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Application of dynamic programming and discrete maximum principle to a problem of optimal operation choice for a robot in assembly processes

by

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This paper deals with application of two optimal control methods: the dynamic programming and the discrete maximum principle to a problem of optimal operation choice in assembly systems with a robot. Discrete stochastic processes are used to model dynamics of such systems. It is shown that both methods give the same resultant optimal algorithm and are in this sense equivalent for the problem.

1. Introduction

Modern industrial systems require more and more flexibility. This need leads usually to a use of industrial robots. There are two possible approaches to meet the requirements of flexibility in such systems. One is to change a software for every new type of production (with the use of robotic systems no or only slight changes of fixtures and transport system are necessary). The other is to introduce a flexible software itself, batch production oriented,

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which is equipped with facilities to react to changes in production. The latter approach is usually also flexible with regard to a random behavior of the process under control [5, 10, 11, 17]. The approach presented in this paper applies to optimization of robotic cells rather than of the overall flexible manufacturing system (FMS). The problem of design of a whole robotic FMS by means of two level approach, based on results of optimization of robotic cells on a lower level was presented in [17]. Here we concentrate on the optimization on the lower level of the robotic assembly cell.

Several approaches have been presented for such systems. One is to use two cameras [3]. The first camera is used for an analysis of a scene of the robotic cell and the second for a comparison with a technical design of a desired product given in three views. As a result the robot "chooses" assembly operations. In [4] a heuristic system of a plan generation is introduced using an actualized state of the assembly process. Another approach is presented in [6]. First a position of an element to be assembled is determined by use of an additional device (reorienting box) and then the element is either grasped by the robot or reoriented in the box until required position is obtained. In all above systems the current state of the assembly process depends only on the previous state, operation executed and random features of the process. (Explanation of this dependence may be found in [13] where a water pump assembly is studied). The above dependence shows that the current operation to be executed by the robot should be chosen based on the information about the current state of the assembly process. This leads to distinguishing three cases of the assembly system. In the first case the robot "is given" an exact information about the state, i.e., either is "informed" by an operator [12] or is equipped with a perfect state recognizer [2, 21]. The second case refers to a situation when the robot "is given" an information about measurements of characteristic features of the current state and "uses" this information directly to choose the assembly operation [14, 15]. In the third case the robot "uses" the same information as in the second case but in another way. First, "recognizes" the current state (not perfect recognition) and based on the recognized state "chooses" the operation to be executed [7, 13].

This paper deals with the second case, i.e., direct operation choice algorithms are under consideration. Assembly systems can be formalized in terms of discrete-time, discrete-state processes, namely controlled Markov chains. Therefore a method to choose operations which can be applied directly is the dynamic programming (DPM). The main interest of the paper is to find if another classical method of optimization of discrete dynamic processes, namely the discrete maximum principle (DMPM), can be used to derive operation choice algorithm (OCA) for assembly processes, and if so, what are conditions of equivalence for obtained algorithms. It is well known that for different dynamic processes, different algorithms can be obtained by DPM or DMPM of different computational complexity.

First results of comparison of DPM and DMPM are given in [16]. A problem of two-state, two-operation and one-measurement process was considered and the equivalence of resultant algorithms was shown. This paper considers a generalization of the above problem to an assembly process of a finite number of states, finite number of possible operations to be executed by the robot and more than one feature of the state to be measured.

In Section 2 a mathematical formalization of assembly processes in terms of controlled Markov chain with incomplete information about the state is presented and the optimal OCA derivation problem is formulated. In Section 3 a detailed solution of the above problem using the DPM is presented and in Section 4 results of applying the DMPM are briefly discussed by transforming the original model of the process. In Section 5 equivalence of numerical algorithms derived by DPM and DMPM is shown. Finally, in Section 6 an example of controlling a machine tool by a robot is described and formalized, and results of application of the optimal OCA are given.

2. Problem formulation

Let us formulate mathematically a problem of OCA derivation for a robot in discrete production processes. A process stage consists of a measurement of a current state characteristic features, a choice of an operation and its execution by a robot. The execution of a following operation yields a change of a current state in respect to a previous state and random features of the assembly process. The state is defined to reflect the main features of the production system, i.e., component elements to be assembled, assembly tools, machines served by the robot, and relevant fixtures etc. These states form a finite set of states. All necessary and possible robot operations to control the process are forming a finite set of operations. With respect to states of the assembly process different features can be measured, e.g., geometrical dimensions of component parts, acting forces and torques etc. At present many of such systems consist of TV cameras providing 2D or 3D vision of a whole assembly scene or its chosen part [7]. Characteristic features of the current state should be chosen very carefully because they strongly influence the process performance quality [14].

