Control
and Cybernetics

VOL. 15 (1986) No. 3-4

A class of VES production functions: properties and estimation results

by

WITOLD CHMIELARZ

Management Faculty of Warsaw University
Szturmowa 3
02-678 Warsaw, Poland

ANDRZEJ STACHURSKI

Systems Research Institute
Polish Academy of Sciences
Newelska 6
01-447 Warsaw, Poland

In this paper a new class of VES production functions is presented. In distinction to the usual approach at the first step the production function formula was constructed which posesses some desired properties. It has a simple form of a sum of linear and nonlinear terms. It is proved that the usual neoclassical requirements for production function are satisfied. The resulting formula on coefficient of elasticity of substitutionis derived at the second step from the assumed production function formula. The minimization method proposed for minimizing the least squares functional is briefly described.

The regression estimates for several sectors of Polish and West German national economies are presented.

1. Introduction

In this paper a new form of VES (variable elasticity of substitution) production function is introduced. It is proved that the commonly known conditions on production functions (see Barkalov, 1981) are satisfied.

Our first attempt was to evaluate the parameters of CES (constant elasticity of substitution) production function in the form used by Raduchel (1979). The results were unsatisfactory. It was done with Polish the national economy data for years 1970–78.

The reasons could be as follows:

i) strong nonlinearities in CES production function,

ii) the assumption of constant elasticity of substitution (in Polish economy of this period quick structural changes were observed).

Because of these reasons an attempt was made to construct the simplest form of production function characterized by variable elasticity of substitution. The simplicity of the introduced formula is desired from the computational point of view. The simpler is the formula, the easier is the minimization of the resulting mean squares functional. The computational needs suggested the formula in the form of two terms — the linear one and the nonlinear one, which allows to reduce, in a sense, the size of the nonlinear regression problem in this case.

So an original approach was adopted. In the first step the production function formula is constructed and then the formula on the coefficient of substitution is derived. In the standard approach the latter dependence is assumed at the first step and at the second step the production function formula is constructed (see e.g. Revankar (1971) or Sato and Hoffman (1968)).

In the next section it is proved that the introduced formula satisfies commonly known conditions on production functions under appropriate assumptions on parameter values. Furthermore, the asymptotic behaviour of this function is discussed.

The results of parameter estimation with annual data for several sectors of Polish and West German national economies are presented. A brief description of the computational method used is also introduced in this paper.

2. Analysis of conditions on production functions

Now, contrary to the usual approach, the formula of production function is first constructed. We try to obtain the simplest formula providing variable elasticity of substitution.

The following "technical" form of production function is proposed:

$$F(K,L)=x_1 K+x_2 L-\sqrt{x_3 K^2+x_4 L^2}\,, \tag{1}$$

where:

K — capital,

L — labour,

x_1, x_2, x_3, x_4 — coefficients to be estimated on the data basis.

The following constraints on the parameters x_1, x_2, x_3, x_4 are assumed:

$$x_3, x_4>0\,, \tag{2}$$

$$x_1\geqslant\sqrt{x_3}\,, \tag{3}$$

$$x_2\geqslant\sqrt{x_4}\,, \tag{4}$$

It is easy to prove that under the above conditions the usual requirements on production functions (taken after Berkalov (1981)) are satisfied:

i) The values of $Y=F(K,L)$ and its first partial derivatives should be positive for all $K, L>0$

$$F(K,L)>0\,, \quad \forall\, K,L>0\,, \tag{5}$$

$$r=\frac{\partial Y}{\partial K}=x_1-\frac{x_3}{\sqrt{x_3+x_4\left(\frac{L}{K}\right)^2}}>0, \tag{6}$$

$$w=\frac{\partial Y}{\partial L}=x_2-\frac{x_4}{\sqrt{x_3\left(\frac{K}{L}\right)^2+x_4}}>0. \tag{7}$$

ii) Conditions on coordinate concavity:

$$r_2=\frac{\partial^2 Y}{\partial K^2}=-\frac{x_3 x_4 L^2}{(x_3 K^2+x_4 L^2)^{3/2}}<0 \quad \text{for all} \quad K, L>0, \tag{8}$$

$$w_2=\frac{\partial^2 Y}{\partial L^2}=-\frac{x_3 x_4 K^2}{(x_3 K^2+x_4 L^2)^{3/2}}<0 \quad \text{for all} \quad K, L>0. \tag{9}$$

iii) The homogeneity condition:

$$F\big(\lambda (K, L)\big)=\lambda F(K, L) \quad \text{for all} \quad K, L>0. \tag{10}$$

