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Interactions between operations research and decision making under uncertainty in economic modelling

by

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The objective of this paper is to present and discuss some aspects of decision analysis in economics, from view-points of operations research and decision making under uncertainty. Our considerations are based on the use of probability distribution function as a quantitative measure of economic events uncertainty. Certainty, risk and uncertainty are assumed to be typical situations in decision processes. Some interactions between physics and economics are described, and uncertainty is shown to be an objective property of economic processes. Operations research is presented as the discipline which provides mathematical apparatus adequate to model and analyse quantitative decision problems under certainty or risk, whereas the decision making under uncertainty theory is introduced as preferring decision situations under non-measurable uncertainty.

1. Introduction

There are considerable difficulties in writing about operations research (OR) and decision making under uncertainty (DMUU) in the context of economic systems modelling. The problem is that both these disciplines are relatively young sciences and so far their boundaries and contents can hardly be deemed rigorously defined. In consequence, in the light of our current knowledge, one cannot probably draw a distinct line between OR and DMUU. Nevertheless, when we talk about OR and DMUU theories, we first of all have to remember that although each of them deals with decisions, it would be a mistake to believe that DMUU falls within the OR domain or conversely. Whereas in OR the trend is to increase the part of problems which require mathematical treatment for a decision to be made, in the case of DMUU, when arriving at a choice, the decision-maker, individual or collective, may take into account other reasons, quantitative or qualitative in nature, not considered in preparing mathematical considerations.

In recent years there has been a wide discussion of methodological problems of OR and DMUU theories (see, for instance, Winkler, 1982; Fishburn, 1982; Keeney, 1982; Pidd, 1985; Wilson, 1985), but a particular question that remains concerns interrelations between the disciplines. It is the purpose of this paper to discuss some aspects of the problem of the applicability of OR and DMUU to economics from the decision making point of view.

2. Determinism and stochasticity in economics

There are two fundamental categories of decision situations known both in the economic science and other disciplines: certainty (determinism) and uncertainty (stochasticity). When the decision-maker has a complete knowledge of the consequences that will follow each action, the decision is certain. When he has not an accurate knowledge about the consequences following each act, the decision is said to be made under uncertainty. In this context it is self-evident that the concept of uncertainty is used to cover a broad class of possible decision situations which one can distinguish between classical determinism (certainty) and complete chaos. From theoretical and practical point of view it would be very useful to be able at least roughly identify the most typical kinds of uncertainty in economi problems. Borch (1968) was apparently the first to elaborate a classification of uncertainty situations accompaned with real economic conditions. It is our intention here to propose a more general classification including that by Borch.

It is well known, that in stochastic mathematics the basic models of uncertain quantities are random variables or random functions. Since any random quantity is completely characterized by its probability distribution function, the decisions under uncertainty fall into one of two categories depending upon the probability distribution function exists or does not exist. When the decision-maker has an accurate knowledge about the probability distribution of the consequences following each act, the decision is said to be made under objective uncertainty or under risk. When he cannot assess definite probabilities to the consequences one says the decision is made under subjective uncertainty.

For a decision-maker to be able to choose the best or at least sufficiently good decision among all possible decisions, he must have the rules of selection and be guided by them. All the decision rules presuppose a determination of the values of the possible consequences following each choice. Most of the decision rules also demand judgments of the likelihood that the different consequences will occur. What can be gathered from this is that the proper process of decision choosing should be proceed by a process of determination of a utility function or a preference scale, according to which the decision-maker can rank the desirability of the possible consequences. In a number of cases it is possible to indicate the strategy for making the best decision, i.e. a choice of the action which leads to the most desired results.

Under other circumstances, there is no such strategy, but there are certain recommendations on how to pose questions in a more reasonable manner, how to construct a suitable mathematical model of the situation, how to study the model, etc.

