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Arbitration Procedures with the Possibility of Compromise [1]

by

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Under final-offer arbitration (FOA), two parties submit final offers, and the arbitrator chooses that which is closer to his or her notion of a fair settlement. There is little incentive to converge under FOA, although combined arbitration (CA), which combines conventional arbitration and FOA, does induce convergence under rather general conditions.

Modifying these two procedures to allow the two players to choose the mean of their final offers, before the arbitrator makes a choice, introduces the possibility of their reaching their own compromise without letting the settlement go to arbitration. This compromise

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will be attractive to risk-neutral players if it coincides with the median of the arbitrator's distribution of fair settlements–assumed to be common knowledge–or if it deviates somewhat from the median and the players are risk averse.

An alternative route to compromise is to modify the procedures to allow the following sequential offers: One player, selected at random, makes an offer; the other player makes a counteroffer *and* must pledge to accept the mean. The first player can then accept or reject the mean; if rejected, FOA or CA is used. This modification, because it allows the players to respond in sequence to each other's offers, better enables them to coordinate their offers, but it introduces an asymmetry into play.

The relative merits of the different procedures in promoting compromise, and practical aspects of their implementation, are assessed.

Keywords: Arbitration, game theory, incomplete information, bargaining.

1. Introduction

Arbitration involves the submission of a dispute to an arbitrator, who makes a judgment about how the dispute is to be resolved. Unlike mediation, the judgment of the arbitrator is binding on the disputants. Traditionally, the arbitrator is free to impose his or her own judgment, but in recent years new arbitration procedures have been proposed that restrict the choices of the arbitrator.

Their purpose is not so much to rob the arbitrator of the freedom to render a fair judgment as to force the disputants to bargain seriously on their own and, if possible, negotiate a settlement. Thereby the need for an arbitrator, and the imposition of a settlement that one or the other side might not agree to on its own, is avoided.

Presumably, a settlement that both sides agree to is better than one that is forced on them by a third party. Not only may the arbitrator not be as well informed about the issues in a dispute as the disputants, but the disputants also may have an incentive to distort these issues in order to try to influence the arbitrator's judgment about what is a fair resolution of their conflict. Indeed, exaggeration, posturing, and even outright deception are commonplace in *conventional arbitration*, in which the arbitrator has free rein to reconcile differences, and each side attempts to make its best case.

Shrewd arbitrators know all this, but still the facts in a dispute may be elusive. In an attempt to elicit more truthful information about the acceptable terms of a settlement, *final-offer arbitration* (FOA) was proposed a generation ago (Stevens, 1966). It is now used to settle public-employee disputes in several states (Freeman, 1986), competitive bids on government contracts (Carrington, 1988; Halloran, 1988), and salary disputes in major league baseball (Chass, 1990), among other applications.

Under FOA, each party submits its final offer for a settlement to an arbitrator, who must choose one final offer or the other. The offer chosen by the arbitrator is the settlement. Unlike conventional arbitration, the arbitrator is not permitted to split the difference, or compromise the offers in any other way. One side or the other "wins" by having its offer chosen; it has recently been proposed that the winner receive, in addition, a bonus (Brams and Merrill, 1991).

Proponents of FOA have argued that it forces the two sides to converge, eliminating the need for a settlement imposed by the arbitrator, as under conventional arbitration. Although FOA undoubtedly dissuades disputants from making outrageous offers — lest they not be chosen by the arbitrator — it offers them little incentive to converge either, as both game-theoretic models and empirical evidence demonstrate (reviewed in Brams, 1990, ch. 3, and Brams, Kilgour, and Merrill, 1991).

But this lack of convergence may as much be a virtue as a vice. Since FOA was adopted in major league baseball in 1975, the possibility of an extreme settlement — favoring either the team owner or the player — has induced both sides to bargain seriously and reach negotiated settlements in most cases. For example, of the 162 major league players who filed for FOA in 1989, 138 (85 percent) negotiated contracts before FOA was actually used (Chass, 1990). As one baseball arbitrator put it, "I'm starting to feel like the atomic bomb. The deterrent effect of me as an arbitrator is enough" (Cronin, 1989, quoting Stephen B. Goldberg).