Function (1) is homogeneous of degree 1. This is a bit restrictive but justified by our aim to simplify the function.

iv) Asymptotic properties of production function.

At this point the asymptotic behaviour of (1), when $K\to 0$, $L\to 0$, $K\to\infty$, $L\to\infty$ is studied. The results obtained are collected in Table 1. One can observe that when constraints (3–4) are satisfied as equalities, function (1) satisfies asymptotic behaviour conditions which are often required of production function formulae. The results collected in Table 1 are consistent with the general results for production functions obtained by Inada (see Otani (1970), cited also in Barkalov (1981)).

During the estimation process conditions (3–4) were treated as inequalities. However, for all cases, for which correct results were obtained with Polish economy data, these conditions are satisfied almost equally.

The first line of Table 1 indicates that for these conditions, together with the unconstrained growth of one of the production factors (with the second one kept constant) production stabilizes at the defined level.

Table 1. Asymptotic values of VES production function for different cases of parameter values

Parameter values		$\lim_{K\to 0+} F(K, L)$	$\lim_{L\to 0+} F(K, L)$	$\lim_{K\to\infty} F(K, L)$	$\lim_{L\to\infty} F(K, L)$
$x_1=\sqrt{x_3}$	$x_2=\sqrt{x_4}$	0	0	$\frac{2x_1 x_2 L}{x_1+\sqrt{x_3}}$	$\frac{2x_1 x_2 K}{x_2+\sqrt{x_4}}$
$x_1=\sqrt{x_3}$	$x_2>\sqrt{x_4}$	$(x_2-\sqrt{x_4})L$	0	$\frac{2x_1 x_2 L}{x_1+\sqrt{x_3}}$	$+\infty$
$x_1>\sqrt{x_3}$	$x_2=\sqrt{x_4}$	0	$x_1-\sqrt{x_3}\,K$	$+\infty$	$\frac{2x_1 x_2 K}{x_2+\sqrt{x_4}}$
$x_1>\sqrt{x_3}$	$x_2>\sqrt{x_4}$	$(x_2-\sqrt{x_4})L$	$(x_1-\sqrt{x_3})K$	$+\infty$	$+\infty$

3. Elasticity of substitution

The elasticity of substitution σ is calculated as a function of the technical equipment of labour z. From the definition of the elasticity of substitution (see Allen, 1970) it follows that:

$$\sigma(z)=\frac{\sqrt{x_3 z^2+x_4}\,(x_1\sqrt{x_3 z^2+x_4}-x_3 z)(x_2\sqrt{x_3 z^2+x_4}-x_4)}{z x_3 x_4 (x_1 z+x_2-\sqrt{x_3 z^2+x_4})}, \tag{11}$$

where:

$$z=\frac{K}{L}. \tag{12}$$

$\sigma(z)$ has positive values for each $z>0$ if conditions (2–4) are satisfied. The proof of this fact is divided into two parts. The denominator in (11) can be written as:

$$M=z x_3 x_4 F(z, 1). \tag{13}$$

$F(z, 1)>0$ since (2–4) hold. Hence M will be positive. Now the numerator in (11) is investigated:

$$N=\sqrt{x_3 z^2+x_4}\,(x_1\sqrt{x_3 z^2+x_4}-x_3 z)(x_2\sqrt{x_3 z^2+x_4}-x_4) \tag{14}$$

It should be proved that the two last factors of N are positive. Inequality

$$x_1\sqrt{x_3 z^2+x_4}-x_3 z>0 \tag{15}$$

can be equivalently transformed into:

$$\sqrt{\frac{x_1^2}{x_3}z^2+\frac{x_4 x_1^2}{x_3^2}}-z>0. \tag{16}$$

It is possible if $x_1, x_3, x_4>0$. Since $x_1\geqslant\sqrt{x_3}$ it implies that (16) holds for all $z>0$.