The assembly process is to be controlled over a finite time horizon of N process stages. Let us use the following notations:

$n = 0, 1, \ldots, N-1$ — the index of the current process stage,

$j \in \{1, \ldots, M\} = S$ — the current process state (given by its index),

S — the set of process state indices,

$k \in \{1, \ldots, r\} = K$ — the currently executed operation (given by its index),
K — the set of indices of possible operations to be executed by the robot,
$x \in X \subset R^m$ — the current measurement of characteristic features of the state,
R^m — the m dimensional Euclidean space.
The first lower index will denote the stage index. A bold $\mathbf{b}_n$ will denote a random variable taking at the stage n its realizations b_n from an appropriate set and $\bar{b}_n \equiv (b_0, \ldots, b_n)$, $\bar{b}_n \in \bar{B}_{n+1} \equiv \prod_{n+1} B$.

The behavior of the assembly process is governed by the set of transition probabilities

$$P(\mathbf{j}_{n+1} = j | \mathbf{j}_n = i, \mathbf{k}_n = k) \equiv p_{ij}^k, \tag{1}$$

where j_n is the state of the process and k_n the operation executed. Initial probabilities of the state j_0 at the stage $n = 0$ are

$$P(\mathbf{j}_0 = j) \equiv p_j, \tag{2}$$

The measurement x_n of the state j_n is given by conditional probability density function vector

$$\bar{f}_n(\mathbf{x}_n = x | \mathbf{j}_n = j, \mathbf{j}_{n-1}, \ldots, \mathbf{j}_0) \equiv f(x|j), \quad j \in S, \tag{3}$$

where $\bar{f}_n$ is the conditional density function of $\mathbf{x}_n$ given the sequence $\bar{j}_n$.

The OCA should take into account the past history of the assembly process and has a general form

$$H_n: \bar{X}_n \times \bar{K}_{n-1} \to K, \text{ i.e.,} \quad k = H_n(\bar{x}_n, \bar{k}_{n-1}). \tag{4}$$

Execution of each of operation by the robot leads to a change of the state. Let us introduce a cost function c_n whose value indicates a local loss incurred by execution of an operation k in a stage n leading to a transition to a state j in a next stage $n+1$. Because of the random character of the process, the following performance index is taken into account

$$Q_N = \underset{\bar{\mathbf{x}}_N, \bar{\mathbf{j}}_N}{E} \sum_{n=0}^{N-1} c_n(\mathbf{j}_{n+1}, \mathbf{k}_n), \tag{5}$$

where $c_n(j, k) \geq 0$, $j \in S$, $k \in K$, $n = 0, \ldots, N-1$. A problem of deriving the optimal OCA can be stated as follows:
Problem P1. Find the OCA for stages $n = 0, \ldots, N-1$ as to minimize (5) when the process is given by (1–3).

3. Application of the dynamic programming

Before we solve the Problem P1 let us introduce three equations (6), (7), (8) which are derived on the basis of the Bayes rule.

$$q_n^j \equiv P\,(\mathbf{j}=j|\bar{\mathbf{x}}_n=\bar{x}_n,\bar{k}_n)=\frac{f\,(x|j)\sum_{i=1}^{M} p_{ij}^k\,q_{n-1}^i}{A_n} \qquad (6)$$

with the initial condition

$$q_0^j \equiv P\,(\mathbf{j}_0=j|\mathbf{x}_0=x)=\frac{f\,(x|j)\,p_0}{A_0},$$

where A_n is the sum of the numerator over $j \in S$.
Further on

$$f_n\,(x|\bar{x}_{n-1},\bar{k}_{n-1})=\sum_{j=1}^{M} f\,(x|j)\,P\,(\mathbf{j}_n=j|\bar{\mathbf{x}}_{n-1}=\bar{x}_{n-1},\bar{k}_{n-1}), \qquad (7)$$

and

$$P\,(\mathbf{j}_n=j|\bar{\mathbf{x}}_{n-1}=\bar{x}_{n-1},\bar{k}_{n-1})=\sum_{i=1}^{M} p_{ij}^k\,q_{n-1}^i. \qquad (8)$$