Evidently, it is the likelihood of divergent final offers under FOA — and the possibility that the other side's might be selected — that puts pressure on both sides to settle on their own, discouraging the actual use of FOA. We called this the *paradox of arbitration* (Brams and Merrill, 1983, p. 940), arguing that "it takes a procedure like FOA, which implements a biased outcome, to get the two sides to abandon it, bargain seriously, and settle their differences on their own."

In arenas, other than baseball, in which FOA has been used, the picture is not so rosy. Thus, in public-employee disputes in which FOA is mandated as the settlement procedure of last resort, it is routinely invoked.

The consequence is usually a lopsided settlement, favoring one side or the other, precisely because the arbitrator is not free to propose a compromise. But giving the arbitrator this freedom introduces the problems, alluded to earlier, that attend conventional arbitration.

One solution that has been proposed to this dilemma is *combined arbitration* (CA), which combines conventional arbitration and FOA in a way that induces the two sides, under rather general conditions, to make the same final offer (Brams and Merrill, 1986; Brams, 1986). Its rules are as follows:

1. Like FOA, two sides submit final offers; at the same time, the arbitrator records his or her judgment of a fair settlement.

2. If the arbitrator's choice falls *between* the two final offers (and the offers do not crisscross), then the offer closer to the arbitrator's judgment becomes the settlement, as under FOA.

3. If the arbitrator's judgment falls *outside* the two final offers (and the offers are not identical, nor do they crosscross), then the arbitrator's judgment is the settlement, as under conventional arbitration.

4. If the offers are identical, then the common offer is the settlement; if the offers crisscross, then the offer closer to the arbitrator's judgment is the settlement, as under FOA.

Rule 3 allows for the imposition of the arbitrator's judgment — and consequently, a settlement more extreme than the final offers of either side. It is especially this rule that induces the parties to converge absolutely, because each party is "protected" on its side by a more extreme arbitrator who favors it. This protection motivates both parties not just to move toward each other but to make identical final offers.

While true in theory, this convergence may not hold in practice. If, in fact, the two offers are not quite convergent but almost so, there is a good chance that the arbitrator's judgment will fall outside them and, therefore, be the settlement that is implemented. Because this settlement will be more extreme than the disputants' offers (either below or above both offers), not only may its reasonableness be questioned but the fairness of the arbitrator may also be challenged.

A related problem with CA is that convergence is to the median of the distribution that the disputants perceive to describe the arbitrator's judgment of a fair settlement (in a game of incomplete information); it is not what the disputants consider fair, or even what the arbitrator might consider fair in any particular instance. Other procedures that have been proposed include sequential arbitration procedures, which give more weight to the arbitrator's actual choice and encourage convergence to it in stages (Brams, Kilgour, and Weber, 1991).

These alternatives to FOA, and FOA itself, suffer from one major shortcoming: they do not permit the disputants to propose — and possibly agree to — *their own* compromise. Either there is no possibility of compromise, as under FOA, or the compromise to which the players converge is dictated by their perception of the arbitrator's distribution (CA) or the arbitrator's actual position that is approached in stages (sequential procedures). By contrast, in this paper we shall analyze variations on both FOA and CA that allow for the possibility of a compromise settlement at the mean of the two sides' final offers.

The plan of the paper is as follows. In section 2 we give conditions under which risk-neutral players in a two-person constant-sum game of incomplete information will accept the compromise settlement, and in section 3 we derive their equilibrium final offers, which turn out to be the same as under FOA and CA. In section 4 we demonstrate how risk aversion may attenuate the knife-edge quality of a player's decision to accept the mean, which enhances the appeal of this compromise.

In section 5 we allow the players to make their offers and acceptance decisions sequentially, according to specific rules, and argue that there is no special advantage conferred on a player by going first or second. In section 6 we compare the simultaneous and sequential versions of FOA and CA (with the compromise option) and conclude that the sequential version is superior because, by permitting more feedback, it better enables the players to learn from each other and coordinate their offers, which is an important aspect of arbitration that has been stressed in other models (Gibbons, 1988).

Because equilibrium offers under sequential FOA are in general more divergent than under sequential CA, they would more likely impel the players to settle through compromise, especially if the players are risk averse. Sequential F0A, therefore, seems to be the best procedure for fostering the kind of compromise settlements that FOA rules out, though it may be invoked more often as a procedure (e.g., in baseball) than FOA now is.

