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Fuzzy vs. non-fuzzy controllers

by

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We argue that the class of all possible fuzzy controllers is the same as the class of all possible non-fuzzy controllers for many control problems. We then argue that for the fuzzy controller to generalize control theory it must be able to process qualitative information.

1. Introduction

Consider a process to be controlled which has m outputs $y(t) = (y_1(t),...,y_m(t))$ and n inputs $u(t) = (u_1(t),...,u_n(t))$. It will not matter what the goals of the controller are and they could include: make $y_1(t)$ equal to target value s_1, make $y_2(t)$ equal to s_2, etc. A non-fuzzy controller determines the change in input vector $c(t)$ from the controller input $y(t)$ and its goals. The process input is then $u(t) = u(t-\Delta) + c(t)$, where Δ is the sampling period, at $t = \Delta, 2\Delta\cdots$. We assume that $y_i(t) \in [-M,M]$ for $1 \leq i \leq m$ and $t \geq 0$, for some $M > 0$, and $c_j(t) \in [-N,N]$ for $1 \leq j \leq n$ and $t \geq 0$, for some $N > 0$.

This non-fuzzy controller (*PI, PID, etc.*) uniquely determines the $c_j(t)$ from the $y_j(t)$ at each sampling time $t = \Delta, 2\Delta, \cdots$. The non-fuzzy controller is simply a function, say *NFC*, from $[-M,M]^m$ to $[-N,N]^n$ and we write

$$c(t) = NFC(y(t)),$$

for $t = \Delta, 2\Delta, \cdots$. Abstractly, a non-fuzzy controller may be any element in

$$F = \{f \mid f: [-M,M]^m \to [-N,N]^n\}.$$

2. Fuzzy controller

We now argue that a fuzzy controller may also be viewed as some element in F. We do not give a detailed discussion of fuzzy controllers (see [10], [11], [12], [13], [14]) but only briefly summarize the following four basic parts of a fuzzy controller.

1. The $y_i(t)$ values may first be scaled by scaling functions S_{oi} before input into the fuzzy control rules. Let $S_{oi}(y_i(t)) = z_i(t) \in [-1,1]$ for $1 \leq i \leq m$ and $t \geq 0$. For example, we could have $z_i(t) = k_i y_i(t)$, $1 \leq i \leq m$, where the k_i are scaling constants.

2. There are fuzzy numbers covering $[-1,1]$, for each input $z_i(t)$, which define the linguistic variables in the fuzzy control rules. The $z_i(t)$ are inputted into the fuzzy control rules, which are then evaluated using the fuzzy numbers, to produce fuzzy sets of output O_j, $1 \leq j \leq n$. The O_j will be fuzzy subsets of $[-1,1]$ ranging from a discrete fuzzy set to higher order fuzzy sets where the elements of an O_j are themselves fuzzy numbers whose support lies in $[-1,1]$.

3. The O_j are defuzzified to a real number in $[-1,1]$, by defuzzifying functions D_j. Let $D_j(O_j) = \delta_j(t) \in [-1,1]$ for $1 \leq j \leq n$ and $t \leq 0$.

4. The $\delta_j(t)$ values may be scaled by scaling functions S_{1j} before input into the process. Let $S_{1j}(\delta_j(t)) = c_j(t) \in [-N,N]$ for $1 \leq j \leq n$ and $t \geq 0$. For example, we could have $c_j(t) = \bar{k}_j\delta_j(t)$, $1 \leq j \leq n$, for non-zero scaling constants $\bar{k}_j$. Process input is $u(t) = u(t-\Delta) + c(t)$, where $c(t) = (c_1(t),\cdots,c_n(t))$, for $t = \Delta, 2\Delta, \cdots$.

The $c_j(t)$ are uniquely determined from the $y_i(t)$ at $t = \Delta, 2\Delta, \cdots$, and therefore the fuzzy controller is also a function, say FC, in F. We write $c(t) = FC(y(t))$ for the fuzzy controller.

We have been able to construct FC for a number of fuzzy controllers ([1], [3], [6], [9], [15]) and also, for certain fuzzy controllers ([2], [7]), we know the limiting structure of FC as the number of fuzzy control rules grows without bound. There are situations where FC can be very complicated but nevertheless, $c(t)$ will be some function of $y(t)$.