Define a value function

$$V_{N-n}\,(\bar{x}_n,\bar{k}_{n-1})\equiv\min_{k_n\in K}\{\underset{\mathbf{j}_{n+1},\,\bar{\mathbf{x}}_{n+1}}{E}[\sum_{l=n}^{N-1} c_l\,(\mathbf{j}_{l+1},k_l)|\bar{\mathbf{x}}_n=\bar{x}_n,\bar{k}_n]\}.$$

Then for the last stage

$$V_1\,(\bar{x}_{N-1},\bar{k}_{N-2})=\min_{k\in K}\{\underset{\mathbf{j},\,\mathbf{x}_N}{E}[c_{N-1}\,(\mathbf{j},k)|\bar{\mathbf{x}}_{n-1}=\bar{x}_{n-1},\bar{k}_{n-1}]\}=$$

$$=\min_{k\in K}\{\sum_{j=1}^{M} c_{N-1}\,(j,k)\,P\,(\mathbf{j}_N=j|\bar{\mathbf{x}}_{n-1}=\bar{x}_{n-1},\bar{k}_{n-1})\}=$$

$$=\min_{k\in K}\{\sum_{j=1}^{M} c_{N-1}\,(j,k)\sum_{i=1}^{M} p_{ij}^k\,q_{n-1}^i\},$$

where equation (8) was used in the last derivation.

Let us introduce two vectors

$$q_n\equiv(q_n^1,q_n^2,\dots,q_n^M),$$

where each component q_n^i is defined by the equation (6) and

$$a_n^k\equiv(a_{n,1}^k,a_{n,2}^k,\dots,a_{n,M}^k),$$

where for the last stage

$$a_{N-1,j}^k\equiv\sum_{j=1}^{M} c_{N-1}\,(j,k)\,p_{kj}^i. \qquad (9)$$

For the last stage we have finally

$$V_1\,(\bar{x}_{N-1},\bar{k}_{N-2})=\min_{k\in K}\{a_{N-1}^k\,q_{N-1}^T\}, \qquad (10)$$

where q^T denotes the transposition of q. The minimization (10) gives the optimal OCA on the last stage, i.e., the dependence of k^* on the value of q_{N-1}. That yields the optimal function

$$a_{N-1}^{k*} = a_{N-1}^{*}(x),$$

which will be used for calculations on the previous stage.

For two last stages, and analogously for all other stages (substituting appropriately n in a place of 2), we have

$$V_2(\bar{x}_{N-2}, \bar{k}_{N-3}) = \min \{ \underset{\mathbf{j}_{N-1}, \mathbf{x}_{N-1}}{E} [c_{N-2}(\mathbf{j}_{N-1}, k) | \bar{\mathbf{x}}_{N-2} = \bar{x}_{N-2}, \bar{k}_{N-2}] +$$

$$+ \underset{x}{E} [V_1(\mathbf{x}, \bar{x}_{N-2}, \bar{k}_{N-2}) | \bar{\mathbf{x}}_{N-2} = \bar{x}_{N-2}, \bar{k}_{N-2}]\} =$$

$$= \min_{k \in K} \{ \sum_{j=1}^{M} c_{N-2}(j, k) P(\mathbf{j}_{N-1} = j | \bar{\mathbf{x}}_{N-2}, \bar{k}_{N-2}) +$$

$$+ \int_X V_1(x, \bar{x}_{N-1}, \bar{k}_{N-2}) f_{N-1}(x | \bar{x}_{N-2}, \bar{k}_{N-2}) \, dx \}.$$

Using (8) and (7) we obtain

$$V_2(\bar{x}_{N-2}, \bar{k}_{N-3}) = \min_{k \in K} \{ \sum_{j=1}^{M} c_{N-2}(j, k) \sum_{i=1}^{M} p_{ij}^{k} q_{N-2}^{i} +$$

$$+ \int_X V_1(x, \bar{x}_{N-1}, \bar{k}_{N-2}) \sum_{j=1}^{M} f(x|j) P(\mathbf{j}_{N-1} = j | \bar{\mathbf{x}}_{N-2} = \bar{x}_{N-2}, \bar{k}_{N-2}) \, dx \}.$$

