Control and Cybernetics

VOL. **20** (1991) No. 1

Controllability of projected control systems.

by

Dumitru MOTREANU

University of Iaşi
Department of Mathematics
6600 Iaşi
Romania

Corresponding to an arbitrary control system, described by a control vector field f, and to a submanifold S of the state space, the notion of projected control system of f on S is introduced. Under a natural transversality hypothesis of f to a fixed submanifold A, local controllability along a reference solution is established for the projected control system of f on every submanifold S which is directed transversally to A in an appropriate sense. This controllability property is shown to be generic with respect to the initial data, the vector field f and the submanifold A.

1. Introduction

Let J and V represent throughout the paper an open interval in R and an open subset V of the Euclidean space R^n, respectively. We are concerned with a nonlinear control system

$$\dot{x}(t) = f(t, x(t), u(t)) \quad \text{for a.e.} \quad t \in J, \tag{1}$$

where the state variable x belongs to V and the control u is in $L^m_\infty(J)$. Here $L^m_\infty(J)$ stands for the Banach space, under the essential–supremum norm, of all (equivalence classes of) measurable, essentialy bounded functions from J into R^m. The mapping $f: J \times V \times R^m \to R^n$ is supposed to be of class C^k, $k \geq 1$. In this case there exists a unique, maximal, absolutely continuous solution $x(t) = x(t, t_0, x_0, u)$ of (1) satisfying the initial condition $x(t_0) = x_0$. Moreover, the solution $x(t)$ is of class C^k with respect to all variables (t, t_0, x_0, u) (cf., e.g., [1]–[3]).

Consider now an arbitrary (embedded) C^{k+1} submanifold S of R^n included in V. We describe a general procedure of inducing from (1) a control system on S. In doing this we employ the geometric notion of projection–valued function used frequently in differential geometry, esspecially in the studies of connections on differentiable manifolds (see [4] and the references herein). For standard terminology concerning the differentiable manifolds we refer to [5]–[7].

To this aim we proceed as follows. The tangent bundle TS of S is a C^k vector subbundle of the trivial bundle $S \times R^n$ over S. Thus there exists a projection-valued function on S which simply means a C^k bundle map $P^S: S \times R^n \to TS$ assigning smootly to each point $x \in S$ a projection operator

$$P^S(x) = P^S(x, \cdot): R^n \to T_x S, \quad P^S(x)^2 = P^S(x), \tag{2}$$

which maps R^n onto the tangent space $T_x S$ of S at x. Various examples of projection–valued functions (2) are given in [4]. They are generally constructed by means of a C^k partition of unity on S. A projection–valued function P^S yields naturally a control system on the submanifold S (this is said to be projected by (1) on S) as it is shown below.

Proposition 1 *Let the control system (1), having controls in $L^m_\infty(J)$, be defined by a C^k mapping $f: J \times V \times R^m \to R^n$, $k \geq 1$. If S is a C^{k+1} submanifold of R^n contained in V which is endowed with a C^k projection–valued function P^S on S, then the mapping $f^S: J \times S \times R^m \to TS$ defined by*

$$f^S(t, x, w) = P^S(x) f(t, x, w), \quad (t, x, w) \in J \times V \times R^n, \tag{3}$$

is a C^k control vector field on S. The corresponding control system on S

$$\dot{y}(t) = f^S(t, y(t), u(t)), \quad \text{for a.e. } \ t \in J, \tag{4}$$

with controls in $L^m_\infty(J)$, is thus well-defined. It is termed to be projected by (1) on S.

PROOF. In view of (2), $P^S(x)$ takes value in T_xS for every $x \in S$. Therefore one has

$$f^S(t,x,w) \in T_xS \quad \text{for all} \quad (t,x,w) \in J \times V \times R^m,$$

so f^S is a control vector field on the submanifold S (see [1]–[3]). The C^k differentiability of f^S follows from relation (3) because the mappings P^S and f have the same property.

The next corollary points out a relevant situation of projected control system.

Corollary 1 *The projected control vector field f^S coincides with the restriction of f to $J \times S \times R^m$ if and only if, for every $(t,w) \in J \times R^m$, the vector field $f(t,\cdot,w): V \to R^n$ is tangent to S, i.e.,*

$$f(t,x,w) \in T_xS \quad \text{for all} \quad x \in S.$$

PROOF. It is a direct consequence of (2) and (3).

