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On an approach to estimation of system parameters

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An approach to estimation of system parameters is described based on consideration of an input error which results from ensuring the same output of the system and its corresponding model previously exposed to the same excitation. The approach is found advantageous in removing the existing restriction to the persistently exciting signal to be used for the estimation purpose. The unbiased parameter estimation obtained by adopting this approach in the case where the measurement vectors are disturbed by additive noise is also discussed and the recursive solution for the on–line computation of the parameter vector is also obtained.

1. Introduction

Whenever the model of a dynamical system is described in a parametric form, the methods of system parameter estimation may in general be seen to consist of two stages [1, 2]. In the first stage, the number of equations of the sytem to be set up for the estimation purpose should be equal to the number of model parameters being sought [3]. In the second stage, the parameters of the model are estimated following an estimation method based on the consideration of some defined errors (output error, output prediction error and equation error [1, 2, 4]) and suitable criteria imposed on these errors used in defining proper cost functions to be minimized. The estimation process requires the measurement of both the input and output vectors for formulation of the estimation problem and hence it becomes necessary to introduce the persistently exciting property [2, 5], putting a restriction as to the form of input signal to be used for the estimation purpose. The Pseudo Random Binary Sequence Input (PRBSI) can, of course, be used to meet the requirement of the persistently exciting property. This finds limited applicability in practice because the parameter estimation in cases of a system like a bio-system or an industrial process-system may demand the use of an actual form of the exciting signal which may not be persistent one making thus the estimation process difficult.

An alternative approach to estimation of the system parameters is presented in this paper. In this approach, the knowledge of the form of the exciting signal to be used for the estimation purpose may be ignored. The approach is based on the use of an error defined as the difference between the system input and a signal which is required to be present at the model input (Requested Input – RI) such that outputs of the sytem and the model are matched. This error may be called an Input Error (IE) which is suitably used in a defined cost function to be minimized for estimating the systems parameters. In the proposed approach, since the IE is defined on the input side of the system, the restriction as to the system input in mathematical manipulation of the estimation problem disappears and thus the approach is applicable to the case of a nonpersistent input too. This approach to the system parameter estimation along with the determination of the IE is described. The influence of the noise in the proposed estimation technique considering the presence of the noise on both the input and output sides of the system is discussed and then an unbiased estimate of the system parameters is found. A recursive solution needed for the purpose of on-line computation of the system parameters is also obtained.

2. Basic approach

The idea of the input error approach to parameter estimation for a continuous-time model of a linear time invariant single input single output (SISO) system is illustrated in Figure 1. Assume that there is no noise involved in the process of estimation. The model–inverse is incorporated in the figure to replace a chosen model whose parameters are usually different from those of the system. When such a model is chosen, it becomes necessary to include a signal at the model input which acts in addition to the system input for ensuring the outputs of the system and the model to be the same. This additional actuating signal (AAS) accounts for the difference between the parameters of the system and the model. By trimming the model parameters, the system parameters can, however, be estimated through minimization of the AAS. The concept of the model–inverse is introduced in the figure to indicate the need of knowledge about the AAS which represents $e^*(t)$ in the sense of the IE.

Some suitable linear dynamical (LD) operators are used on the output side of the system to generate the time derivatives of the output signal [1]. This LD operator may be any of linear filtering operations or spectral chracterizations of signals or method/modulating functions [1]. There, the LD operation means a multiplication of each term of the SISO model with known functions (the number of the known functions is equal to the number of the parameters to be estimated) and integration of the products over the period of available data with a repetition of the process for all modulation functions to yield a system of independent equations for the estimation purpose [10, 11, 12]. The method function technique, however, suggests a means of avoiding the direct time derivative measurment problem. In a disrete–time mode approach, the LD operator may be seen as an A/D converter which facilitates the appropriate measurements required for formulation of the parameter estimation equations. One LD operator is also used at the input side of the system to convert the input signal to the respective domain present at the model–inverse output.

A general form of the continuous–time model is considered as:

$$\sum_{j=0}^{n} a_j \frac{d^j y_M^*(t)}{dt^j} = \sum_{j=0}^{n} b_j \frac{d^j u_M^*(t)}{dt^j} \tag{1}$$

where the measurement values of the j–th derivatives of the output and input the model are denoted by $\frac{d^j y_M^*(t)}{dt^j} = y_{Mj}^*(t)$ and $\frac{d^j u_M^*(t)}{dt^j} = u_{Mj}^*(t)$ respectively with an assumption of the availability of the data for the parameter estimation.

If the model–inverse exists, (1) may be rewritten for the k–th stage of the LD in absence of noise with $b_0 = 1$ as:

$$u_{Mk}^*(t) = \sum_{j=0}^{n} a_j y_{kj}^*(t) - \sum_{j=1}^{n} b_j u_{Mkj}^*(t) \tag{2}$$

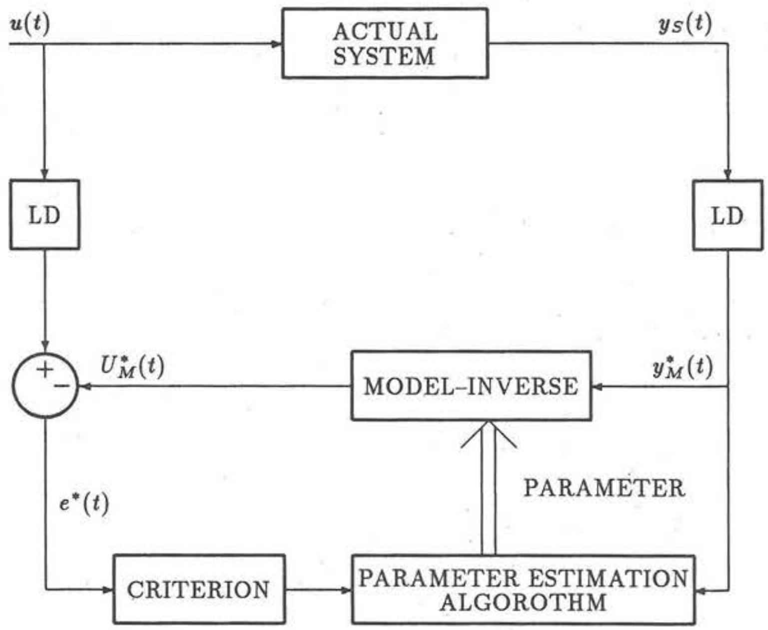

Figure 1. Block diagram of the input error approach.

where the measurement values of the j–th derivatives of the system is denoted by $y^*_{kj}(t) = \frac{d^j y^*_k(t)}{dt^j}$ at the k–th stage of the LD.