2. When Should Risk-Neutral Players Accept a Compromise?

Under *modified FOA*, after the final offers have been submitted but before the arbitrator announces a decision, both players are given the opportunity to accept (Ac) or reject (Re) the mean of their offers. If either player rejects the mean, FOA is implemented — that is, the arbitrator announces a decision which must be one or the other of the two final offers. If, on the other hand, both players accept, the mean becomes the settlement, and the arbitrator's choice is never revealed.

We begin by analyzing the conditional strategies of acceptance or rejection, given that the offers have been made and announced. We assume throughout that the offers are a and b and that both parties view the arbitrator's notion of a fair settlement as a continuous random variable with probability density f and distribution function F, with $F' = f$ (or at least one-sided derivatives of F for each x) and, without loss of generality, the median at 0.

THEOREM 1 *Let B be the focal player and p = the probability (subjective estimate by B) that A* accepts. *Under modified FOA, if both players are risk neutral, B accepts if his or her offer is farther from 0 and* rejects *if it is nearer. If the offers are equidistant from 0, then B is indifferent between accepting and rejecting.*

PROOF: B's expected payoff under FOA (as well as A's, because the game is constant-sum) is

$$\begin{aligned} g(a,b) &= aF(m) + b[1 - F(m)] \\ &= b - (b-a)F(m), \end{aligned}$$

where the mean offer, m, is $(a+b)/2$. Under modified FOA, assume B accepts. Then B's expected payoff is

$$E_B(Ac) = [b - (b-a)F(m)][1-p] + pm,$$

or a weighted average of B's payoff under FOA and that under mutual acceptance of m. On the other hand, if B rejects, $E_B(Re) = g(a,b)$.

The difference

$$
\begin{aligned}
E_B(Ac) - E_B(Re) &= pm - p[b - (b-a)F(m)] \\
&= p[m - b + (b-a)F(m)] \\
&= p[-(b-a)/2 + (b-a)F(m)] \\
&= p(b-a)[F(m) - 1/2] \qquad \begin{array}{l} >0 \text{ if } m > 0 \\ <0 \text{ if } m < 0. \end{array}
\end{aligned} \tag{1}
$$

Thus, B accepts if his or her offer is more extreme, and rejects if it is less extreme, than A's. ■

Note that this result is independent of $p, 0 \leq p \leq 1$. Hence, at least in principle, only one side accepts, so that FOA (rather than m) is implemented. In the zero-probability event that $m = 0$, the players will be indifferent between accepting and rejecting, creating only a knife-edge opportunity for compromise.

Next we consider *modified CA*, which incorporates into CA the accept/reject option.

THEOREM 2 *Let B be the focal player and p = B's subjective probability that A accepts. Assume further that the arbitrator's distribution is symmetric and strictly unimodal — that is, $f(x)$ is strictly decreasing as x recedes from 0 in both directions. Under modified CA, if both players are risk neutral, B accepts if his or her offer is farther from 0 and rejects if it is nearer.*

PROOF: In Brams and Merrill (1986, p. 1354), we show that A and B's expected payoff under CA is

$$g(a,b) = (a-b)F(m) + \int_a^b F(x)dx,$$

where $m = (a+b)/2$. Under modified CA, assume B accepts. Then B's expected payoff is

$$E_B(Ac) = [(a-b)F(m) + \int_a^b F(x)dx][1-p] + pm.$$

If B rejects, the expected payoff is $E_B(Re) = g(a,b)$. The difference

$$E_B(Ac) - E_B(Re) = p[m + (b-a)F(m) - (b-a)\int_a^b F(x)dx/(b-a)]$$

$$= p\{m + (b-a)[F(m) - \int_a^b F(x)dx/(b-a)]\} \quad \begin{array}{l} > \ 0 \text{ if } m > 0 \\ < \ 0 \text{ if } m < 0. \end{array} \tag{2}$$

To justify these last inequalities, it suffices to show that, if $m > 0$, then $F(m) \geq$ the average of $F(x)$ for x in the interval (a, b). Note first that if $m > 0, |m+t| > |m-t|$ for $t \geq 0$. Because F is symmetric and unimodal, $F'(x)[= f(x)]$ decreases as $|x|$ increases, so that

$$F'(m+t) \leq F'(m-t) \quad (t \geq 0).$$

Thus, for $x \geq 0$:

$$\int_0^x F'(m+t)dt \leq \int_0^x F'(m-t)dt,$$

so that, using the substitution $u = -t$ in the second integral,

$$F(m+x) - F(m) \leq F(m) - F(m-x).$$

In turn,

$$\int_0^{(b-a)/2} [F(m+x) - F(m)]dx \leq \int_0^{(b-a)/2} [F(m) - F(m-x)]dx,$$

or

$$\int_0^{(b-a)/2} F(m+x)dx + \int_0^{(b-a)/2} F(m-x)dx \leq (b-a)F(m),$$

so that

$$1/(b-a) \int_a^b F(x)dx \leq F(m). \tag{3}$$

Hence, $F(m)$ exceeds the average of $F(x)$ in (a, b), as desired.

