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Modelling and supporting of the consensus reaching process using fuzzy preference relations

by

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In the paper the problem of supporting a consensus-reaching-oriented session with the experts is dealt with. Main elements of the consensus reaching process are identified. Two approaches in the domain of decision support known in the literature are discussed as to what extent they meet requirements of our model of the process. Finally, our own proposal for support is sketched.

1. Introduction

Successful decision making requires more and more competence. Rarely, a single decision maker is able or willing to choose the course of action without any support from experts. Usually, in non-trivial cases, opinions of the experts are quite different. Hence, a session is often organized letting the experts to exchange opinions about an issue under consideration. Opinions of the experts may change due to new information obtained during the discussion. In an optimistic case these changes lead to more similar opinions. The decision maker expects from the experts as clear as possible an advice as to what decision should be made. If opinions of the experts are identical at the end of the session, there is no problem. As it is a rather unusual situation, decision maker needs such an aggregated opinion which can be rationally deemed as a representation of different opinions of the experts. Certainly, the higher the agreement among the experts the more reliable is a given aggregate of opinions.

Evidently, some support from decision analysts is possible and necessary. It requires a kind of model of considered situation to be constructed. Such models are studied in the framework of group decision making theory. First of all, opinions of experts must be somehow modelled. One of the classical approaches, which we adopt in our work too, is employing of the preference relation defined on the set of possible decisions (alternatives). Group decision theory concentrates on the problem: given set of individuals' preferences what decision should be made. We include group opinion formation as an important element of our model but not the only one. In our model we try to capture main features of what we call consensus reaching process (abbreviated in what follows as CRP) rather than of decision process. We want to model and support the session with experts as a process. We try to do this in a holistic way, but we do not pretend to encompass all factors - certainly we exclude from our analysis psychological and sociological aspects of CRP. We propose, beside group opinion formation, two more tools of support, namely consensus degree measuring and discussion supporting.

In section 2 we present our model of the CRP including all the elements mentioned above. In the next section advantages of fuzzy preference relation as a representation of the individuals' opinions are discussed. In section 4 we compare two approaches known in the literature which can be treated as models of CRP. Finally, in section 5 we propose methods to support the elements of our model of CRP.

2. The structure of the consensus reaching process

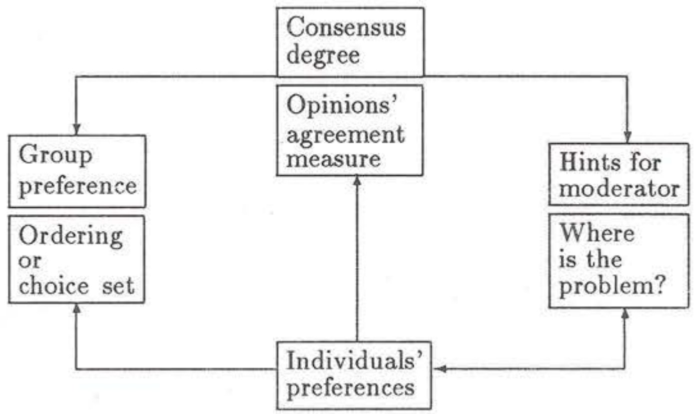
The subject of our consideration is a following decision problem. There is a finite set of feasible decisions (options, alternatives). Decision maker wants to consult experts. Hence, a session with the experts is organized to learn their preferences on the set of the decisions. These preferences reflect knowledge of the experts. In non-trivial cases these opinions will be usually different. A person supervising the session, called in the sequel the moderator, after finding discrepancy of opinions tries to persuade the experts to discuss their preferences.

The moderator points out the most controversial options, i.e. options as to which the experts definitely disagree. After a discussion experts again express their preferences. In most cases, due to the exchange of information (taking into consideration new criteria, more precise description of possible decisions etc.), these opinions are closer one to another or at least different than in the beginning. The session is ended if the preferences are consent enough or if there is no more time for the discussion. In the other case the moderator initiates a next stage of the discussion. When the session is over its results are presented to the decision maker. These results can take different form. Basically it is a kind of ordering of the possible decisions (ordering is meant here very generally, i.e. from simple discerning "good" decision and "bad" ones up to the full, linear ordering of all decisions under consideration). This ordering is to reflect the preferences of the group of the experts as a whole. Depending on the problem statement and the requirements of the decision maker this ordering may include all possible decisions, only "the highest ranked" options or only one, the best alternative.