We now argue that: (1) given a NFC there is a FC so that $FC = NFC$; and (2) given a FC there is a NFC so that $NFC = FC$.

1. Suppose we have $NFC(y(t)) = c(t)$. We can choose different scaling functions S_{0i} and S_{1j}, we can use different fuzzy numbers and fuzzy control rules, and we can employ many different types of defuzzifiers D_j to construct an FC. With so many different variables to change we should be able to build an FC equal to NFC. First find non-zero scaling constants $\bar{k}_j$ so that $(c_j(t) / \bar{k}_j) \in [-1,1]$ for $1 \leq j \leq n$ and $t = \Delta, 2\Delta, \cdots$. Then we choose the D_j so that $D_j(O_j) = c_j(t) / \bar{k}_j$ for $1 \leq j \leq n$ and $t = \Delta, 2\Delta, \cdots$. Finally, we set $S_{1j}(\delta_j(t)) = k_j\delta_j(t)$ for $1 \leq j \leq n$ and $t = \Delta, 2\Delta, \cdots$ and we have $FC = NFC$.

2. Suppose we have $c(t) = FC(y(t))$. Since any element in F is a non-fuzzy controller we choose NFC in F to equal FC.

We have argued that any fuzzy controller is a function in F and once this function has been identified it can be used as a non-fuzzy controller to control the process and no fuzzy control rules need to be executed. We have also argued that given any non-fuzzy controller we can construct a fuzzy controller with the same control output.

3. Future Fuzzy Controller

For fuzzy controllers to truly generalize non-fuzzy control theory they must be able to process qualitative data. Qualitative output from the process would be in the form of linguistic variables which would first be translated into fuzzy sets before input into the fuzzy controller. Fuzzy set output from the fuzzy controller could be translated into linguistic variables to become the qualitative input to the process.

Machines in the system do not directly produce fuzzy data for input into the fuzzy controller. The $y_i(t)$ are usually obtained from measurements on the process and they will contain measurement error. It is not difficult to incorporate these uncertainties into the $y_i(t)$ by making them into fuzzy numbers [4]. If some of the $y_i(t)$ are fuzzy numbers, then the fuzzy control rules must be able to accept fuzzy number input [8] to produce the fuzzy sets O_j for output. Also, some of the $y_i(t)$ may come from human operators working within, or observing, the process. The $y_i(t)$ coming from humans would be in the form of linguistic variables that would be translated to fuzzy subsets of $[-1,1]$ for input into the fuzzy controller [5].

Machines in the system do not yet accept fuzzy commands, however humans within the process can be trained to act correctly given fuzzy instructions. Some of the output fuzzy sets O_j will be defuzzified for input into the machines in the process while others will be translated into linguistic commands for the human operators in the system [5]. Therefore, we must allow for the possibility that a fuzzy controller will not defuzzify any output but instead translate to fuzzy commands understood by the process.

When we allow for fuzzy input and output the fuzzy controller becomes a function from $\overline{[-M,M]}^m$ to $\overline{[-N,N]}^n$ where (1) $\overline{[-M,M]}$ is $[-M,M]$ together with a class of fuzzy subsets of $[-M,M]$; and (2) $\overline{[-N,N]}$ is $[-N,N]$ plus a collection of fuzzy subsets of $[-N,N]$. Since $[-M,M] \subset \overline{[-M,M]}$ and $[-N,N] \subset \overline{[-N,N]}$ fuzzy controllers will now generalize non-fuzzy controllers.

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Regulatory rozmyte i regulatory nierozmyte

W pracy pokazuje się, że w wielu zagadnieniach sterowania klasa wszystkich możliwych regulatorów rozmytych jest taka sama, jak klasa wszystkich możliwych regulatorów nierozmytych. Pokazuje się następnie, że aby uogólnić teorię sterowania regulator rozmyty musi być w stanie przetwarzać informację jakościową.

Размытые и неразмытые регуляторы

В работе доказывается, что во многих проблемах управления класс всех возможных размытых регуляторов такой же как и класс всех возможных неразмытых регуляторов. Затем показано, что для обобщения теории управления необходимо, чтобы размытый регулятор был в состоянии обработывать качественную информацию.