Analogically inequality

$$x_2\sqrt{x_3 z^2+x_4}-x_4>0 \tag{17}$$

is equivalent to:

$$\sqrt{\frac{x_3 x_2^2}{x_4^2}z^2+\frac{x_2^2}{x_4}}>1, \tag{18}$$

if $x_2, x_3, x_4>0$.

Thus the next assumption implies that (18) holds for all $z\neq 0$. One can conclude that $\sigma(z)>0$ for all $z>0$, when conditions (3–4) are satisfied (inequalities (2) are sharp).

In this paper we neglected the problem of economical interpretation of the parameters of the introduced model. Our main purpose was to overcome the computational difficulties with parameter estimation and to present the results obtained.

An attempt in this direction was made by Chmielarz and Stachurski (1983), although up till now we are not fully satisfied with it. Let us present it briefly.

The following transformed production function was used:

$$F(K, L) = \alpha\,[\delta K + (1-\delta)\,L - \beta\,\sqrt{\omega K^2 + (1-\omega)\,L^2}\,] \tag{19}$$

where the following identities hold:

$$x_1 = \alpha\delta, \tag{20}$$

$$x_2 = \alpha\,(1-\delta), \tag{21}$$

$$x_3 = \alpha^2\,\beta^2\,\omega, \tag{22}$$

$$x_4 = \alpha^2\,\beta^2\,(1-\omega). \tag{23}$$

The parameter β was called "substitution parameter" since the elasticity of substitution δ and β are correlative: an increase of the value of β causes the decrease of the elasticity of substitution (under the assumption that α, δ and ω are constant). δ and ω were simply called parameters of labour and capital participation in production, appearing in linear and nonlinear parts of production function (19) accordingly.

For parameter α a little more doubtful name "efficiency parameter" was adopted.

4. Brief description of parameter estimation method

For estimation purposes the usual least squares method is used. Therefore the resulting minimization problem is:

$$\min_{x=(x_1, x_2, x_3, x_4)} \quad f(x) = \sum_{i=1}^{NT} \,[Y_i - x_1\,K_i - x_2\,L_i + \sqrt{x_3\,K^2 + x_4\,L^2}\,]^2 \tag{24}$$

subject to:

$$x_3,\, x_4 \geqslant 0\,, \tag{25}$$

$$x_1 \geqslant \sqrt{x_3}\,, \tag{26}$$

$$x_2 \geqslant \sqrt{x_4}\,, \tag{27}$$

where NT denotes the number of observations (data) of production Y and production factors K and L. Constraints (25–27) are implied by the conditions on production function.

A special, two-stage optimization algorithm making use of the specific structure of the problem is proposed (see Figure 1).

The upper-level problem is solved by means of the Hook-Jeeves (1961) method with discrete step (see also Bazaraa and Shetty (1979)), which belongs to the group of simple search methods. A simple modification was introduced into it: a great number is taken as the functional f_g value when x_3 or x_4 is negative. Generally, this way of taking constraints into account is not recommended, but the Hook-Jeeves method is here used only for local optimization. Furthermore the boundary of the