For ensuring the output of the model to be the same as delivered by the system, the chosen model would ask for a RI as given in (2), which can be written in the standard matrix form as:

$$u^*_{Mk}(t) = m_k^T p \tag{3}$$

where $p = [a_0 \ldots\ldots a_n \mid b_1 \ldots\ldots b_n]^T$ is the parameter vector to be estimated and m_k is the vector

$$m_k = [y^*_{k0}(t) \ldots y^*_{kn}(t) \mid -u^*_{Mk1}(t) \ldots -u^*_{Mkn}(t)]^T \tag{4}$$

It is seen form (4) that $y^*_{k0}(t) \ldots y^*_{kn}(t)$ is the set of system output data which are supplied by the LD operators and the set $u^*_{Mk1}(t) \ldots u^*_{Mkn}(t)$ is directly supplied by the model–inverse.

Since the parameters of the model–inverse differ from those of the system, the RI is not the same as the system input. Thus an input error results at the k–th stage as given by:

$$e_k^* = u_k^* - u_{Mk}^* \tag{5}$$

where u_k^* is supplied by the LD operator at the input of the system.

From (3), a set of equations for $k = 1, 2, \ldots, N$ stages is formed and written in the matrix form as:

$$U_{MN} = M_N P \tag{6}$$

where $U_{MN} = [u_{M1}^* \, u_{M2}^* \ldots\ldots u_{MN}^*]^T$ and $M_N = [m_1^T \,|\, m_2^T \,|\, \ldots\ldots \,|\, m_N^T]^T$.

A set of input error equations is also formed for (5) with $k = 1, 2, \ldots, N$. Referring to (6), this set of the input error equations is written in the matrix form as:

$$\epsilon_N^* = U_N - M_N P \tag{7}$$

where $\epsilon_N^* = [e_1^* \, e_2^* \ldots\ldots e_N^*]^T$ and $U_N = [u_1^* \, u_2^* \ldots\ldots u_N^*]^T$.

A cost function is defined as:

$$J(p) = \sum_{k=1}^{n} (e_k^*)^2 = (\epsilon_N^*)^T \cdot (\epsilon_N^*) = (U_N - M_N P)^T (U_N - M_N P) \tag{8}$$

The parameters are to be estimated in such a way that the cost function is minimized. The necessary condition for minimization of the cost function is $\left.\frac{\partial J(p)}{\partial p}\right|_{\hat{p}=p} = 0$, where $\hat{p}$ stands for estimated value of p. With reference to $J(p)$ in (8), the normal equation of the problem in this case is obtained as:

$$(M_N^T M_N)\hat{p} = M_N^T \cdot U_N \tag{9}$$

The solution of the above equation is the Least Square (LS) estimate $\hat{p}_{LS}$ which is obtained as:

$$\hat{p}_{LS} = (M_N^T M_N)^{-1} M_N^T U_N \tag{10}$$

It is seen form (4), (6) and (10) that the necessary data, required to be supplied by LD operators for the estimation purpose, are the set of system output data and the measured value of the system input. It is also seen that $\hat{p}_{LS}$ may be obtained if the inverse of the $(2n+1) \times (2n+1)$ matrix $M_N^T M_N$

exists. Recalling the structure of the matrix M_N of (6) and referring to m_k of (4), one may write:

$$M_N^T M_N = \left[\begin{array}{c} y_{10}^* \cdots\cdots\cdots y_{N0}^* \\ \vdots \;{\scriptstyle (n+1)\times N}\; \vdots \\ y_{1n}^* \cdots\cdots\cdots y_{Nn}^* \\ \hline -u_{M11}^* \cdots\cdots\cdots -u_{MN1}^* \\ \vdots \quad {\scriptstyle n\times N} \quad \vdots \\ -u_{M1n}^* \cdots\cdots\cdots -u_{MNn}^* \end{array}\right] \times \qquad (11)$$

$$\times \left[\begin{array}{c|c} y_{10}^* \cdots\cdots\cdots y_{1n}^* & -u_{M11}^* \cdots\cdots\cdots -u_{M1n}^* \\ \vdots \;{\scriptstyle N\times(n+1)}\; \vdots & \vdots \quad {\scriptstyle N\times n} \quad \vdots \\ y_{N0}^* \cdots\cdots\cdots y_{Nn}^* & -u_{MN1}^* \cdots\cdots\cdots -u_{MNn}^* \end{array}\right]$$

For a sufficiently large number of stages ($N \to \infty$), (11) can be rewritten, with introduction of the corresponding correlation functions [6], as:

$$M_N^T M_N = N\times \qquad (12)$$

$$\times \left[\begin{array}{ccc|c} R_{y^*y^*}(0) & \cdots\cdots\cdots & R_{y^*y^*}(n) & 0\cdots\cdots\cdots 0 \\ \vdots & & \vdots & -R_{y^*u_M^*}(1)\cdots\cdots\cdots -R_{y^*u_M^*}(n) \\ \vdots & {\scriptstyle (n+1)\times(n+1)} & \vdots & \vdots \;{\scriptstyle (n+1)\times n}\; \vdots \\ R_{y^*y^*}(n) & \cdots\cdots\cdots & R_{y^*y^*}(0) & -R_{y^*u_M^*}(n)\cdots\cdots\cdots -R_{y^*u_M^*}(1) \\ \hline 0 & -R_{u_M^*y^*}(1)\cdots\cdots & -R_{u_M^*y^*}(n) & R_{u_M^*u_M^*}(1)\cdots\cdots\cdots R_{u_M^*u_M^*}(n) \\ \vdots & {\scriptstyle n\times(n+1)} & \vdots & \vdots \quad {\scriptstyle n\times n} \quad \vdots \\ 0 & -R_{u_M^*y^*}(n)\cdots\cdots & -R_{u_M^*y^*}(1) & R_{u_M^*u_M^*}(n)\cdots\cdots\cdots R_{u_M^*u_M^*}(1) \end{array}\right]$$

From (10), it is seen that the LS estimates obtained in this case has the same form as that obtained in the case where the errors are not defined on the input side of the system [1]. The parameter estimation method demand, however, the existence of the matrix $(M_N^T M_N)^{-1}$ in all the cases. In the case where the errors are not defined on the input side of the system, the same matrix contains the elements obtained from auto and cross co–relations of the input and ouput data of the system and the input signal must therefore be persistently existed so that $(M_N^T M_N)^{-1}$ exists. It is now seen from (12) that the input data of the system are replaced by those of the model in the newly formed matrix. Even though the system input is not persistently existed, the output of the system is always persistently existed. Since the system output acts as the input of the model–inverse, its output is also persistently existed and hence $(M_N^T M_N)^{-1}$

always exists. Thus the proposed IE approach is found to be applicable to the use of any form of the input signal for the estimation purpose. The advantage of the proposed approach over the existing ones (where the defined error is not on the input side of the system) is that in the nonpersistently exciting signal case, a unique solution to the estimation problem can be found by adopting the present approach while the use of the generalized inverse of the matrix due to nonexistence of $(M_N^T M_N)^{-1}$ in the known approaches does not provide a unique solution.