Uncertainty is an objective feature of economic systems. There is uncertainty about the present state of the system, uncertainty about the response of the system to policy measures, and uncertainty about future events (Kendrick, 1981). If all uncertainty vanishes, we have a deterministic system, but determinism is too rough approximation in economic problems. Unfortunately, though it is well known that real economic systems are dynamic, nonlinear and stochastic, their mathematical models which have been developed in recent decades are at least dynamic and nonlinear, but mostly deterministic. However, while talking about deterministic economic models, one should remember that classical determinism assumes an unambiguous and inflexible link between present and future, in the same way as between past and present. In other words, classical determinism hypothesizes that if the state of an economic system as well as all the actions applied to it are known at some instant of time, we can precisely predict the state of the system at any subsequent instant. But is the assumption of deterministic economies realistic? The key to understanding of the mode of operation of an economic system is to be found in the economic laws resulting from the relations of production, distribution exchange and consumption specific to that system. Economic laws are continually repeated relationship between various elements of economic processes, they operate in the course of those processes. Economic laws are objective (i.e. they operate independently of human will and consciousness) and stochastic (i.e. the law of large numbers constitutes the specific form in which they manifest themselves). The knowledge of economic laws ensures planned and balanced development of the economy. The control of the operation of economic laws consists in the creation of consitions in which social causes set in movement by men produce the results intended by men (Lange, 1963).

To understand the behavior of the economic systems we resort to mathematical models. Each of economic models is only an approximation to reality and provides a simplification that can be useful in appropriate circumstances to the study of the economic processes. Different models may be appropriate for different aspects of economy but a major problem is to develop the stochastic view in economics. Meanwhile, it has been a long tradition in economics that models are deterministic. What are the reasons of the deterministic way of thinking in economics? In a very interesting paper Tintner (1975), one of the pioneers of the stochastic approach to economic processes and systems modelling, we find the following explanation: "Deterministic models ought to be interpreted as dealing with the mathematical expectations (or other measures of central tendency) of the stochastic processes which really make up the true description of any given economy. The relationship is about the same; as with the relationship between classical deterministic and modern stochastic phycics. If deterministic physical relations are still usefull, e.g. in engineering applications, this is due to some law of large numbers. But since we typically have a very small number of observations, it is somewhat doubtful if the conditions of the law of large numbers are fulfilled with economic data". In the context of these remarks Tintner proposes to follow the example of modern physics and consider economic phenomena from a stochastic point of view, i.e. to treat economic variables as random quantities.

A transition from deterministic to stochastic economics presupposes a rejection of the basic ideas of classical physics. It is known, that from the view-point of classical physics there are two primitive dynamic variables of every physical phenomena: energy and entropy. In classical physics we make use of the laws of conservation of energy and momentum. The law of conservation of energy is a consequence of homogenity of time (independence of the course of a physical process of the moment choosen as the starting point of the process). The law of conservation of momentum is a consequence of the uniformity of space (all point in space are physically equivalent). In other words, energy and momentum can be considered as integrals of motion, whose conservation is a consequence of the corresponding homogenity of time and the uniformity of space. From the point of view of classical physics economic processes are known to have individuality since it is always possible in principle to enumerate them and observe the behaviour of any of them. In this case, however alike two economic processes may be, they are never identical and can always be distinguished. In contradiction to classic physics, in modern physics two microeconomic processes of the same type should be treated as absolutely identical. It is understood that the identity of microeconomic processes does not exclude the possibility of their differentiation on the basis of different states in which these processes may be found. The rejection of the classical individualization of an economic process as a physical object is quite fundamental. Although the concepts of energy and momentum as basic dynamic variables of economic processes are carried over to stochastic economics, nevertheless these concepts are now seen differently with a reconsideration of the previous interconnections, and the limitations imposed by the uncertaintly relations.

It seems that modern physics should be treated as one of the theoretical foundations of the stochastic economics. In comparison with deterministic economics, stochastic economics considers the properties of economic processes on a deeper and more fundamental level. In Bereziński (1980) some main concepts and ideas of statistical physics in application to economic systems modelling are presented. The concepts of an economic system phase state, phase space, inertia and storage are introduced and the Gibs distribution as a mathematical tool for analyzing probabilistic-statistical properties of phase space is discussed. Economic system development is considered to be open irreversible process and two types of its characteristics are distinguished, extensive and intensive ones. Entropy, energy and information are regarded as basic, primitive parameters of economic systems. The laws of energy conservation and entropy production in terms of economic systems are formulated and discussed. As a result a methematical stochastic model of the economics system is constructed in which the economy is considered to be a physical system influenced by random factors, with two input flows represented by stream of energomatter and that of organization. In the economic system processes of synergy and dissipation of the energy take place. The economic system momentum is observed as useful output of the system.