Ordering is the most highly aggregated form of results of the session with experts. Interesting information can be also in the preferences of particular experts and in the report about the running of the session. They are especially essential when consensus has not been obtained.

The conclusion from the above description is that during the session we can observe a consensus reaching process. In the framework of this process the final opinions of either particular experts or the group as a whole are formulated. The subject of our work is modelling and supporting of this process. The underlined group of words in the above text describe the main elements of the consensus reaching process. Figure 1 shows their role and relations among them.

Figure 1. Scheme of the consensus reaching process



The first element, preferences of the particular experts, form basic data in the process. During the conventional discussion they are articulated by evaluative expressions in natural language. Usually they are supported by some rational arguments. Often the opinions are ridden with uncertainty and imprecision. For the processing of preferences they must be somehow modelled. Most often the expert is required to give an ordering of alternatives - from the most preferred to the least one. It corresponds to establishing a certain relation on the set of options. There are assumed some properties of such relation. The preferences can be modelled also by using the concept of an utility function. Obviously, the preferences (their representation) are indispensable for consensus measuring and group preference formation. In the case of dissensus they should be modified by particular experts.

A consensus degree is meant to measure the extent of agreement among opinions of the experts. Consensus reaching is a main, most essential condition for ending the session. The agreement among experts constitutes a good starting point for the determination of a proper decision. Then one can claim that the group as a whole represents a certain opinion which is close enough to the opinions of the particular experts. The classical concept of consensus as an unanimous agreement is impractical and inconvenient. In a real decision making context, where there are considered above 10 possible alternatives, a full agreement of opinions is impossible. On the other hand, such an agreement is neither needed. It is enough to define consensus as, for instance, an agreement of the majority of experts as to the majority of alternatives. The second drawback of the classical consensus definition is its dichotomy: there is consensus or there is not.To control the process of consensus reaching we need some measure which allows us to check if the opinions get closer one to another. The ways to overcome this difficulties are discussed in section 5.1.

Figure 1 points out two more functions of a consensus degree. First, the changes of its value during the experts' discussion can be a good hint for the diagnosis of dissensus. On the other hand, a consensus degree can be interpreted as a certain index of the group preference implied by experts' preferences . The higher the consensus degree the higher the reliability of group preference as a real representation of the opinion of the group as a whole. One must note, however, that the agreement in opinions does not guarantee the existence of a "best" alternative - among the experts there can be an unanimous agreement as to the fact that all decisions are indifferent for them. Assuming reasonable dimensions of the decision problem (for example 10 alternatives and 10 experts) it is not a trivial task to measure agreement among the experts.

The third important element of the consensus reaching process is the organization of experts' discussion in a way assuring, if possible, a quick unification of their opinions. It means either the pointing out of alternatives as to which there is the greatest disagreement or revealing subgroups of experts having similar preferences. Without such an information the computed consensus degree is not enough informative and gives no hint for the experts as to what changes in their preferences would be most useful to reach consensus. Such an auxiliary information, even assuming moderate dimensions of the decision problem, is not easy to be obtained. It is especially true when the group is expected to propose a kind of ordering of all possible alternatives. In Section 4, on the example of two CRP modelling methods, we discuss some ways proposed to solve this problem. In Section 5 we sketch our proposals how to cope with it.

Figure 1 shows a relation between the elements of the CRP. The consensus degree and the individuals' preferences make a starting point for generating hints as to the subject of a further discussion. These hints, in turn, constitute a feedback information for experts and are used by them during the modification of preferences in succeeding stages of the discussion.

