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Modeling and optimizing fuel consumption and pollutant emissions in a car

by

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With reference to data provided for an Alfa Romeo car we describe the construction of a mathematical model for engine performance. The total fuel consumption over a European cycle is minimized under driveability constraints and legal limits on the total emissions of HC, CO, NOx. The obtained solution significantly improves the previously best known performance. Sensitivity analysis indicates robustness of the model and stability of the determined solution.

1. Introduction

Among the tasks that presently face the automotive industry those of reducing fuel consumption (in view of energy conservation policies) and of keeping pollutant emissions as low as possible (in view of environmental and health considerations) are of great importance. There are many ways of dealing with these problems, including better aerodynamical design, use of new materials and new fuels, innovative concepts of engine (for instance hybrid thermo-electrical engines) and so on. In this paper we shall only consider how the use of mathematical techniques can make the electronic control of certain engine parameters more efficient. We shall assume that the car is provided with sensors, which measure certain state parameters and input the measured data to a microcomputer. The microcomputer elaborates the data according to the criteria described further on in order to determine optimal values of certain control parameters, to be then inputted to suitable actuators. Feedback action to take into account delays and errors is not considered here, being however of course of great importance in practice.

The described research has been performed in a period of four years in the framework of the Progetto Finalizzato Trasporti of the National Science Council of Italy (CNR). It has been done in strict collaboration with the Italian automotive industry Alfa Romeo, which has provided data relating to a 2000 cubic centimeter engine. Further developements are expected with the introduction of additional state and control parameters. Commercial applications are expected in the nineties.

2. The mathematical problem of optimal electronic engine control

The multiobjective problem of reducing fuel consumption and pollutant emissions has been set by the automotive industry as the following single object optimization problem. Let the engine be characterized at time t by a set of state parameters $s = s(t)$ and a set of control parameters $u = u(t)$. Let $f(s(t), u(t), t)$ be the rate of fuel consumption at time t; note that the variable t appears as an independent argument, since f is not a univariate function of s and u, its value depending also on the trajectory through which the point $s(t)$, $u(t)$ is reached. Let similarly $f_i(s(t), u(t),t)$ be the rates of emission of certain I pollutants, $i = 1,..,I$. Let $Q = Q(s(t),t)$ be the region of implementable values for the control $u(t)$. Let S be a given trajectory of duration T in the space (s,t). Let c_i, $i = 1,.., I$, be the maximum admitted values of the total emissions of the i-th pollutant along S. Then we are given the following Problem P1:

$$\underset{u(t)}{\text{minimize}}\; F = \int_o^T f(s(t), u(t), t)\, dt$$

subject to: $s(t) \in S;\ u(t) \in Q$

$$\int_o^T f_i(s(t), u(t), t)\, dt \leqslant c_i \qquad i = 1,\dots I$$

Problem P1 in practice is simplified as follows:

— the history dependence in f and f_i is neglected, say f and f_i are assumed to be functions only of $s(t)$ and $u(t)$

— the region Q is given in the form of lower and upper bounds for $u(t)$, say $u_1(t) \leqslant u(t) \leqslant u_2(t)$

— the integrals are approximated by a summation over a finite number N of points along the cycle, so that instead of evaluating the function $u(t)$ for $0 \leqslant t \leqslant T$ the problem is reduced to evaluate the N controls $u(t_1),\ldots u(t_N)$. Therefore we obtain the following simplified Problem P2:

$$\underset{u(t_j)..\,u(t_N)}{\text{minimize}} \qquad F = \sum_{j=1}^{N} f(s(t_j), u(t_j))\, dt_j$$

subject: to: $s(t_j) \in S$

$$u_1(t_j) \leqslant u(t_j) \leqslant u_2(t_j)$$

$$\sum_{j=1}^{N} f_i(s(t_j), u(t_j))\, dt_j \leqslant c_i$$

$$(i = 1,\ldots I;\ j = 1,..\ N)$$

If the control u is a vector of dimension q, then Problem P2 is a mathematical programming problem of $n = Nq$ variables and $2Nq + I$ constraints, once the cycle S is given and assuming that the functions f, f_i are known.

