.

Финансовый подход к операционным инвестициям

В работе анализируется проблему операционных инвестиций исходя из предположения деятельности в условиях идеального рынка. Показано применение „модели выбора портфеля" для описания инвестиционной деятельности фирмы. Определено политику распределения данной суммы денег между многие роды продукционной деятельности. Рассмотрено проблему закупки на склад запасных продуктов необходимых для поддержки продукции. Представлено вкратце возможное расширение области исследований, что сделало бы возможным учёт добавочных элементов неуверенности.