The purpose of this paper is to investigate the controllability properties of the projected control system (4) in connection with system (1). Corollary 1 shows that this problem can be regarded as an extension of the question of describing the controllability of system (1) in the submanifold S. We mention that the problem of the controllability in a prescribed region of the state space has been considered. Local controllability in a given cone is discussed, for example, in [8].

More precisely, we are concerned with local controllability of the projected control system (4) along a reference solution. This is a strong type of controllability introduced by Hermes [9]. Notice that the Hermes' approach, based on the analiticity assumption of the control vector field with respect to the state variable, cannot be applied to system (4) where only the C^k differentiability, $k \geq 1$, is supposed.

For the sake of clarity we recall the notion of local controllability along a reference solution in the case of the projected control system (4). If (t_0, x_0) is a fixed point in $J \times S$ and u is a fixed control in $L_\infty^m(J)$, let $t \to y(t) = y(t,t_0,x_0,u)$ designate the unique, maximal, absolutely continuous solution of (4) satisfying the initial condition

$$y(t_0) = y(t_0,t_0,x_0,u) = x_0. \tag{5}$$

The attainable set $\mathcal{A}(t_0,x_0;t)$ of system (4) from $(t_0,x_0) \in J \times S$ at time $t \in J$, $t > t_0$, is defined by

$$\mathcal{A}(t_0,x_0;t) = \{y(t,t_0,x_0,u) | u \in L_\infty^m(J) \text{ provided } y(t,t_0,x_0,u) \text{ exists}\}. \tag{6}$$

More restrictively than (6), let us introduce, for each constant $\varepsilon > 0$ and each control u_0, the following subset of $\mathcal{A}(t_0, x_0; t)$

$$\mathcal{A}_{\varepsilon,u_0}(t_0, x_0; t) = \{y(t, t_0, x_0, u) \in \mathcal{A}(t_0, x_0; t) |\ \|u - u_0\| < \varepsilon\}. \tag{6'}$$

In (6'), as opposed to (6), only the small controls near u_0 are neaded. The control system (4) is said to be locally controllable along a reference solution $t \to y(t, t_0, x_0, u)$ if the following condition is verified

$$y(t, t_0, x_0, u) \in \text{int } \mathcal{A}(t_0, x_0, t) \tag{7}$$

for all $t > t_0$ in the domain of $y(\cdot, t_0, x_0, u)$. Abreviation "int", used in (7), means the interior in the submanifold S. If instead of (7) we postulate

$$y(t, t_0, x_0, u) \in \text{int } \mathcal{A}_{\varepsilon,u_0}(t_0, x_0; t), \quad t > t_0, \tag{7'}$$

for every $\varepsilon > 0$, the system (4) is called locally controllable along the reference solution $y(\cdot, t_0, x_0, u)$ by arbitrarily small controls.

We treat local controllability along a reference solution by arbitrarily small controls in the case of the projected control system (4) using essentially the transversality theory (see [5]–[7]). The main idea is to take into account the position of the control vector field f entering (1) relative to the submanifold S. The result obtained is stated in Theorem 1. It is important that Theorem 1 holds for all submanifolds S having at x_0 the same tangent space. This fixed position of the tangent space is determined by means of a "precised" submanifold A of R^n.

Then, we study the genericity of the controllability property described in Theorem 1 relative to all parameters entering the initial value problem (4), (5). Recall that a property is called generic on a topological space X if it holds on a residual set, that is, on an intersection of a countable family of open dense subsets of X. Namely, we formulate in Theorem 2 an effectively computable sufficient condition in order that the controllability result of Theorem 1 be a generic property with respect to the initial data (t_0, x_0). Then, Theorem 3 shows genericity of this controllability property with respect to the control vector field f taken as a parameter. The density and the openness in the generic result of Theorem 3 are considered relative to the (strong) C^k Whitney topology on the set $C^k(J \times V \times R^m; R^n)$ ([6], [7]) under identification in Section 1 between the control system (1) and the corresponding control vector field f. Finally, in Theorem 4 a generic result on the controllability property of Theorem 1 with respect to the C^k submanifold A is proved. The argument in proving Theorems 2–4 relies on the parametric transversality theorem, The Thom transversality theorem and the Abraham's transversality theorem (see [5]–[7]).

The use of the transversality theory in solving controllability problems appeared in different papers. For instance, Medved [10] studied the genericity of the complete controllability for linear parametrized control systems. Other applications of the transversality theory in different control problems have been given by the present author in [11]–[13].