It may be mentioned that only one LD operator is required at the input side of the system to convert the system input signal to the respective domain of the model–inverse output while in the existing methods, the number of the LD operators is greater than one and usually equal to the system order excepting the case of discrete–time model approach. Further, the LD at the input of the system may be removed if $(\mathrm{LD})^{-1}$ is used on the output side of the model–inverse to retransfer the RI signal from its measured value. The use of $(\mathrm{LD})^{-1}$ would, however, result in a creation of some extra noise due to the nonhomogeneity of the operators.

3. Determination of the Input Error (IE)

The scheme for parameter estimation with error defined on the input side of linear time invariant SISO model is shown in Figure 2. It is presently assumed that the noise signals $r_S(t)$ and $r_M(t)$ are absent in the scheme.

In this scheme, the error measure is given by:

$$e^*(t) = u^*(t) - u_M^*(t) \tag{13}$$

The measure $u^*(t)$ of the input signal is obtained from the LD with noise $r_I(t)$ at its input. Thus

$$u^*(t) = \mathrm{LD}\,[u(t) + r_I(t)] \tag{14}$$

General form of the model–inverse with $b_0 = 1$ can be written from (1) as:

$$u_M^*(t) = \sum_{j=0}^{n} a_j \frac{d^j y^*(t)}{dt^j} - \sum_{j=1}^{n} b_j \frac{d^j u_M^*(t)}{dt^j} \tag{15}$$

Thus, the transfer function of the model–inverse becomes:

$$F_M(s) = \frac{U_M^*(s)}{Y^*(s)} = \frac{\sum_{j=0}^{n} a_j \cdot s^j}{1 + \sum_{j=1}^{n} b_j \cdot s^j} \tag{16}$$

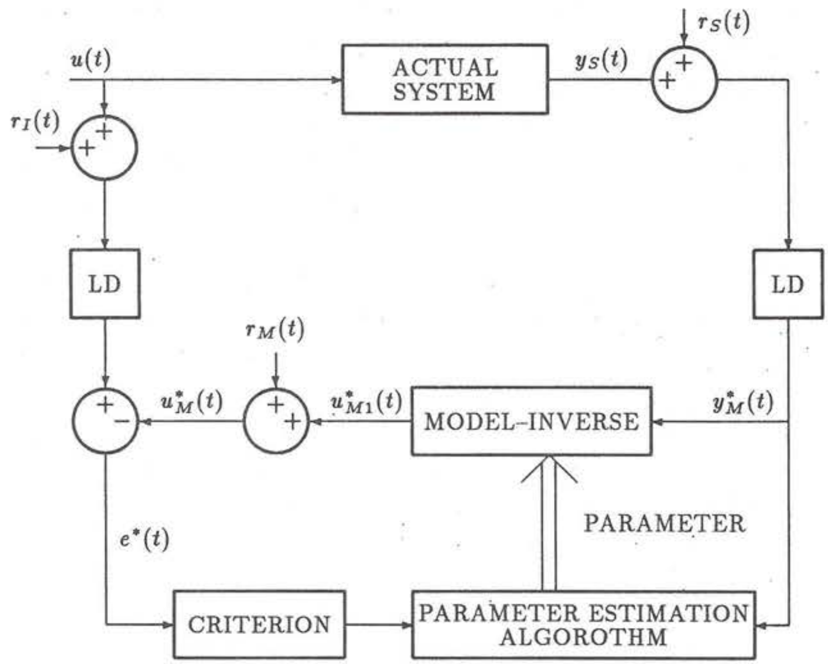

Figure 2. Parameter estimation scheme – definition of the input error and influence of noise.

If the model–inverse includes a stochastic noise, the stochastic part of the model–inverse has a transfer function of the form:

$$G_n(s) = \frac{R_I(s)}{W(s)} \tag{17}$$

The noise $r_I(t) = L^{-1}[R_I(s)]$ in the model–inverse is assumed to be present due to the white noise $w(t) = L^{-1}[W(s)]$ and the transfer function $G_n(s)$ which can take different forms depending upon the model structure [1, 4].

In a complete matching condition, one can obtain:

$$U^*(s) = U^*_M(s) + G_n(s) \cdot W^*(s) \tag{18}$$

where $W^*(s)$ is the measured value of the white noise signal $w(t)$ in Laplace domain and can be interpreted as:

$$W^*(s) = G_n^{-1}(s) \cdot [U^*(s) - U^*_M(s)] \tag{19}$$

The Laplace transform of the white noise can be obtained by taking the inverse linear operation of $W^*(s)$ as:

$$W(s) = (\mathrm{LD})^{-1}\left\{G_n^{-1}(s)\cdot[U^*(s) - U_M^*(s)]\right\} \tag{20}$$

The white noise $w(t)$ in time domain can be obtained by taking the inverse Laplace transform of $W(s)$ given in (20) and hence the measured value of the white noise in time domain $w^*(t)$ is supposed to be obtained from the LD as:

$$w^*(t) = \mathrm{LD}\left\{\mathrm{L}^{-1}\left[(\mathrm{LD})^{-1}[G_n^*(s)\cdot(U^*(s) - U_M^*(s))]\right]\right\} \tag{21}$$

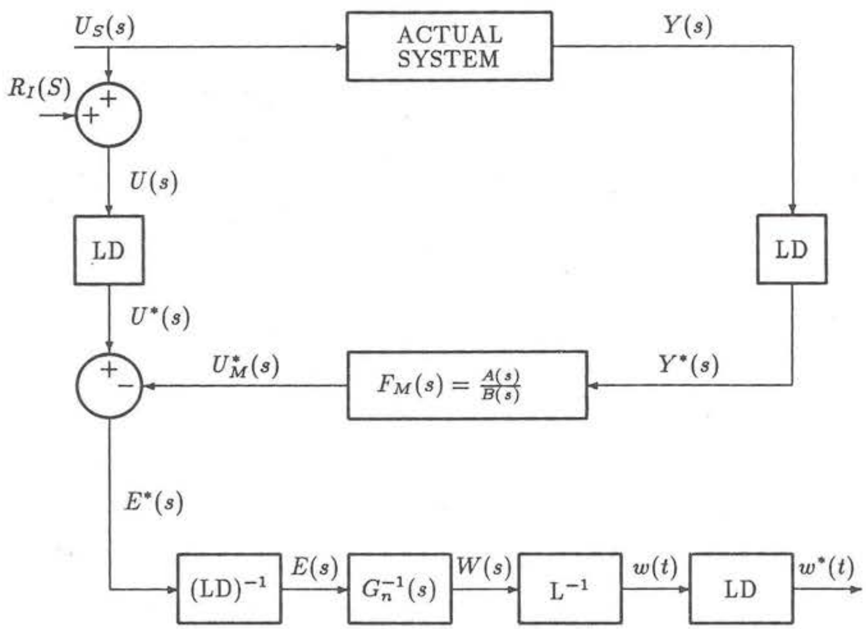

Figure 3. Block diagram for determination of the input error.