Thus, B accepts if $m > 0$, i.e., if b is more extreme than a, the same result as obtained for modified FOA. Again this result is independent of p as long as $p > 0$. ■

To illustrate Theorem 2, if the distribution is uniform on $[0, 1]$, then the CA expected payoff increases by the constant $1/2$ from the value for a distribution on $(-\infty, \infty)$, i.e.,

$$g(a, b) - (a-b)F(m) + \int_a^b F(x)dx + 1/2.$$

It follows that, because this increment becomes a decrement when compared with m, the difference

$$E_B(Ac) - E_B(Re) = p[m - 1/2] \quad \begin{array}{l} > 0 \text{ if } m > 1/2 \\ < 0 \text{ if } m < 1/2. \end{array}$$

We conclude that mutual acceptance is never optimal, except when $m = 0$ and the players are indifferent: either one player or the other will reject m (never both), unless the two offers are equidistant from the median of 0.

3. Equilibrium Strategies under Modified FOA and CA

Next we turn to the question of what offers are optimal for A and B. Assume that the probability, $p = p(a, b)$, that A accepts and the probability, $q = q(a, b)$, that B accepts depend on a and b, and that these probabilities are common knowledge. Although Theorems 1 and 2 show that either $p = 0$, $q = 1$ or $p = 1$, $q = 0$ are optimal, except when $m = 0$ and there is indifference, they say nothing about what the values of a and b are in equilibrium.

THEOREM 3 *If both players are risk neutral, then the Nash equilibria for modified F0A are identical to those for FOA.*

PROOF: Denote $r(a, b) = p(a, b)q(a, b)$. We showed in the proof of Theorem 1 that the expected payoff of modified FOA is

$$g(a, b) = r(a, b)m + [1 - r(a, b)][b - (b - a)F(m)], \tag{4}$$

where $m = (a + b)/2$. If $a = -b$, then $m = 0$ and $b - (b - a)F(m)$, the payoff under FOA, is also 0. It follows that $g(a, b) = 0$, just as under FOA. Theorem 1 implies that, for $a \neq -b$, either $p(a, b) = 0$ or $q(a, b) = 0$. It follows that $r(a, b) = 0$, so that $g(a, b) = b - (b - a)F(m)$. Thus, in either case, $g(a, b)$ is identical to the payoff under FOA, so that both modified FOA and FOA have the same Nash equilibria. ∎

THEOREM 4 *If both players are risk neutral and f is symmetric and strictly unimodal, then the Nash equilibria for modified CA are identical to those under CA.*

PROOF: From Theorem 2, the expected payoff of modified CA is

$$g(a, b) = r(a, b)m + [1 - r(a, b)][(a - b)F(m) + \int_a^b F(x)dx]. \tag{5}$$

By an argument similar to that in the proof of Theorem 3, it can be shown that the payoff functions for modified CA and CA are identical, so that the Nash equilibria are likewise identical. ∎

4. Risk Aversion

Now assume that one of the players, say B, is risk averse with utility function $u, u' > 0$ and $u'' < 0$. Some implications of risk aversion for player strategies under FOA have been analyzed by Wittman (1986) and Brams and Merrill (1991). Under modified FOA, the expected payoffs for B if A accepts (Ac) or rejects (Re) are:

$$E_B(Ac) = \{u(a)F(m) + u(b)[1 - F(m)]\}[1 - p] + pu(m)$$

and

$$E_B(Re) = u(a)F(m) + u(b)[1 - F(m)].$$

Hence, the difference is

$$\begin{aligned} E_B(Ac) - E_B(Re) &= pu(m) - p\{u(a)F(m) + u(b)[1 - F(m)]\} \\ &= p\{u(m) - [F(m)u(a) + [1 - F(m)]u(b)]\} \qquad (6) \\ &= p\{u(m) - x\}, \end{aligned}$$