3. Operations research and decision making

A number of areas of mathematics are closely related to problems of decision making, but in spite of many succesful applications of mathematical methods to different decision problems the general methodology of the decision making processes still remaind a problem to be solved. There is a growing interest in mathematical models of economic systems under uncertainty as well as in using specialized computer systems as tools supporting analysis and planning of economic phenomena.

While talking about decision making methodology in the context of economic problems, let us remember that the decision situations can be categorized into three groups: quantitative, qualitative and mixed (both qualitative and quantitative) OR is the discipline which provides mathematical apparatus adequate to model and analyse quantitative decision problems. If the relations between variables in decision problem are expressed qualitatively only, then the appropriate approach is heuristic programming. The focus in heuristic approach is in gaining knowledge from an expert or group of experts. In the third situation, when we deal with qualitative and quantitative relations, the systems analysis methods have to be used. The systems approach combines both the mathematical methods and heuristic programming, and assumes uncertainty to be an inherent property of decision situation and its environment.

Operations research is an area of great potential for decision making in economic systems. Referring to the term OR one implies the application of quantitative, mathematical methods to prepare optimal decisions, by finding optimal solutions for OR problems. According to Kleitman (1982), a typical problem in OR has the following aspects:

1) The situation, in which there are various limitations what can be done, and various merits and demerits of allowable courses of action;
2) A model, in which variables are introduced, the limitations reduced to constraint equations or inequalities, and the benefits described by an objective function to be maximized;
3) Techniques, for finding solutions to the "mathematical program" defined by the model, which hopefully yield recommended actions.

The ingredients of a traditional procedure for OR problems solving are the following: (1) an individual who can be identified over time; (2) a series of environments; (3) in each environment a set of actions which the individual can choose; (4) possible states of nature occurring later than the series of environments in which specified "outcomes" may or may not occur; (5) a set of numbers which measure the probabilities that the outcome occurs, given that a certain action has been chosen in a specific environment. It should be emphasized that the environment of a set of action choices consists of all conditions sufficient to arrive at (5), the conditional probabilities of outcomes given a specific action. The model assumes that such an environment exists for a given moment of time (Churchman, 1961).

In developing a model, the complexity of mathematical tools may be various depending on a type of operation, objectives of the study and accuracy of input data. Both analytical and numerical, deterministic and uncertain (with different kinds of measurable uncertainty) models are widely used in OR and the calculations that make the process of decision making easier are the subject matter of this discipline.

As it was said above, when dealing with decision making under objective uncertainty, the approach of OR can succesfully be used to find optimal or at least sufficiently good solutions for decision problems. Notice, that rather rarely economic decisions are taken in conditions of objective uncertainty which assumes that the unknown factors of decision process are random variables or random functions, whose probabilistic-statistical characteristics are known or can be estimated with the aid of mathematical statistics.

As we know, random quantities are completely characterized by their probability distribution functions, and objective uncertainty makes the situation almost certain if the random quantities involved are supplied with the respective probabilities Probability is an objective characteritic of random, statistically stable events. Only such random events are studied in probability theory and in mathematical statistics. Thus, the statistician who gives the rules for taking a decision, and the decision--maker who wpplies these rules must remember that it is under statistical stability conditions that the concept of probability of the occurrence of the event is introduced as the limit of the frequency of its occurrence. For this reason, methods of mathematical statistics should be applied reasonably and with care, and then scientifically justified probabilistic-statistical methods will become an useful tool in the hands of the decision-maker.

The theory of probability is defined as the domain of mathematics that treats of mathematical models of random phenomena which possess the property of stability of frequencies (Gnedenko, 1976). It should be mentioned, that what is generally realized is that economic time series are relatively rapidly changing and cannot be assumed to be samples of unchanging populations. In order to apply statistical methods we need long runs under essentially constant conditions.