Using (8) again we obtain

$$V_2(\bar{x}_{N-2}, \bar{k}_{N-3}) = \min_{k \in K} \{ \sum_{j=1}^{M} \sum_{i=1}^{M} c_{N-2}(j, k) q_{N-2}^{i} +$$

$$+ \int_X \sum_{j=1}^{M} \sum_{i=1}^{M} V_1(x, \bar{x}_{N-1}, \bar{k}_{N-2}) p_{ij}^{k} f(x|j) q_{n-2}^{i} \, dx \}. \tag{11}$$

Substituting (10) and (6) into the definitions of $V_1(\,)$ and q_{N-1}^{j}, respectively, we obtain

$$V_2(\bar{x}_{N-2}, \bar{k}_{N-3}) = \min_{k \in K} \left\{ \sum_{j=1}^{M} \sum_{i=1}^{M} c_{N-2}(j, k) p_{ij}^{k} q_{N-2}^{i} + \right.$$

$$\left. + \int_X \sum_{j=1}^{M} \sum_{i=1}^{M} p_{ij}^{k} q_{N-2}^{i} f(x|j) \sum_{j=1}^{M} a_{N-1,j}^{*} \frac{\sum_{i=1}^{M} f(x|j) p_{ij}^{k} q_{N-2}^{i}}{\sum_{j=1}^{M} \sum_{i=1}^{M} f(x|j) p_{ij}^{k} q_{N-2}^{i}} dx \right\} =$$

$$= \min_{k \in K} \{ \sum_{i=1}^{M} a_{N-2,i}^{k} q_{N-2}^{i} \},$$

so finally

$$V_2(\bar{x}_{N-2}, \bar{k}_{N-3}) = \min_{k\in K} \{a^k_{N-2}\, q^T_{N-2}\}, \tag{12}$$

where

$$a^k_{n,i} \equiv \sum_{j=1}^{M} \left(c_n(j, k) + g^k_{n+1,j}\, p^k_{ij}\right), \tag{13a}$$

and

$$g^k_{n,j} \equiv \int_X a^*_{n,j}(x)\, f(x|j)\, dx, \tag{13b}$$

is a Riemann integral. Existence of the last integral is guaranteed by the Riemann integral definition since values $a^*_{n,j}(x)$ are bounded for $n = 0, ..., N-1$, $j \in S$ and all $x \in X$. To get the final form of (12) two identical terms in the numerator and the denominator were reduced.

The minimization (12) gives the optimal OCA on the stage $N-2$, i.e., dependence of k^* on the value of q_{N-2}. That yields the optimal function

$$a^{k*}_{N-2} = a^*_{N-2}(x),$$

which is then used for calculations on stage $N-3$.

Optimal operations k^*_n are determined in the same way for other stages as functions of q_n. For the on-line control, based on the current measurement x_n, previously executed operation k^*_{n-1}, and stored value of q_{n-1}, the value of q_n is determined using (6). Then comparison of this value with stored sets of q_n (obtained from (12)) gives the optimal operation to be executed on the stage n. To the end let us denote the optimal OCA (12–13) as OCA1.

4. Application of the discrete maximum principle

The discrete maximum principle [8] cannot be applied directly to the assembly process problem P1. The reasons are: the discrete set of states S and the form of the performance index (5). To apply the discrete maximum principle to the Problem P1 let us first define a one-stage expected cost

$$\tilde{c}_n(q_n, k_n) \equiv \mathop{E}_{\mathbf{j}_{n+1}} [c_n(\mathbf{j}_{n+1}, k_n) | \bar{\mathbf{x}}_n = \bar{x}_n, \bar{k}_n] = \sum_{j=1}^{M} c_n(j, k) \sum_{i=1}^{M} p^k_{ij}\, q^i_n. \tag{14}$$

The last equations is derived on the basis of simple operations on a sum operator, Bayes rule, and definitions (1) and (6). The performance index (5) can be now rewritren in the following form

$$Q_N = \mathop{E}_{\bar{\mathbf{x}}_N} \sum_{n=0}^{N-1} \tilde{c}_n(q_n, k_n). \tag{15}$$

Define a new state variable vector

$$q_n \equiv (q_n^1, q_n^2, \ldots, q_n^M),$$

where the recursion (6) is treated as a state transformation, i.e.,

$$q_n^i = h_n^i(x_n, q_{n-1}, k_{n-1}), \qquad i \in S. \tag{16}$$

The following problem can be now formulated:

Problem P2. Find the OCA for stages $n = 0, \ldots, N-1$ as to minimize (15) when the process is given by (16).