The remaining of the paper is organized as follows. Section 2 is devoted to the study of local controllability along a reference solution by arbitrarily small controls in the case of the projected systems (4). Section 3 contains our generic results for the controllability property discussed in Section 2. Some final remarks are presented in Section 4.

2. Local controllability along a reference solution

The following preliminary result will be useful in the sequel.

Proposition 2 *Let $f: J \times V \times R^m \to R^n$ be a C^k mapping, $k \geq 1$, let A be a C^k submanifold of R^n and let (t_0, x_0, w_0) be a point in $J \times V \times R^m$ with $q = f(t_0, x_0, w_0)$ belonging to A. Then the vector spaces $D_3 f(t_0, x_0, w_0)(R^m)$ and $T_q A$ are transverse in R^n, i.e.,*

$$D_3 f(t_0, x_0, w_0)(R^m) + T_q A = R^n, \tag{8}$$

if and only if for every C^{k+1} submanifold S of V containing x_0, endowed with a C^k projection–valued function P^S such that

$$\ker P^S(x_0) = T_q A, \tag{9}$$

the projected control vector field $f^S: J \times S \times R^m \to TS$ introduced in (3) has a surjective partial derivative $D_3 f^S(t_0, x_0, w_0): R^m \to T_{x_0} S$.

PROOF. Notice that Proposition 1 ensures the existence of $D_3 f^S(t_0, x_0, w_0)$ as a linear map from R^m into $T_{x_0} S$. Let us first assume that equality (8) is satisfied. Applying the projection $P^S(x_0)$ to both sides of it and using (9) one gets

$$D_3 f^S(t_0, x_0, w_0)(R^m) = T_{x_0} S. \tag{10}$$

Conversely, assume that relation (10) holds. Then, combined with (3), it implies

$$D_3f(t_0,x_0,w_0)(R^m)+\ker P^S(x_0)=R^n$$

In view of (9), this is just the equality (8). The proof is complete. ∎

Remark 1 *Formula (8) expresses the transversality of the mapping $f(t_0,x_0,\cdot)$: $R^m\to R^n$ to the C^k submanifold A of R^n at the point w_0 (cf. [5]-[7]). Conditions (8) and (9) imply the following restrictions on the dimensions of the manifolds*

$$\dim A+\dim S=n\le m+\dim A \tag{11}$$

involved. Relation (11) imposes dimension restrictions in the application of the below Theorem 1.

We state now our main result concerning the local controllability along a reference solution by arbitrarily small controls of the projected system (4).

Theorem 1 *Let $f\colon J\times V\times R^m\to R^n$ be a C^k mapping with $k\ge 1$, let A be a C^ksubmanifold of R^n and let (t_0,x_0,w_0) be a point in $J\times V\times R^m$ such that $q=f(t_0,x_0,w_0)$ belongs to A and the transversality condition (8) is verified. Then, if S is a C^{k+1} submanifold of V containing x_0, endowed with a C^k projection-valued function P^S satisfying (9), there exist positive constants δ_1, δ_2 such that the projected control system (4) is locally controllable along any reference solution $t\to y(t,t_0,x_0,u)$ by arbitrarily small controls provided the control u verifies the condition*

$$\|u(t)-w_0\|<\delta_1 \quad \text{for a.e.} \quad t_0<t<t_0+\delta_2. \tag{12}$$

PROOF. Let S be a fixed C^{k+1} submanifold of V as in the statement. Observe that it suffices to prove the existence of the constants $\delta_1,\ \delta_2>0$ such that

$$y(t,t_0,x_0,u)\in \operatorname{int}\mathcal{A}_{\varepsilon,u}(t_0,x_0;t) \tag{13}$$

for all controls u satisfying (12), for all t with $t_0<t<t_0+\delta_2$ and for all constants $\varepsilon>0$ (Notations in (13) are according to (6') and (7')). Indeed, let us suppose that (13) holds and let t be any instant in the domain of $y(\cdot,t_0,x_0,u)$. Choosing a time t_1 with $t_0<t_1<t_0+\delta_2$, we can write for an arbitrary control v the relation

$$y(t,t_0,x_0,v)=y(t,t_1,y(t_1,t_0,x_0,v),v) \tag{14}$$

Since the mapping $y(t, t_1, \cdot, u)$ is a diffeomorphism, the formulae (13) and (14) prove the claim.