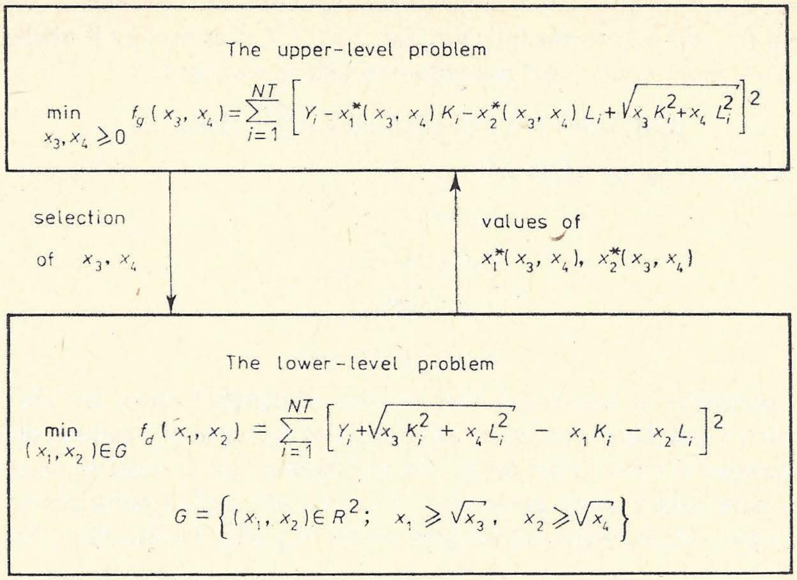

Fig. 1. Two-level optimization scheme for parameter estimation of VES production function

feasible set $\{(x_3, x_4) \in R^2;\ x_3, x_4 \geq 0\}$ is an isoquant of the functional f_g. The starting point was found by an introductory search on the simple net. We accepted the starting point (x_3^0, x_4^0) if it belonged to the interior of the feasible set and $f_g\,(x_3^0, x_4^0)$ was lower than the f_g value on the boundary — $\{(x_3, x_4) \in R^2;\ x_3=0 \text{ or } x_4=0\}$ (the second one is equal to the corresponding optimal value of the least squares functional for the linear model).

The lower-level problem with fixed values of (x_3, x_4) is a quadratic programming problem with lower bounds on variables. Its Hessian is the variance-covariance matrix with positive entries in our case, which is positive definite if the linear independance of observations is assumed (see, for instance Pawłowski (1969)). A special algorithm using the properties of the lower-level problem was applied for finding its optimal solution.

The aim of such division of independent variables was to reduce the total computational effort. It allows to use the specific properties of the problem and to reduce its size. At the upper level the parameters appearing nonlinearly in formula (1) were used while at the lower level the parameters which appear linearly.

5. Some estimation results for the VES production function

5.1. Data set description

The estimation was carried out on the basis of annual data for Polish and West German economies. The data for Polish economy for consecutive years 1970–1978 were taken from the official Statistical Yearbooks 1971–1979. Division of Polish national economy into the following nine sectors was assumed:

1) energy,
2) metal and machinery,
3) chemistry,
4) construction materials,
5) forestry and timber,
6) light manufacturing,
7) food and agriculture,
8) construction,
9) services.

The data for West German economy for consecutive years 1950–1974 were taken from Oest (1979) and from other materials obtained from Oest. The following division of West German national economy into nineteen sectors (as in Oest (1979)) was assumed:

1) agriculture,
2) energy,
3) mining,
4) construction materials,
5) food,
6) textile industry,
7) garment industry,
8) timber, paper and print,
9) chemistry,
10) metallurgy,
11) metal industry,
12) transport machinery,
13) machine construction,
14) construction,
15) trade,
16) transportation,
17) housing,
18) governmental sector,
19) services.

The capital stock values for West Germany were taken from Table 5 in Oest, 1979. At the same source sectoral capital coefficients were found in Table 7 ($q_i = K_i/Y_i$ for each year i). Hence, by means of tables 5 and 7 the values of production were obtained. The labour values were taken from another table obtained independently from Oest.

The capital stock and production values were given in constant prices: for Poland in prices from 1977 and for West Germany from 1962.

In the computational process the following units were assumed:

i) for Polish economy:
   — capital stocks and production — 1000 million zlotys,
   — labour — 1000 million zlotys (the number of employees in each sector was multiplied by the average wage per year),

ii) for West German economy:
   — capital stocks and production — 1000 million DM,
   — labour — 1 million of employees (the scale was chosen to obtain the values numerically comparable with the values of the capital stock).