The fourth element of the CRP is group preference formation (solving of the decision problem). This is an aspect of the decision making most widely discussed (most often the sole aspect) in the bibliography of the subject. There are many concepts of a reasonable, in some sense, solution (see for example [1, 10, 11, 16]). There is a lot of known theoretical results, beginning from the famous Arrow's Theorem. Although the problem of generating the solution has been thoroughly studied, the relations with the other elements of the CRP are not clear. Without doubt the individuals' preferences constitute a fundamental information for group preference determination. The consensus degree points out as to what extent the group preference is an adequate aggregation of the individuals' preferences. The hints for the moderator as to the focus of a further discussion should take into account the method selected for group preference formation.

The application of a computer based system to support CRP meant as described above should be fruitful. Firstly, it ensures that all, often non-trivial, computations are correctly performed. Secondly, it makes it possible to capture all relations existing among the particular elements of the process. Finally, it makes it much easier to generate a report from the session. There are several possible ways to employ a computer system during the experts' session. Such a system can encompass the whole CRP being responsible for the whole communication between the experts and for the proper running of all elements of the process mentioned above. Our requirements as to the system are more modest - we expect it to only support a traditionally run session. The communication between the experts goes freely according to rules accepted. The system follows changes in the individuals preferences appearing during the session, measures the consensus degree, points out what is a main obstacle to reach consensus and, finally, proposes how to rationally aggregate the opinions of all participants into a group preference.

3. Fuzzy preference relation as a representation of vague opinion

A crucial issue in decision making is the modelling of preferences. Let A be a finite set of possible alternatives (decisions,options). The preferences can be represented as a subset $C \subset A$ consisting of preferred alternatives (choice set), as relation defined on the set A or as utility function. We consider the most general form of preferences representation, that is a relation. We assume that preferences can be vague. Hence, we employ the concept of a fuzzy relation (sometimes called a valued binary relation) to allow for this imprecision in the expression of preferences. The idea of a fuzzy preference relation is not new. Blin & Whinston [5] proposed fuzzy binary relation as being an ideal form to represent preferences of the group as a whole, assuming each individual provides his or her own preferences as a crisp preference relation. There are also many papers studying applications of a fuzzy relation as a model of individuals' preferences.

Formally, a fuzzy preference relation is a fuzzy set in $A \times A$ of whose membership function is:

$$\mu_R : A \times A \longrightarrow [0,1]$$

where $\mu_R(a_i, a_j)$ denotes the degree of preference of alternative a_i over a_j as felt by the given individual. The particular values of this membership function are given the following interpretation:

$$\mu_R(a_i,a_j) = \begin{cases} 1 & \text{if } a_i \text{ is definitely preferred over } a_j, \\ c\in (0.5,\ 1) & \text{if } a_i \text{ is slightly preferred over } a_j, \\ 0.5 & \text{if there is no preference (indifference),} \\ d\in (0,\ 0.5) & \text{if } a_j \text{ is slightly preferred over } a_i, \\ 0 & \text{if } a_j \text{ is definitely preferred over } a_i. \end{cases} \quad (1)$$

These values can be given also another interpretation: $\mu_R(a_i, a_j) > 0.5$ means that a_i is definitely preferred over a_j and the particular value describes only the intensity of this preference (see [16]). If card A is small enough, as considered here, R may be represented by a matrix $R = [r_{ij}]$, $r_{ij} = \mu_R(a_i, a_j)$. Usually some assumptions are made about properties of a fuzzy preference relation. Most widely accepted are the reciprocity and transitivity (for several definitions possible see [27]) . We employ a more general form of a fuzzy relation and require it to be only reciprocal, that is $\mu_R(a_i, a_j) + \mu_R(a_j, a_i) = 1, i \neq j$. Such a fuzzy relation is called the fuzzy tournament [17, 22]. The value $\mu_R(a_i, a_i), \forall i$, is irrelevant for our considerations.

The transitivity of the preference relation is a rather controversial issue. We do not assume it, because there are well known examples of decision problems where preferences need not to be transitive .