The assertion (13) is a local problem on the manifold S around the point x_0. Hence, via a local chart of S at x_0, we may suppose that

$$S = R^p \cap V, \tag{15}$$

where $p = \dim S \le n$ and R^p is identified to $R^p \times 0 \subset R^n$.

The hypotheses stated allow to apply Proposition 2 to deduce that $D_3 f^S(t_0, x_0, w_0)$ maps R^m onto $T_{x_0}S = R^p$. Then there exist positive constants δ_1, δ_2 such that the solution $t \to y(t, t_0, x_0, u)$ be defined on $t_0 \le t < t_0 + \delta_2$ for each control u satisfying (12), and the below equality be verified

$$D_3 f^S(t, y(t, t_0, x_0, u), u(t))(R^m) = R^p \tag{16}$$

for every time t with $t_0 \le t < t_0 + \delta_2$ and every control u as in (12).

Fix now a time $t_1 \in J$ with $t_0 \le t_1 < t_0 + \delta_2$ and a control u that verifies (12). It is well-known that the mapping $y(t_1, t_0, x_0, \cdot): L^m_\infty(J) \to S$ is differentiable of class C^k, $k \ge 1$ (cf. [1]–[3], [14]). Under the identification (15), its derivative, which in fact equals $D_4 y(t_1, t_0, x_0, u): L^m_\infty(J) \to R^p$, is given by

$$D_4 y(t_1, t_0, x_0, u)(v) = z(t_1, t_0, x_0, u, v), \quad v \in L^m_\infty(J), \tag{17}$$

where $t \to z(t) = z(t, t_0, x_0, u, v)$ denotes the solution of the initial value problem

$$\begin{aligned} \dot z(t) &= D_2 f^S(t, y(t, t_0, x_0, u), u(t)) z(t) \quad + \\ &+ \; D_3 f^S(t, y(t, t_0, x_0, u), u(t)) v(t) \quad \text{for a.e.} \quad t_0 \le t < t_0 + \delta_2 \\ z(t_0) &= 0. \end{aligned} \tag{18}$$

In (18) we have a time–dependent, linear control system. According to the criterion of the complete controllability (see [14, p. 186–187], or [15, p. 99]), system (18) is completely controllable on the closed interval $t_0 \le t \le t_1$ if and only if the equality

$$D_3 f^S(t, y(t, t_0, x_0, u), u(t))^T \Phi(t_0, t)^T q = 0 \quad \text{for a.e.} \quad t_0 \le t \le t_1, \tag{19}$$

with $q \in R^p$, implies $q = 0$. Here the superscript T means the matrix transpose, and $\Phi(t, s)$ denotes the fundamental matrix solution of the homogeneous linear system corresponding to (18) with $\Phi(s, s)$ equal identity. Using relation (16), it follows readily that from (19) one obtains $q = 0$. Consequently, the complete controllability on $t_0 \le t \le t_1$ of the linear control system (18) is checked. By formula (17) this is equivalent to the surjectivity of the linear map $D_4 y(t_1, t_0, x_0, u): L^m_\infty(J) \to R^p$.

We are thus in a position to apply the Graves' theorem [16] to the C^1 mapping $y(t_1, t_0, x_0, \cdot): L^m_\infty(J) \to R^p$. The theorem quoted ensures that the mapping $y(t_1, t_0, x_0, \cdot): L^m_\infty(J) \to S$ is locally open at any control u having property (12). This clearly implies the assertion (13) for $t = t_1$, which concludes the proof.

The example below describes a typical situation where Theorem 1 applies.

Example 1 *Consider the C^1 control system (1) on the open set $V \subset R^n$ with a fixed time interval J. Let (t_0, x_0, w_0) be a point in the domain of the mapping $f: J \times V \times R^m \to R^n$ and let A be the linear manifold $A = f(t_0, x_0, w_0) + E$, where E is the orthogonal complement of the linear space $D_3 f(t_0, x_0, w_0)(R^m)$ in R^n. Letting S be an arbitrary C^2 submanifold of V such that $x_0 \in S$ and*

$$T_{x_0} S = D_3 f(t_0, x_0, w_0)(R^m),$$

and letting $P(x): R^n \to T_x S$ denotes the orthogonal projection for every $x \in S$, Theorem 1 applies to the corresponding projected control system (4).

