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Optimal control of F-MM II

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The quadratic optimal control problem for the second Fornasini-Marchesini model of 2-D linear, stationary, discrete system is considered. Using two methods of transformation for system and performance index, the problem of finding the optimal sequence of control vectors is reduced to equivalent mathematical programming problem.

1. Introduction and problem formulation

The second Fornasini-Marchesini model (F-MM II) of 2-dimensional linear, stationary, discrete system is given by equations [5]:

$$\begin{aligned} x(k+1, l+1) = A_1\, x(k, l+1) + A_2\, x(k+1, l) + \\ + B_1\, u(k, l+1) + B_2\, u(k+1, l), \\ y(k, l) = Cx(k, l), \end{aligned} \tag{1}$$

where $(k, l) \in N \times N$, (N — set of natural numbers) $x \in R^n$ is a state vector, $u \in R^m$ is an input vector and $y \in R^p$ is an output vector, $A_1,\ A_2 \in R^{n \times n}$, $B_1,\ B_2 \in B^{n \times m}$ are real matrices.
Boundary conditions for the F-MM II model are given as sequences:

$$\begin{aligned} x(i, 0) &= x_{i0} \quad \text{for} \quad i = 0, 1, \ldots, \\ x(0, j) &= x_{0j} \quad \text{for} \quad j = 1, 2, \ldots, \end{aligned} \tag{2}$$

For this model we will consider the following optimal problem: choose the sequence $[(r+1)(s+1)-2]$ control vectors $u(i, j)$, so that the quadratic performance index:

$$J_{r,s}(\{x(i,j)\},\{u(i,j)\}) = \sum_{(i,j)\in D_{r,s}\setminus(C_1\times\{0\}\cup\{0\}\times C_2)} x^T(i,j)\cdot Q(i,j)\cdot x(i,j) + \\ + \sum_{(i,j)\in D_{r,s}\setminus\{(0,0),(r,s)\}} u^T(i,j)\cdot P(i,j)\cdot u(i,j), \tag{3}$$

is minimized.

Where $C_1 = \{0, 1, ..., r\}$, $C_2 = \{0, 1, ..., s\}$ and $D_{r,s} = C_1 \times C_2$ (symbol "x" denotes here the Cartesian product). It is assumed, that $Q(i,j)$ are $n \times n$ — dimensional, symmetric, nonnegative definite matrices, and $P(i,j)$ are $m \times m$ — dimensional, symmetric, positive definite matrices. This problem was considered for different models of 2-D systems in papers [1], [2], [3].

A similar problem, the so called the minimum energy control problem was formulated and solved by Klamka in [7].

2. Problem transformation

We want to reduce the presented problem to some mathematical programming problem, namely:

Problem (*)

Minimize

$$f(z) = \langle z, Hz \rangle, \tag{4}$$

(symbol $\langle \cdot,\cdot \rangle$ denotes inner product) with the following constraint:

$$Rz = c, \tag{5}$$

where z, c — vectors, H, R — matrices of appropriate dimensions; H is symmetric, nonnegative definite matrix.

I method:

Let us define vector z in the following manner:

$$z^T = [x^T(1,1), x^T(2,1), ..., x^T(r,1), x^T(1,2), x^T(2,2), ... \\ ..., x^T(r,2), ..., x^T(1,s), x^T(2,s), ..., x^T(r,s), u^T(1,0), u^T(2,0), ... \\ ..., u^T(r,0), u^T(0,1), u^T(1,1), u^T(2,1), ..., u^T(r,1), ... \\ ..., u^T(0,s), u^T(1,s), u^T(2,s), ..., u^T(r-1,s)], \tag{6}$$

z is $\{rsn + [(r+1)(s+1)-2]\, m\}$ — dimensional vector.