5.2. Introductory examination of data

At the beginning of calculations an introductory examination of the available data set was carried out for each sector independently. The linear regression parameters were estimated, i.e. for the following model:

$$Y=x_1\, K+x_2\, L. \tag{28}$$

For this purpose the ordinary least squares method was used. The estimated parameter values are collected in Table 2, which contains also:
— the standard deviation S_L of the linear regression estimation:

$$S_2^L=\left(\sum_{i=1}^{NT} Y_i-\tilde{Y}_i)^2\right)\Big/(NT-2), \tag{29}$$

where:

$$\tilde{Y}_i=\tilde{x}_1\, K_i+\tilde{x}_2\, L_i, \quad i=1, \ldots, NT, \tag{30}$$

— sample correlation coefficient $r_{Y\tilde{Y}}$ between the observed Y and the estimated $\tilde{Y}$ values of production (see (30)):

$$r_{Y\tilde{Y}}=\frac{\dfrac{1}{NT-1}\displaystyle\sum_{i=1}^{NT}(Y_i-\bar{Y})(\tilde{Y}_i-\bar{\tilde{Y}})}{\left[\dfrac{1}{NT-1}\displaystyle\sum_{i=1}^{NT}(Y_i-\bar{Y})^2\right]^{1/2}\left[\dfrac{1}{NT-1}\displaystyle\sum_{i=1}^{NT}(\tilde{Y}_i-\bar{\tilde{Y}})^2\right]^{1/2}} \tag{31}$$

where:

$$\bar{Y}=\frac{1}{NT-1}\sum_{i=1}^{NT} Y_i, \quad \bar{\tilde{Y}}=\frac{1}{NT-1}\sum_{i=1}^{NT} \tilde{Y}_i. \tag{32}$$

For sectors: 2–6 and 9 of Polish economy and 4, 7–9, 13 and 16–17 of West German economy the introductory search mentioned already in section 4 failed. We have not found any feasible point with lower value of mean squares functional than the corresponding optimal value for the linear model. For some of them the linear model is very good one (the coefficient $r_{Y\tilde{Y}}$ is almost equal to one). There exist also sectors with negative values of $\tilde{x}_1$ or $\tilde{x}_2$. In all cases it is hardly expected that a nonlinear concave model could be better.

Deeper analysis of data set was neglected since our main purpose was to overcome the existing difficulties in parameter estimation for production functions.

Table 2. Results of linear regression analysis

Number of sector	Parameters		S_L	r_{YY}
	x_1	x_2		
	Polish economy			
1	0.5286	0.1433	43.2560	0.6052
2	1.0515	3.2219	11.7878	0.9957
3	0.4435	8.9837	22.9054	0.6170
4	1.1468	−2.8110	7.4611	0.6644
5	1.4953	−0.9192	10.2603	0.5541
6	5.7397	−7.3263	20.0781	0.5349
7	0.4670	2.0830	54.6886	0.9766
8	1.1549	3.6914	14.8395	0.9922
9	0.4539	0.3623	54.2433	0.6009
	West German economy			
1	0.1100	2.9891	0.9438	0.5337
2	0.1431	5.3642	0.7852	0.5918
3	0.1598	14.2941	0.3391	0.7566
4	0.6317	−0.3130	0.5033	0.7987
5	0.7147	3.5744	1.2528	0.6469
6	0.3243	3.1909	0.6048	0.6537
7	0.5428	7.0567	0.4673	0.9672
8	0.6432	0.0467	0.7793	0.7840
9	0.6910	−4.2260	1.6860	0.4820
10	0.2790	2.1349	1.7342	0.6430
11	0.2689	10.9437	1.4759	0.5503
12	1.2049	0.9768	1.0276	0.6168
13	0,6781	2.1305	2.5764	0.7881
14	0.6076	8.7171	3.2751	0.6530
15	0.3967	6.3090	3.2501	0.6401
16	0.2542	−3.4700	1.4565	0.8135
17	0.0363	−18.4086	0.4723	0.6518
18	0.0324	10.2606	1.3789	0.6437
19	0.2324	5.6606	4.5158	0.6626

5.6. Description of the estimation results

In this section only these sectors are considered for which the introductory search was successful. The starting values (x_3^0, x_4^0) were found. For these sectors the calculations were successfully continued by means of the Hook-Jeeves method at the upper-level problem (described in section 4). Table 3 contains the estimated parameter values $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4$ and furthermore:

— the standard deviation S of the nonlinear estimation

$$S^2 = \frac{1}{NT-4} \sum_{i=1}^{NT} (Y_i - \hat{Y}_i)^2 \tag{33}$$

and

$$\hat{Y}_i = \hat{x}_1 K_i + \hat{x}_2 L_i - \sqrt{\hat{x}_3 K_i^2 + \hat{x}_4 L_i^2}, \quad i=1, ..., NT. \tag{34}$$

— sample correlation coefficient $r_{Y\hat{Y}}$ calculated according to the formula (31) with $\tilde{Y}$ replaced by $\hat{Y}$.
— values of the t-Student statistic, i.e.:

$$t_0 = \frac{\bar{z}}{S_z} \sqrt{NT-1}, \tag{35}$$

where:

$$z_i = Y_i - \hat{Y}_i; \quad \bar{z} = \frac{1}{NT} \sum_{i=1}^{NT} z_i; \quad S_z^2 = \frac{1}{NT-1} \Big[\sum_{i=1}^{NT} z_i^2 - \frac{1}{NT} \Big(\sum_{i=1}^{NT} z_i\Big)^2\Big].$$

This last value allows to test the hypothesis that the expectation of the error $z = Y - \hat{Y}$ is equal to zero under the assumption that the error distribution is normal (see Afifi and Azen (1979)).

Comparison of Table 2 with Table 3 shows that for each sector represented in Table 3 the sample correlation coefficients $r_{Y\hat{Y}}$ are positive and greater than the corresponding $r_{Y\tilde{Y}}$. So one can conclude that for these sectors the optimal nonlinear model (1) is better than the optimal linear one (formula (28)). For some of them it is indeed very good (especially for some sectors of West German economy and for the first sector of Polish economy) — the value of $r_{Y\hat{Y}}$ is nearly one.

Table 3. Results of nonlinear regression analysis

Number of sector	x_1	x_2	x_3	x_4	S	$r_{Y\hat{Y}}$	t_0
			Polish economy				
1	0.6689	23.8369	0.4474	568.2000	48.0001	0.9958	−0.3548
7	1.4714	7.8447	2.1650	61.5400	64.7094	0.9766	−0.6094
8	5.7871	9.3978	33.4912	88.3190	15.3717	0.9923	−0.0608
			West Germany economy				
1	1.0763	3.5025	0.9489	12.2679	0,9688	0.8072	0.0137
2	0.5106	5.9656	0.1360	35.5880	0.8203	0.8219	−0.2636
3	0.6583	54.1578	0.4337	2933.0641	0.2688	0.8569	−0.2725
5	0.8896	193.6651	0.7915	37506.1572	1.2626	0.8368	−0.0899
6	0.7056	10.6617	0.2639	113.6737	0.5828	0.6982	−0.4611
10	0.3745	76.0985	0.1402	5790.9761	1.7361	0.8340	−0.5427
11	1.4770	27.0297	2.1813	730.6049	1.4549	0.7471	−0.2961
12	1.4430	92.6200	2.0823	8578.4669	0.9994	0.8204	−0.6609
14	3.3102	18.5556	9.2405	344.3123	1.9044	0.8139	−0.3465
15	1.1996	28.0722	1.4391	788.0503	1.6378	0.8117	−1.5507
18	0.7263	14.9358	0.5275	223.0770	0.9293	0.8199	0.2250
19	0.7004	24.0362	0.4905	577.7413	4.0445	0.8461	−0.5903

Let us furthermore present the results of testing the hypothesis H_0 that the expectation of $Z=Y-\hat{Y}$ is equal to zero. For this purpose it is assumed that z has a normal distribution with unknown σ^2. Then t determined by formula (35) has Student t-distribution (see Afifi and Azen (1979)) with $NT-1$ degrees of freedom. Let $\theta=E(z)$ be the expectation of z. H_0: $\theta=0$. As the alternative hypothesis $H_1: \theta \neq 0$ is used. Then P-value is

$$P=2Pr\left(t(\nu)>|t_0|\right). \tag{36}$$

Hypothesis H_0 is rejected if $P<\lambda$, where λ denotes the level of significance. Instead of (36) one can check equivalently whether t_0 belongs to the interval:

$$\left(-t_{1-\lambda/2}(NT-1),\, t_{1-\lambda/2}(NT-1)\right), \tag{37}$$

where $t_{1-\lambda/2}(NT-1)$ denotes the critical value with the level of significance equal λ.