The definition of the input error can be illustrated using (19)–(21) and is diagramatically shown in Figure 3, where the transfer function of the model–inverse $F_M(s)$ is expressed by the ratio of $A(s)$ to $B(s)$. The equation (21) implies that the deviation $(U^*(s) - U_M^*(s))$ of the system input $U^*(s)$ from its deterministic behaviour $U_M^*(s)$ may be seen due to operation of white noise $w^*(t)$ on a system consisting of LD and $G_n^*(s)$.

4. The noise treatment

The scheme of parameter estimation in presence of noise for a SISO continuous–time model is discussed with reference to Figure 2, where the noise signals $r_S(t)$ and $r_M(t)$ are considered in the absence of the noise signal $r_I(t)$. The error measure in such a scheme is given by

$$e^*(t) = u^*(t) - u_M^*(t) \tag{22}$$

The measure of the output signal $y_S(t)$ of the actual system or the input signal of the model–inverse is usually contaminated with the noise $r_S(t)$. Thus

$$y_M^*(t) = \text{LD}\ \{y_S(t) + r_S(t)\} = y_S^*(t) + r_S^*(t) \tag{23}$$

where $r_S^*(t)$ arises out of application of white noise to a system consisting of LD. When LD operator stands for a linear filter, the LD operation may be characterized by an appropriate transfer function [1]. In the method using orthogonal functions, an approximate and algebraical relation between the output and input of the system may be obtained utilizing the one–shot operation matrix for repeated integration [13]. It is not possible to describe the LD operator in the usual form of transfer function in S–domain since the LD in this case corresponds to a chain of integrators in the LS approximation sense.

$r_S^*(t)$ can generally be expressed in terms of the white noise $w^*(t)$ having a certain variance $\sigma_{w^*}^2$ in the following dynamical equation:

$$r_S^*(t) = \sum_{i=1}^{n} e_i \frac{d^i w^*(t)}{dt^i} - \sum_{i=1}^{n} f_i \frac{d^i r_S^*(t)}{dt^i} + w^*(t) \tag{24}$$

where, in the case of orthogonal functions expansion system, the i–th derivatives of $w^*(t)$ and $r_S^*(t)$ are the products of the i–th order of the operational matrix for single stage integration with $w^*(t)$ and $r_S^*(t)$ respectively. The forms of the operational matrix for single stage integration with respect standard systems of orthogonal functions are given in detail in [1]. In the case of Poisson moment functional method, the i–th derivatives of $w^*(t)$ and $r_S^*(t)$ are the output of the i–th stage of a chain of filters driven by $w^*(t)$ and $r_S^*(t)$, respectively. It is, however, mentioned in [14] that both the methods of the orthogonal functions expansion and the Poisson moment function are considerably immune to the zero mean additive noise and that the noise accentuating direct time derivative operation is elegantly avoided in both these methods.

The noise $r_M(t)$ at the output of the model–inverse may be assumed to be present due to application of the white noise signal to the LD operator at the input of the actual system and a part of the inherent model–inverse noise (stochastic noise). This noise $r_M(t)$ is considered to be a random process appearing at the output of the general filter driven by a white noise source. The measured value of $r_M(t)$ can be expressed in time domain as:

$$r_M^*(t) = \sum_{j=1}^{n} d_j \frac{d^j w^*(t)}{dt^j} - \sum_{j=1}^{n} c_i \frac{d^j r_M^*(t)}{dt^j} + w^*(t) \tag{25}$$

where $w^*(t)$ is assumed to be the same as the white noise used in (24).

The expected value of the system parameters are to be determined from noise free measurement. Assume that there exists a model–inverse whose parameters are the expected system parameters. This model–inverse can be represented as:

$$u_{M1}^*(t) = \sum_{j=0}^{n} a_j \frac{d^j y_S^*(t)}{dt^j} - \sum_{j=1}^{n} b_j \frac{d^j u_{M1}^*(t)}{dt^j} \tag{26}$$

In the case of a perfect matching ($e^*(t) = 0$), one can write

$$u^*(t) = u_M^*(t) = u_{M1}^*(t) + r_M^*(t) \tag{27}$$

where $r_M^*(t)$ is affecting only $u_{M1}^*(t)$ but not the derivatives of $u_{M1}^*(t)$ since the latter are directly supplied by the model–inverse which is now noise–free.

Referring to (23)–(26), equation (27) can be written as:

$$\begin{aligned} u_M^*(t) = & \sum_{j=0}^{n} a_j \frac{d^j y_M^*(t)}{dt^j} - \sum_{j=1}^{n} b_j \frac{d^j u_M^*(t)}{dt^j} \\ & - \sum_{j=0}^{n} a_j \frac{d^j w^*(t)}{dt^j} + \sum_{j=1}^{n} d_j \frac{d^j w^*(t)}{dt^j} - \sum_{j=0}^{n} a_j \sum_{i=1}^{n} e_i \frac{d^{i+j} w^*(t)}{dt^{i+j}} \\ & - \sum_{j=1}^{n} c_j \frac{d^j r_M^*(t)}{dt^j} + \sum_{j=0}^{n} a_j \sum_{i=1}^{n} f_i \frac{d^{i+j} r_S^*(t)}{dt^{i+j}} + w^*(t) \end{aligned} \tag{28}$$

If low pass filters are used at the output side of the model–inverse so that $\frac{d^k w^*(t)}{dt^k} = 0$ and $\frac{d^k r_S^*(t)}{dt^k} = 0$ for $k > n$, the fifth and the seventh term of (28) can be expressed as:

$$\sum_{j=0}^{n} a_j \sum_{i=1}^{n} e_i \frac{d^{i+j} w^*(t)}{dt^{i+j}} = \sum_{j=1}^{n} \sum_{i=0}^{j-1} a_i e_{j-1} \frac{d^j w^*(t)}{dt^j} \tag{29}$$

$$\sum_{j=0}^{n} a_j \sum_{i=1}^{n} f_i \frac{d^{i+j} r_S^*(t)}{dt^{i+j}} = \sum_{j=1}^{n} \sum_{i=0}^{j-1} a_i f_{j-1} \frac{d^j r_S^*(t)}{dt^j} \tag{30}$$

The matrices Δ, E and F, each of the dimension $(n+1)\times(n+1)$, are defined as:

$$\Delta = \left[\begin{array}{c|c} 0 & I_n \\ \hline 0 & 0 \end{array}\right],$$

$$E = \begin{bmatrix} e_1 & 0 & 0 & \dots\dots\dots & 0 \\ e_2 & e_1 & 0 & \dots\dots\dots & 0 \\ \vdots & & & & \vdots \\ e_n & e_{n-1} & \dots\dots\dots & e_1 & 0 \\ 0 & 0 & \dots\dots\dots & 0 & 0 \end{bmatrix}, \qquad (31)$$

$$F = \begin{bmatrix} f_1 & 0 & 0 & \dots\dots\dots & 0 \\ f_2 & f_1 & 0 & \dots\dots\dots & 0 \\ \vdots & & & & \vdots \\ f_n & f_{n-1} & \dots\dots\dots & f_1 & 0 \\ 0 & 0 & \dots\dots\dots & 0 & 0 \end{bmatrix},$$

and an $(n+1)$ vector w is defined at the k–th stage as:

$$w = [w^*_{k0}\ w^*_{k1}\ \dots\dots\ w^*_{kn}]^T \qquad (32)$$

The noise vector, $\bar{w} = [w^*_{k1}\ w^*_{k2}\ \dots\dots\ w^*_{kn} \mid 0]^T$ involved in (28) can be expressed in term of w and Δ as: $\bar{w} = \Delta w$.

