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Modeling and optimization of international economic cooperation via fuzzy mathematical programming and cooperative games*)

by

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A new "soft" model of international economic cooperation is proposed. It is essentially based on the multiperson cooperative game philosophy in which the participating countries (players) form coalitions (groups of countries) and then choose appropriate economic strategies to maximize a benefit (payoff, in fact their shares of some commodity). To account for an inherent "softness" of this problem we apply fuzzy mathematical (linear) programming to model the economic behaviour of both individual countries and their coalitions. This makes it possible to introduce "soft" (imprecisely defined or fuzzy) aspiration levels for both the objective function values and the satisfaction of constraints. This replaces the conventional strict optimization and constraint satisfaction which are unnecessarily rigid and unrealistic in practice in our context. Assuming such a "soft" model of economic behaviour we use tools of multiperson cooperative games and derive as a solution the *C*-core which is coalitionally stable.

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1. Introduction

International stability is presently one of top concerns both at the level of individual countries, blocks of countries, and international agencies and organizations. It is not only a prerequisite for a successful development of mankind but even "to be or not to be" for the today's world. Issues related to the international stability are therefore thoroughly studied by both governmental agencies, and scholarly and research institutions.

The economic aspects are here certainly of utmost importance because economy is what determines the strength of each country, standard of living, etc. hence has a decisive impact on the country's political, military, social, etc. situation.

Among the economic aspects those related to the international economic cooperation are particularly important as cooperation, if appropriately carried out, may significantly help raise the living standard, increase efficiency due to the effect of scale, make countries less vulnerable to economic perturbations, etc.

This paper is concerned with modeling of the international economic cooperation. Our point of departure is a series of papers [1], [2], [3], [4] in which a cooperative-game-theoretic approach has been employed.

Here we present a "softening" of that approach. A rationale is that since the economic cooperation problems concern some highly aggregated, often not clear-cut entities and relations on which data are rarely fully available and reliable (e.g., because they concern the future), strict requirements of the conventional models as to the precision of data are not realistic in many cases and should be allieviated.

Fuzzy sets theory [18] which provides formal means for representing imprecise concepts and relations, e.g., given by linguistic terms ("around 5", "much more than 10", ...) is used. In particular, we use fuzzy mathematical programming to model the economic behaviour of both the individual countries and their coalitions. Basically, fuzzy mathematical programming makes possible to "soften" too strict and unrealistic requirements to seek exact optima of the objective function(s) and to exactly satisfy the constraints. This is replaced by the requirements to attain some (possibly imprecisely defined) aspiration levels on the values of the objective function(s) and constraints. This is certainly more realistic in practice.

"Softening" of model and tools to deal with a variety of nontrivial real world problems involving diverse economic, social, political, environmental, etc. aspects — which are characteristic for the international economic cooperation considered here — has been strongly advocated in systems analysis [7], [16] and we think that fuzzy sets may well serve this purpose (see, e.g., [11] for an extensive review of fuzzy sets theory and its applications in a large spectrum of fields).

We propose here first a fuzzy mathematical (linear) programming model describing the economic behaviour of a country and a coalition of them. Next we present a cooperative-game-theoretic model of the economic cooperation between the countries which is based on the aforementioned fuzzy means. In Appendix, to make the paper self-contained we briefly sketch fuzzy sets theory and fuzzy mathematical programming.

2. Modeling international economic cooperation via multiperson cooperative games and fuzzy mathematical programming

The point of departure is here [1], [2], [3], [4], [8] where the international economic cooperation is viewed as a multiperson cooperative game in which the participating countries tend to form their best suited coalitions and to choose their best economic strategies, i.e. yielding a best economic effect.

We will now first "soften" the model governing the economic behaviour of both the individual countries and their coalitions, and then incorporate this model into a cooperative game.

2.1. A "soft" model of economic behaviour using fuzzy mathematical programming

Denote by: $M' = \{m\} = \{1, ..., M\}$ the set of countries in question, $K' = \{k\} = \{1, ..., K\}$ the set of sectors (individual economic activities), $N' = \{n\} = \{1, ..., N\}$ the set of commodities (goods) being produced and consumed, and $I' = \{i\} = \{1, ..., I\}$ the set of resource types.