The form of the state transformation (6) yields that q_n is the sufficient statistics of the state of the assembly process j_n [20]. Moreover, the performance index is easily to be seen as a rewritten equation (5), so the Problem P2 is identical to the Problem P1 [1]. Define an auxiliary variable vector

$$\psi_n \equiv (\psi_n^1, \psi_n^2, \ldots, \psi_n^M)',$$

where

$$\psi_n^i \equiv -\frac{\partial \tilde{c}_n(q_n, k_n)}{\partial q_n^i} + \sum_{j=1}^{M} \frac{\partial h_n^j}{\partial q_n^i} \psi_{n+1}^j. \tag{17}$$

Define the Hamiltonian

$$H_{n-1} \equiv H_{n-1}(\psi_n^*, x_n, q_{n-1}, k_{n-1}) \equiv -\tilde{c}_{n-1}(q_{n-1}, k_{n-1}) + \\ + h_n(x_n, q_{n-1}, k_{n-1})\, \psi_n^* \tag{18}$$

where ψ_n^* is the value of ψ_n for the optimal operation k_n^*.

Problem P2 is solved if the maximum over the set K of the expected values of the Hamiltonian is found for each stage $n = 0, \ldots, N-1$ [8, 19]

$$\max_{k \in K} \{ \underset{\mathbf{x}_{n+1}, \ldots, \mathbf{x}_N}{E} [H_n(\psi_{n+1}^*, \mathbf{x}_{n+1}, q_n, k) | \bar{\mathbf{x}}_n = \bar{x}_n, \bar{k}_n] \}.$$

To the end, the above OCA (16–19) will be denoted as OCA2.

5. Equivalence of OCA1 and OCA2

The equivalence of OCA1 and OCA2 will be shown in a sense of the identical form of these algorithms. The same form yields the same results, i.e., the same on-line operation choice as well as the same computational complexity.

Let us first show the equivalence of both algorithms for the last stage. By definition $\psi_N = 0$. From (18) we have

$$H_{N-1} = -\tilde{c}_{N-1}(q_{N-1}, k_{N-1}),$$

and so, by (14)

$$\underset{x_N}{E}\,[H_{N-1}|\bar{\mathbf{x}}_{N-1}=\bar{x}_{N-1},\bar{k}_{N-1}]=-\sum_{j=1}^{M}c_{N-1}(j,k)\sum_{i=1}^{M}p_{ij}^{k}\,q_{N-1}^{i}. \tag{20}$$

By maximizing this value over $k\in K$ we obtain the same result as for the dynamic programming (7) and so for the last stage OCA1 and OCA2 are equivalent. Let us denote the maximum of the expected value (20) as EH_{N-1}^{*}. For the stage $N-2$ we have from (17)

$$\psi_{N-1}^{i}=\frac{\partial H_{N-1}}{\partial q_{N-1}^{i}}=-\frac{\partial\tilde{c}_{N-1}(q_{N-1},k_{N-1})}{\partial q_{N-1}^{i}}=-\sum_{j=1}^{M}c_{N-1}(j,k)\,p_{ij}^{k}. \tag{21}$$

From (18) we have

$$H_{N-2}=-\tilde{c}_{N-2}(q_{N-2},k_{N-2})-\sum_{j=1}^{M}c_{N-1}(j,k^{*})\sum_{i=1}^{M}p_{ij}^{k*}\,q_{N-1}^{i}=$$

$$=-\tilde{c}_{N-2}(q_{N-2},k_{N-2})-EH_{N-1}^{*}, \tag{22}$$

and

$$\underset{x}{E}\,[H_{N-2}|\bar{\mathbf{x}}_{N-2}=\bar{x}_{N-2},\bar{k}_{N-2}]=-\tilde{c}_{N-2}(q_{N-2},k_{N-2})-$$

$$-\int_X EH_{N-1}^{*}\,f_{N-1}(x|\bar{x}_{N-2},\bar{k}_{N-2})\,dx=\tilde{c}_{N-2}(q_{N-2},k_{N-2})-$$

$$-\int_X EH_{N-1}^{*}\sum_{j=1}^{M}f(x|j)\sum_{i=1}^{M}p_{ij}^{k_{N-2}}\,q_{N-2}^{i},$$

where in the last derivation equations (7) and (8) were used. Define

$$V_{N-n}(\bar{x}_n,\bar{k}_{n-1})\equiv EH_n^{*}\equiv\underset{\mathbf{x}_{n+1}}{E}\,[H_n^{*}|\bar{\mathbf{x}}_n=\bar{x}_n,\bar{k}_{n-1}]. \tag{23}$$