In the case that we have studied, with data relating to the Alfa Romeo engine, both the state and control parameters are two-dimensional vectors, consisting in the first of the speed of rotation and torque, and in the second of the spark advance angle and the air/fuel ratio. The cycle S is assumed to be the European cycle, a trajectory in the space of the state variables that attempts to simulate the average behaviour of a European car (there exist similar American and Japanese cycles). Of the many pollutants emitted by a car, only three have been considered, following current EEC legislation, say HC (which is cancerogenous), CO (dangerous to the respiratory system) and NOx (a major culprit of acid rains inter alia). The limits c_i are those fixed presently by EEC legislation. The cycle has been discretized in N = 16 discretization points (working points), chosen, with the value of the weights dt_j, with engineering criteria. For an analysis of the discretization effects on the European cycle see Rudzinska and Spedicato [1].

3. Construction of the engine functions

The functions f and f_i are not known in explicit form and they are not easily computable by solving suitable dynamical equations (some authors have proposed such equations; see Spedicato [16] for a review of this and related problems). In practice they are defined or estimated through data collected in laboratory. There the engine is set running at a certain point s_j in the space of the

Table 1

Data on spline fitting

Point	Spline degree				Number of knots				Average relative error			
	Fuel	HC	CO	Nox	Fuel	HC	CO	NOx	Fuel	HC	CO	NOx
1	3	3	3	3	2	3	3	3	.76	1.3	17.6	1.5
2	idem				1	1	3	3	.24	.86	.68	.48
3					1	2	3	2	.32	.43	1.3	1.2
4					2	2	2	3	.54	2.3	6.1	2.2
5					1	1	2	2	.32	.36	18.9	1.9
6					1	1	2	1	.13	.43	178.3	10.3
7					1	2	2	2	.14	.32	73.9	3.7
8					1	2	1	2	.28	1.3	52.5	22.1
9			4		2	2	1	3	.43	1.0	64.1	1.7
10					2	2	1	3	.20	.64	31.6	1.2
11					0	1	1	2	.86	2.5	99.9	3.4
12					0	2	1	3	.52	1.3	48.8	2.1
13					1	2	1	3	.22	.16	71.9	.31
14					1	1	1	2	.15	.51	38.1	1.0
15					1	1	1	2	.31	1.0	70.1	1.9
16					1	1	2	2	.28	2.2	38.3	2.7

Problem formulation and ecemplary references state variables. Then several values of the control parameters are implemented, defining a more or less regular grid in the control space. The values of the functions f, f_i are then measured.

To reduce history effects, the measurements are made after warming up the engine. The result of the experiments is the availability of a two dimensional table (in our case consisting of about one hundred data) for each function and each working point.

Once the tables are obtained, there are two possible approaches to Problem P2. One is to select the best sequence of implemented controls, say the one that minimizes F subject to the constraints. This approach can be realized using discrete optimization techniques. The other one is to use data to construct a contiuous mapping of the functions and then apply contiuous optimization techniques. This is the approach that has been mainly followed in the recent literature and that we too have adopted.

Most researchers have used linear regression techniques to estimate the functions. For instance Powell et al. [2] have used the old well known BMD package for linear regression (using Legendre polynomials). Due to the availability since 1982 of the very efficient package DASL developed at the Physical National Laboratory in Teddington, UK, for one- and two-dimensional data fitting by Chebichev polynomials or splines, we have decided to make use of it. After extensive testing the conclusion was that fitting by splines was preferable, since approximation of the data of comparable quality could be obtained with splines using lower degree polynomials. The degree and the knot location of the splines were determined on heuristic criteria, since statistically based automatic criteria were not available. Incidentally the use of the splines was instrumental in discovering some serious problems in the first given set of data (outliers and irregular working of the engine in certain parameters areas); quite remarkably such problems had gone unnoticed in an independent fitting of the data by another analyst team using linear regression techniques. In Table 1 we give the used degree of splines and the average relative error in the fitted data for the Alfa Romeo data.

4. Computing optimal control values

Problem P2, with f, f_i obtained by a smooth fitting of the data, constitutes a nonlinear programming problem, of moderately large size in the case considered by us (the number n of variables is 32 and the number p of inequality constraints is 67). This problem can be solved quite straightforwardly using existing optimization packages. If more variables were included (in future research we plan to include exhaust gas recycle ratio and continuous transmission ratio) or more points along the cycle were considered, or also more refined definition of the driveability region Q were given, then the increase in the dimension would make convenient to exploit the structure of the problem to reduce the computational cost. A possible way of taking into account the structure is the following. Consider the first order Kuhn-Tucker conditions of problem P2:

$$\left[\nabla u_j f(s_j, u_j) + \sum_{i=1}^{3} \lambda_i \, u_j f_i(s_j, u_j) \, dt_j\right] + \mu_{j1} + \mu_{j2} = 0 \tag{1}$$

