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Mathematical questions of modelling and control in water distribution problems

by

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A model of water flow in channels of an irrigation system, characterized by simplicity of relations, is suggested. The method of level forecast is developed on the basis of such a model permitting realization of control of an irrigation system given various information about water demands.

1. Introduction

The paper presents a proposal of a method of water flow control in an irrigation system under various information situations. The method is based on the theory of differential games, which makes it possible to construct the control of a player in the absence of reliable information about the future actions of an opponent.

The irrigation system is a complex system which can comprise open channels which are divided by partition structures into reaches, storage basins and other arrangements. The principal function of the system is the water supply to different consumers with minimal water transport and operation expenditures.

In most of cases the basic control of hydro–engineering facilities of medium and large scale is performed from the central master station, and water intake facilities are controlled directly by water consumers. That is why it is necessary to divide all the hydro–engineering facilities in the system according to the criterion of water supply and water intake into two groups. The first group contains objects which are controlled from the central master station and which provide water supply and its corresponding parameters (basic elements, pumping plants, partition structures). The second group comprises the facilities for water intake from the system.

We shall say that the first group of installations is controlled by a player–ally (P) and the second group by a player–opponent (E). Such division makes it possible to apply methods of the theory of differential games to description of the process of irrigation system operation. Water consumers who control the second group of installations change the requests given earlier and, consequently, the future actions of the opponent are unknown or are known with some error.

The aim of the paper is to construct an active method of irrigation system control under some uncertainty of water consumption, sudden changes of water consumers' requests and in emergency situations, this method being oriented towards the use of a real–time hardware. The paper extends the results obtained in [1–3].

2. Linear Model of Water Flow in Irrigation System

We establish the relation between water consumption and water levels at particular points of channels. With the help of some assumptions we obtain rather simple relations which make it possible in future to solve the problem of keeping the levels within the specified limits. The simplified model does not result in the marked divergence from reality since it assumes the feedback in the form of constantly introduced real values of water levels.

2.1. Water Flow in One Reach

Let us use the following notation: $h(x,t)$ is water level at point x at time t; l is the length of reach ; $x = 0$ ($x = l$) are coordinates of the upper (lower) reach; $Q(t)$ ($-q(t)$) is the discharge of water flowing through the upper (lower) part of facility, $Q(t)$ is the intensity of the water source, $q(t)$ is the intensity of flow, so that $Q(t) \geq 0$ and $q(t) \geq 0$.

We shall admit that a reach has a prismatic rectangular channel with a horizontal bed. Assume that up to the instant $t = 0$ there was a uniform steady–state flat with level H in the reach.

Let point $x = 0$ ($x = l$) be the location of a source (runoff) of water which changes the level at the given point without consideration of influence from some other sources, according to $h_0(t) + H$ ($h_l(t) + H$). The flow of water in channel without consideration of partition structures is described by the St.–Venant equation [4,5]. For their solution we use the theory of low–amplitude waves [4].

The solution obtained gives the value of wave velocity upstream and downstream. We denote these velocities by ω_+ and ω_-, respectively. Then $\tau_0 = l/\omega_+$ is the time of passage of the wave from the upper to the lower reach. Therefore hereafter we shall consider the level $h(l,t)$ at point $x = l$.

If we neglect the reflection of waves at channel partitions, the level at point $x = l$ can be determined by formula

$$h(l, t + \tau_0) = \alpha h_0(t) + h_l(t + \tau_0) + H \tag{1}$$

where $t \geq 0$, and $\alpha \leq 1$ is a coefficient of damping of the wave which has passed from point $x = 0$ to point $x = l$.

We define more exactly the initial conditions assuming that

$$Q(t) = 0, \quad t \leq 0; \qquad q(t) = 0, \quad t \leq \tau_0. \tag{2}$$

In this case $h(l,t) = H$.

Consider the case of a dispersed consumer. Assume that at points x_i, $i = 1, \ldots, n$, there are water intake facilities which change water levels at given points without consideration of the rest of consumers according to functions $h_i(t) + H$.

Then

$$h(l, t + \tau_0) = \alpha_0 h_0(t) + h_l(t + \tau_0) + \sum_{i=1}^{n} \alpha_i h_i(t + \frac{x_i}{\omega_+}) + H, \tag{3}$$

where α_i is the dumping coefficient of the wave passing from point $x = x_i$ to point $x = l$.