In order to transform the performance index $J_{r,s}$ to the form (4), let introduce the following notation:

$$H = \operatorname{diag}[Q(1,1), Q(2,1), ..., Q(r,1), Q(1,2), ..., Q(r,2), ... \\ ..., Q(1,s), Q(2,s), ..., Q(r,s), P(1,0), P(2,0), ... \\ ..., P(r,0), P(0,1), P(1,1), ..., P(r,1), ...$$

$$\ldots, P(0, s), P(1, s), \ldots, P(r-1, s)]. \qquad (7)$$

It is easy to observe, that H is square $\{rsn+[(r+1)(s+1)-2]\, m\}$ — dimensional, symmetric, nonnegative definite matrix. Similarly, we can express the equation (1) described F-MM II model with boundary conditions (2) in the following equivalent form:

$$Rz = c, \quad \text{where} \quad R = [R_1 \mid R_2], \qquad (8)$$

$$R_1 = \left[\begin{array}{c|c|c|c|c}
\begin{matrix} I\ 0 \ldots 0\ 0 \\ -A_1\ I \ldots 0\ 0 \\ \ldots \\ 0\ 0 \ldots -A_1\ I \end{matrix} & 0 & \vdots & 0 & 0 \\
\hline
\begin{matrix} -A_2\ 0 \ldots 0\ 0 \\ 0 -A_2 \ldots 0\ 0 \\ \\ 0\ 0 \ldots 0 -A_2 \end{matrix} & \begin{matrix} I\ 0 \ldots 0\ 0 \\ -A_1\ I\ \ 0\ 0 \\ \\ 0\ 0 \ldots -A_1\ I \end{matrix} & \vdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\hline
0 & 0 & \vdots & \begin{matrix} -A_2\ 0 \ldots 0\ 0 \\ 0 -A_2 \ldots 0\ 0 \\ \ldots \\ 0\ 0 \ldots 0 -A_2 \end{matrix} & \begin{matrix} I\ 0 \ldots 0\ 0 \\ -A_1\ I \ldots 0\ 0 \\ \ldots \\ 0\ 0 \ldots -A_1\ I \end{matrix}
\end{array}\right]$$

$$\qquad \underbrace{\qquad\qquad}_{r} \quad \underbrace{\qquad\qquad}_{r} \qquad \underbrace{\qquad\qquad}_{r} \quad \underbrace{\qquad\qquad}_{r}$$

$$R_2 = \left[\begin{array}{c|c|c|c|c|c}
\begin{matrix} -B_2\ 0 \ldots 0\ 0 \\ 0 -B_2 \ldots 0\ 0 \\ \ldots \\ 0\ 0 \ldots 0 -B_2 \end{matrix} & \begin{matrix} -B_1\ 0 \ldots 0\ 0 \\ 0 -B_1 \ldots 0\ 0 \\ \ldots \\ 0\ 0 \ldots -B_1\ 0 \end{matrix} & 0 & \vdots & 0 & 0 \\
\hline
 & \begin{matrix} 0 -B_2 \ldots 0\ 0 \\ \ldots \\ 0\ 0\ 0 \ldots 0 -B_2 \end{matrix} & \begin{matrix} -B_1\ 0 \ldots 0\ 0 \\ \ldots \\ 0\ 0 \ldots -B_1\ 0 \end{matrix} & \vdots & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\hline
0 & 0 & 0 & \vdots & \begin{matrix} 0 -B_2\ 0 \ldots 0 \\ 0\ 0 -B_2 \ldots 0 \\ \ldots \\ 0\ 0 \ldots 0 -B_2 \end{matrix} & \begin{matrix} -B_1\ 0 \ldots 0 \\ 0 -B_1 \ldots 0 \\ \ldots \\ 0\ 0 \ldots -B_1 \end{matrix}
\end{array}\right]$$

$$\qquad \underbrace{\qquad\qquad}_{r} \quad \underbrace{\qquad\qquad}_{r+1} \quad \underbrace{\qquad\qquad}_{r+1} \qquad \underbrace{\qquad\qquad}_{r+1} \quad \underbrace{\qquad\qquad}_{r}$$

Hence, R_1 is $(rsn \times rsn)$ — dimensional matrix, R_2 is $\{rsn \times [(r+1)(s+1)-2]\, m\}$ — dimensional matrix. It is easy to show that vector c has the form:

$$c^T = [c_1^T, c_2^T, \ldots, c_{s-1}^T, c_s^T], \tag{9}$$

where

$$c_1 = \begin{bmatrix} A_1\, x(0,1) + A_2\, x(1,0) \\ A_2\, x(2,0) \\ \vdots \\ A_2\, x(r,0) \end{bmatrix}, \quad c_i = \begin{bmatrix} A_1\, x(0,i) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad i = 2, 3, \ldots, s \tag{9a}$$

c_i for $i = 1, 2, \ldots, s$ is (nr) — dimensional vector.