In our opinion, owing to its generality, the fuzzy tournament resolves most problems related to a proper representation of preferences. If one wants to articulate his or her preferences as a classical (non-fuzzy) relation he can only simply use numbers 0 and 1 as values of the membership function. One can easily express indifference between two alternatives i and j by setting the value $\mu_R(a_i, a_j)$ to be equal 0.5. Hence we can properly represent (fuzzy) weak order. On the other hand, the indifference relation need not be transitive. One can set $\mu_R(a_i, a_j)$ and $\mu_R(a_j, a_k)$ equal to 0.5 but $\mu_R(a_i, a_k)$ equal to 1.0. Another problem is an adequate modelling of incomparability. As yet we do not provide for some special representation of it. The only way to express it is to use the value 0.5 - the same as for indifference. The fuzzy tournament is also very convenient in the following situation. An expert can have definite preferences only with respect to a subset of alternatives, B. Then he can set $\mu_R(a_i, a_j)$ according to his or her feelings for $a_i, a_j \in B$, $\mu_R(a_i, a_j) = 0.5$ for $a_i, a_j \notin B$ and $\mu_R(a_i, a_j) = 1$ for $a_i \in B$ and $a_j \notin B$.

These examples demonstrate the usefulness of pairwise comparison using the terms "preferred" and "indifferent". Intermediate values of μ_R can be interpreted as a representation of vagueness in experts' opinions or intensity of preferences. For example, if an expert, using the standard linear order, expresses his or her preferences among three alternatives as (a, b, c), it can reflect two quite different situations. One, when the expert strongly believes that alternative a is the best one, and he prefers only a little b over c. In the second situation, option a is preferred only a little to b and b only a little to c. These two situations can be easily differentiated using a fuzzy relation: in the first case we set $\mu_R(a, b) = 1$, $\mu_R(a, c) = 1$, $\mu_R(b, c) = x$, and in the second case $\mu_R(a, b) = \mu_R(b, c) = x, \mu_R(a, c) = y$, where $0.5 < x \ll 1.0$ and $y > x$ (see [18]) for a concept of software dealing with fuzzy orders).

We claim that the fuzzy tournaments capture some characteristic features of the individual preferences. The above examples give a certain intuitive justification for this claim.

4. Characterization and comparison of two models of the CRP

Here, we will discuss two approaches to modelling of the CRP known in the literature. In the next section we will propose our own approach.

We describe here the methods to support consensus reaching proposed by Ragade [20, 21] and Lehrer and Wagner [15]. They are different in many aspects. The former approach encompasses all the elements of the CRP we mentioned in the previous section, however it is rather of a descriptive type (a model) than a method to support a real session with the experts. The latter approach, in turn, focuses on forming the group preference. The experts are expected to provide, beside their individual preferences, also certain other parameters used to establish a consensual preference. In both approaches the form of decision problem differs from our basic definition but it can be easily adopted for our purposes to make the method work correctly.

4.1. Ragade's approach

This approach is meant as a way of understanding and evaluating any information-processing system with the human beings as part of it. The central notion is the so-called purposeful information processing activity. Opinions of particular members of the group form fuzzy profiles, i.e. vectors of membership values. Originally, they are meant as a characterization of a considered entity in a certain property space but can be easily interpreted as a fuzzy preference relation. There is considered a set of operators on profiles such as negation, union, concentration etc. It is assumed that members of the group can form profiles concerning the same entity in different property spaces. Consequently, there are considered transformations of profiles for each pair of the session's participants. There are proposed four definitions of transformations and interrelation among them.

Communication in the group causes changes in the profiles. Each member of the group has its own strategy of taking into account opinions of the others. This strategy is modelled as a rule of forming a profile on the basis of his or her own profile and profiles incoming from other experts. Ragade proposes six basic rules. There is one rule, called the weighted consensus, which defines a new profile as a linear combination of the old one and the profiles of the others. Moreover, the profiles of other participants of the session may be modified using the operators on profiles, as mentioned earlier. Profiles may be split into parts being transformed according to different schemes.

As a model of consensus formation Ragade proposes a system where the above mentioned elements meet the following conditions

- all experts choose the same transformation operator
- all experts use the same profile formation rule, namely the weighted consensus, with the weights being constant during the whole session.

It gives rise to 24 possible models of consensus formation. Ragade proves that, generally, six of them lead to null consensus, i.e. the profiles at stage infinity are zero vectors (it is proved assuming that property spaces of the experts are not identical).