By substitution of equations (14) and (23) we get equations (11–12) obtained by the dynamic programming. Therefore the OCA1 and OCA2 are equivalent for the stage $N-2$. For the stage $N-3$ and analogously for other stages we have from (22) and (17)

$$\psi_{N-2}^{i}=\frac{\partial H_{N-2}}{\partial q_{N-2}^{i}}=-\frac{\partial\,[\tilde{c}_{N-2}(q_{N-2},k_{N-2}^{*})+EH_{N-1}^{*}]}{\partial q_{N-2}^{i}}=$$

$$=-\frac{\partial\,[\tilde{c}(q_{N-2},k_{N-2}^{*})+\sum_{j=1}^{M}c_{N-1}(j,k_{N-1}^{*})\sum_{i=1}^{M}p_{ij}^{k_{N-1}^{*}}q_{N-1}^{i}]}{\partial q_{N-2}^{i}}.$$

Let us first calculate $\partial q_n^j/\partial q_{n-1}^i$

$$\frac{\partial q_n^j}{\partial q_{n-1}^i}=\frac{\partial\left[\dfrac{f(x|j)\sum_{l=1}^{M}p_{lj}^{k}\,q_{n-1}^{l}}{\sum_{m=1}^{M}f(x|j)\sum_{l=1}^{M}p_{lm}^{k}\,q_{n-1}^{l}}\right]}{\partial q_{n-1}^{i}}$$

$$= \Big[f(x|j)\Big[\sum_{m=1}^{M} p_{ij}^{k} f(x|m) \sum_{l=1}^{M} p_{lm}^{k} q_{n-1}^{l} -$$

$$- \sum_{m=1}^{M} f(x|m)\, p_{im}^{k} \sum_{l=1}^{M} p_{ij}^{k} q_{n-1}^{l}\Big]\Big]/C_{n-1},$$

where

$$C_{n-1} \equiv \Big[\sum_{m=1}^{M} f(x|m) \sum_{l=1}^{M} p_{lm}^{k} q_{n-1}^{l}\Big]^2.$$

Therefore we have

$$C_{N-2}\,\psi_{N-2}^{i} = -C_{N-2} \sum_{m=1}^{M} c_{N-2}(m, k_{N-2}^{*})\, p_{im}^{k_{N-2}^{*}} -$$

$$- \sum_{s=1}^{M} c_{N-1}(s, k_{N-1}^{*}) \sum_{m=1}^{M} p_{ms}^{k_{N-1}^{*}} f(x|m) \cdot$$

$$\cdot \Big[\sum_{m=1}^{M} p_{im}^{k_{N-2}^{*}} f(x|m) \sum_{l=1}^{M} p_{lm}^{k_{N-2}^{*}} q_{N-2}^{l} -$$

$$- \sum_{m=1}^{M} p_{im}^{k_{N-2}^{*}} f(x|m) \sum_{l=1}^{M} p_{lm}^{k_{N-2}^{*}} q_{N-2}^{l}\Big],$$

so finally we have

$$\psi_{N-2}^{i} = -\sum_{m=1}^{M} c_{N-2}(m, k)\, p_{im}^{k}, \tag{24}$$

which has the same form as for the stage $N-2$. From (18) we have

$$H_{N-3} = -\tilde{c}_{N-3}(q_{N-3}, k_{N-3}) - \sum_{j=1}^{M} c_{N-2}(j, k_{N-2}^{*}) \sum_{i=1}^{M} p_{ij}^{k_{N-2}^{*}} q_{N-2}^{i},$$

so

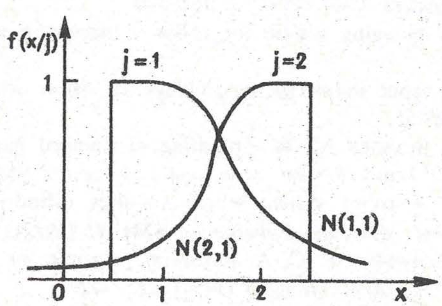
$$H_{N-3} = -\tilde{C}_{N-3}(q_{N-3}, k_{N-3}) - EH_{N-2}^{*}.$$