Using (29)–(32), equation (28) can be written for the k–th stage of the LD operation in the matrix form as:

$$u^*_k = [y^T_{Mk} \mid -u^T_{MIk}]p + [-w^T \mid -r^T_{Mk} \mid r^T_{Sk}]\cdot\theta + w^*_k \qquad (33)$$

where $y_{Mk} = [y^*_{Mk0}\ y^*_{Mk1}\ \dots\dots\ y^*_{Mkn}]^T$, $u_{MIk} = [u^*_{Mk1}\ u^*_{Mk2}\ \dots\dots\ u^*_{Mkn}]^T$, $r_{Mk} = [r^*_{Mk1}\ r^*_{Mk2}\ \dots\dots\ r^*_{Mkn}]^T$ and $r_{Sk} = [r^*_{Sk1}\ r^*_{Sk2}\ \dots\dots\ r^*_{Skn}]^T$.

The system parameter p and the noise parameter θ are given through:

$$p = [a \mid b]^T, \quad \theta = [a + \Delta^T E a - \Delta^T d \mid c \mid Fa]^T \qquad (34)$$

where $a = [a_0\ a_1\ \dots\dots\ a_n]$, $b = [b_1\ b_2\ \dots\dots\ b_n]$, $c = [c_1\ c_2\ \dots\dots\ c_n]$, and $d = [d_1\ d_2\ \dots\dots\ d_n]$ and the measurement vector in general is: $[y^T_{Mk} \mid -u^T_{Mk} \mid -w^T \mid -r^T_{Mk} \mid r^T_{Sk}]$.

In the case where the model-inverse-error-measurement (MIEM) $w^*(t)$ is uncorrellated with the measures of the input $y^*_M(t)$ and output $u^*_M(t)$ of the model–inverse and both the noise $r_S^*(t)$ and $r_M^*(t)$ are absent, equation (33) can be rewritten in the standard matrix form as:

$$u_k^* = m_k^T \cdot p + w_k^* \tag{35}$$

where p is given in (4), m_k is given in (3).

From(35), a set of equations for $k = 1, 2, \ldots, N$ stages is formed and written as:

$$U_N = M_N \cdot p + W_N \tag{36}$$

where M_N and U_N are the same as given in (6) and (7) respectively and $W_N = [w_1^* \; w_2^* \; \ldots\ldots \; w_n^*]^T$.

In presence of noise, equation (36) is required to be solved under the condition that the sum of squares of the MIEM is minimized. For this purpose, a cost function which depends on parameter vector p is defined as:

$$J(p) = \sum_{k=1}^{N} w_k^{*2} = W_N^T \cdot W_N \tag{37}$$

In other words, the parameter vector p is required to be found in such a way that $J(p)$ is minimized. Substituting the vector of the MIEM from (36) into (37) and setting partial derivative $\frac{\partial J(p)}{\partial p} = 0$, one gets the LS estimate of the parameter vector $\hat{p}_N$ as:

$$\hat{p}_N = (M_N^T M_N)^{-1} M_N^T U_N \tag{38}$$

where the subscript N of $\hat{p}$ indicates that the estimated value is calculated on the basis of the measurements up to N stages.

Assume that MIEM is a white noise of zero mean with standard deviation σ_w^2, so that the expected value of the estimated vector $\hat{p}_N$ becomes:

$$\lim_{N \to \infty} \varepsilon[\hat{p}] = \lim_{N \to \infty} \varepsilon[(M_N^T M_N)^{-1} M_N^T U_N] \tag{39}$$

Referring to (36), the above equation can be rewritten as:

$$\lim_{N \to \infty} \varepsilon[\hat{p}] = \varepsilon[p] + \lim_{N \to \infty} \varepsilon[(M_N^T M_N)^{-1} M_N^T W_N] \tag{40}$$

Under the assumptions mentioned above, the second term on the right hand side of (40) becomes zero, that is, for a very large number of stages N, the estimated value of the parameter vector is negligibely biased. Thus

$$\lim_{N \to \infty} \varepsilon[\hat{p}] = p \tag{41}$$

For a large value of N, the parameter vector has a deviation given by:

$$\hat{p}_N - p = (M_N^T M_N)^{-1} M_N^T W_N \tag{42}$$

The variance is then obtained as:

$$\varepsilon[(\hat{p} - p)^T(\hat{p} - p)] = \sigma_{w^*}^2 \cdot (M_N^T M_N)^{-1} \tag{43}$$

where $\sigma_{w^*}^2$ is a measure of the standard deviation σ_w^2.

It is seen that with the use of proper low pass filters at the output side of the model–inverse, the noise figure in the LS estimate obtained by the IE approach remains the same as obtained by adopting the existing error approaches [1, 7].

In the present case, i.e. when the components of the measurement vector are corrupted by additive noise as expressed in (23) and (24), the LS estimate given in (38) becomes biased even for $N \to \infty$. The reason for the presence of the bias can, however, readily be found and removed by following the same ways as followed in building the algorithms given in [15–17] for a known value of the noise statistics. Three well known methods for obtaining an unbiased estimate of the system parameter are: Instrumental Variable (IV) [17-20], Maximum Likelihood (ML) [2, 19, 21] and Generalized Least Squares (GLS) [7, 19, 22]. The GLS method considered in this paper is briefly given below.

Rewritting (33) as:

$$u_k^* = [y_{Mk}^T \mid -u_{MIk}^T]p + v_k^*(t) \tag{44}$$

where $v_k^*(t)$ is the noise at the output of the model–inverse and represents the bias part of the LS estimate of p. It can readily be shown that the bias is given by the expected value of $[(M_N^T M_N)^{-1} M_N^T V_N]$ where V_N is a set values of $v_k^*(t)$ for $k = 1, 2, \ldots, N$. To reduce the bias term to zero, it is necessary to reduce V_N to an uncorrelated noise so that the elements of M_N be independent of $v_k^*(t)$. Presence of $v_k^*(t)$ in the value of $u_k^*(t)$ can always be fitted as closely as desired to an ARMA model by taking the requisite number of terms. Thus, at the k–th stage $v_k^*(t)$ may be expressed in terms of a white noise $\xi_k^*(t)$ other than that used in (24) and (25) as:

$$v_k^*(t) = \sum_{l=1}^{q} \alpha_l \frac{d^l \xi_k^*(t)}{dt^l} - \sum_{l=1}^{q} \beta_l \frac{d^l v_k^*(t)}{dt^l} + \xi_k^*(t) \quad \text{where} \quad q > n \tag{45}$$

which can be written in the standard matrix form as:

$$v^* = \chi^T \mu + \xi^* \tag{46}$$

where $\chi_k = [\xi^*_{k1}\ \xi^*_{k2}\ \ldots\ldots\ \xi^*_{kq}\ |\ -v^*_{k1}\ \ -v^*_{k2}\ \ldots\ldots\ -v^*_{kq}]^T$, and $\mu = [\alpha_1\ \alpha_2\ \ldots\ldots\ \alpha q\ |\beta_1\ \beta_2\ \ldots\ldots\ \beta_q]^T$ – which may be named process noise parameter (this parameter vector is involved in the process of eliminating the bias given by noise).