Therefore, using notations (8) and (9), we expressed our optimal problem described in section (1) in the equivalent mathematical programming problem (*).

II method

The similar transformation can be obtained, using the general response formula for the second Fornasini-Marchesini model.

This formula has the form [6]:

$$\begin{aligned} x(k,l) = \sum_{i=1}^{k} A^{k-i,l-1} [A_2\, x(i,0) + B_2\, u(i,0)] + \\ + \sum_{j=1}^{l} A^{k-1,l-j} [A_1\, x(0,j) + B_1\, u(0,j)] + \\ + \sum_{i=1}^{k} \sum_{j=1}^{l} (A^{k-i-1,l-j} B_1 + A^{k-i,l-1} B_2)\, u(i,j), \end{aligned} \tag{10}$$

For convenience we write:

$$G(k-i, l-j) = A^{k-i-1,l-j} B_1 + A^{k-i,l-j-1} B_2. \tag{11}$$

For this F-MM II model they introduced a state-transition matrix $A^{i,j}$ [6], [8]:

$$\begin{aligned} & A^{i,j} = A_1\, A^{i-1,j} + A_2\, A^{i,j-1} \quad \text{for} \quad (i,j) \geqslant (0,0), \\ & A^{0,0} = I,\; A^{-i,j} = A^{i,-j} = A^{-i,-j} = 0 \quad \text{for} \quad (i,j) > (0,0). \end{aligned} \tag{12}$$

It is interesting to note that the matrix $A^{i,j}$ can be also calculated according to the following formula:

$$A^{i,j} = A^{i-1,j} A_1 + A^{i,j-1} A_2. \tag{13}$$

In our problem the natural way is to assume z as a sequence of boundary condition in $(C_1 \times \{0\} \cup \{0\} \times C_2)$ and vectors $u(i,j)$ in rectangle $D_{r,s}$

$$\begin{aligned} z^T = [x^T(1,0), x^T(2,0), \ldots, x^T(r,0), x^T(0,1), x^T(0,2), \ldots \\ \ldots, x^T(0,s), u^T(1,0), u^T(2,0), \ldots, u^T(r,0), u^T(0,1), \end{aligned}$$

$$u^T(0,2), \dots, u^T(0,s), u^T(1,1), u^T(2,1), \dots, u^T(r,1), \dots$$
$$\dots, u^T(1,s), u^T(2,s), \dots, u^T(r-1,s)]. \tag{14}$$

Hence z is an $\{(r+s)n+[(s+1)(r+1)-2]m\}$ — dimensional vector. It will be convenient to express formula (10) in the following form:

$$x(k,l) = M(k,l)z. \tag{15}$$

Comparison of (10) and (15) gives us the general form of the matrix $M(k,l)$:

$$\begin{aligned}
M(k,l) = [&A^{k-1,l-1}.A_2, A^{k-2,l-1}.A_2, \dots, A^{1,l-1}.A_2, A^{0,l-1}.A_2, \overbrace{0,\dots,0}^{r-k},\\
&A^{k-1,l-1}.A_1, A^{k-1,l-2}.A_1, \dots, A^{k-1,1}.A_1, A^{k-1,0}.A_1, \overbrace{0,\dots,0}^{s-l},\\
&A^{k-1,l-1}.B_2, A^{k-2,l-1}.B_2, \dots, A^{1,l-1}.B_2, A^{0,l-1}.B_2, \overbrace{0,\dots,0}^{r-k},\\
&A^{k-1,l-1}.B_1, A^{k-1,l-2}.B_1, \dots, A^{k-1,1}.B_1, A^{k-1,0}.B_1, \overbrace{0,\dots,0}^{s-l},\\
&G(k-1,l-1), G(k-2,l-1), \dots, G(1,l-1), G(0,l-1), \overbrace{0,\dots,0}^{r-k},\\
&G(k-1,l-2), G(k-2,l-2), \dots, G(1,l-2), G(0,l-2), \overbrace{0,\dots,0}^{r-k},\\
&\vdots\\
&G(k-1,1), G(k-2,1), \dots, G(1,1), G(0,1), \overbrace{0,\dots,0}^{r-k},\\
&G(k-1,0), G(k-2,0), \dots, G(1,0), \underbrace{0,\dots,0}_{(r-k+1)}, \underbrace{0,\dots,0}_{(s-l)r-1}].
\end{aligned} \tag{16}$$