where $x = F(m)u(a) + [1 - F(m)]u(b)$ is a weighted average of $u(a)$ and $u(b)$. If $m > 0$, then $F(m) > 1/2$, so x is closer to $u(a)$. It follows that

$$x \leq [u(a) + u(b)]/2 \leq u(m), \qquad (7)$$

because u is increasing and u is concave, respectively. The difference given by (6) is therefore non-negative. Thus, B should accept if $m > 0$. If $m < 0$, the first inequality in (7) reverses, and $x \leq u(m)$ if m is negative but sufficiently close to zero. Hence, B should accept if his or her offer is slightly more reasonable than A's, but reject if it is much more reasonable.

We conclude that, for modified FOA, if either side is risk averse that side should accept if its offer is more extreme or slightly less extreme and reject only if its offer is much less extreme. If both sides are risk averse, both should accept if the offers are symmetric or nearly so, with the more leeway the more risk averse.

In the following example, we determine the crossover point at which $E_B(Ac) - E_B(Re)$ changes sign, i.e., at which the optimal strategy changes from accept to reject. Suppose f is uniform on $[0, 1]$ and the utility function for B is $u(x) = \sqrt{x}$. Suppose $a = 0$ and $0 < b \leq 1$. At the crossover point, b:

$$\begin{aligned} E_B(Ac) - E_B(Re) &= p\{u(m) - [F(m)u(a) + [1 - F(m)]u(b)]\} \\ &= p\{\sqrt{b/2} - [(b/2)(0) + [1 - b/2]\sqrt{b}]\} \\ &= p\{\sqrt{b/2} - [(2 - b)/2]\sqrt{b}\} = 0. \end{aligned}$$

Solving for b, we obtain:

$$b = 2 - \sqrt{2} = .586,$$

which represents a substantial reduction from the FOA Nash equilibrium offer of 1 for B. Note that if $p = 0$ (i.e., A is certain to reject), any b yields $E_B(Ac) = E_B(Re)$, making B's choice irrelevant. The assumption that A is certain to reject, however, seems unreasonable near a crossover point.

This last result for the uniform distribution on $[0, 1]$ can be generalized to any utility function of the form $u(x) = x^p$. By the argument above, we obtain

$$b = (2^p - 1)/(2^{p-1}).$$

In fact, if $z = u(1/2) = (1/2)^p$, then $b = 2(1 - z)$. Hence, $db/dz = -2$, i.e., the rate of reduction of the optimal offer, b, as B moves toward risk aversion (and $p \to 0$, so z increases) is twice the movement of the value, $u(1/2)$. For example, if $p = .9$, then $b = .928$ for a reduction of .072 from 1, which is about twice the deviation of $u(1/2) = (0.5)^{0.9} = .536$ from 0.5.

5. Equilibrium Strategies under Sequential FOA and CA

The rules of negotiation need not be symmetrical between the parties in order to ensure symmetrical optimal strategies. In fact, such strategies can be achieved with the following asymmetrical sequential procedure. One player, selected at random (say, B), makes the first offer, b. A, knowing b, counters with an offer, a, *and* must pledge to accept $m = (a + b)/2$. Then B can either accept or reject m; if rejected, FOA or CA is used. We call these sequential procedures, based on FOA and CA, *sequential FOA* and *sequential CA*. The game tree of sequential FOA is shown in Figure 1.

THEOREM 5 *Under sequential FOA, if both players are risk neutral, B accepts if his or her offer is farther from* 0 *and rejects if it is nearer.*

PROOF: To determine B's optimal strategy, after the offers of a and b have been made, note that the expected payoff to B for acceptance is $E_B(Ac) = m$ and for rejection, $E_B(Re) = b - (b - a)F(m)$. Hence,

$$\begin{aligned} E_B(Ac) - E_B(Re) &= m - b + (b - a)F(m) \\ &= (b - a)[F(m) - 1/2] \quad \begin{array}{l} > 0 \text{ if } m > 0 \\ < 0 \text{ if } m < 0. \end{array} \end{aligned} \tag{8}$$

So B accepts if $m > 0$ (i.e., b is more extreme than a) and rejects if $m < 0$ (i.e., b is less extreme than a). B is indifferent if $m = 0$ (symmetric offers). ■