Assume the matrices of technological coefficients to be: $A^m = (a^m_{nk})$ for the unit consumption (use) of commodity n in sector k in country m, $B^m = (b^m_{nk})$ for the unit production of commodity n in sector k in country m, and $D^m = (d^m_{ik})$ for the unit use of resource i in sector k in country m. Indices m, n, k and i range over the sets M', N', K', and I' in the sequel, if not otherwise specified.

The economic strategy of country m, which is the decision variable, is a vector $U^m = (u^m_1, ..., u^m_k, ..., u^m_k)^T$ where u^m_k is the production volume of sector k in country m.

The function

$$F^m(U^m) = \sum_{k=1}^{K} \sum_{n=1}^{N} (b^m_{ik} - a^m_{ik})\, u^m_k, \tag{1}$$

is called the production system operator of country m and yields an economic effect resulting from strategy U^m.

We introduce now the constraints to which the economy is subjected.

They are given as the following arrays.

$X^m = ((\underline{x}_1^m, \bar{x}_1^m), \ldots, (\underline{x}_n^m, \bar{x}_n^m), \ldots, (\underline{x}_N^m, \bar{x}_N^m))$ is the volume of commodity n in country m which is available for consumption. This should be meant as in Fig. 1, i.e.

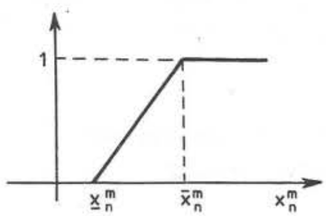

Fig. 1

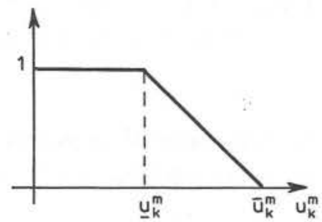

Fig. 2

we wish to attain the consumption level of commodity n in country m at least equal to $\bar{x}_n^m$ which is our satisfaction level (satisfaction = 1); the consumption level may be lower than $\bar{x}_n^m$ although the lower the less desirable ($x_n^m \downarrow$ satisfaction$\downarrow$), and $\underline{x}_n^m$ is the lowest possible consumption level.

And analogously, $\underline{\bar{U}}^m = ((\underline{u}_1^m, \bar{u}_1^m), \ldots, (\underline{u}_k^m, \bar{u}_k^m), \ldots, (\underline{u}_K^m, \bar{u}_K^m))$ is the array representing the limitations of production capacities in the particular sectors of country m to be meant as in Fig. 2, i.e. sector k can freely produce up to capacity $\underline{u}_k^m$, it can produce more although it is undersirable, and never more than $\bar{u}_k^m$.

Likewise, $Z^m = ((\underline{z}_1^m, \bar{z}_1^m), \ldots, (\underline{z}_i^m, \bar{z}_i^m), \ldots, (\underline{z}_I^m, \bar{z}_I^m))$ gives the limitations on the resources $i = 1, \ldots, I$ available to country m. Thus we can freely use resource i up to $\underline{z}_i^m$, we can eventually use somewhat more (e.g., by emergency importation or withdrawal from strategic stock) but never more than $\bar{z}_i^m$.

Assume, for simplicity, that the above constraints on the consumption level, production capacity, and resource availability are the only ones to be dealt with, i.e. we neglect those on labor force, feasible technologies, etc. They can be accounted for in a similar manner.

A feasible economic strategy is therefore one which satisfies:

— the production capacity constraints

$$0 \leqslant U^m \lessapprox \underline{\bar{U}}^m, \tag{2a}$$

or

$$0 \leqslant u_k^m \lessapprox \{\underline{u}_k^m, \bar{u}_k^m\}, \tag{2b}$$

where "$\lessapprox \{\cdot, \cdot\}$" is meant as given by (A7) in Appendix, i.e. it is satisfied to some degree, between 0 and 1

$$\begin{aligned} \mu_{km}^1(u_k^m) &= 1 && \text{for} \quad u_k^m \leqslant \underline{u}_k^m, \\ &= 1 - (u_k^m - \underline{u}_k^m)/(\bar{u}_k^m - \underline{u}_k^m) && \text{for} \quad \underline{u}_k^m < u_k^m < \bar{u}_k^m, \\ &= 0 && \text{for} \quad u_k^m \geqslant \bar{u}_k^m, \end{aligned} \tag{3}$$