$M(k,l)$ is an $n \times \{(s+r)n+[(r+1)(s+1)-2]m\}$ — dimensional matrix. Let us introduce vector X:

$$X^T = [x^T(1,1), x^T(2,1), \dots, x^T(r,1), \dots, x^T(1,s), x^T(2,s), \dots$$
$$\dots, x^T(r-1,s), x^T(r,s)]. \tag{17}$$

We want to express the vector X as a function of z:

$$X = M \cdot z. \tag{18}$$

Where M has the form:

$$M = \begin{bmatrix} M(1,1) \\ M(2,1) \\ \vdots \\ M(r,1) \\ M(1,s) \\ \vdots \\ M(r,s) \end{bmatrix}, \tag{18a}$$

and is an $rsn \times \{(s+r)n+[(r+l)(s+l)-2]m\}$ — — dimensional matrix

Therefore, using notations (14)–(18) we can express the performance index (3) in the following equivalent and more convenient form:

$$J_{r,s}\{X, U\} = X^T QX + U^T PU = z^T M^T QMz + z^T Nz =$$
$$= z^T [M^T QM + N]\, z = z^T Hz = \langle z, Hz\rangle, \qquad (19)$$

where

$$U^T = [u^T(1,0), \ldots, u^T(r,0), u^T(0,1), \ldots, u^T(0,s), u^T(1,1), \ldots$$
$$\ldots, u^T(r-1,s)],$$

$$Q = \operatorname{diag}[Q(1,1), Q(2,1), \ldots, Q(r,1), \ldots, Q(1,s), Q(2,s), \ldots$$
$$\ldots, Q(r,s)],$$

$$P = \operatorname{diag}[P(1,0), P(2,0), \ldots, P(r,0), P(0,1), P(0,2), \ldots$$
$$\ldots, P(0,s), P(1,1), \ldots, P(r,1), \ldots, P(1,s), \ldots, P(r-1,s)],$$

$$N = \left[\begin{array}{c|c} 0_{(r+s)n} & 0 \\ \hline 0 & P \end{array}\right], \qquad (20)$$

Q and P are symmetric, square matrices of the dimensions: rsn and $[(r+1)(s+1)-2]\, m$, respectively. H is also symmetric, square $\{(r+s)\, n+ +[(r+1)(s+1)-2]\, m\}$ — dimensional, nonnegative definite matrix.
The constraints have here the form:

$$Rz = c.$$

Where

$$R = \left[\begin{array}{ccccc|c} I & 0 & 0 & \ldots & 0 & \\ 0 & I & 0 & \ldots & 0 & \\ & & & \ldots & & 0 \\ 0 & 0 & 0 & \ldots & I & \end{array}\right]$$ Is an $(r+s)\, n \times \{[(r+1)(s+1)-2]\, m+(r+s)\, n$ — dimensional matrix, (21)

and

$$c^T = [x_{10}^T, x_{20}^T, \ldots, x_{r0}^T, x_{01}^T, x_{02}^T, \ldots, x_{0s}^T]. \qquad (22)$$

3. The problem solution

The necessary and sufficient condition for optimal problem (*) of mathematical programming is existence of solution for equation [4, Th. 1, page 146]:

$$\begin{bmatrix} -H & R^T \\ R & 0 \end{bmatrix} \begin{bmatrix} z \\ \varphi \end{bmatrix} = \begin{bmatrix} 0 \\ c \end{bmatrix}. \qquad (23)$$

Note, that the optimal solution (23) is unique if and only if the matrix in the left-hand side is non-singular. Otherwise, solution is not unique, and any solution of the equation (23) is an optimal solution of our problem.