The relation between parameters of θ and μ can be expressed through the transfer function matrix [1], whose elements are:

$$\begin{aligned}
F_{\xi w} &= \frac{L(\xi^*)}{L(w^*)} \\
&= \frac{(1-a_0)+\sum_{j=1}^{n}(-a_j+d_j-\sum_{i=0}^{j-1}a_i e_{j-i})s^j}{1+\sum_{l=1}^{q}\alpha_l s^l} &\quad (47.a)\\
F_{vw} &= \frac{L(v^*)}{L(w^*)} \\
&= \frac{(1-a_0)+\sum_{j=1}^{n}(-a_j+d_j-\sum_{i=0}^{j-1}a_i e_{j-i})s^j}{-\sum_{l=1}^{q}\beta_l s^l} &\quad (47.b)\\
F_{vr_M} &= \frac{L(v^*)}{L(r^*_M)} = \frac{\sum_{j=1}^{n}c_j s^j}{\sum_{l=1}^{q}\beta_l s^l} &\quad (47.c)\\
F_{vr_S} &= \frac{L(v^*)}{L(r^*_S)} = \frac{\sum_{j=1}^{n}(\sum_{i=0}^{j-1}a_i f_{j-i})s^j}{-\sum_{l=1}^{q}\beta_l s^l} &\quad (47.d)\\
F_{\xi r_M} &= \frac{L(\xi^*)}{L(r^*_M)} = \frac{-\sum_{j=1}^{n}c_j s^j}{1+\sum_{l=1}^{q}\alpha_l s^l} &\quad (47.e)\\
F_{\xi r_S} &= \frac{L(\xi^*)}{L(r^*_S)} = \frac{\sum_{j=1}^{n}(\sum_{i=0}^{j-1}a_i f_{j-i})s^j}{1+\sum_{l=1}^{q}\alpha_l s^l} &\quad (47.f)
\end{aligned}$$

A set of equations from (46) for $k = 1, 2, \ldots, N$ is formed and written as:

$$V_N = X_N\mu + \Xi_N \qquad (48)$$

where $V_N = [v^*_1\ v^*_2\ \ldots\ v^*_N]^T$, $X_N = \begin{bmatrix} \chi_1^T \\ \vdots \\ \chi_N^T \end{bmatrix}$ and $\Xi_N = [\xi^*_1\ \xi^*_2\ \ldots\ \xi^*_N]^T$

The LS estimate of μ is given by:

$$\mu = (X_N^T X_N)^{-1} X_N^T V_N \qquad (49)$$

This LS estimate of the process noise parameter is found to be unbiased and allows a transformation of the corrupted data leading to an unbiased estimate of the system parameter vector p. At the k–th stage, the transformation is made by replacing the measured value of the system input $u_k^*(t)$ and the measured input set of the model–inverse $y_M^*(t)$ by:

$$u_k^F(t) = \sum_{l=1}^{q} \alpha_l \frac{d^l u_k^*(t)}{dt^l} - \sum_{l=1}^{q} \beta_l \frac{d^l u_k^F(t)}{dt^l} + u_k^*(t) \qquad (50.a)$$

$$y_{Mkj}^F(t) = \sum_{l=1}^{q} \alpha_l \frac{d^l y_{Mkj}^*(t)}{dt^l} - \sum_{l=1}^{q} \beta_l \frac{d^l y_{Mkj}^F(t)}{dt^l} + y_{Mkj}^*(t) \qquad j = 0, 1, \ldots, n \qquad (50.b)$$

The set of equations (44) written for $k = 1, \ldots, N$ reduces then to:

$$U_N^F = M_N^F \cdot p + \Xi_N \qquad (51)$$

where Ξ is given in (48), $U_N^F = [u_1^F(t)\ u_2^F(t)\ \ldots\ u_N^F(t)]^T$ and $M_N^F = [m_1^{FT} \mid \ldots \mid m_N^{FT}]^T$ with $m_k^F = [y_{k0}^F(t)\ \ldots\ y_{kn}^F(t) \mid -u_{Mk1}^*(t)\ \ldots\ -u_{Mkn}^*(t)]^T$.

The unbiased estimate of the parameter vactor $\hat{p}_{Nu}$ is then given as the LS estimate of (51), which is

$$\hat{p}_{Nu} = (M_N^{FT} M_N^F)^{-1} M_N^{FT} U_N^F \qquad (52)$$

where subscript "u" of $\hat{p}_N$ stands for the unbiased estimate of up to N stages of k. Thus, after obtaining an LS estimate of p from (38), the process of mutual improvement between the estimation of the system parameters and the disturbances can be performed by iterating (48), (49) and (52). An unbiased estimate of the system parameters can be obtained when V_N reaches a minimum. Knowing the LS estimate $\hat{p}_N$ and the GLS estimate $\hat{p}_{Nu}$ from (38) and the iteration mentioned earlier, respectively, the proper noise parameters (c_i, d_i, e_i, f_i) can also be estimated.

It may be mensioned that a transformation in the GLS procedure is made in (50) which has the meaning of a filter process. It can then be stated that if some suitable filters are used at the outputs of the system and the model–inverse as well as at the input of the system, the unbiased parameter can be obtained as its LS estimate. The GLS procedure is, however, a quasilinear formulation of a nonlinear estimation problem, the convergence in solution is therefore not asured unless very restrictive assumptions are made regarding the closeness of the original estimate of the parameters to their actual values [20].

It is noted that the unbiased parameter estimate could also be obtained in different ways. First, by employing the parameter estimation for a multiple input single output ARMAX model [18], described in (33), the system parameter p and the noise parameter θ could be obtained. Knowing p and θ, the proper noise parameters could also be obtained. Second, a cost function $J(p)$ defined in (37) would be minimized subject to the constraint in (33). For this, the standard Lagrangian or Hamiltonian method could be used in which the unbiased estimate as well as the proper noise parameters could be obtained simultaneously.

5. Recursive solution

The estimated value of the parameter vector at the k–th stage is obtained from (52) by replacing N by k.