— the resource availability constraint

$$D^m U^M \lesseqgtr Z^m, \tag{4a}$$

or

$$w_i^m = \sum_{k=1}^{K} d_{ik}^m u_k^m \lesseqgtr \{\underline{z}_i^m, \overline{z}_i^m\}, \tag{4b}$$

which is satisfied to degree

$$\begin{aligned} \mu_{im}^2 (w_i^m) &= 1 && \text{for} \quad w_i^m \leqslant \underline{z}_i^m, \\ &= 1-(w_i^m-\underline{z}_i^m)/(\overline{z}_i^m-\underline{z}_i^m) && \text{for} \quad \underline{z}_i^m < w_i^m < \overline{z}_i^m, \\ &= 0 && \text{for} \quad w_i^m \geqslant \overline{z}_i^m, \end{aligned} \tag{5}$$

— the consumption level constraint

$$(B^m - A^m)\, U^m \gtrsim X^m, \tag{6a}$$

or

$$g_n^m = \sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m)\, u_k^m \gtrsim \{\underline{x}_n^m, \overline{x}_n^m\}, \tag{6b}$$

which is satisfied to degree

$$\begin{aligned} \mu_{nm}^3 (g_n^m) &= 1 && \text{for} \quad g_n^m \geqslant \overline{x}_n^m, \\ &= 1-(\overline{x}_n^m-g_n^m)/(\overline{x}_n^m-\underline{x}_n^m) && \text{for} \quad \underline{x}_n^m < g_n^m < \overline{x}_n^m, \\ &= 0 && \text{for} \quad g_n^m \leqslant \underline{x}_n^m. \end{aligned} \tag{7}$$

We can also distinguish some representative commodity, say x_1^m, and express the consumption of other commodities, $x_2^m, \ldots, x_N^m$, with respect to x_1^m using the vector

$$h^m = (h_1^m, \ldots, h_n^m, \ldots, h_N^m), \tag{8}$$

where $h_n^m = x_n^m / x_1^m$; h^m is called the consumption structure.

In this case (6b) becomes

$$\sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m)\, u_k^m = h_n^m \sum_{k=1}^{K} (b_{1k}^m - a_{1k}^m)\, u_k^m \gtrless \{\underline{x}_n^m, \overline{x}_n^m\}. \tag{9}$$

An economic strategy of country m, $U^m = (u_1^m, \ldots, u_K^m)^T$, is clearly feasible to some degree which is here assumed to be

$$\begin{aligned} \mu_m^S (U^m) = \min \big(&\min (\mu_1^1 (u_1^m), \ldots, \mu_{Km}^1 (u_K^m)), \min (\mu_{1m}^2 (w_i^m), \ldots \\ &\ldots, \mu_{Im}^2 (w_I^m)), \min (\mu_{1m}^3 (g_1^m), \ldots, \mu_{Nm}^3 (g_N^m))\big), \end{aligned} \tag{10}$$

Notice that since we assume the degree of feasibility equal to the least degree of satisfaction of the particular constraints, this may be viewed to reflect some "satisfy-first" attitude which seems to be appropriate for most problems in our context.

An economic strategy is now called l-feasible if $\mu_m^S(U^m) = l$, i.e. it satisfies each constraint at least to degree l.