4. Example

Let us consider the F-MM II system with the following properties:

$$n=2, \quad m=1, \quad (r,s)=(2,1),$$

$$A_1=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}, \quad A_2=\begin{bmatrix}-1 & 0\\ 1 & 1\end{bmatrix}, \quad B_1=\begin{bmatrix}1\\ 1\end{bmatrix}, \quad B_2=\begin{bmatrix}-1\\ 0\end{bmatrix},$$

$$x_{10}=x_{20}=\begin{bmatrix}0\\ 1\end{bmatrix}, \quad x_{01}=\begin{bmatrix}1\\ 0\end{bmatrix},$$

and the quadratic performance index (3) with matrices:

$$Q(i,j)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \quad P(i,j)=1.$$

In the first method of transformation we have

$$z^T=[x^T(1,1), x^T(2,1), u(1,0), u(2,0), u(0,1), u(1,1)],$$

$$z\in R^8,$$

$$H=\operatorname{diag}[1,1,1,1,1,1,1,1,]=I_8,$$

$$R=\begin{bmatrix}1&0&0&0&1&0&-1&0\\ 0&1&0&0&0&0&-1&0\\ 0&1&1&0&0&1&0&-1\\ -1&0&0&1&0&0&0&-1\end{bmatrix}, \quad c=\begin{bmatrix}0\\ 2\\ 0\\ 1\end{bmatrix}.$$

The matrix equation (23) yields the following set of equations:

$$-x_0(1,1)+\varphi_1-\varphi_4=0,$$

$$-x_1(1,1)+\varphi_2-\varphi_3=0,$$

$$-x_0(2,1)+\varphi_3=0,$$

$$-x_1(2,1)+\varphi_4=0,$$

$$-u(1,0)+\varphi_1=0,$$

$$-u(2,0)+\varphi_3=0,$$

$$-u(0,1)-\varphi_1-\varphi_2=0,$$

$$-u(1,1)-\varphi_3-\varphi_4=0,$$

$$x_0(1,1)+u(1,0)-u(0,1)=0,$$

$$x_1(1,1)-u(0,1)=2,$$
$$x_1(1,1)+x_0(2,1)+u(2,0)-u(1,1)=0,$$
$$-x_0(1,1)+x_1(2,1)-u(1,1)=1.$$

The solution has the form:

$$\varphi_1=\frac{-14}{41}\qquad x(1,1)=\begin{bmatrix}x_0(1,1)\\x_1(1,1)\end{bmatrix}=\begin{bmatrix}\frac{-29}{41}\\ \frac{39}{41}\end{bmatrix}\qquad u(1,0)=\frac{-14}{41},$$
$$\varphi_2=\frac{57}{41}\qquad u(1,0)=\frac{-18}{41},$$

$$\varphi_3=\frac{-18}{41}\qquad x(2,1)=\begin{bmatrix}x_0(2,1)\\x_1(2,1)\end{bmatrix}=\begin{bmatrix}\frac{-18}{41}\\ \frac{15}{41}\end{bmatrix}\qquad u(0,1)=\frac{-43}{42},$$
$$\varphi_4=\frac{15}{41}\qquad u(1,1)=\frac{3}{41}.$$

The value of performance index $J_{2,1}$:

$$J_{2,1}=x^T(1,1)\,Q(1,1)\,x(1,1)+x^T(2,1)\,Q(2,1)\,x(2,1)+$$
$$+u^T(1,0)\,P(1,0)\,u(1,0)+u^T(2,0)\,P(2,0)\,u(2,0)+$$
$$+u^T(0,1)\,P(0,1)\,u(0,1)+u^T(1,1)\,P(1,1)\,u(1,1)=$$

$$=\left[\frac{-29}{41},\frac{39}{41}\right]\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\frac{-29}{41}\\ \frac{39}{41}\end{bmatrix}+\left[\frac{-18}{41},\frac{15}{41}\right]\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\frac{-18}{41}\\ \frac{15}{41}\end{bmatrix}+$$
$$+\left(\frac{-14}{41}\right)^2+\left(\frac{-18}{41}\right)^2+\left(\frac{-43}{41}\right)^2+\left(\frac{3}{41}\right)^2=3.1463415.$$