3. Generic controllability via transversality

As in the preceding sections, the nonlinear control system (1) is given by a C^k mapping $f: J \times V \times R^m \to R^n$, $k \geq 1$, where J and V represent an open interval of R and an open subset of R^n, respectively, and the controls u belong to $L^m_\infty(J)$.

For a fixed (possibly with boundary) C^k submanifold A of R^n and for a point $(t_0, x_0) \in J \times V$, we are concerned with the below controllability property which expresses the conclusion of Theorem 1.

(P_{f,A,t_0,x_0}) For every point $w_0 \in R^m$ and for every (possibly with boundary) C^{k+1} submanifold S of V endowed with a projection function P^S provided $x_0 \in S$, $q = f(t_0, x_0, w_0) \in A$ and formula (9) holds, there exist positive constants δ_1 and δ_2 such that the projected control system (4) is locally controllable along any reference solution $t \to y(t, t_0, x_0, u)$, with u satisfying (12), by arbitrarily small controls.

We refer to Section 1 for the meaning of all notions in the above formulation.

Our first generic result deals with the change of the initial data (t_0, x_0) in (5).

Theorem 2 *Assume that the C^k mapping $f: J \times V \times R^m \to R^n$ and the C^k submanifold A of R^n satisfy the hypotheses*

(i) $k > \max(0, n + \dim A - m)$;

(ii) the mapping f is transverse to the submanifold A, that is, for every $(t, x, w) \in J \times V \times R^m$ with $a = f(t, x, w) \in A$ the following condition holds

$$Df(t, x, w)(R \times R^n \times R^n) + T_a A = R^n \tag{20}$$

Then the set $G_{f,A}$ of all points $(t_0, x_0) \in J \times V$ admitting the property (P_{f,A,t_0,x_0}) contains a residual set in $J \times V$, so it is dense in $J \times V$

If the above holds and, in addition, the following condition

(iii) the submanifold A is closed in R^n and the mapping $(t_0, x_0) \in J \times V \to f(t, x, \cdot) \in C^k(R^m, R^n)$ is continuous with respect to the (strong) C^k Whitney topology on $C^k(R^m, R^n)$,

then $G_{f,A}$ contains an open dense subset of $J \times V$

PROOF. The assumptions (i) and (ii) assure that the parametric transversality theorem as stated in [7, p. 79–80], can be utilized for the mapping f taking the space of parameters to be $J \times V$. It follows that the set of points $(t, x) \in J \times V$ with $f(t, x, \cdot): R^m \to R^n$ transverse to the sumbanifold A includes a residual subset $M_{f,A}$ of $J \times V$. Hence, if (t_0, x_0) is an arbitrary point of $M_{f,A}$, then equality (8) is valid for every $w_0 \in R^m$ with $q = f(t_0, x_0, w_0) \in A$. Theorem 1 applies thus to any point in $M_{f,A}$, so $M_{f,A}$ is included in $G_{f,A}$. Therefore $M_{f,A}$ is the residual set required in the first part of the theorem.

After adding hypothesis (iii), the openness part of the parametric transversality theorem implies that the points (t, x) for which $f(t, x, \cdot): R^m \to R^n$ is transverse to A form an open subset of $J \times V$. Because, as it was shown above, this set is contained in $G_{f,A}$, the proof is complete. ■

Corollary 2 *If the C^k mapping $f: J \times V \times R^m \to R^n$, with k satisfying condition (i) in Theorem 2, is a submersion, i.e., it has a surjective derivative $Df(t, x, w): R \times R^n \times R^m \to R^n$ at every point $(t, x, w) \in J \times V \times R^m$, then the conclusion of Theorem 2 is true for every C^k submanifold A of R^n.*

PROOF. It suffices to observe that the submersion hypothesis implies the validity of relation (20) for each C^k submanifold A of R^n. ■

Remark 2 *The explicit description of the continuity assumed in (iii) of the Theorem 2 requires the following: given any continuous and positive function* $\delta: R^m \to R$ *and any integer* r *with* $0 \le r \le k$*, one has*

$$\|D_3^r f(t,x,w) - D_3^r f(s,y,w)\| < \delta(w) \quad \text{for all} \quad w \in R^m \tag{21}$$

whenever (s,y) *is sufficiently close to* (t,x) *in* $J \times V$*. In (21) notation* $D_3^r f$ *means the* r*-th order partial derivative of* f *with respect to the third variable, while* $\|\cdot\|$ *stands for the norm of an* r*-multilinear map.*

Let us now vary in the problem (P_{f,A,t_0,x_0}) the vector field f besides the initial data (t_0, x_0).