The objective function is the production system operator (1) to be maximized. This is replaced by a more realistic requirement

$$F^m(U^m) \gtrsim \underline{F}^m, \tag{11a}$$

or

$$y_k^m = \sum_{k=1}^{K} \sum_{n=1}^{N} (b_{ik}^m - a_{ik}^m)\, u_k^m \geqslant \{\underline{f}^m, \bar{f}^m\}, \tag{11b}$$

which is satisfied to degree

$$\begin{aligned} \mu_m^0(U^m) &= 1 && \text{for} \quad y_m \geqslant \underline{f}^m, \\ &= 1-(\bar{f}^m - y^m)/(\bar{f}^m - \underline{f}^m) && \text{for} \quad \underline{f}^m < y^m\, y\, \bar{f}^m, \\ &= 0 && \text{for} \quad y^m \leqslant \underline{f}^m, \end{aligned} \tag{12}$$

to be meant similarly as in Fig. 1, i.e. we are fully satisfied with $F^m(U^m) \geqslant \bar{f}^m$, partially satisfied with $\underline{f}^m < F^m(U^m) < \bar{f}^m$, and $F^m(U^m) \leqslant \underline{f}^m$ is unacceptable.

An economic strategy is said to be l-optimal if $\mu_m^0(U^m) = l$.

For country m the problem is now to find an optimal strategy U^{m*} and a maximal l, l^*, such that it yields the highest feasibility and optimality degrees of U^m.

Due to Appendix, this can be represented by a fuzzy linear programming (LP) problem with the fuzzy objective function (11) and the fuzzy constraints (2), (4) and (6), which is equivalent to the following nonfuzzy LP problem: find $U^{m*} = (u_1^{m*}, \ldots, u_K^{m*})^T$ and l^* such that

$$l \to \max$$

s.t.:

$$\begin{gathered} l(\bar{u}_k^m - \underline{u}_k^m) + u_k^m \leqslant \bar{u}_k^m, \\ l(\bar{z}_i^m - \underline{z}_i^m) + \sum_{k=1}^{K} d_{ik}^m u_k^m \leqslant \bar{z}_i^m, \\ l(\bar{x}_n^m - \underline{x}_n^m) + \sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m)\, u_k^m \geqslant \bar{x}_n^m, \\ 0 \leqslant l \leqslant 1, \\ u_k^m \geqslant 0. \end{gathered} \tag{13}$$

Evidently, the consumption level constraints may be replaced by (9) to explicitly involve the consumption structure.

2.2. A multiperson cooperative game model of international economic cooperation involving fuzzy linear programming

The economic cooperation between the countries whose economies are described by the model developed in the previous subsection is now modeled in terms of a multiperson cooperative game. This game is assumed to be an ordered pair $G=(M, v)$. M is the set of players (individual countries), and

$$2^M = \{S : S \subset M\}, \tag{14}$$

is the set of coalitions, i.e. subsets of countries; in particular, $S=\emptyset$, $S=M$, and $S=\{m\}$, i.e. an empty coalition (no coalition), the whole set M, and each individual countries acting separately, respectively.

A function, called the characteristic function of the game,

$$v: 2^M \to R;\ v(\emptyset)=0, \tag{15}$$

is defined which associates with each coalition S an economic effect, $v(S)$ — i.e. the value of (1) — that can be assured by the coalition no matter what the conduct of other players (not belonging to this coalition) is.

The algorithm for determining $v(S)$ (see, e.g., [3], [4]), but now taking into account the "soft" model of the economy of each country presented in Section 2.1, is

$$v(S) = F^*(S) = \sum_{m \in S} \sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m)\, u_k^{m*};\ v(\emptyset) = 0, \tag{16}$$

where $U^{m*} = (u_1^{m*}, \ldots, u_K^{m*})$, $m \in S$, are the optimal economic strategies of the countries belonging to coalition S, obtained by solving the following fuzzy LP problem

$$l \to \max,$$

s.t.:

$$\begin{gathered}
l(\bar{u}_k^m - \underline{u}_k^m) + u_k^m \leqslant \bar{u}_k^m; \quad m \in S, \\
l\Big(\sum_{m\in S} \bar{z}_i^m - \sum_{m\in S} \underline{z}_i^m\Big) + \sum_{m\in S}\sum_{k=1}^{K} d_{ik}^m u_k^m \leqslant \sum_{m\in S} \bar{z}_i^m, \\
l\Big(\sum_{m\in S} \bar{x}_n^m - \sum_{m\in S} \underline{x}_n^m\Big) + \sum_{m\in S}\sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m u_k^m \geqslant \bar{x}_n^m, \\
0 \leqslant l \leqslant 1, \\
u_k^m \geqslant 0.
\end{gathered} \tag{17}$$

We can obviously introduce into the consumption level constraints the consumption structure h as in (9).

