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Game Theoretic Analysis of Bargaining Models

by

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Bargaining models define a special class of strategic games. In the first part we investigate bargaining games with unique and multiple equilibria as well as with and without incomplete information. It is argued that the more natural bargaining models are typically games with incomplete information and multiple equilibria. The second part of our paper is devoted to a special bargaining problem, namely how to sell a lemon, i.e. a used car of bad quality. We analyse the game of asymmetric incomplete information by using the concept of uniformly perfect equilibria in pure and mixed strategies. It is shown that the multiplicity of uniformly perfect equilibria is a generic phenomenon.

Part 1. Game theoretic analysis of bargaining models

1. Introduction

Bargaining theory can be partitioned into a normative and a descriptive discipline. Descriptive bargaining theories are designed to explain experimentally observed or field data. Here we will consider such attempts only occasionally and concentrate on **normative bargaining theory** which tries to determine the individually rational bargaining behavior. Now every bargaining situation involves at least two strategically interacting bargaining parties with at least partially divergent interests, i.e. bargaining problems form a special class of **strategic game**. Since game theory tries to define the individually rational decision behavior in games, we mainly apply **game theory** to games which describe essential aspects of bargaining problems.

In former times bargaining models were often represented by their **characteristic function** which assigns to every subgroup of bargaining parties the set of feasible payoff vectors which this subgroup can guarantee its members by an appropriate binding agreement. This assumes that every bargaining party can commit itself to every possible future behavior, i.e. unilateral deviations from an agreement can be excluded.

Bargaining situations where all interacting parties have unrestricted commitment power form a rather special class of bargaining problems. In general, some moves can be decided in advance and others not. Furthermore, it may very well depend on the institutional aspects of a certain party, e.g. it may be an organization or an individual, whether binding commitments are possible or not. This is the reason why we restrict ourselves to **strategic bargaining games** which do not require unrestricted commitment power but allow for all possible degrees of self commitment, which furthermore, may vary between bargaining parties. The analysis of strategic bargaining games is sometimes called the noncooperative approach to bargaining. Here 'noncooperative' simply means that all strategic moves, regardless whether they are cooperative acts or not, are individual decisions.

From a game theoretic point of view there is no need to develop a special bargaining theory, i.e. a theory of individually rational decision behavior in bargaining games. On the contrary, every ad hoc–theory for a special class of games may rely on rationality requirements which does not make sense for other classes of games. Denying the need to develop a special theory for strategic bargaining games means, of course, that bargaining theory has essentially two major tasks, to model bargaining situations as strategic games and to apply game theory in order to derive the individually rational bargaining behavior. A strategic bargaining game should try to capture all strategically relevant aspects of the actual situation. Whereas purely analytic studies of classes of bargaining games try to incorporate such aspects in the simplest possible way, numerically specified bargaining games can be of a more complex nature. Due to recent developments in computer software one can hope to study also more complex bargaining games analytically.

The following three chapters of the first part of our paper study different classes of bargaining games. Bargaining games with unique subgame perfect equilibria, which received a lot of attention in the literature (see, for instance, Osborne and Rubinstein, 1990, Part I, Bester, 1989 and Rubinstein, 1982), are discussed in Section 2 whereas in Section 3 it is argued and demonstrated that bargaining games with more than one equilibrium may appear more natural. In Section 4 we consider bargaining games with **incomplete or private information** where at least one bargaining party is not sure about the others' types, i.e. one has to bargain with somebody without knowing for sure what he is trying to achieve, which strategies he can choose etc.

In the second part of our paper we describe a situation where an owner of a used car which may be of poor quality, i.e. **a lemon**, is bargaining with somebody interested in buying this car and where the quality of this car is only known to its owner. Due to the sequential decision process for this bargaining game with private information one can distinguish **pooling equilibria**, meaning that the owner of a good car and the one of a bad car behave in the same way, and **signaling equilibria** with decisions revealing the true quality of the car. In our Conclusions we summarize our results and indicate some of its limitations.