For Polish economy with $NT=9$ and the level of significance $\lambda=0.1$ the critical value is

$$t_{1-\lambda/2}(8)=1.833$$

and for West German economy with $NT=25$ and $\lambda=0.1$ one obtains accordingly:

$$t_{1-\lambda/2}(24)=1.711.$$

Hence with the level of significance $\lambda=0.1$ for each of the sectors represented in Table 3 it is justified to assume that hypothesis H_0 is true.

The comparison of standard deviations S and S_L is not so optimistic. For all three Polish economy sectors and sectors 1, 2, 5 and 9 of West German economy the value of S_L is lower than S. The corresponding mean squares functional values were lower for the nonlinear model. But in this case we have more parameters — 4 while in the linear case only 2.

Table 3 contains only the estimated values $\hat{x}_i$ $i=1, 2, 3, 4$. Let us quote for some sectors the corresponding $\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\omega}$ (see section 3 of this paper):

— construction sector of Polish economy

$$\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\omega}=(15.1849, 0.7268, 0.3811, 0.2749)$$

— construction sector of West German economy:

$$\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\omega}=(21.8658, 0.8599, 0.1514, 0.0261).$$

The parameter $\hat{\alpha}$ is for some sectors greater than one. One could recognize it as an error. Recall though, the units in which production factors are measured. If these units are not appropriately chosen then it is obvious that $\hat{\alpha}$ can be greater than one.

The values of the elasticity of substitution obtained are not constant. This justifies our assumption of variable elasticity of substitution for the analysed data.

Our purpose was to overcome the difficulties connected with parameter estimation for production functions. We chose an original way of attaining our aim. A new form of production function with variable elasticity of substitution is constructed starting from the assumed form of VES production function, and not from the formula on the elasticity of substitution as is done usually. At the same time we took into account the future computational method for solving the resulting least squares problem. Our approach seems to be successful. We obtained acceptable estimates for three sectors of Polish economy (from nine considered) and for twelve sectors of West German economy (from nineteen considered). Let us notice for instance that the investigations with CES production function carried out by Tylec and Woroniecka (1981) on the same data from Polish economy were worse. The obtained estimation results were correct only for the construction sector.

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Pewna klasa funkcji produkcji VES: właściwości i wyniki estymacji

W niniejszej pracy przedstawiono nową klasę funkcji produkcji VES. W przeciwieństwie do zwykle stosowanego podejścia w pierwszym kroku skonstruowano funkcję produkcji posiadającą pewne pożądane własności. Ma ona prostą postać sumy składnika liniowego. Udowodniono, że spełnia ona zwykłe neoklasyczne wymagania nakładane na funkcje produkcji. Wynikowy wzór na stopę elastyczności substytucji jest wyznaczany w drugim kroku z założonej postaci funkcji produkcji. Opisano w skrócie metodę minimalizacji zaproponowaną dla minimalizowania funkcjonału najmniejszych kwadratów.

Przedstawiono wyniki estymacji parametrów wprowadzonej funkcji produkcji dla kilkunastu sektorów gospodarki narodowej Polski i RFN.

Один класс производственных функций VES: свойства и результаты оценки

В данной работе представлен новый класс производственных функций типа VES. Противоположно к обще применяемому подходу в первом шаге построена формула производственной функции, обладающая некоторыми желаемыми свойствами. У нее простой вид линейного и нелинейного слагаемого. Показано, что она удовлетворяет обычным неоклассическим требованиям к производственным функциям. Результирующая формула нормы эластичности субституции получена на втором шаге из принятой формулы производственной функции. Описан вкратце метод минимизации, предложенный для нахождения минимума функционала наименьших квадратов.

Представлены результаты оценки параметров введенной производственной функции для нескольких секторов народной экономики Польши и ФРГ.