Control
and Cybernetics
VOL. 19 (1990) No. 3–4

Active control of elastic wing of an aircraft

by

T. K. SERAZETDINOV

Kazan Aviation Institute
ul. Karla Marksa 10
420084 Kazan, USSR

The paper raises and solves the problem of reaching the given terminal state or damping oscillations of the wing by means of an actively controlled surface. It is assumed that constraints imposed on the norm of control are to be met in the minimal time.

1 Introduction

A contemporary aircraft is a complex mechanical system, comprised of interacting rigid and elastic elements. Its individual parts, e.g. a wing, tail of a plain, missile body, have the form of thin elongated bodies and during the flight they are subjected to large deformations and vibrations. The interaction of aircraft deformations with aerodynamic and control forces can result in undesired phenomena, such as oscillations, loss in the flight stability, flutter. To overcome these unwanted phenomena some precautions, in the main of passive nature, are undertaken. They are as follows: improving the construction rigidity or forward shift of the line of wing center of mass by means of disposing additional loads close to the leading edge. However, these measures are not always effective. Hence at the present time the problem of active damping of elastic vibrations of a wing or other parts of an aircraft becomes the subject of study. Of particular interest is the problem of increasing the critical speed of wing flutter by active damping of elastic vibrations of a wing.

Consideration presented gives rise to the following problem: given the shape and the speed of construction under examination and its equations of motion, one has to determine the desired control signal, i.e. such that vibrations are damped.

The problem of active control of elastic constructions is very complicated and to solve it fundamental theoretic and experimental investigations are to be carried on. The paper is concerned only with theoretic time-optimal problems of active damping or critical damping of elastic vibrations of a wing.

Elastic vibrations of a construction are described by partial differential or integro-differential equations and refer to distributed parameter systems. Problems of control of distributed-parameter systems as well as those related to a number of applications are generalized in the monography [1]. In this monography the problem of analytic construction of an optimal controller for damping vibrations of the elastic axis of a ballistic missile or elastic wing is discussed. In particular, it is shown that: (i) there exists a possibility of the significant increase in the critical speed of flutter by the use of the control surface of an aileron type (ii) the fluid vibrations occurring in the tank of a mother-missile can be optimally damped (iii) the elastic vibrations of a wing under random disturbances can be time-optimally damped.

In the book [2] and in other publications one can find the analysis of the following problems: (i) the mathematical description of an elastic spacecraft considered as a distributed-parameter plant (ii) the optimal synthesis of control in deterministic and stochastic systems (iii) the adaptive optimal control (iv) the estimation of system state (v) the question of filtration in distributed-parameter systems and elastic spacecrafts.

The paper deals with a generalization of the time-optimal control problem formulated for distributed-parameter systems as well as the application of this approach to the optimal damping of elastic vibrations of a wing.

2 Formulation of the problem and a method of solving it

The coupled flexural vibrations of a wing arising from the action of forces: elastic, aerodynamic and those brought about by actuators are described, in the non-dimensional form, by the following system of differential equations [3]

$$\begin{gathered} m\frac{\partial^2 y}{\partial t^2} - m\sigma\frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial}{\partial t^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) - Y - F_1\delta = 0, \\ I\frac{\partial^2 \varphi}{\partial t^2} - m\sigma\frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial t}\left(GI_p\frac{\partial \varphi}{\partial x}\right) - M - F_2\delta = 0, \\ t > 0, \qquad x \in (0,1), \end{gathered} \tag{1}$$

where t – time; x, y – the coordinates of points of the elastic axis of the wing; φ – the angle of rotation of the wing cross-section about the elastic axis; m –the distributed mass of wing cross-section; σ – the distance of the center of mass of the wing cross-section to the wing elastic axis; EI, GI_p – the flexural and torsional wing rigidity, respectively; Y, M – the distributed lift force and aerodynamic moment about the wing elastic axis, respectively; δ – the control angle.

$$F_i = \begin{cases} F_{i0} & \text{at the points, where there is the movable surface.} \\ 0 & \text{at the points, where there is no the movable surface.} \end{cases}$$

Given a controllable system described by the integral equation

$$\Psi_i(x,t) = \int_0^t \int_0^1 K_i(x,t,\xi,t')u(\xi,t')d\xi dt', \qquad i = \overline{1,n}, \tag{2}$$

where: $\Psi_i(x,t)$ – the functions representing the state of process at the time instant $t > 0$ and at the point $x \in [0,1]$, ($\Psi_i \in L_q$, $q \geq 1$); $u = u(\xi,t)$ – the control function ($u \in L_p$, $p \geq 1$, $\frac{1}{p} + \frac{1}{q} = 1$); $K_i = K_i(x,t,\xi,t')$ – the kernel of the equation ($K_i \in L_q$, $q \geq 1$).