Theorem 3 *Assume that the (possibly with boundary)* C^k *submanifold* A *of* R^n *verifies the condition (i) in Theorem 2. Then the set* $C^k(J \times V \times R^m, R^n)$ *of all* C^k *mappings from* $J \times V \times R^m$ *into* R^n*, endowed with the (strong)* C^k *Whitney topology, contains a residual subset* F_A *consisting of mappings* f *such that the conclusion of Theorem 2 holds for the corresponding set* $G_{f,A}$*. In particular,* F_A *is a dense subset of* $C^k(J \times V \times R^m, R^n)$*. Moreover, if* A *is closed in* R^n*, then* F_A *is a dense open subset of* $C^k(J \times V \times R^m, R^n)$*.*

Proof. Denote by F_A the set of mappings in $C^k(J \times V \times R^m, R^n)$ which are transverse to the C^k submanifold A of R^n, that is, those mappings that satisfy condition (ii) in Theorem 2. The (elementery) Thom transversality theorem (see [7, p. 74]) implies that F_A is residual in $C^k(J \times V \times R^m, R^n)$. Applying Theorem 2 for each element of F_A we obtain the first part of the result. The density of F_A follows from the property of $C^k(J \times V \times R^m, R^n)$ to be a Baire space with respect to the (strong) C^k Whitney (see [7, p. 62]). In the case when A is closed, the openness transversality theorem (see [5]–[7]) shows that F_A is also open. This completes the proof. ■

We end this section with a generic result concerning the case where A is changed smoothly in the problem (P_{f,A,t_0,x_0}) within a suitable class of submanifolds.

Let M denotes the smooth Banach manifold (in fact an open subset of a Banach space) of all C^k functions $g: R^n \to R^p$, $k \ge 1$, with compact support and having $0 \in R^p$ as a regular value. The topology of M is induced by the C^k topology on $C^k(R^n, R^p)$. If g is an element of M, let us put

$$A = g^{-1}(0). \tag{22}$$

Theorem 4 *Assume that the mapping* $f: J \times V \times R^m \to R^n$ *is differentiable of class* C^k *with* $k > \max(m+1, 2n-m-p)$. *Then* M *contains a residual (hence dense) subset* M_f *satisfying the property: if* $g \in M_f$ *the set* $G_{f,A}$ *with* A *given by (22), as described in Theorem 2, fulfils the conclusion of Theorem 2.*

PROOF. Notice that A in (22) is a C^k submanifold of R^n, because $0 \in R^p$ is a regular value of g. It is straightforward to check the transversality of the mapping $(g,t,x,w) \in M \times J \times V \times R^m \to g(f(t,x,w)) \in R^p$ to the one-point submanifold $\{0\} \subset R^p$. Therefore the Abraham's transversality theorem ([5, p. 48]) may be invoked. We deduce that we can take M_f to be the set of $g \in M$ so that $0 \in R^p$ is a regular value of the composition gf. An easy calculation shows that the residual set M_f consists of those R^p-valued functions g for which the mapping f is transverse to the corresponding submanifolds A (see (22)). The result is now derived by applying Theorem 2 to each submanifold A of R^n of type (22) with $g \in M_f$. ■

Remark 3 *Theorems 1–4 are valid, with obvious modifications in the statements, if the state space* V *is an* n*–dimensional differentiable manifold instead of an open set of* R^n. *Also, Theorems 1–4 remain essentially true if the control space* R^m *is replaced by a differentiable manifold. This is due to the fact that we use arbitrarily small controls and to the possibility of applying the Whitney embedding theorem.*

4. Concluding remarks

A geometric method is described for inducing from a general control system (1) a new control system (4) (called projected) on a submanifold of the state space. The construction is based on the notion of projected–valued function (2).

Under a general and effectively computable condition (8) one proves a criterion of local controllability along a reference solution, by arbitrarily small controls, for the projected control system (4).

This controllability result can be regarded as the solution of a problem (P_{f,A,t_0,x_0}) which depends on the control vector field f of system (1), the submanifold A precising the position of the submanifold S on which one projects (1) and the initial condition (t_0, x_0). Using the transversality theory one studies the behaviour of the controllability property of system (4) relative to all parameters entering the problem. One obtains in this way three generic results corresponding to the specified parameters. The physical meaning of a such generic result is two fold: the property is stable with respect to small perturbations of the parameter (the openness part); (ii) each value of the parameter can be approximated by values satisfying the property (the density part).