If we use notation

$$h_l^*(t) = h_l(t) + \sum_{i=1}^{n} \alpha_i h_i(t + \frac{x_i}{\omega_+} - \tau_0),$$

then we can say that some "generalized" water consumer being at point $x = l$ changes the level at point $x = l$ according to the function $h_l^*(t) + H$.

Thus the case of dispersed consumer is reduced to the case when a consumer is situated at point $x = l$.

2.2. Relationship between level and Water Flow. Method of Exponents

By extending the method of exponents [1,4] we relate the water flow to values of levels by means of the following formulas:

$$\frac{d}{dt}h_0(t) = c_1 Q(t) - k_1 h_0(t) \tag{4}$$

$$\frac{d}{dt}h_l(t) = -c_2 q(t) - k_2 h_2(t) \tag{5}$$

$c_i > 0$, $k_i > 0$, $i = 1, 2$. Conditions (2) guarantee that $h_0(0) = 0$, $h_l(\tau_0) = 0$. Hence

$$h_0(t) = \int_0^t e^{-k_1(t-\tau)} c_1 Q(\tau) d\tau \tag{6}$$

$$h_l(t) = \int_0^t e^{-k_2(t-\tau)} c_2 q(\tau + \tau_0) d\tau \tag{7}$$

Now, let us determine the coefficient $k_0^1 = k_1/c_1$ from the following considerations. The discharge of water running into the reach should be equal to the amount of water carried by the formed wave. Therefore

$$Q(t) = \omega_+ h_0(t) b,$$

where b is the width of the channel.

Since the equality should be satisfied at $Q = const$, then

$$\omega_+ b c_1 \int_0^t e^{-k_1(t-\tau)} d\tau = 1$$

or

$$k_0^1 = \frac{k_1}{c_1} = \omega_+ b(1 + e^{-k_1 t}).$$

Hence, with rather great t we obtain $k_0^1 \approx \omega_+ b$. Similarly we define

$$k_0^2 = \frac{k_2}{c_2} = \omega_- \times b.$$

We simplify the model assuming that $c_1 = c_2 = c$. Then the coefficients k_0^i are determined by coefficients k_i. The case of $k_1 \neq k_2$ is considered in [1]. When solving numerous practical problems we may assume that $k_1 = k_2 = k$, which makes it possible to avoid cumbersome formulas and reasoning.

Thus, by formulas (6) and (7) we obtain

$$h(l, t + \tau_0) = H + \int_0^t e^{-k(t-\tau)} c[\alpha Q(\tau) - q(\tau + \tau_0)]d\tau. \tag{8}$$

2.3. The Balance Method

Formula (8) makes it possible to determine the level at point $x = l$ for $t \geq \tau_0$ under condition (2) which is very often not satisfied. With regard for this we define more exactly the value H with the help of some balance relations. In formula (8) $h(l, \tau_0) = H$, i.e. H should be the level value at time τ_0.

When the condition (2) is not satisfied, $h(x, \tau_0) = const$. In this case we say that H is the average level in the reach at time τ_0. Assume that $H = H(\tau_0)$. Determine the value $H(\tau_0)$ through volumes of inflowing and outflowing water.

In irrigation systems the levels are measured at the upper and lower ends of the reach. Therefore in practice the values $H(\tau_0)$ should be interpreted as a level at the lower end of the reach at time τ_0. We relate $H(\tau_0)$ to the level $H(0)$ at the lower reach at moment $t = 0$.

If before, with the help of the St.–Venant equations, we took into account the wave–like character of the process of water flow, we now neglect the waves and assume that in the reach the movement of water is steady. We say that in this case the relation between the water volume V in the reach and the water level at its end which is specified by the function $H = H^*(V)$ is known together with the inverse function $V = V^*(H)$.