Having the Green function constructed for the equations (1), they can be reduced to the integral equation (2). Under such a transformation the functions $\Psi_i = \Psi_i(x,t)$ can be expressed as linear functions depending upon $y = y(x,t)$, $\varphi = \varphi(x,t)$ and their time derivatives.

Let us assume that $\alpha_i = \alpha(x,t) \in A \subset L_q$ are given functions and at some time instant $t = T > 0$ the following condition has to be satisfied

$$[\Psi_i(x,t)]_{t=T} = \alpha_i(x,T) \tag{3}$$

It is assumed that if there is no constraints imposed on the norm of control $||u||_{L_p}$, then any element of the set A can be reached at an arbitrary time instant $T \geq 0$.

The problem 1. For a system described by the equation (2) one has to determine such a control rule $u = u(x,t)$ that at the time instant $t = T > 0$ the given conditions (3) and constraints imposed on the norm of control $||u||_{L_p} \leq l$, where l some positive number, are satisfied.

The problem 2. For a system described by the equation (2) one has to determine such a control rule $u = u(x,t)$ that the given conditions (3) and constraints imposed on the norm of control $||u||_{L_p} \leq l$, where l some positive number, are met in the minimal time $t = T > 0$.

3 Necessary and sufficient conditions for solving the problems stated

Let us assume that the functions $\alpha_i(x,t)$ and $K_{L_i}(x,T,\xi,t')$ are such that the equations (2) and (3) can be solved with respect to $\Psi_i(x) \in L_p$ for every $T \in (0,+\infty)$.

Theorem 3.1 [1] *The necessary and sufficient condition for the existence of a solution $u = u(x,t) \in L_p$ to the equations (1) and (2) subject to the constraint $||u||_{L_p} \leq l$ ($l = const > 0$) at some $T \in (0,\infty)$ is that for all possible choices of the vector function $\lambda(x) = \{\lambda_1(x),\cdots,\lambda_n(x)\}$, $\lambda_i(x) \in L_p$ the following inequality holds*

$$\left|\int_0^1 \sum_{i=1}^n \lambda_i(x)a_i(x,T)dx\right| \leq lF(\lambda), \tag{4}$$

where

$$F(\lambda) = \left\{\int_0^T \int_0^1 \left|\int_0^1 \sum_{i=1}^n \lambda_i(x)K_i(x,T,\xi,t')dx\right|^q d\xi dt'\right\}^{\frac{1}{q}}. \tag{5}$$

A control rule, for which the assumptions of the theorem hold true, is of the form

$$u(\xi,t') = \frac{1}{(F[\lambda,t])^q}\left|K(\xi,t')\right|^{q-1} sign\left[K(\xi,t')\right], \tag{6}$$

where

$$K(\xi, t') = \int_0^1 \sum_{i=1}^{n} \lambda_i(x) K_i(x, T, \xi, t') dx. \tag{7}$$

The Lagrange multipliers λ_i satisfy the following equations

$$q \int_0^T \int_0^1 |K(\xi, t')|^{q-1} K_j(y, T, \xi, t') sign\,[K(\xi, t')]\, d\xi dt' - \lambda_0 \alpha_j(y) = 0, \tag{8}$$

$$\int_0^1 \sum_{i=1}^{n} \lambda_i(x) \alpha_i(x, T) dx = 1, \tag{9}$$

$$\frac{1}{q} + \frac{1}{p} = 1 \qquad \text{and} \qquad p \geq 1, \qquad q \geq 1,$$

where $||u||_{L_q} = \frac{1}{F(\lambda)} \leq l$.

To make use of the control rule (6), one has to solve the equations (7) to (9) with respect to λ_i $(i = 0, 1, \cdots, n)$ or the Lagrange multipliers are to be directly determined from the condition that $F(\lambda)$ is to be minimized subject to the constraint (9), where $F(\lambda)$ is defined by the expression (5). The multipliers λ_i derived in such a way are substituted into the formula (7). The formulae (6) and (7) allow to construct a control rule providing a solution to the Problem 1. The equations (7) to (9) are nonlinear; in order to solve them a modified Newton method of tangents is applied. It should be emphasized that the relations (6) hold for $q = 1$ too.

Theorem 3.2 [1] *The smallest positive root T of the equation*

$$\frac{1}{l} = F(\lambda, t) \tag{10}$$

yields a solution to the time optimal problem.

To determine control providing a solution to the time optimal problem one has to solve the equations (7) to (9) and construct a control rule such that the condition (10) of Theorem 2 holds.

4 Approximate method of solving the time optimal problem for an elastic wing

When examining elastic vibrations of a wing, the system of equations (1) and the boundary conditions are usually transformed into an infinite system of ordinary differential equations. Further this system is approximated by a finite one.