References

[1] ALBRECHT F., Topics in control theory, Springer–Verlag, New York, 1968.

[2] GRASSE K.A., Controllability and accessibility in nonlinear control systems, Ph.D. Dissertation, University of Illinois, 1979.

[3] GRASSE K.A., Perturbations of nonlinear control systems, *SIAM J. Control Optimiz.* **19** (1981), 203–220.

[4] HAAHTI H., Projection–valued functions and their linear connections on surface in a Banach space. *Proceedings on the second Finish–Polish summer school in complex analysis (Jyväskylä, 1983). Ber. Univ. Jyväskylä.* **28** (1984), 15–40.

[5] ABRAHAM R., ROBBIN J., Transversal mappings and flows., W.A. Benjamin, New York, 1967.

[6] GOLUBITSKY M., GUILLEMIN V., Stable mappings and their singularities. Springer–Verlag, New York, 1973.

[7] HIRSCH M.W., Differential topology, Springer–Verlag, New York, 1976.

[8] SZTAJNIC J., WALCZAK S., On controllability of smooth and nonsmooth systems. *Bull. Soc. Sci. Lettres Lodz. Vol. 36,* **28** (1986), No. 38, 1–11.

[9] HERMES H., On local controllability. *SIAM J. Control Optimiz.* **20** (1982), 211–220.

[10] MEDVED M., On genericity of complete controllability in the space of linear parametrized control systems. *Czechoslovak Math. J.* **33 (108)** (1983), 167–175.

[11] MOTREANU D., Existence for minimization with nonconvex constraints, *J. Math. Anal. Appl.*, **117** (1986), 128–137.

[12] MOTREANU D., Generic finitness in solving optimal control problems. *Anal. Polon. Math.* **49** (1988), 135–145.

[13] MOTREANU D., Strong accessibility with respect to a submersion, *Problems Control Inform. Theory* **1** (1990), 45–54.

[14] LEE E.B., MARKUS L., Foundations of optimal control theory, John Wiley & Sons, New York, 1967.

[15] CONTI R., Differential equations and control. Academic Press, New York, 1976.

[16] GRAVES L.M., Some mapping theorems. *Duke Math. J.* **17** (1950), 111–114.

Additional readings suggested by the anonymous referee:

[1] LOBRY C., Dynamical polysystems and control theory. In: Geometric Methods in Systems Theory, ed. by D.Q. Mayne and R.W. Brockett, Reidel, Dordrecht 1973, 1–42.

[2] SUSSMANN H.J., Some properties of vector fields that are not affected by small perturbations. *J. Diff. Eqs.* **20** (1976), 292–315.

[3] AEYELS D., Generic observability of differentiable systems, *SIAM J. Control. Opt.* **19** 1981, 595–603.

[4] JAKUBCZYK B., PRZYTYCKI F., Singularities of k–Tuples of Vector Fields. Diss. Math. CCXIII, Polish Sci. Publ., Warsaw, 1984.

[5] TCHOŃ K., On Stable and Typical Properties of Control Systems. Wrocław Technical Univ. Publ., Wrocław, 1986.

Sterowalność rzutowanych układów sterowania

W artykule wprowadzono pojęcie układu sterowania f rzutowego na S, określonego dla dowolnego układu sterowania opisanego przez pole wektorowe sterowań f i dowolnej podrozmaitosci S przestrzeni stanów. Przy naturalnym założeniu

o transwersalności f względem ustalonej podrozmaitości A wykazano lokalną sterowalność wzdluż rozwiązania odniesienia dla rzutowego układu sterowania f na każdej podrozmaitości S, która jest w określonym sensie skierowana transwersalnie do A. Pokazano, że własność sterowalności jest generyczna względem danych początkowych, pola wektorowego f i podrozmaitości A.

Управляемость проектируемых систем управления

В статье вводится понятие f проективной на S системы управления для случая произвольной системы управления описываемой вeкторным полем управлений f и произвольного подмногообразия S пространства состояний. При естественной предпосылке о трансверсальности f по отношению к определенному подмногообразию A, доказано существование локальной управлаемости вдоль решения отнесения для проектируемой системы управления f на каждом подмногообразии S, которое в определенном смысле направлено трансверсально к A. Показано, что свойство управляемости генеративно по отношению к начальным данным, векторному полю, подмногообразию A.