Notice that $v(\{m\})$, $m \in M$, is some characteristic feature of country m which can be found by solving (13).

Function $v(S)$ is superadditive, i.e. $v(S) \geqslant \sum_{m \in S} v(\{m\})$. Intuitively, it can be justified as follows: the aspiration levels on lowest (highest) possible levels are obtained by summing up over $m \in S$ their individual counterparts. Thus if it happens that in one country a constraint limit is only partially used, it may be used by other countries of the coalition alleviating their limits, hence leading to a possibly higher economic effect.

Evidently, the maximal joint economic effect is for $S = M$, i.e. when all the countries cooperate, and is equal to $v(M)$ given by solving (17) for $S = M$.

If $\hat{X} = (\hat{x}_1, \ldots, \hat{x}_N)$ is the vector of global volumes of the consumption commodities, then the problem of international exchange, assuming a known and constant consumption structure h, is to determine the payoff vector $X = [x_1, \ldots, x_m, \ldots, x_M]$ which gives the share of x_1 (the basic, or representative commodity) allocated to country m, $m = 1, \ldots, M$.

Evidently

$$\sum_{m=1}^{K} x_m = \hat{x}_1 . \tag{18}$$

Notice that in (18) the ammount of $\hat{x}_1$ is assumed strictly given as opposed to the "soft" limits ($\bar{x}_n^m$ and $\underline{x}_n^m$) in (6). A "soft" definition of $\hat{x}_1$ (by $\underline{x}_1$ and $\hat{\bar{x}}_1$) is also possible but is beyond the scope of this paper.

Among many solution concepts in the game considered here, the C-core, $C(v)$, has proven very useful (see, e.g., [3], [4]). In our case it is defined as

$$C(v) = \{X : \sum_{m \in S} x_m \geqslant v(S), S \in 2^M ; \sum_{m=1}^{M} x_m = v(M)\} . \tag{19}$$

If $C(v) \neq \emptyset$, it contains therefore coalitionally rational (stable) solutions (the payoff vectors, i.e. the shares of x_1) in the sense that the players (countries) cannot do any better by leaving S.

3. Concluding remarks

We feel that the proposed model may help more realistically model important problems of international economic cooperation. First, it explicitly accounts for an inherent "softness" of these problems in an intuitively appealing and computationally tractable way. Second, it proposes to use some "softer" derivations of the tools of cooperative game theory and its solution concepts which have already proven to be useful for analysing problems from the consiered class.

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Appendix

Our notations and concepts related to fuzzy sets and fuzzy linear programming are standard (cf. [11]).

A fuzzy set A in a universe of discourse $X = \{x\}$, written $A \subsetneq X$, is represented by, and often equated with its membership function $\mu_A : X \to [0, 1]$ such that $\mu_A(x)$ states to what degree x belongs to A: from 0 for full nonbelongingness to 1 for full belongingnes, through all intermediate values.

The basic operations on fuzzy sets are:
— the complement (corresponding to "not")

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad \text{for each} \quad x \in X, \tag{A1}$$

— the intersection (corresponding to "and")

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \quad \text{for each} \quad x \in X, \tag{A2}$$

— the union (corresponding to "or")

$$\mu_{A+B}(x) = \max(\mu_A(x), \mu_B(x)) \quad \text{for each} \quad x \in X. \tag{A3}$$

Fuzzy linear programming is here meant in the sense of Zimmermann [19] who replaces the conventional linear program ($c^T x \to \min$, subject to: $Ax \leqslant b$, $x \geqslant 0$; $x \in R^n - \emptyset$, $c \in R^m$, A is an $n \times n$ matrix of real elements) by

$$\begin{cases} c^T x \lesssim z, \\ Ax \lesssim b, \\ x \geqslant 0, \end{cases} \tag{A4}$$

to be read as that the value of the objective function $c^T x$ should be possibly less than an aspiration level z, and the constraints should be possibly well fulfilled.