Now, we will briefly discuss a relation between our model of the CRP and the one due to Ragade. The latter is more general. According to Ragade, the supporting of the decision-making process is only one of many possible applications. Anyway this model can be quite easily interpreted in terms appropriate for our approach. A fuzzy preference relation can be seen as a vector of membership values. It is so because we do not assume any special properties of this relation (except reciprocity). Property spaces of all experts are identical. Hence, there is no need for the transformations of profiles. There is no consensus gauging in Ragade's approach. Consensus is meant traditionally as a full agreement. This proposal does not consider group opinion's formation as in our model. Its role plays the individual's opinion after reaching consensus.

One can give a twofold interpretation of Ragade's approach, namely a normative and descriptive one. In the former case it is a kind of an automaton. The individuals must articulate their initial preferences, choose rules of forming a profile, optionally involving a subset of operators on the profiles. Then, automatically, a new profile for each individual can be determined. Ragade studies the convergence of such an algorithm only for a special selection of rules. Hence, in this case, we should treat Ragade's approach as a certain class of algorithms to form a group opinion. Some of these algorithms are identical to other known in the literature. Generally, this approach in the form presented does not make it possible to support the discussion what is the main postulate of our work. In this context, the second, descriptive interpretation of Ragade's approach seems to be more interesting. Then, it is a certain model of behavior of each member of the group on succeeding stages of the discussion. It is meant to describe, using the rules of forming a profile, the way the experts change their opinions. Operators on the profiles represent a way of preliminary processing of profiles of other individuals.

4.2. Lehrer and Wagner's approach

This approach was originally conceived in a probabilistic context [15]. Its authors developed and gave theoretical foundations for the algorithm presented earlier by DeGroot [6] which in turn is based on a work of Stone [26]. The problem considered may be formulated as follows. Each member of the experts' group is requested to articulate his or her belief as to the occurrence of some event (truth of some hypothesis). This belief is to be expressed as a subjective probability of this event. It is assumed that the articulation of belief degrees is preceded by full discussion among the experts. Then, it is said that the group is in the state of dialectical equilibrium. The authors' claim is that consensus formation should exploit all the knowledge of the group about an issue considered and nothing more. They deem this knowledge to be twofold. One, a standard part of it, consists of the experts belief degrees about the issue under consideration. Second feature of this knowledge is the evaluation, given by each expert, of the competence of all members of the group. This evaluation is formulated as a weight assigned to each individual. So, the input to the consensus formation process consists of a vector of belief degrees and a matrix of weights given by each expert to all other experts. Then, the authors argue that to be rational, each expert should update his or her probability assignment by taking a weighted average of all the probabilities assigned to the proposition (event) in question. Having done this updating each participant of the session has got a new probability assignment to the proposition under consideration. The authors claim that each member of the group should continue this averaging process until the moment when the degrees of belief of all of them are equal.

Mathematically it is formulated as follows. Let us assume a group consisting of n individuals. Each member i assigns a subjective probability p_i^0 to the proposition in question. The weights form an $n \times n$ matrix W. The entry w_{ij} $(i, j = 1, \ldots, n)$ indicates a weight (respect's degree, competence evaluation coefficient) that i gives to j. The constraints on w_{ij} are:

$$0 \leq w_{ij} \leq 1$$

$$\sum_j w_{ij} = 1 \quad , \forall i$$

The updating of opinions is done through the multiplication $P^1 = WP^0$ where P^0 is a vector of initial subjective probabilities, and P^1 is a vector of updated subjective probabilities. The process is continued by forming $P^2 = WP^1 = W^2P^0, P^3 = WP^2 = W^3P^0$ and so on. Then, after some iteration all entries in P^k become identical. This statement is true only if matrix W meets a rather mild condition. Namely, there must be at least one column of the matrix where all entries are greater than 0 (this condition comes from the Markov chains theory). Stating it in another way, there must be a participant who is assigned a non-zero weight by all members of the group.