We can therefore determine $H(\tau_0)$ by formula

$$H(\tau_0) = H^*(V^*(H(0)) + \int_{-\tau_0}^{0} Q(\tau)d\tau - \int_0^{\tau_0} q(\tau)d\tau) \tag{9}$$

If the assumption about the reach geometry is valid, then

$$H(\tau_0) = H(0) + \frac{1}{lb}\left[\int_{-\tau_0}^{0} Q(\tau)d\tau - \int_{0}^{\tau_0} q(\tau)d\tau\right].$$

The real reach, however, has a slope, and its walls have a slant, i.e. the reach is some prism. If we assume that if the movement is steady then the water surface is horizontal, then functions H^* and V^* can be found in the analytical form. If necessary, following [2], we may find a curve of free surface by solving particular differential equations numerically. These problems are, however, beyond the scopes of our paper.

2.4. Water Flow in a Cascade of Reaches

Consider a cascade consisting of N reaches divided by partition structures (PS). Let us number the reaches in increasing number downstream. Assume that the water is pumped into the first reach by a pumping station, and Q_i is its current discharge.

Let the i–th reach ($i \geq 2$) be situated between the i–th and $(i+1)$–th PS (PS_i and PS_{i+1}); the first reach ends with the 2–nd PS; Q_i is the discharge of water flowing through PS_i, $i = 2, \dots, N$ (we say that $Q_{N+1} \not\equiv 0$); the i–th consumer with discharge $-q_i$ is situated at the end of the i–th reach; $H_i(t)$ is the level at the end of the i–th reach at time t; τ_i is the time of passage of a wave from the beginning to the end of the i–th reach; α_i is the coefficient of dumping when the wave passes through the i–th reach; k_i and c_i are coefficients obtained in Section 2 for the i–th reach; H_i^* and V_i^* are functions considered in Section 2 for the i–th reach; l_i is the length of the i–th reach; $h_i(x,t)$ is a level in the i–th reach.

It follows from Section 2.2 that the level at the end of the i–th reach can be represented in the form

$$h_i(l_i, t+\tau_i) = H_i(\tau_i) + \int_{0}^{t} e^{-k_i(t-\tau)} c_i[\alpha_i Q_i(\tau) - Q_{i+1}(\tau+\tau_i) - q_i(\tau+\tau_i)]d\tau, \tag{10}$$

where

$$H_i(\tau_i) = H_i^*\left(V_i^*(H_i(0)) + \int_{-\tau_i}^{0} Q_i(\tau)d\tau - \int_{0}^{\tau_i}[Q_{i+1}(\tau) + q_i(\tau)]d\tau\right) \tag{11}$$

The quantity Q_i, enters into the i–th equation with the argument τ, and into the $(i-1)$–th – with the argument $\tau+\tau_{i-1}$. That is why to solve the control problem we correlate the equation (10) in such a way that the arguments of Q_i are identical.

For each m $(1 \le m \le N)$ we write a set of equations similar to (10) into which Q_m enters with the argument τ

$$h_i(l_i, t+\theta_m^i) = H_i(\theta_m^i) + \int_0^t e^{-k_i(t-\tau)} c_i[\alpha_i Q_i(\tau+\theta_m^{i-1})$$
$$-Q_{i+1}(\tau+\theta_m^i) - q_i(\tau+\theta_m^i)]d\tau, \tag{12}$$
$$\text{at} \quad m=1, i=1,\ldots,N,$$

where $\theta_m^i = \sum_{j=1}^i \tau_j$, $\theta_m^{m-1}=0$, $\theta_m^{m-2} = -\tau_{m-1}$, $i=m-1,\ldots,N$. It follows from (11) that

$$H_i(\theta_m^i) = H_i^*\left(V_i^*(H_i(0)) + \int_{-\tau_i}^{\theta_m^{i-1}} Q_i(\tau)d\tau - \int_0^{\theta_m^i}[Q_{i+1}(\tau)+q_i(\tau)]d\tau\right) =$$
$$= H_i^*\left(V_i^*(H_i(\theta_{m+1}^i)) + \int_{\theta_{m+1}^{i-1}}^{\theta_m^{i-1}} Q_i(\tau)d\tau - \int_{\theta_{m+1}^i}^{\theta_m^i}[Q_{i+1}(\tau)+q_i(\tau 0)]d\tau\right) \tag{13}$$