Game Tree of Sequential FOA

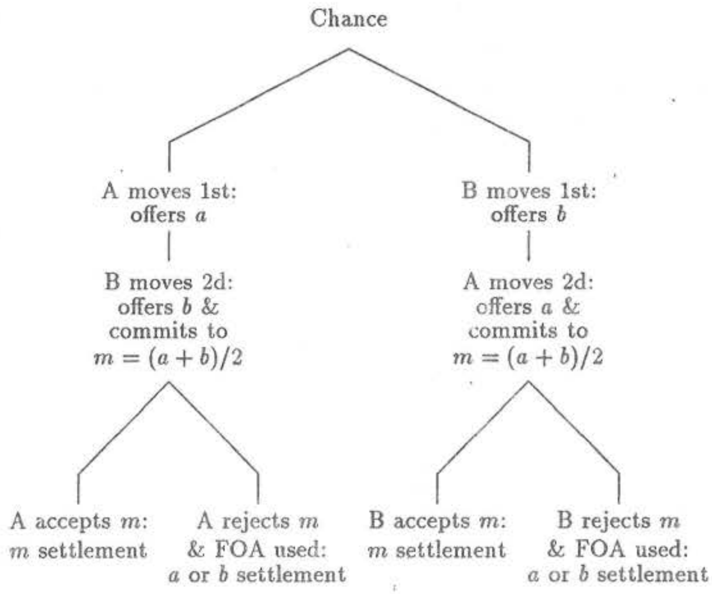

Fig. 1

Assuming that B will accept or reject as above in the final stage, we can determine A's best response in the previous stage to B's original offer of b. For fixed b, the payoff for A's counteroffer of a is

$$g(a,b) = \begin{cases} m & \text{if} \quad m \geq 0 \\ b - (b-a)F(m) & \text{if} \quad m < 0. \end{cases}$$

We next determine what offers of B, and counteroffers of A, constitute a Nash equilibrium.

THEOREM 6 *If both players are risk neutral, and a Nash equilibrium exists for FOA, then $(-b, b)$ is a Nash equilibrium under sequential FOA, for any offer b greater than or equal to B's pure-strategy Nash equilibrium under FOA. No other Nash equilibria exist.*

Proof: Without loss of generality, assume that $f(0) = 1$, so that the pure-strategy Nash equilibrium under FOA is $(-1/2, 1/2)$ (Brams and Merrill, 1983, p. 930). Given b, for any $a \geq -b$, $g(a, b) = m \geq 0$, so A's best offer in this range is $a = -b$, which ensures A a payoff of 0. For the range $a \leq -b$, consider two cases. First, suppose $b < 1/2$, i.e., less than the Nash equilibrium offer under FOA. If A counters with its Nash equilibrium offer of $a = -1/2$, then the payoff is negative (favorable to A) or at worst 0. (Computer calculation indicates that, for many distributions, A can do even better by choosing a value of a slightly less than $-1/2$.) Second, if $b \geq 1/2$, and $a \leq -b$, then the FOA payoff favors B, and A's best response is $a = -b$. To see this, set $a = -b - \delta$ for some $\delta \geq 0$. First note that inequality (4) in Brams and Merrill (1983, p. 929) implies the following inequality:

$$F(-\delta/2) = 1/2 - \int_{-\delta/2}^{0} f(t)dt \leq 1/2 - \frac{\delta/2}{1+\delta} = \frac{1}{2(1+\delta)}. \tag{9}$$

Thus, using (9) for the inequality below, we obtain

$$\begin{aligned} g(-b-\delta, b) &= b - (2b+\delta)F(-\delta/2) \\ &\geq b - \frac{2b+\delta}{2(1+\delta)} = \frac{b+\delta b - b - \delta/2}{1+\delta} \\ &= \delta(b-1/2)/(1+\delta) \geq 0 \end{aligned}$$

because $b \geq 1/2$. Thus, any more extreme deviation by A from $-b$ is favorable to B. In summary, if B chooses $b < 1/2$, B may lose (payoff < 0), but any offer $b \geq 1/2$ should induce a symmetric counteroffer from A for which the payoff is 0. Thus, B cannot do better by deviating from any given offer, b. Hence, any offer $b \geq 1/2$ is optimal for B; A's best counteroffer is $-b$. ■

We now determine similar optimal strategies for sequential CA.