Now, let us compare Lehrer and Wagner's proposal and our model of the CRP. In the former one the input is given as a subjective probability of a proposition in question and a weight of respect for each expert in the group. Let us consider a matrix representing a fuzzy preference relation employed in our approach. The values of its elements belong to the interval $[0, 1]$. No dependencies between the elements are assumed, except for the reciprocity. Hence, each entry, taken separately, can be treated as a subjective probability. So, we can put forward the same arguments as Lehrer and Wagner and apply their algorithm to obtain a consensual group preference relation. We only need to extend the input data with the weight coefficients.

Basically, Lehrer and Wagner's approach supports only one element of the CRP meant according to our definition from section 2, namely the group preference formation. The absence of a discussion supporting element is a consequence of a general idea of this approach - the Lehrer and Wagner's algorithm is to be used in a state of dialectical equilibrium. This means that the discussion has been completed, all arguments presented etc.. The authors claim that an application of their method is a natural, only rational extension of the discussion, and the results should be unconditionally accepted by the members of the group. It is emphasized that giving weights of respect for partners in the group leaves no way to reject the computed final subjective probability. It really seems that the so obtained aggregate of opinions based not only on individuals' opinions is more convincing. However it involves a rather ambiguous and troublesome concept of respect for other experts. Hence, considering subsequent iterations of the Lehrer and Wagner's algorithm as a continuation of the discussion, we can interpret this algorithm as a tool supporting the discussion. This is a rather specific support precisely determining the shape of preference relations of all participants on each step.

No consensus measure is considered in the framework of this approach. It is so, because of a very simple form of preferences (i.e. single value of subjective probability) in the original statement of the problem. Hence it is rather difficult to conceive any non-trivial measure different from the standard deviation. If the method is to be applied in the case of fuzzy preference relations some kind of a consensus measure could be very useful. The moderator could more reasonably accept a given state of discussion as a state of equilibrium knowing, for example, that "almost all of the experts agree in their opinion as to the most pairs of options".

5. Basic elements of the proposed system for consensus reaching support

Now, we would like to present briefly the concept of the system supporting all the elements of CRP as it was defined in section 2. This is the concept behind the microcomputer system described in [9] and [12]. We employ the fuzzy preference relation to represent the experts' preferences. Its advantages and arguments for it are presented in section 3. In the succeeding subsections we describe our proposals as to the consensus degree gauging and group discussion directing. For group preference forming we are going to adopt one (or a class) of known algorithms. It requires further study of relations among particular elements of the process.

5.1. Consensus degree

Classically, consensus is meant as a full, unanimous agreement of all members of a given group. Sometimes such a consensus is really needed and there is no escape from the definition formulated above. However, quite often such a definition is too rigid and impractical. This is especially true in the context of experts, not decision makers, discussion. Then the aim is more to reveal the opinions of experts as to the issues under consideration than to give an unambiguous answer to the problem. Certainly, the more these opinions are consensory the better, i.e. it is much easier to make a reasonable decision on their basis.

In section 2 we pointed out two drawbacks of the classical definition, i.e. its rigidness and dichotomy. When fuzzy preference relation (FPR) started to be used as a model of expert's preferences, a kind of incoherence between the elasticity of FPR and the rigidness of the classical concept of consensus was observed. Spillman, Spillman and Bezdek [2, 3, 4] proposed how to remove this incompatibility. Their work was motivated mostly by uselessness of the classical definition in the case of monitoring decision making in a small group. They needed to trace how far from consensus the group was on the subsequent stages of the process. In their early works they defined three types of consensus, each represented by a different consensus preference matrix. As a preference matrix of the group they used the average preference matrix. The difference between the group preference matrix and one of the consensus preference matrices was proposed as a measure of consensus (originally - *a distance from consensus*). Disadvantages of this approach were soon recognized: a simple averaging application and dependence of the consensus measure on the choice of consensus type. Hence in [23, 24, 25] they proposed a new measure of consensus. To define it they use α-cuts of fuzzy relation. Then, for each pair of experts their agreement degree is computed, which is proportional to the number of 1's appearing in the same places in the corresponding matrices. These agreement degrees form a new matrix. Now it is computed what is the proportion of 1's in this matrix. This way we obtain a consensus measure for a given α. The final value of the consensus measure proposed is calculated by numerical integration over α.