Quantities Q_i, are bounded below by zero, and above – by some maximal discharge $Q_i^{\max}$ which can change with time. The quantity $Q_i^{\max}$ is bounded above by the capacity of the i–th reach Q_i^+ and depends on levels at the end of the $(i-1)$–th reach and at the beginning of the i–th reach. In the case of low–amplitude wave we can say that the level at the beginning of the i–th reach linearly depends upon the level at the end of the reach. Levels H_i at time θ_m^i enter into the formula (12). Define at the same moments the value $Q_i^{\max}$ by formula

$$Q_i^{\max}(\theta_m^i) = \min\{Q_i^+, f_i(e_i^{\max}, H_{i-1}(\theta_m^i), H_i(\theta_m^{i+1}))\}, \tag{14}$$

where

$$f_i(e, x_1, x_2) = \varepsilon_i e\sqrt{2g(x_1 - \delta_i x_2)};$$

and ε_i, $e_i^{\max}$, δ_i – are known constants. The function f_i gives at the known values H_{i-1} and H_i the relationship between the height of the shield e and the corresponding water discharge. As a rule the water discharge is not directly determined, and only the value e is measured.

Thus, in the present paper, two fairly different models are used for reduction of potential errors. The first model, based on St.–Venant equations takes into account the wave–like character of the process. However during its construction a number of assumptions have been made for the sake of simplicity, which can bring about discrepancy with reality. The second model is therefore called for. It takes into account the volumes of inflowing and outflowing water and makes it possible to introduce the feedback owing to periodic change of water levels in reaches or water volumes in basins.

3. The Level Forecast Method In Irrigation System Control

3.1. Mathematical Problem Statement

The model described permits formulation of the problem of keeping the levels at the end of each reach and volumes in storage basins within the given limits. The problem will be solved for the time interval $[0,T]$. The question of the choice of control period T will be discussed below.

We consider the cascade of reaches described in Section 2.4. Let H_i^- and H_i^+ be admissible values of lower and upper levels in the i–th reach. Then for each m $(1 \leq m \leq N)$ and any $t \in [0,T]$ a system of inequalities

$$H_i^- \leq h_i(l_i, t+\theta_m^i) \leq H_i^+, \quad i = \max\{1, m-1\}, \ldots, N \tag{15}$$

should be satisfied.

The value $t_* = \sum_{i=1}^{N} \tau_i$ is the time of transient processes in the irrigation system. The player E informs beforehand about his/her control, i.e. about the vector of orders $(q_1(t), \ldots, q_N(t))$, for the time interval of length not less than t_*.

The vector of orders $\bar{q}_m(\tau) = \{q_i(\tau+\theta_m^i), i = \max\{1, m-1\}, \ldots, N\}$ and the vector of player controls $\bar{Q}_m(\tau) = \{Q_i(\tau+\theta_m^{i-1}), i = \max\{1, m-1\}, \ldots, N\}$ enter into the m–th system of inequalities. By virtue of a number of reasons the orders change as time goes on, i.e. functions $q_i(\tau)$ change and, therefore, information about the vector $\bar{q}_m(\tau)$ can also change at each time instant τ. That is why when choosing the vector $\bar{Q}_m(\tau)$ at time τ we shall use only information about the vector $\bar{q}_m(\tau)$, and the information about values $\bar{q}_m(\theta)$, $\theta > \tau$, is considered unknown. Thus we come to the differential relation game whose dynamics is described by the equation (10), terminal set is specified by inequalities (15), and $\bar{Q}_m$, $\bar{q}_m$ are controls of players P and E, respectively.

3.2. Derivation of Basic Inequalities

Assume that

$$H_i(\theta_m^i) \in [H_i^-, H_i^+]$$

It follows from (6) that in order to satisfy inequalities (15) for all $t \in [0, T]$ it is sufficient that for all $t \in [0, T]$ the inequalities

$$H_i^- \le H_i(\theta_m^i) + \int_0^T e^{-k_i(T-\tau)} d\tau c_i[\alpha_i Q_i(t + \theta_m^{i-1}) - Q_{i+1}(t + \theta_m^i)$$
$$-q_i(t + \theta_m^i)] \le H_i^+, \quad i = \max\{1, m-1\}, \dots, N, \tag{16}$$

be satisfied.