This becomes, if we denote $B = (cA)^T$ and $d = (zb)^T$,

$$\begin{cases} Bx \lesssim d \\ x \geqslant 0 \end{cases} \tag{A5}$$

or, for each i-th/j-th element of the respective vectors

$$\begin{cases} (Bx)_i \lesssim d_i & i = 1, \dots, m+1, \\ x_j \geqslant 0 & j = 1, \dots, n, \end{cases} \tag{A6}$$

To formalize "$\lesssim$" we construct the function

$$\begin{aligned} \mu_i(x) &= 1 && \text{for} \quad Bx_i \leqslant d_i \\ &= 1 - ((Bx)_i - d_i)/p_i && \text{for} \quad d_i < (Bx)_i < d_i + p_i \\ &= 0 && \text{for} \quad (Bx)_i \geqslant d_i + p_i \end{aligned} \tag{A7}$$

to be meant as: we are fully satisfied ($=1$) if $(Bx)_i$ does not exceed our satisfaction level d_i, we are fully dissatisfied ($=0$) if it exceeds $d_i + p_i$, and we are partially satisfied if it between d_i and $d_i + p_i$.

The problem (A4) of fuzzy linear programming becomes now

$$\min(\mu_1(x), \dots, \mu_{m+1}(x)) \to \max_{x \geqslant 0}, \tag{A8}$$

which is equivalent to (see, e.g., [11, 19])

$$\begin{cases} w \to \max \\ \text{s.t.: } wp_i + (Bx)_i \leqslant d_i + p_i \quad i = 1, \dots, m+1 \\ 0 \leqslant w \leqslant 1 \\ x \geqslant 0 \end{cases} \tag{A9}$$

which can be solved by any commercial LP package.

For details and a formal analysis, see, e.g., [11]; for a collection of works presenting the newest approaches to fuzzy linear programming, see the respective chapters in [13].

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Modelowanie i optymalizacja międzynarodowej współpracy gospodarczej za pomocą programowania matematycznego rozmytego i gier kooperacyjnych

Zaproponowano nowy „miękki” model współpracy międzynarodowej, oparty na idei wieloosobowej gry kooperacyjnej. Kraje uczestniczące w grze (gracze) tworzą koalicje (grupy krajów), a następnie wybierają takie strategie ekonomiczne, które maksymalizują korzyści-wypłaty (faktycznie dokonują oni podziału pewnego dobra). Z uwagi na „miękkość” tego problemu stosuje się model programowania matematycznego rozmytego (liniowego) dla opisu ekonomicznego zachowania się zarówno poszczególnych krajów, jak ich koalicji. Umożliwia to wprowadzenie „miękkich” (nieprecyzyjnie określonych lub rozmytych) poziomów aspiracji dla wartości funkcji celu i ograniczeń. Zastępuje to tradycyjną ścisłą optymalizację ze ścisłymi ograniczeniami, która jest niepotrzebnie zbyt rygorystyczna i nierealna w praktyce. Przyjmując taki „miękki” model ekonomicznego zachowania się, stosujemy narzędzia teorii wieloosobowych gier kooperacyjnych i znajdujemy rozwiązanie w postaci *C*-jądra, które jest koalicyjnie stabilne.

Моделирование и оптимизация международного экономического сотрудничества с помощью математического размытого программирования и кооперативных игр

Предлагается новая „мягкая” модель международного сотрудничества, основанная на идее кооперативной игры со многими участниками. Страны, участвующие в игре (игроки) создают коалиции (группы стран), а затем выбирают такие экономические стратегии, которые максимизируют пользу — выплату (фактически производят распределение некоторых благ). Учитывая „мягкость” этой задачи используется модель математического размытого (линейного) программирования для описания экономического поведения как отдельных стран, так и их коалиций. Это позволяет вводить „мягкие” (неточно определенные либо размытые) уровни стремлений для значений функции цели и ограничений. Это заменяет традиционную точную оптимизацию со строгими ограничениями, которая без надобности слишком требовательна и нереальна в практике. Принимая такую „мягкую” модель экономического поведения используем инструменты теории кооперативных игр со многими участниками и находим решение в виде С-ядра, которое является коалиционно стабильным.