This approach overcomes the rigidness of the classical definition. One can now say how far from consensus the group is. Using it the moderator knows whether the group gets closer to consensus at a given stage of discussion or quite opposite.

In our project of the CRP supporting system we propose, following [7, 8], another concept of the definition and measuring of consensus. Actually, the approach given above is more elastic and practical having as a result a number from the interval [0,1] instead of a number from the set {0,1}. But the very definition of consensus is not changed. For the question: "Is there a consensus among the experts" we obtain an answer on the base of multivalued logic rather than on the binary one. But, to say if this question (or rather the answer to it) is true or not we must assume a certain designated value in logic. A natural and frequent choice is number one. Hence, we are still able to say that there is consensus only if all the experts agree as to everything. Otherwise we will say that there is consensus to some extent.

In [8] there is proposed a fundamental change of the concept of consensus. It is made possible to formulate a "soft" definition of consensus, each time depending on the context in which it is to be used. So, we can define that there is consensus if "almost all the experts agree as to most pairs of options". Then if we ask if there is consensus, we will obtain an answer "yes(=1)" not only when all experts agree as to everything.

Now, we will sketch the algorithm; for details see [8, 9]. A crucial role in the proposed method plays the concept of a linguistic quantifier. It is used to give a meaning (truth value) to such expressions from natural language like "almost all" or "most". In the algorithm Zadeh's interpretation of a linguistic quantifier is employed. There have to be defined two such quantifiers, which are used to aggregate information contained in experts' preferences. It is a task of the moderator to set parameters of the quantifiers, i.e. to define the meaning of consensus in the context of the decision problem under consideration. After these and a few other parameters of the algorithm are settled and preference matrices are input, the computer can calculate the degree of consensus among the experts. For every pair of experts their matrices are compared element by element. For each element an index of agreement between two experts is calculated. Then these indices are aggregated using a linguistic quantifier corresponding, for example, to the phrase "almost all pairs of options" in the definition of consensus. As a result we obtain a degree of agreement between a pair of experts. When we complete these steps for each pair of experts we must again aggregate the degrees of agreement obtained using another linguistic quantifier corresponding , for example, to the phrase "Most experts". This way the degree of consensus among the experts is obtained.

Let us sum up the arguments for employing the concept of consensus proposed in [8]. It is a most sophisticated approach known in the literature. It enables the organizer of a discussion to define consensus specifically to the requirements of the decision problem considered. This approach is numerically efficient. It can be further developed. For example, it would be useful to check what is the degree of consensus among experts as to the alternatives which are highly preferred by the group as a whole. It is a non-trivial task. It can be solved only in the context of a given algorithm for group preference formation. The relation between these two elements of the CRP must be further studied.

5.2. Directing group discussion

What we mean by this term was stated in section 2. As such, this the feature of the CRP is evidently least discussed in the literature. Most attention in this area is paid to the problem how to make easier the communication between members of the group using computers. But this means dealing with technical, general rules of communication rather than with problems specific to a given decision problem.

We will not be very specific in presenting our proposal for supporting a discussion. We will rather point out directions in which to go.

First of all we need some method to point out the most controversial alternatives. Then the discussion can be focused on them leading hopefully to a better agreement. The simplest solution of this task is a statistical information about the individuals' preferences. For example, we can give for each pair of options the mean value and standard deviation, arranging numbers in the descending order of the latter value. Statistical data are often misleading, but in the context of our problem they can be quite useful. The moderator is able to quickly identify the most controversial pairs of options. Then he can more precisely inspect raw data, i.e. preferences of the experts for the pairs of options suspected to be troublesome.

It is also possible to employ more sophisticated techniques of data aggregation. We can, for example, apply Yager's approach [28], which makes it possible to evaluate expressions of the form:

"Most experts agree that option i is strongly preferred to option j"

or

"Most experts agree in preferences as to the pair of options i and j"

In the above, the underlined clauses can be changed making statement more (less) precise and the same time less (more) true (in the sense of multivalued logic).