Since $k_i > 0$, the system (16) follows from the system

$$\frac{k_i}{c_i}[H_i^- - H_i(\theta_m^i)] \le \alpha_i Q_i(t + \theta_m^{i-1}) - Q_{i+1}(t + \theta_m^i) - q_i(t + \theta_m^i) \le$$
$$\frac{k_i}{c_i}[H_i^+ - H_i(\theta_m^i)], \quad i = \max\{1, m-1\}, \dots, N. \tag{17}$$

Besides, we should take into account the constraint on controls

$$0 \le Q_i(t + \theta_m^{i-1}) \le Q_i^{\max}(\theta_m^{i-1}), \quad i = \max\{1, m-1\}, \dots, N. \tag{18}$$

Assume that values $H_i(\theta_m^i)$ are known, then the problem of Section 3.1 is solved in the following manner. For each $t \in [0, T]$ we search for $Q_m(t)$ as a solution of the m–th system of inequalities (17). Since $Q_i(t)$ enters into $Q_m(t)$ as a component then by solving N systems of inequalities of the kind of (17) we shall find the required controls for any interval $[0, T]$.

3.3. Forecast of Levels

In inequalities (17), (18) we find values $H_i(\theta_m^i)$ which represent some future water levels in reaches. Forecasting of these levels and the solutions of corresponding problems (17), (18) constitute the essence of the method at level control and forecast. When determining $H_i(\theta_m^i)$, $q_i(t)$ will be considered to be known over the interval $[0, t_*]$. At $m = N$ we rewrite the system of inequalities (17), (18) in the form

$$\frac{k_{N-1}}{c_{N-1}}[H_{N-1}^{-} - H_{N-1}(0)] \le \alpha_{N-1}Q_{N-1}(t-\tau_{N-1}) - Q_N(t) - q_{N-1}(t) \le$$
$$\le \frac{k_{N-1}}{c_{N-1}}[H_{N-1}^{+} - H_{N-1}(0)]. \qquad (19)$$
$$\frac{k_N}{c_N}[H_N^{-} - H_N(\tau_N)] \le \alpha_N Q_N(t) - q_N(t+\tau_N) \le \frac{k_N}{c_N}[H_N^{+} - H_N(\tau_N)]$$
$$0 \le Q_N(t) \le Q_N^{\max}(\tau_N)$$

where

$$H_N(\tau_N) = H_N^*(V_N^*(H_N(0)) + \int_{-\tau_N}^{0} Q_N(\tau)d\tau - \int_{0}^{\tau_N} q_N(\tau)d\tau)$$

is the known value.

The function $Q_{N-1}(t-\tau_{N-1})$ is defined by the past control and, therefore, is known over the interval $[0, \tau_{N-1})$. Thus, when solving the system (19) for the interval $[0, \tau_{N-1})$ according to a particular criterion we may define the control Q_N in the interval $[0, \tau_{N-1})$. This control is used afterwards for the $(N-1)$–th system.

Now consider the m–th system for $m > 1$. Assume that values $H_i(\theta^i_{m+1})$ have been already calculated and after solving the $(m+1)$–th system the controls Q, $i = m+1, \ldots, N$ are found for the interval $[\theta^{i-1}_{m+1}, \theta^{i-1}_m)$, respectively. Then, having known the past control Q_m in the interval $[-\tau_m, 0)$ we may determine the values $H_i(\theta^i_m)$ by formula (13). The values $Q_{m-1}(t-\tau_{m-1})$ determined by the past control enter into the m–th system. The given function is known in the interval $[0, \tau_{m-1})$ and now when solving the m–th system we may find the controls Q_i, $i = m, \ldots, N$ for intervals $[\theta^{i-1}_m, \theta^{i-1}_{m-1})$, respectively. The information obtained is used as the initial one for solution of the $(m-1)$–th system.

Continuing the process, we define $H_i(\theta^i_1)$ from the 2–nd system. As a result we shall know the values H_i in inequalities (17), (18) for all $m = 1, \ldots, N$. Then for each $t > 0$ we solve the 1–st system, using the above criteria and we use the value $Q_1(t)$ for solution of the 2–nd system into which Q_1 enters with delay. Similarly for $t \ge 0$ we solve the m–th system and use $Q_m(t)$ for solution of the $(m+1)$–th system.

Thus the controls $Q_m(t)$ for the interval $[0, T]$ are determined. Then the instant T is taken as the initial one, new levels H_i are measured, and the choice of control similar to the described above for some subsequent interval $[T, T+T']$ is made.