When a new measurements are available, at the $(k+1)$–st stage, after the filtering via (50), using the present estimate of process noise parameter μ_k so that:

$$\begin{aligned} u^F_{(k+1)}(t) &= \sum_{l=1}^{q} \alpha_{lk} \frac{d^l u^*_{(k+1)}(t)}{dt^l} - \sum_{l=1}^{q} \beta_{lk} \frac{d^l u^F_{(k+1)}(t)}{dt^l} + \\ &\quad + u^*_{(k+1)}(t) \end{aligned} \tag{53.a}$$

$$\begin{aligned} y^F_{M(k+1)j}(t) &= \sum_{l=1}^{q} \alpha_{lk} \frac{d^l y^*_{M(k+1)j}(t)}{dt^l} - \sum_{l=1}^{q} \beta_{lk} \frac{d^l y^F_{M(k+1)j}(t)}{dt^l} + \\ &\quad + y^*_{M(k+1)j}(t) \quad \text{for} \quad j = 0, \ldots, n \end{aligned} \tag{53.b}$$

and the matrix M^F_k and the input vector U^F_k become M^F_{k+1} and U^F_{k+1} respectively, which can be expressed as:

$$M^F_{(k+1)} = \left[\frac{M^F_k}{m^{FT}_{(k+1)}} \right], \quad U^F_{(k+1)} = \left[U^{FT}_{(k+1)} \mid u^F_{(k+1)} \right]^T \tag{54}$$

where $m^F_{(k+1)} = [y^F_{M(k+1)0} \ \cdots \ y^F_{M(k+1)n} \mid -u_{M(k+1)1} \ \cdots \ -u_{M(k+1)n}]^T$ and $u^F_{(k+1)}$ is given in (53).

At the $(k+1)$–st stage, the parameter vector is established as:

$$\hat{p}_{(k+1)} = (M^{FT}_{(k+1)} M^F_{(k+1)})^{-1} M^{FT}_{(k+1)} U^F_{(k+1)} \tag{55}$$

The Kalman gain vector is defined in this case as:

$$q_{(k+1)} = (M^{FT}_{(k+1)} M^F_{(k+1)})^{-1} \cdot m^F_{(k+1)} \tag{56}$$

Since $(M^{FT}_{(k+1)}M^F_{(k+1)})^{-1}$ is a symmetric matrix which is invertible, employing the matrix inversion lemma $(A+BC)^{-1} = A^{-1} - A^{-1}B(I+CA^{-1}B)^{-1}CA^{-1}$ and using the partition form of the $M^F_{(k+1)}$ matrix in (54), the Kalman gain vector can be expressed in the terms of the available quantities as:

$$q_{(k+1)} = \frac{(M_k^{FT}M_k^F)^{-1}\cdot m^F_{(k+1)}}{1+m^{FT}_{(k+1)}(M_k^{FT}M_k^F)^{-1}\cdot m^F_{(k+1)}} \tag{57}$$

The predicted input is chosen for the $(k+1)$ stage as:

$$u^F_{(k+1),k} = m^{FT}_{(k+1)}\hat{p}_k \tag{58}$$

Based on the parameter estimation up to the k–th stage, a prediction error on the input side of the system for the $(k+1)$–st stage is obtained as:

$$\hat{\eta}_{(k+1)} = u^F_{(k+1)} - u^F_{(k+1),k} \tag{59}$$

The estimated value of the parameter vector at the $(k+1)$–st stage given in (55) can be written in terms of the available quantities at the k–th stage as:

$$\hat{p}_{(k+1)} = \hat{p}_k + q_{(k+1)}\cdot\hat{\eta}_{(k+1)} \tag{60}$$

The update of the matrix consisting of the transformed auto and cross correlation becomes:

$$(M^{FT}_{(k+1)}M^F_{(k+1)})^{-1} = (M_k^{FT}M_k^F)^{-1} - q_{(k+1)}m^{FT}_{(k+1)}(M_k^{FT}M_k^F)^{-1} \tag{61}$$

It remains to update the value of $\mu_{(k+1)}$ on the basis of new observation. Since the process noise parameter μ is an unbiased LS estimate, as mentioned earlier, the update is given by the LS recursion:

$$\mu_{(k+1)} = \mu_k - \mu_k\chi_{(k+1)}[1+\chi^T_{(k+1)}\mu_k\chi_{(k+1)}]^{-1}\chi^T_{(k+1)}\mu_k \tag{62}$$

where $\mu_k = [\alpha_{k1}\ \alpha_{k2}\ \dots\ \alpha_{kq}\ |\ -\beta_{k1}\ -\beta_{k2}\ \dots\ -\beta_{kq}]^T$ and $\chi_{(k+1)} = [\xi^*_{(k+1)1}\ \xi^*_{(k+1)2}\ \cdots\xi^*_{(k+1)q}\ |\ -v^*_{(k+1)1}\ -v^*_{(k+1)2}\ \cdots\ -v^*_{(k+1)q}\ ]^T$.

The equations (57)–(62) along with the equations (53) represent the generalized recursive solution in the input error approach to the parameter estimation problem. This solution is found suitable to develop an algorithm for on–line computation of the parameters. The available algorithms to compute the recursive GLS estimate [7, 18, 19] obtained by following the approaches where the error equations are not formed on the input side of the system may also be applied in this case. Suitable interfaces are required to facilitate the measurements of the input and output data of the system. The index k in the recursive equations directly indicates the number of sampling instants or the order of an orthogonal function or the Poisson moment functional transformation, etc., depending on the method used in the primary stage to generate the vector m_k, the input and the output measures.

6. Numerical example

The use of a discrete technique in adopting the proposed IE method to estimate the parameter for a continuous-time model under a nonpersistently exciting condition is described in the following example.

A linear, time invariant first order SISO system is considered as:

$$0.4\frac{dy(t)}{dt} + 3y(t) = u(t) \tag{63}$$

with an initial condition $y(0) = 0$.

The response of the system to the unit step input is:

$$y(t) = \frac{1}{3}(1 - e^{\frac{-3t}{0.4}}) \tag{64}$$

A linear, time invariant SISO first order model is chosen as:

$$a_1 \cdot \frac{dy(t)}{dt} + a_0 \cdot y(t) = b_0 \cdot \mathrm{RI}(t) + b_1 \cdot \frac{d\mathrm{RI}(t)}{dt} \tag{65}$$

having the initial condition as: $y(0) = \mathrm{RI}(0) = 0$.

For this particular model, $\mathrm{RI}(t)$ can also be found as:

$$\mathrm{RI}(t) = \frac{a_0}{3b_1} \cdot 1(t) + \frac{(a_1 b_0 + a_0 b_1)}{b_0(3b_1 - 0.4b_0)} e^{\frac{-b_0 t}{b_1}} + \frac{3a_1 - 0.4a_0}{3(0.4b_0 - 3b_1)} e^{\frac{-3t}{0.4}} \tag{66}$$

From (65), the transfer function of the model–inverse in S–domain is obtained as:

$$F(s) = \frac{\mathrm{RI}(s)}{Y(s)} = \frac{a_0 + a_1 s}{b_0 + b_1 s} \tag{67}$$

which can be written in Z–domain using the Tustin transformation as:

$$f(z) = \frac{\mathrm{RI}(z)}{Y(z)} = \frac{(a_0 T + 2a_1) + (a_0 T + 2a_1)z^{-1}}{(b_0 T + 2b_1) + (b_0 T + 2b_1)z^{-1}} \tag{68}$$

where T is the sampling time.