Similar information may be gained by considering rankings of the alternatives occurring implicitly in the experts' preferences. It may happen that some of them are ranked more or less equally by the overwhelming majority of experts. If it is so, the next stages of the discussion should be focused on other alternatives. The main problem in this approach is how to properly determine the rankings. It may be done by the introduction of an appropriate distance in the space of fuzzy preference relation [3, 29, 13, 14]. The way these rankings are obtained should be compatible with the method used for group preferences formation. Instead of rankings we can consider undominated sets of options or more generally sets of alternatives being values of a selected choice function (see for example [1]).

More general support of a discussion consists in the confrontation of experts with solutions (choice subsets or group preference relations) proposed by known algorithms for social welfare function or choice function. In the most optimistic case the experts can adopt one of the proposed solutions or they can, at least, accept one of them as a good point for further discussion. Among the proposed solutions there should be, of course, the one to be chosen in the group preference formation module.

The above mentioned proposal moves the discussion to the space of subsets or rankings of alternatives. One can support the discussion analogously in the original space of preference relations. We can confront the experts with fuzzy preference relations being a solution to the following problem:

> "Find a profile maximizing a consensus degree and simultaneously being maximally similar to the initial profile"

where a profile is meant as a set of fuzzy preference relations of all experts. Each expert can estimate what changes in his or her preferences would positively influence the consensus degree in the group. Next, he or she must decide if such a change is acceptable for him. Such a decision should be made after a further discussion with other members of the group. This discussion can really change the opinion of a given expert as to the particular options. The difficulty in such an approach consists in how to properly define the similarity and how to solve the above formulated problem.

Another information about the group opinion may be gained by the application of cluster analysis of preference matrices (see for example [19]). This way we can study the structure of the group. Generally speaking, we can identify subgroups of experts whose members have similar opinions one to another and, on the other hand, rather different opinions between subgroups. There can be many different "configurations" of opinions in the group. For example, a clear majority of experts may agree quite well and only a few can be of different opinion. It corresponds to the situation where consensus degree is high, but here we obtain also information about "outsiders", i.e. experts who disagree with the opinion of a majority. A rather different situation may also happen where there are two quite homogeneous subgroups which strongly disagree witch each other.

This knowledge about the structure of preferences in the group can be very helpful. Firstly, it can make a further discussion easier and more efficient. It works in the "space" of experts rather than alternatives so it is complementary, in some sense, to the approaches presented earlier. Secondly, it can be directly useful for the decisionmaker. Especially when from discussion there eventually emerge a few subgroups of experts having internally very close opinions, while the intergroup similarity is low. It is a situation where a consensus degree is very low and the group preference is not reliable. The decisionmaker can be then supported by group preferences of the subgroups revealed - not only by information that there is no consensus and by raw data, i.e. the individuals' preference matrices.

The problem is that the generally stated purpose of cluster analysis can be formulated in a few different ways. It should be studied how it can and/or should be related to other parts of the system supporting the CRP.

6. Concluding remarks

We have presented a model of the consensus reaching process which is deemed to be taking place during the session with experts. In the framework of the group decision making theory only one element of this process is considered, namely the group opinion formation. We also pointed out a few possible ways to model the other elements of the process.

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Modelowanie i wspieranie procesu osiągania consensusu z użyciem rozmytych relacji preferencji

W pracy rozważono zagadnienie wspomagania sesji ekspertów ukierunkowanej na osiągnięcie consensusu. Wyodrębniono główne elementy procesu osiągania consensusu. Omówiono dwa znane z literatury podejścia z dziedziny wspomagania decyzji pod kątem spełniania przez nie wymagań zapropowanego modelu. Na koniec naszkicowano własną propozycję wspomagania procesu.

Моделирование и автоматизирование процесса достижения соглашения при использовании размытых соотношений предпочтения

В работе рассматривается вопрос автоматизирования сессии экспертов направленой на достижение соглашения. Выделены основные элементы процесса достижения соглашения. Рассмотрены известные из литературы подходы к проблеме автоматизирования принятия решения с точки зрения удовлетворения требований предлагаемой модели. В заключение оговорены собственные предложения по автоматизированию этого процесса.