The instant of switching T is chosen on the basis of the following reasons. First, T can be an instant of the change of information about requests. Second, the value of T should be limited by the value depending on the irrigation system structure such that within the interval $[0, T]$ the error of the wave model is kept within the admissible limits.

3.4. Solution of the system of inequalities

Let us describe a method of solution of the system (17), (18),. when the time instant t is fixed. Consider the case when $m = 1$ and write inequalities (17), (18) without arguments for functions Q_i. Then we have

$$\begin{aligned} & a_1 \leq \alpha_i Q_1 - Q_2 \leq b_1 \\ & \dots\dots\dots\dots\dots\dots \\ & a_{N-1} \leq \alpha_{N-1} Q_{N-1} - Q_N \leq b_{N-1} \\ & a_N \leq \alpha_N Q_N \leq b_N \\ & \beta_i \leq Q_i \leq \gamma_i, \quad i = 1, \dots, N. \end{aligned} \tag{20}$$

where α_i, b_i, β_i, γ_i are some constants.

Let us put

$$\begin{aligned} c_N &= \max\{a_N/\alpha_N, \beta_N\}, \quad d_N = \min\{b_N/\alpha_N, \gamma_N\} \\ c_m &= \max\{(a_m + c_{m+1})/\alpha_m, \beta_m\}, \quad d_m = \min\{(b_m + d_{m+1})/\alpha_m, \gamma_m\} \\ m &= N-1, \dots, 1. \end{aligned}$$

Conditions $c_m \leq d_m$, $m = 1, \dots, N$, guarantee the existence of solutions to inequalities (20). Let Q_1^0 be some point from $[c_1, d_1]$. If we have no additional information about Q_1 we can put $Q_1^0 = \frac{1}{2}(c_1 + d_1)$. After putting Q_1^0 in the first couple of inequalities (20), we obtain

$$\alpha_1 Q_1^0 - b_1 \leq Q_2 \leq \alpha_1 Q_1^0 - a_1$$

Using the latter inequalities we can exact c_2, d_2 and put

$$c_2^0 = \max\{c_2, \alpha_1 Q_1^0 - b_1\}, \quad d_2^0 = \max\{d_2, \alpha_1 Q_1^0 - a_1\}.$$

Choose some point Q_2^0 out of $[c_2^0, d_2^0]$. We can put, as before, $Q_2^0 = \frac{1}{2}(c_2^0 + d_2^0)$ and determine c_3^0 and d_3^0.

By continuing this process we will find Q_i^0, $i = 1, \ldots, N$, which will be the solution of (20).

4. Conclusions

1. The method of water level forecast can be used with different information about the actions of water consumers. It can be used in the long-term planning, in emergency situations and for the sudden changes of orders.

2. The choice of optimal values of discharge of water flowing through hydro-engineering structures is reduced to solution of some systems of linear inequalities. This makes it possible to use the method presented for control of rather complex irrigation systems which have a tree-like structure and contain storage basins along with reaches and pump stations as well as with partition structures.

3. Computation-oriented constructs used in this method are analytical, reduced to rather simple formulas. This permits realization of the method on a personal computer.

4. The method is based on different models of water flow ("wave" and "balance" ones). If necessary, account may be taken of the dependence of the time of wave run on levels and water discharge.

5. The method uses, whenever possible, information about the irrigation system: data about levels in upper and lower reaches, information about future water consumption, probability characteristics of errors in future orders, data about change of orders and about current water distribution as well as about past control of the irrigation system.

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Matematyczne problemy w modelowaniu i sterowaniu zagadnień rozdziału wody

W pracy zaproponowano model przepływu wody w kanałach systemu nawadniania, charakteryzujący się prostotą relacji. Na podstawie tego modelu opracowano metodę prognozowania poziomów wody, pozwalającą na realizację sterowania systemami nawadniania mając dane różne informacje o zapotrzebowaniach na wodę.

Математические вопросы моделирования и управления в задачах рацпределения воды

В работе предлагается модель потока воды в каналах оросительной системы, характеризующаяся простыми соотношениями. На основе этой модели разработан метод прогнозирования уровней воды, позваляющий реализовать управление оросительной системой на основе информации о предиолагаемом потреблении воды.