It was shown that the forms of OCA1 and OCA2 are identical under the definition (23). Problem P2 is equivalent to Problem P1. Therefore two presented methods applied to Problem P1 give same results and are equivalent. From above derivations it follows that OCA1 and OCA2 are of equal computational complexity.

6. Example — inspection in a machine tool cell

It is impossible to construct a simple two-state illustrative assembly process. Therefore let us consider a machine tool cell where service as well as inspection is due to an industrial robot. The robot serves machine tool by grasping elements to be machined from the input belt conveyor. It fixes them into machine tools and disposes the ready elements onto the output belt conveyor. The service program is deterministic and is based mainly on signals of machining programs completeness. It is possibe to use the idle time of the robot to inspect either machined parts or tools. During the process of machining the value of an appropriate force affecting the tool can be used to decide whether to proceed machining or to exchange the tool. Usually the force is compared with a certain fixed value and on this base the decision is made [23]. However, a dynamic behavior of a deterioration process of the tool as well a measurement noise are not taken into account (see [9], [21] for description of the machine tool deterioration process dynamics and randomness).

Let us fix a certain time period δt as a length of each stage $n = 0, \ldots, N-1$. There are two operations of the robot to be considered, namely $k = 1$ — "proceed machining" and $k = 2$ — "change tool", and two states of each tool, namely $j = 1$ — "good state" and $j = 2$ — "breakdown state" (tool blunted). Assume that a measurement of only one feature, namely acting force, is made at the beginning of each time period. Let us assume the following values of initial probabilities: $p_1 = 1$ and $p_2 = 0$, and of transition probabilities $p^1_{11} = 0.8$, $p^1_{12} = 0.2$, $p^1_{11} = 0.0$, $p^1_{22} = 1.0$, $p^2_{11} = 1.0$, $p^2_{12} = 0.0$, $p^2_{21} = 0.9$ $p^2_{22} = 0.1$. Conditional density functions are given in Fig. 1. Let us assume the local costs: $c\,(1,1) = 1.0$, $c\,(2,1) = 5.0$, $c\,(1,2) = 10.0$, $c\,(2,2) = 21.1$.



For the time horizon of five stages the results are following:
— for stages $n = 0$, 1, 2 and $n = 4$ the optimal operation is $k = 1$ (proceed),
— for the stage $n = 3$

$$k^* = \begin{cases} 1 \text{ for } q^1_3 < 0.66 \\ 2 \text{ for } q^1_3 \geqslant 0.66 \end{cases}$$

Change of the tool should be done only at the beginning of the third stage if $q^1_3 \geqslant 0.66$. However, measurements should be performed at each stage for computation of q_3^1. The optimal operation for the first and the last stage is obviously "to proceed".

7. Conclusions

A lot of flexible industrial processes may be modeled as discrete stochastic processes. Such a description is chosen for a robotic assembly process with finite number of states and operations, where the information about the current state is incomplete. Two different methods have been applied to solve the underlying problem of optimal choice of a sequence of operations. It was shown that both the dynamic programming approach and discrete maximum principle approach lead to the same optimal operation choice algorithm. The resultant algorithm was applied to a simple two-state, two-operation, one-measurement problem. Results shows the great simplicity of the on-line algorithm.

References

[1] Bertsekas D., Dynamic Programming and Stochastic Control, Academic Press, New York 1976.

[2] Bubnicki Z., Reyman G., Staroswiecki M., Djeghaba M. The control in the assembly system with two robots, Proc. Int. Conf. Systems Engineering 4th, Coventry 1985.

[3] Ejiri M. et al. A prototype intelligent robot that assembles objects from plane drawings, *IEEE Trans. Computers*. **C-21** (1972) 2, 161–170.

[4] Fahlman S. E. A planning system for robot construction tasks, *Artificial Intelligence*, **5** (1974) 1, 1–49.

[5] Ferretti M. Un debut industriel: assemblage et vision artificiel. *Le Nouvel Automatisme*, **31** (1982), 49–55.