At any sampling instant n, the value of RI can be expressed:

$$\mathrm{RI}(n) = a_0^* y(n) + a_1^* y(n-1) - b_1^* \mathrm{RI}(n-1) \tag{69}$$

where $a_0^* = (a_0T + 2a_1)/(b_0T + 2b_1)$, which is written in the matrix form
$a_1^* = (a_0T - 2a_1)/(b_0T + 2b_1)$
$b_1^* = (b_0T - 2b_1)/(b_0T + 2b_1)$
as:

$$\mathrm{RI}(n) = m_n^T p^* \tag{70}$$

where $m_n = [y(n)\ y(n-1)\ -\mathrm{RI}(n-1)]^T$ and $p^* = [a_0^*\ a_1^*\ b_1^*]^T$.

The IE is given by:

$$\mathrm{IE}(n) = u(n) - \mathrm{RI}(n) \tag{71}$$

The LS estimate of the system parameter vector P^* is obtained through minimization of a cost function $J = \sum_{n=k}^{N} \mathrm{IE}^2(n)$ as:

$$p_{LS}^* = (M_N^T M_N)^{-1} M_N^T U_N \tag{72}$$

where $M_N = [m_k\ m_{k+1}\ \ldots\ m_N]^T$ and $U_N = [u(k)\ u(k+1)\ \ldots\ u(N)]^T$.

The transfer function in S–domain can be obtained from its Z–domain version as:

$$F(s) = \frac{\mathrm{RI}(s)}{y(s)} = \frac{(a_0^* - a_1^*) + s\frac{T}{2}(a_0^* + a_1^*)}{(1 + b_1^*) + s\frac{T}{2}(1 - b_1^*)} \tag{73}$$

The model parameters become then the system parameters which are:

$$a_{0S} = (a_{0L}^* + a_{1L}^*),\ a_{1S} = \frac{T}{2}(a_{0L}^* - a_{1L}^*),\ b_{1S} = \frac{T}{2}(1 - b_{1L}^*),$$

and

$$b_{0S} = (1 + b_{1L}^*)$$

where a_{0L}^*, a_{1L}^* and b_{1L}^* are the components of p_{LS}^* and subscript S of a_0, a_1, b_0 and b_1 stands for the system parameters. The above expressions are valid for the parameter transformations from Z to S domain only.

Three different models each with $b_0 = 1$ are considered. The output response $y(t)$ is computed using (64) and the RIs for the said models are computed by the use of (66). Using the computed data for $y(t)$ and RI, the parameter vector p_{LS}^* is computed for each model with the help of (72) referring to (70) in a recursive algorithm (1000 sampling data and sampling time $T = 0.1$ sec.) and then the system parameters are computed using (73). The results are shown below.

Model parameters	Elements of p^*_{LS}	System parameters
$a_0 = 6.5$ $a_1 = 2.4$ $b_1 = 2.5$	$a^*_{0L} = 41.50817$ $a^*_{1L} = -38.50782$ $b^*_{1L} = 1.89091E-04$	$a_{0S} = 3.000351$ $a_{1S} = 0.4000799$ $b_{0S} = 1.000189$ $b_{1S} = 4.99055E-03$
$a_0 = 4.5$ $a_1 = 4.2$ $b_1 = 4.5$	$a^*_{0L} = 41.49649$ $a^*_{1L} = -38.48789$ $b^*_{1L} = 2.94428E-05$	$a_{0S} = 3.008602$ $a_{1S} = 0.3999219$ $b_{0S} = 1.000029$ $b_{1S} = 4.999853E-03$
$a_0 = 2.5$ $a_1 = 8.2$ $b_1 = 0.5$	$a^*_{0L} = 41.45624$ $a^*_{1L} = -38.42302$ $b^*_{1L} = -2.40637E-05$	$a_{0S} = 3.033222$ $a_{1S} = 0.3993963$ $b_{0S} = 0.9999759$ $b_{1S} = 5.000012E-03$

It is seen that although the widely different models are chosen, values of the system parameters obtained from this computation agree well.

7. Conclusion

The proposed IE approach to estimate the system parameter is found to be applicable to any form of input signal and to overcome the restriction to the persistently exciting signal to be used for the estimation purpose existing in the approaches to date where the order of the persistency of the input should be twice the order of the system whose parameters are to be estimated [19]. This approach hence satisfies the demand of practical cases where the system parameters are asked to be uniquely estimated with the use of the actual form of the input signal which may not be persistent. The approach can thus be successfully applied in the cases where the system does not permit the use of any test signal including even PRBSI for estimation of its parameters. The influence of noise considering its presence on both sides of the model–inverse is discussed and it is shown that an unbiased system the GLS estimation as obtained in the known approaches considering the noise present only on the output side of the model. The true value of the system parameters can, therefore, be determined in the same way as used in the existing approaches by the use of proper filters. The form of the generalized recursive solution for facilitating the on–line computation of the system parameters in the case of corrupted data is also found to be the same in both existing and the proposed approaches if some suitable interfaces are used to supply the necessary data. Thus, the algorithms developed, applicable to the existing approaches, may also be used for computations in the present approach.

The IE approach can also be extended to estimation of parameters of a reduced-order system with two advantages over the existing techniques. First, any form of the input signal can be used and second, the outputs of the system to be reduced and of the reduced-order system are totally matched which is of great significance in the case of a regulator and a projective control.

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O pewnym podejściu do estymacji parametrów systemu

Opisano metodę estymacji parametrów opartą na uwzględnieniu błędu na wejściu wynikającego z zapewnienia tego samego wyjscia układu i jego modelu przy tym samym pobudzeniu. Metoda ta nie wymaga stosowania na wejsciu sygnałów pobudzających odpowiednio wysokich rzędów. Rozważono także nieobciążoną estymację parametrów przy pomocy tej metody w przypadku, gdy wektory pomiarów są zakłócone przez szum addytywny, jak rownież otrzymano rozwiązanie rekurencyjne dla wyliczania wektora parametrów na bieżąco.

О некотором подходе к оценке параметров системы

Описан метод оценки параметров, основанный на учете ошибки на входе, вытекающей из обеспечения одинакового выхода системы и ее модели при том же возбуждении. Этот метод не требует использования на входе возбуждающих сигналов соответетвующе высокого порядка. Рассматривается также несмещенная оценка параметров с помощью этого метода в случае, когда векторы измерений искажены аддитивным шумом. Получено рекуррентное решение для текущего вычисления вектора параметров.