Being an interdisciplinary science OR has been strongly influenced by systems theory. The systems approach in the OR field is materialized as a treatment of an object being optimized as a part (i.e. as a subsystem) of a larger system within which the subunit undergoes an evaluation of the influence it may impose on the performance of the whole system. Some examples of interdependencies between OR ans systems approach as well as some areas for potential future interactions between the two disciplines in the field of economics discusses Intriligator (1979).

When considering decision-making problems from the view-point of OR, one should note that the decision making proper does not fall within the OR domain. It is a decision maker or, what is more often, a group of persons who is entrusted with the final decision making and who bears the responsibility for the choice. In arriving at a choice, the decision makers may and usually take into account other factors, quantitative or qualitative in nature, not considered in preparing mathematical recommendations (Keeney, 1982; Wentzel, 1983).

4. Decision making under statistically unstable uncertainty

In the preceeding chapter we have assumed that the decision-maker knows the probabilities with which the different states of the environment will occur. It is important to remember that in real life, the decision maker will often argue that he does not really know these probabilities, and that he cannot use programming based on probability calculus. An economic quantity can be considered to be random only if we are able to make our observations repeatidly and indefinitely under identical conditions (i.e. the chance of an economic event represented by the concrete value of quantity under consideration is the same). However, many decisions made in economic planning (e.g. investment decisions related to the construction of large projects) are in the nature of once-over decisions in which the law of large numbers does not operate. We cannot repeat an economic situation indefinitely under identical conditions. In short, in practice we are faced in the economy not with pure probabilistic-statistical situations, but with a mixture of statistically stable and a number of statistically unstable decision situations.

The mixed decision problems do not fall into the OR domain. It is a manager or a group of persons who is entrusted with the final decision choosing and who bears the responsability for the choice. Thus, economic decisions are made under statistically unstable uncertainty and are encumed with some subjectivism. At present the main effort in the field of decision analysis has been directed towards the production of both theoretical and practical tools for supporting decision processes under mixed uncertainty. Beside the work on the improvement of decision models there has been, during the last two decades, an extensive research on the description how the events are valuated by the decision maker, how important the events are for him, how certain he is about his probability judgement, and how much he feels that he himself can influence the likelihoods of occurrence of the events. Survey and extensive bibliography of existing approaches to solving these problems are gigen in Sevon (1978). The fundamental result is that in a decision under uncertainty the rule for a rational choice is to select the action which leads to the most preferred set of consequences.

As regards their development and use, the methods of the DMUU theory are closely related to the theory of large systems and to the methods and means of mathematics, operations research, management science, computer science, psychology, philosophy etc. This interdisciplinary nature of DMUU has important implications for both theory and practice of decision making. An important feature of decision analysis is that it has axiomatic foundation. According to Keeney (1982) and Winkler (1982), there are four main steps in decision analysis:

1) Modelling a decision-making problem, i.e. problem structuring;
2) Modelling uncertainty, i.e. assessing the possible impact of each alternative;

3) Modelling preferences, i.e. determining preferences (values) of decision makers;

4) Modelling competitive and group decisions, i.e. evaluating and comparing alternatives.

The are some interesting results concerning axiomatic approach to DMUU. Pratt, Raiffa and Schlaifer (1964) present the following axioms that suggest how to conduct a decision analysis:

Axiom 1: Generation of alternatives and identification of consequences (at least two alternatives can be specified and possible consequences of each alternative can be identified.

Axiom 2: Quantification of judgement and preference (the relative likelihoods, i.e. probabilities, of each possible consequence that could result from each alternative can be specified as well as the relative desirability, i.e. utility, for all the possible consequences of any alternative can be defined).

Axiom 3: Comparison of alternatives, transitivity of preferences and substitution of consequences. Comparison of alternatives indicates that if two alternatives would each result in the same two possible consequences, the alternative yielding the higher chance of the preferred consequence is preferred. Transitivity of preferences implies that if one alternative is preferred to a second alternative and if the second alternative is preferred to a third alternative, then the first alternative is preferred to the third alternative. Substitution of consequences indicates that if an alternative is modified by replacing one of its consequences with a set of consequences and associated probabilities that is indifferent to the consequence being replaced, then the original and the modified alternatives should be indifferent.