Theorem 7 *Under sequential CA, if both players are risk neutral and the arbitrator's distribution is symmetric and strictly unimodal, B should accept if his or her offer is farther from 0 and reject if it is nearer.*

Proof: Given B's offer, b, and A's counteroffer, a, $E_B(Ac) = m$ and

$$E_B(Re) = (a-b)F(m) + \int_a^b F(x)dx.$$

Thus,

$$\begin{aligned}E_B(Ac) - E_B(Re) &= m + (b-a)F(m) - (b-a)\int_a^b F(x)dx/(b-a)\\ &= m + (b-a)\left[F(m) - \int_a^b F(x)dx/(b-a)\right] \quad \begin{array}{l} > 0 \text{ if } m > 0 \\ < 0 \text{ if } m < 0. \end{array}\end{aligned}$$

Thus, B accepts if $m > 0$ and rejects if $m < 0$ (and is indifferent if $m = 0$), exactly as under sequential FOA. ■

It follows that, given offers a and b, the expected payoff to the players is

$$g(a,b) = \begin{cases} m & \text{if} \quad m \geq 0 \\ (a-b)F(m) + \int_a^b F(x)dx & \text{if} \quad m < 0. \end{cases}$$

We next determine what values of a and b constitute a Nash equilibrium.

THEOREM 8 *Under sequential CA, if both players are risk neutral and f is symmetric and strictly unimodal, then $(-b, b)$ is a Nash equilibrium for any offer $b \geq 0$. No other Nash equilibria exist.*

PROOF: For any offer $b \geq 0$, A must counteroffer with $a = -b$. For if A moves closer to 0, the payoff m is favorable to B. If, on the other hand, A moves farther from 0, the payoff is determined by the rules of CA, which favor the party nearer 0, namely B.

Because B can thus expect a payoff of 0 for any non-negative offer, b, B has no motivation to deviate from any such offer. Hence, any non–negative offer is optimal for the first player, B; A's optimal response is a symmetric counteroffer. ■

If both sides are risk averse, the first player may choose the most conservative of its optimal strategies. Thus, under sequential FOA, B would choose $b = 1/2$ in order to minimize the uncertainties of a possible one-sided settlement; under sequential CA, B would choose $b = 0$. On the other hand, if the first player knows that its opponent is as risk averse as it is, it might well choose an extreme offer to put greater pressure on the opponent to compromise. It is somewhat paradoxical, perhaps, that a player might escalate the level of risk so as to steer an opponent away from a risky choice and toward the compromise m.

6. Which Procedure Is Best?

To recapitulate, we have analyzed two variations on FOA and CA that allow for the possibility of compromise at the mean of the two final offers. Theoretically, the possibility of compromise would seem more needed for FOA, because the equilibrium strategies under this procedure are — in terms of the arbitrator's distribution — generally two or more standard deviations apart (Brams and Merrill, 1983), whereas under CA there is convergence at the median (Brams and Merrill, 1986). In practice, however, the opportunity for compromise may be attractive under both procedures.

It is clearly so under modified FOA, because the divergent equilibrium final offers under this procedure are the same as under FOA. However, the possibility of a compromise at the mean might impel the disputants more often to invoke modified FOA than FOA. With the expectation that they will compromise in the end, why should they negotiate their differences beforehand?

Arguably, the mean is tantamount to a negotiated settlement, because the arbitrator never intervenes in the process. To be sure, it is the arbitrator's presence that induces the players to compromise — unless they both agree, the arbitrator will intervene.

If there is not compromise and the arbitrator intervenes, then one player will have reason to feel regret, because the arbitrator's FOA choice will be worse for one (and better for the other) than the mean. On the other hand, because the arbitrator's judgment is not revealed if the players agree to the compromise, the players do not know who would have won, so they can feel no regret.

To help preserve the integrity of the arbitrator's judgment under modified FOA, we recommend that his or her judgment be recorded before the players make their final offers. If the compromise is not chosen by the disputants, the disputants could rest assured that the arbitrator's decision was independent of the two final offers; the winning offer would simply be the offer closer to the arbitrator's judgment. Thereby, the possibly undue influence of the actual offers on the arbitrator's judgment is precluded.

Under modified FOA, the arbitrator's intervention probably would be quite rare. This possible advantage of modified FOA, however, is to be weighed against the likelihood that this procedure would be invoked more frequently than FOA — albeit less as a last-resort gamble and more to try to induce the compromise.