In the second method of transformation we have:

$$z^T=[x^T(1,0),x^T(2,0),x^T(0,1),u(1,0),u(2,0),u(0,1),u(1,1)]\in R^{10}.$$

The matrix (18) M here has the form:

$$M=\begin{bmatrix}M(1,1)\\M(2,1)\end{bmatrix}\quad\text{for}\quad X^T=[x^T(1,1),x^T(2,1)].$$

Auxiliarily we calculate:

$$A^{1,0}=A_1\,A^{0,0}+A_2\,A^{1,-1}=A_1.\,I+A_2.\,0=A_1,$$
$$G(1,0)=A^{0,0}.\,B_1+A^{1,-1}B_2=B_1,$$
$$M(1,1)=[A^{0,0}.\,A_2,0,A^{0,0}.\,A_1,A^{0,0}.\,B_2,0,A^{0,0}.\,B_1,0]=$$

$$=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$M(2,1)=[A^{1,0}.\,A_2,A^{0,0}.\,A_2,A^{1,0}.\,A_1,A^{1,0}.\,B_2,A^{0,0}.\,B_2,A^{1,0}.\,B_1,G(1,0)]$$

$$=\begin{bmatrix} -1 & -1 & -1 & 0 & -1 & 0 & 0 & -1 & -1 & 1 \\ -1 & 0 & 1 & 1 & 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix},$$

$$P=Q=I_4, \quad N=\begin{bmatrix} 0_{6.6} & 0 \\ 0 & I_4 \end{bmatrix},$$

$$H=M^T QM+N=\begin{bmatrix} 4 & 2 & 0 & -1 & 2 & 2 & 2 & 1 & 0 & -2 \\ 2 & 2 & 1 & 0 & 2 & 0 & 0 & 1 & 2 & -1 \\ 0 & 1 & 2 & 1 & 1 & -1 & -1 & 1 & 2 & 0 \\ -1 & 0 & 1 & 1 & 0 & -1 & -1 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 2 & 0 & 0 & 1 & 2 & 1 \\ 2 & 0 & -1 & -1 & 0 & 2 & 2 & 0 & -2 & -1 \\ 2 & 0 & -1 & -1 & 0 & 2 & 3 & 0 & -2 & -1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 2 & 2 & 1 & 2 & -2 & -2 & 1 & 5 & 0 \\ -2 & -1 & 0 & 1 & -1 & -1 & -1 & -1 & 0 & 3 \end{bmatrix}$$

$$R=\left[\begin{array}{cccccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 1 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 1 & 0 & 0 & & 0 & & \\ 0 & 0 & 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 0 & 0 & 1 & & & & \end{array}\right] \quad c=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix},$$

The solution of matrix equation (23) gives:

$$u(1,0)=\frac{-14}{41} \quad u(0,1)=\frac{-43}{41}$$

$$u(2,0)=\frac{-18}{41} \quad u(1,1)=\frac{3}{41}$$

The optimal trajectory we have from (10), and finally $J_{2,1}=3.1463415$.

5. Remark

In the optimal problem for the F-MM II system, considered in section (1), we can add a condition concerning the terminal state vector. The obtained results, however, will be analogical as presented in this paper.

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Sterowanie optymalne F-MM II

W pracy przedstawiono metodę rozwiązywania zadania sterowania optymalnego przy kwadratowym wskaźniku jakości dla systemu liniowego, dyskretnego, stacjonarnego, dwuwymiarowego opisanego II modelem Fornasiniego-Marchesiniego. Metoda ta polega na sprowadzeniu zadania sterowania do pewnego zadania programowania matematycznego. Wyniki zilustrowano przykładem liczbowym.

О некоторой задаче оптимального управления для второй модели 2-D Форнасини-Марчесини

В работе представлен метод решения задачи оптимального управления, в случае квадратного показателя качества, двухмерной, линейной, стационарной, дискретной системой, описываемой II-ой моделью Форнасини-Марчесини. Этот метод состоит в сведении задачи управления к некоторой задаче математического программирования. Результаты иллюстрируются численным примером.