One of the essential features of DMUU processes in economics is that the rational decisions have to be judged simultaneously from the view-point of several different and usually conflicting objectives. The most fundamental results referring to this problem present for instance Keeney and Raiffa (1976), Hwang and Masud (1979) and Hwang and Yoon (1981).

Decision making under uncertainty is now mostly computer-aided. In computer-aided decision systems use is made of computers in combination with interactive methods, and computerized model. A unifying approach to the issue of using computers and computerized mathematical models for decision analysis and support is presented by Wierzbicki (1983).

5. Concluding remarks

Our discussion of the interplay between OR and DMUU in the context of economic systems modelling may be summarized as follows:

1) There are three categories of decision situations: certainty, risky and uncertainty;

2) Uncertainty is an objective feature of economic processes and systems;

3) Traditional models of economic systems are at least nonlinear and dynamic but still deterministic;

4) It is necessary to follow the example of modern physics and consider economic phenomena from a stochastic point of view;

5) In comparison to deterministic economics, stochastic economics considers the properties of economic processes on a deeper and more fundamental level;

6) Mathematical models of economic systems are tools supporting decision-making in economics;

7) Decision situations in economics can be classified into quantitative, qualitative and mixed. Operations research is the discipline which provides mathematical apparatus adequate to model and analyse quantitative decision problems under certainty or risk;

8) From the OR point of view, the decision making proper does not fall within the OR domain;

9) In DMUU two components of uncertainty factor coexist: objective and subjective. The DMUU theory provides a methodological framework for solving decision processes with objective and subjective uncertainties;

10) An important feature of decision analysis under uncertainty is that it has axiomatic foundation;

11) As a rule, DMUU processes in economics are multi-objective and processes with conflicting objectives;

12) Decision making under uncertainty is now mostly computer-aided;

13) There is a growing interest in quantitative (OR) models of economic systems under uncertainty. The models can be used as mathematical tools supporting analysis and planning of economic processes.

It must be emphasised that these conclusions applies not only to decision making in economics and are methodological rather than methodical.

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Interakcje między teorią badań operacyjnych i teorią podejmowania decyzji w warunkach nieokreśloności a modelowanie ekonomiczne

Celem artykułu jest przedstawienie i dyskusja niektórych aspektów analizy decyzyjnej w ekonomii, z punktu widzenia teorii badań operacyjnych i teorii podejmowania decyzji w warunkach nieokreśloności. Rozważania są oparte na założeniu, że funkcja rozkładu prawdopodobieństwa jest ilościową miarą nieokreśloności zdarzeń ekonomicznych. Założono, że pewność, ryzyko i nieokreśloność są typowymi sytuacjami decyzyjnymi w ekonomii. Opisane są pewne związki między fizyką a teorią modelowania ekonomicznego, z których wynika, że nieokreśloność jest obiektywną właściwością procesów ekonomicznych. Badania operacyjne są traktowane jako dyscyplina, która dostarcza modeli matematycznych odpowiednich dla analizowania ilościowych zadań decyzyjnych w warunkach pewności i ryzyka, podczas gdy teoria podejmowania decyzji w warunkach nieokreśloności jest wprowadzona jako dyscyplina preferująca sytuacje decyzyjne, którym towarzyszy niemierzalna nieokreśloność.

Интерактивные связи между теорией операционных исследований, теорией принятия решений в условиях неопределенности и экономическим моделированием

Целью статьи является представление и обсуждение некоторых аспектов анализа принятия решений в экономике, с точки зрения теории операционных исследований и теории принятия решений в условиях неопределенности. Рассмотрение основано на предпосылке что

функция распределения вероятности является количественной мерой неопределенности экономических событий. Предполагается, что уверенность, риск и неопределенность являются типичными ситуациями принятия решений в экономике. Описаны некоторые связи между физическими явлениями и теорией экономического моделирования, из которых вытекает, что неопределенность является объективным свойством экономических процессов. Операционные исследования принимаются как отрасль, которая разрабатывает математические модели, соответствующие для количественного анализа задач принятия решений в условиях уверенности и риска, в то ремя когда теория принятия решений в условиях неопределености вводится как отрасль дающая предпочтение ситуациям, которым сопутствует неизмеряемая неопределенность.