The constraint on the deflection angle of movable surface can be reduced to the assumption that $q = 1$ and $|u| \leq l$. Under these conditions the system of equations (7) to (9) can be approximately written in the form

$$\begin{array}{c} \int_0^T K_i(T,t) sign \sum_{i=1}^{2N} \lambda_i K_i(T,t) dt - \lambda_0 \alpha_{Ti} = 0, \\ \sum_{j=1}^{2N} \lambda_j \alpha_{jj} = 1, \qquad i = 1, 2, \cdots, 2N, \end{array} \tag{11}$$

where N – the number of wing vibration frequencies taken into account.

To construct the optimal control rule, one has to solve the system of equations (11) with respect to λ_i $(i = 0, \cdots, 2N)$ first. However, this system is nonlinear due to the fact that λ_i enter into the *sign* expression. It is suggested to solve the system under consideration by a linearization method at the vicinity of $\lambda_i = \lambda_i^{(k)}$, where $\lambda_i^{(k)}$ – the value of the k-th approximation of λ_i. As a result the following system to be solved is obtained:

$$\begin{array}{l} \alpha_i^{(k)} - \lambda_0 \alpha_{Ti} + \sum_{j=1}^{2N} K_{ij} \delta\lambda_j = 0, \quad i = 1, \cdots, 2N \\ \alpha_0^{(k)} + \sum_{j=1}^{2N} \alpha_{Tj} \delta\lambda_j = 0, \end{array} \tag{12}$$

where

$$\alpha_i^{(k)} = \int_0^T K_i(T,t) sign K^{(k)}(t) dt$$

$$K^{(k)}(t) = \sum_{j=1}^{2N} K_j(T,t) \lambda_j^{(k)}$$

$$\alpha_0^{(k)} = \sum_{j=1}^{2N} \alpha_{Tj} \lambda_j^{(k)} - 1$$

$$K_{ij}^{(k)} = - \sum_{s=1}^{S_0} \left[sign K^{(k)} \right]_{-s} \frac{2K_i(T,t) K_j(T,t)}{\left[\frac{dK^{(k)}}{dt} \right]_{t_s}}$$

$$\left[\frac{dK^{(k)}}{dt} \right]_{t_s} = \sum_{j=1}^{2N} \left[\frac{dK_i(T,t)}{dt} \right]_{t=t_s} \lambda_i^{(k)}$$

where t_s – the time instant of control switching; $\left[sign K^{(k)} \right]_{-s}$ – the sign of the function $K^{(k)}(T,t)$ at points $t < t_s$ belonging to the left-hand side neighbourhood of the point t_s, $s = 1, 2, \cdots, S_0$; S_0 – the total number of switching points in the time interval $(0, T)$. The switching points t_s correspond to values of t such that $K^{(k)}(t)$ equals to zero.

5 Solution procedure

To accomplish the time-optimal control, one has to make at every time instant measurements of: (i) the initial deflection of the elastic axis $y(t)$; (ii) the precession angle of this axis $\varphi(t)$ as well as their velocities $\dot{y}(t)$ and $\dot{\varphi}(t)$. On the basis of these measurements the coefficients α_{Ti} are determined.

The kernel of equation $K_i(T,t)$ depends upon the wing parameters and is computed in advance. This kernel is given in the form of an analytic function or a table.

When solving the system of equations (11) with respect to λ_i for distinct values of T, one can find the smallest value of T among solutions to the equation (10). This is a solution to the time-optimal problem and λ_i is the value of Lagrange multiplier corresponding to the time-optimal control. Substitution of the obtained value of λ_i into the control rule results in the optimal value of control angle.

If external disturbances were absent, then the control rule derived in such a way would be optimal with respect to ideal damping of measured disturbances of the initial state. However, in the course of flight disturbances of different character, e.g. gust of wind, non-uniformity of the atmosphere etc., exert their influence. For this reason the initial deviations should be measured continuously and use to solve Problems I and II.

Numerical experiments carried out have shown that the optimal value of T is a fraction of a second. However, the possibility of solving the problem discussed in on-line mode, i.e. during the aircraft flight, requires a considerable effort to work out computation procedures effective enough to accomplish this objective.

References

[1] Serazetdinov T. K. Optimization of distributed-parameter systems, (in Russian), Nauka, Moscow, 1977.

[2] Diegtiariev G. L., Serazetdinov T. K. Theoretical fundamentals of optimal control of elastic spacecrafts, (in Russian), Mashinostroienie, Moscow, 1986.

[3] Kolesnikov L. S., Sukhov W. N. Elastic aircraft as a controlled plant, (in Russian), Mashinostroienie, Moscow, 1974.

[4] Smirnova E. T. On time-optimal problem in distributed-parameter systems, (in Russian). In: Process optimization in aerospace technology, KAI Publ., Kazan, 1976, pp. 56-62.