Modified FOA is not the only way to forge a compromise. Indeed, we think that sequential FOA would not only facilitate a compromise; it also better approximates a negotiated settlement, because it enables the players to receive more feedback and thereby better coordinate their offers.

This feedback is especially important in situations in which the arbitrator's distribution may not be common knowledge, or the players may disagree on the distribution. If this is the case, their divergent offers under modified FOA may not be symmetrical (with respect to a single distribution): one offer may be more extreme than the other.

If the player who makes the less extreme offer is sufficiently risk averse, he or she may still accept the compromise. Often, however, the dearth of information will be too great an obstacle for the players to overcome, resulting in asymmetric offers. As a consequence, one player might reject the compromise because it is not even-handed, whereas a more symmetric set of offers would have led to a compromise that both players would accept.

This problem is solved, in large part, by sequential FOA. Whether the first player's offer is its FOA equilibrium or a more extreme equilibrium offer, the second player can respond with a symmetric counteroffer on the other side of the arbitrator's distribution that makes compromise at least as appealing as FOA. If the first player is risk-averse — or more risk-averse than the second player — then the second player might even be able to shade its counteroffer so that compromise slightly favors it but, nevertheless, will be accepted by the first player.

This prerogative of the second player, however, does not necessarily give it an advantage under sequential FOA. True, the game is constant-sum, which in general never helps, and may hurt, the player who moves first. But this disadvantage for the first player is counterbalanced by the fact that the second player must make a commitment to the compromise, which the first player in turn may or may not accept.

The first player, in addition, has leeway in how extreme it wishes to make its offer. By contrast, the second player can do no better than match this initial offer with a symmetric counteroffer if the players are risk neutral. Paradoxically, a risk-averse player may choose to make a very extreme initial offer, which an equally risk averse player will match, simply to try to promote compromise. There is nothing like two preposterous offers, with neither player thinking it can risk the other player's offer being chosen, to make the compromise all but irresistable.

Because the Nash equilibrium strategies under CA are convergent offers at the median (if the arbitrator's distribution is symmetric and strictly unimodal), one might legitimately ask why the possibility of compromise needs to be built into CA. Even if the two offers are not quite convergent but almost so, the two sides could be given an opportunity in a penultimate stage — after their offers are announced but before the arbitrator's judgment is revealed — to resolve their (small) differences (Brams and Merrill, 1986).

In this manner CA, instead of allowing for only one compromise (i.e., the mean of the offers) in its modified or sequential incarnations, would allow the players to negotiate any settlement at a penultimate stage. By giving them this opportunity, the sting of implementing the arbitrator's (possibly) more extreme choice — if their negotiations fail — would be alleviated.

It is difficult to say whether CA with no specific compromise, or modified or sequential CA with a specific compromise, would be better at facilitating negotiated settlements. What we can say, though, is that because the equilibrium offers under modified CA will be less extreme than under modified FOA, modified FOA is probably better at inducing the mean as a compromise.

For the same reasons, sequential FOA is probably better than sequential CA at inducing the mean as a compromise. Although the players have equilibrium strategies of being as extreme as they like under both sequential procedures, convergent strategies at the median are in equilibrium under sequential CA but not under sequential FOA. It seems likely, therefore, that sequential FOA will in practice lead to more divergent equilibrium offers than sequential CA.

For reasons already given, we believe that divergent offers are probably beneficial in promoting the mean as a compromise settlement. Coupled with the fact that sequential FOA better allows the players to coordinate their offers than does modified FOA, we see it as a superior procedure.

That FOA is now widely used should make a variation on it more palatable than adopting an entirely new procedure like CA or its variations. Of course, sequential FOA is a considerable extension of FOA. The fact that it involves (1) an initial offer, (2) a counteroffer with a commitment to accept the mean as a compromise, and (3) acceptance or rejection of the mean by the initial offerer will be harder for practitioners to grasp.

Nevertheless, we think they will quickly see that it enables them to learn from each other's prior decisions and thereby better coordinate their subsequent choices than FOA or modified FOA. Given that offers have been more or less symmetric on the first two rounds, if the first player refuses to compromise on the final round, he or she may face severe disapprobation if the FOA outcome turns out to be worse than the compromise — making this player think twice about turning down the compromise. It therefore seems likely that compromise will emerge as the prominent solution under sequential FOA, especially if the offers are relatively extreme and the players are risk averse.

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