[6] Grossman D. D., Blasgen M. W. Orienting mechanical parts by computer controlled manipulator. *IEEE Trans. System. Man. and Cybernetics*, **SMC-5** (1975), 561–565.

[7] Kelley R. et al. A robot system which acquires cylindrical workpieces from bins, *IEEE Trans. System. Man. and Cybernetics*, **SMC-12** (1982), 204–213.

[8] Kushner H. J., Schweppe F. C. A maximum principle for stochastic control systems, *J. Math. Analysis and Applications*, **8** (1964), 287–302.

[9] Luss H. Maintenance policies when deterioration can be observed by inspections, *Operations Research*, **24** (1976), 359–366.

[10] Nevins J. L., Whitney D. E. Assembly research. *Automatica*, **16** (1980), 6, 595–613.

[11] Paul R. P. Robot Manipulators. Mathematics, Programming, and Control, MIT Press, Cambridge, 1980.

[12] Reyman G. Adaptive operations choice for a robot in the assembly system with perfect state recognition, Mathematik. Modellierung und Optimierung von Systemen, ZLOMHF, Zwickau, (to appear).

[13] Reyman G. State recognition algorithm for robot assembly control, Proc. 7th Int. Conf. Pattern Recognition, Montreal 1984.

[14] Reyman G. Zastosowanie teorii rozpoznawania obrazów do sterowania optymalnego w dyskretnych systemach stochastycznych. *Postępy Cybernetyki*, **8** (1985), 103–116.

[15] REYMAN G. Wybór operacji z uczeniem w procesie montażowym z robotem sterującym, Proc. 5th Conf. Automatyzacja Dyskretnych Procesów Przemysłowych, Kozubnik, 1986.
[16] REYMAN G., SCHÄFER W., TRIPPLER G. Application of dynamic programming and maximum principle to the inspection and repair problem, Proc. 4th Int. Conf. Systems Engineering, Coventry 1985.
[17] REYMAN G. Design of assembly systems. *The Int. J. Advanced Manufacturing Technology*, **2** (1987) 13–21.
[18] ROSEN C. A. Material-handling robots for programmable automation, Proc. IFAC Int. Symp. Information and Control in Manufacturing Technology, Tokyo 1977.
[19] SCHÄFER W., TRIPPLER G., REYMAN G. Zur Optimalsteuerung discreter stochastischer Prozesse mit einem Maximumprinzip, Wissenschaftliche Zeitschrift der TH Leipzig (to appear).
[20] SMALLWOD R. D., SONDIK E. J. The optimal control of partially observable Markov processes over a finite horizon, *Operations Research*, **21** (1973), 1071–1088.
[21] STAROSWIECKI M., DJEGHABA M., BAYART M., REYMAN G. Decision problems in a felxible assembly cell using robot cooperation, Proc. 4th Int. Conf. Systems Enginnering, Coventry 1985.
[22] WHITE C. C. A Markov quality control process subject to partial observation, *Management Science*, **23** (1977), 843–852.
[23] ZEPPELIN W. Automatischer Werkzeugwechsel nach Verschliess an NC-Drehmaschinen, *TZ für Metallbearbeitung*, **3** (1984), 1–8.

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Zastosowanie programowania dynamicznego i dyskretnej zasady maksimum do zadania optymalnego wyboru operacji robota w procesie montażu

Artykuł dotyczy zastosowania dwóch metod sterowania optymalnego: Programowania dynamicznego i dyskretnej zasady maksimum, do zadania optymalnego wyboru operacji w systemach montażowych z wykorzystaniem robotów. Dynamika tych systemów opisana jest przy pomocy dyskretnych procesów stochastycznych. Pokazano, że obie metody dają ten sam algorytm wynikowy i są w tym znaczeniu równoważne dla rozpatrywanego zadania.

Применение динамического программирования и дискретного метода принципа максимума к задаче оптимального выбора операций робота в процессе монтажа

Статья касается применения двух методов оптимального управления: динамического программирования и дискретного принципа максимума, к задаче оптимального выбора операций в монтажных системах, использующих роботы. Динамика этих систем описана с помощью дискретных стохастических процессов. Показано, что оба метода дают тот же результирующий алгоритм и в этом смысле эквивалентны для рассматриваемой задачи.