$$\lambda_i \left(\sum_{j=1}^{N} f_i(s_j, u_j) \, dt_j - c_i \right) = 0 \tag{2}$$

$$\mu_{j2} \quad (u_j - u_{2j}) = 0 \tag{3}$$

$$\mu_{j1} \quad (u_{1j} - u_j) = 0 \tag{4}$$

$$\lambda_i \geqslant 0 \tag{5}$$

$$\mu_{j1}, \mu_{j2} \geqslant 0 \tag{6}$$

where $i = 1,2,3, j = 1,\dots N, s_j = s(t_j), u_j = u(t_j), u_{1j} = u_1(t_j), u_{2j} = u(t_j)$ and λ_i, μ_{j1}, μ_{j2} are the Lagrange multipliers associated respectively with the total pollutant emissions and the lower and upper bounds in the control parameters. Inspection of the above conditions shows that the control parameters at different discretization points are coupled only through the multipliers λ_i. If these multipliers were known, the controls at the j-th working point could be obtained independently by dropping conditions (2) and (5) (under the usual function conditions which guarantee uniqueness of the Lagrange multipliers). Observe that the remaining equations are just the Kuhn-Tucker conditions of the following problem P3:

$$\underset{u_j}{\text{minimize}}\ F = (f(s_j, u_j) + \sum_{i=1}^{3} \lambda_i f_i(s_j, u_j)) \, dt_j$$

subject to: $u_{1j} \leqslant u_j \leqslant u_{2j}$

Note also that in such a case the weight dt_j does not influence the solution, so that it can be actually removed from the definition of F. The considered use of the structure implies that N problems of dimension two (in our case) have to be solved, instead of a single problem of dimension $2N$. This results generally in a great reduction in the computational cost.

The considered simplification has been adopted in the past (see for instance Powell et al. [2]), when efficient software was not available. The initial estimate of the λ_i's was set to zero (this being equivalent to putting no limits on the pollutant emissions) and the multipliers were then increased according to some heuristics until the total amout of the emissions, computed once the optimization was performed, were below the legal limits. More sophisticated ways of updating the multipliers could clearly be devised in the framework of the augmented Lagrange multipliers or recursive quadratic programming algorithms used for the optimization.

After experimenting with different optimization packages, our final choice was in favour of the code OPRQP in the package OPTIMA developed at the Numerical Optimization Centre of the Hatfield Polytechnic, UK, and based upon the recursive quadratic programming algorithm of Biggs [3]. The code had performed very well in the experiments of Schittkowsky [4] and has confirmed its robustness and efficiency (coupled with simplicity of use and remarkably short length of the program) in the case under consideration. The code was run in single precision on a Gould computer at Alfa Romeo, producing a solution generally in about half a minute for the several tried starting points. We considered eleven starting points generated as follows. One consisted of the optimal point known to Alfa Romeo and presently implemented on the engine. Ten more points were obtained by randomly generating $3 \cdot 10^5$ points (a task which took a few hours CPU time) inside the driveability region and memorizing, among the points that satisfied also the limits on the emissions, those which gave the least fuel consumption. Quite remarkably, none of the points generated in this way, gave a lower fuel consumption than that associated with the Alfa Romeo point, a result clearly representative of the „dimensionality curse" that affects random algorithms for optimization.

From all the considered starting points the algorithm has found the same solution giving, with respect to the previous best point known to Alfa Romeo, a 6% reduction in fuel consumptioon, a 20% reduction in HC, a 55% reduction in CO, but a 20% increase in NOx. In Table 2 we give the following information: initial value of consumption and emission for the first five starting points (P1 is the Alfa Romeo point), values in the computed optimal solution (point PF) and legal EEC limits (last column). While it is not possible to claim that the problem has a unique solution, the obtained convergence to the same point is in agreement

Table 2

Starting point and final point function values

Function	P1	P2	P3	P4	P5	PF	Bounds
Fuel	275.3	275.8	276.2	276.2	276.2	252.5	
HC	3.37	3.1	3.51	3.30	3.48	2.93	12.68
CO	28.08	23.4	41.92	31.17	44.65	10.06	155.12
NOx	8.15	8.2	8.63	7.8	8.1	10.65	18.21

Note: the units are grams per hour and kilowatt with previous findings (Powell, private communication). We should however remark that when OPRQP was applied to the problem with the functions evaluated from the first set of data (which, as said before, had irregularities detected through the use of the splines) different starting points produced different solutions!

In the determined solution no constraint is active. By sufficiently reducing the limits on the pollutants, active constraints appear in both the emission and driveability constraints. For instance, when the legal limit for NOx is reduced from 18.21 to about 10.60 it becomes active. Similarly HC becomes active when it is reduced from 12.80 to about 2.90 (all other limits left unchanged). If the limit for NOx is further reduced to about 9.90, no solution is found close to the solution corresponding to a slightly higher values of the limit (but a solution with higher fuel consumption exists in another region). This fact seems to indicate that with more stringent emission policies multimodality and discontinuity problems are expected. Multimodality is also expected when additional control parameters are included (see Radtke et al. [5] for the existence of two solutions when transmission ratio is included).

Once the optimal solution was obtained, we applied sensitivity analysis techniques to evaluate its stability and the robustness of the model. First Monte Carlo techniques were used to evaluate the effect of perturbation in the data and in the solution. Randomly generated errors with Gaussian distribution, zero mean and different variances were added to the data to simulate variability among cars. The perturbed data were then fitted with the same set of splines previously determined and the corresponding optimal control parameters were recomputed. Their expected variation was evaluated by averaging over a large set (three hundred) of perturbed data. The results indicate that the model is robust, stronger variations appearing only in the expected value of CO. We consider the high sensitivity of CO a possible consequence of the poor quality of the Alfa Romeo data for CO, as clearly appears from Table 1 (actually a closer inspection of the data shows the presence of a physically unacceptable peak for large values of the rotation speed).

Random variations in the optimal control parameters were then introduced in similar way, by treating the two controls either seperately or together. The expected variations in fuel consumption and emissions were again evaluated by averaging on a large number (four hundred) of cases. The solution appears to be stable from the point of view of consumption and emissions, again with a higher sensitivity of CO (its variation being one order larger). An additional interesting result was obtained when the sensitivity of our solution was compared with that of the previous best Alfa Romeo point, which turned out to be generally more sensitive by a factor of two. Overall it appears that an improvement over the old policy can be still obtained in presence of errors up to one per cent in the actuators (errors well below the technologically and commercially feasible limits).

The results of sensitivity analysis by the described Monte Carlo technique have been confirmed also by using deterministic techniques as those discussed by McKeown [6]; see Finardi [7] for details.

Finally an estimate of the optimal control parameters over the whole engine working region has been obtained by fitting the optimal controls in the discretized European cycle with orthogonal polynomials and interpolating splines, see Spedicato and Vespucci [8]. The fitting indicates that a smooth control surface is obtained.

5. Conclusions

In this paper we have synthesized the work done at the University of Bergamo in collaboration with Alfa Romeo on the optimal control of engine parameters (this work is fully documented in several technical reports, see for instance [9], [10], [11], [12], [13]). The results have indicated that significant improvements can be obtained by using the described techniques. In the next future we expect to apply these techniques to more control variables. Also the developement of on line model building and optimization is envisaged, making use of the greater power offered by parallel processors. For preliminary results in this direction, see Dixon and Spedicato [14], [15].

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Modelowanie i optymalizacja zużycia paliwa i emisji spalin w samochodach

W artykule przedstawiono metodę optymalnej kalibracji silnika samochodu osobowego pod kątem zmniejszenia zużycia paliwa w Cyklu Europejskim z uwzględnieniem dopuszczalnych norm na emisje związków toksycznych orraz ograniczeń zapewniających poprawną pracę silnika. Opisany model matematyczny opracowano na podstawie danych eksperymentalnych dostarczonych przez Alfa Romeo. W wyniku optymalizacji uzyskano znaczną poprawę pracy silnika. Analiza wrażliwości wykazała dobre własności modelu i stabilność otrzymanego rozwiązania.

Моделирование и оптимизация потребления горючего и выделение выхлопных автомобильных газов

В статье представлен метод оптимальной каллибровки двигателя легкового автомобиля, с точки зрения потребления горючего в Эвропейском Цикле, с учетом допустлимыцх норм выделения токсических соединений, а также ограничений обеспечивающих исправную работу двигателя. Описанная математическая модель разработана на основе экспериментальных данных, предоставленных фирмой Альфа Ромео. В результате оптимизации достигнуто значительное улучшение работы двигателя. Анализ чувствительности выявил хорошие свойства модели и устойчивость полученного решения.