2. Bargaining models with a unique equilibrium

In reality one cannot vary a decision variable continuously. Furthermore, all action spaces are bounded. In a game a player can therefore choose only among finitely many possibilities. As a consequence a bargaining game can be respresented by a **game tree**, i.e. a connected graph consisting of nodes and branches without loops and with a special node o, called the origine of the game tree. To illustrate this we consider a simple bargaining game with two bargaining parties or players 1 and 2 who can distribute a positive monetary amount c among themselves. The rules are those of **ultimatum bargaining**, i.e. one party, e.g. player 1, can propose how to divide c whereas the other party, player 2, can either accept this proposal or not. An obvious discrete version for such a situation will allow for all possible demands d_1 by player 1 which satisfy

$$d_1 = mg \text{ with } m \in \mathbb{N} \text{ and } 0 \leq d_1 \leq c, \tag{2.1}$$

i.e. all possible demands d_1 are integer multiples of a positive smallest money unit g. For our graphical illustration we assume that d_1 can assume only three values, namely $d_1 = g$, $d_1 = c/2$, and $d_1 = c - g$, where it is implicitly assumed that c is an even integer multiple of g.

If player 2 accepts player 1's proposal, player 1 receives d_1 and player 2 gets the residual amount $c - d_1$, i.e. $(d_1, c - d_1)$ is the payoff vector for the play starting with the proposal d_1 which then is accepted by player 2. In case that 2 rejects player 1's demand for d_1, both players receive 0–payoffs since they did not agree on how to divide c. Thus player 2 is confronted with the ultimatum either to accept the proposal $(d_1, c - d_1)$ or to choose conflict with 0-payoffs.

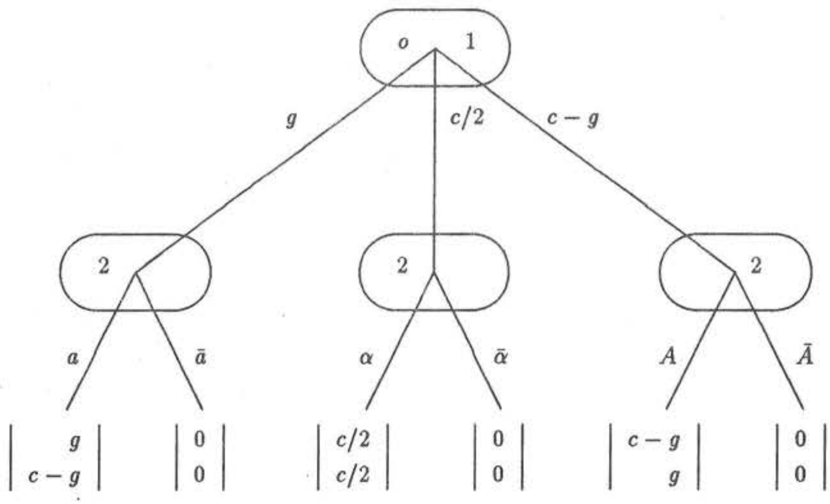

Figure 2.1. An ultimatum bargaining game with three possible demands

For the special case of just three possible demands $d_1 = g$, $c/2$, and $c - g$ the game is graphically visualized in Figure 2.1. A play starts at the top decision node o, which is the origine of the game tree, with player 1's move $d_1 = g$, $c/2$, or $c - g$. Knowing d_1, what is graphically illustrated by encircling every decision node of player 2 following d_1, player 2 can accept d_1, these are the moves a, α, or A, or reject d_1, these are the moves $\bar{a}$, $\bar{\alpha}$, or $\bar{A}$. Since a **play** is a sequence of moves from o to an endpoint (a node without downward pointing branches), **evalutation of plays** can be specified by attaching a payoff vector to every endpoint, i.e. to every lowest node of the game tree. The upper (lower) component of a payoff vector is player 1's (2's) payoff for this respective play.

A strategy s_i of player i must define a move for every information set of player i, i.e. in the graphical illustration for every set of encircled decision nodes of player i. For an ultimatum bargaining game s_1 is simply player 1's demand d_1 whereas a strategy s_2 of player 2 has to state for all possible demands d_1 whether 2 will accept them or not. In the game of Figure 2.1 player 1 has three strategies s_1, whereas player 2 has $8 = 2^3$ possible strategies s_2.

A strategy vector $s = (s_1, \ldots, s_n)$ of an n–person game is an **equilibrium** if no player i can gain by unilaterally deviating from s, i.e. every equilibrium strategy s_i is a best reply to the equilibrium strategies s_j ot the other players $j \neq i$. Since every non–equilibrium expectation $s = (s_1, \ldots, s_n)$ by its very definition induces at least one player to deviate from s, only équilibria quality as selfstabilizing behavioral expectations. The solution of a bargaining game therefore has to be an equilibrium.

A **subgame** of a game is a subtree of the game tree which is informatically closed, i.e. every information set, which contains a decision node of this subtree, contains only such decision nodes. Since a strategy s_i specifies a move for all information sets of player i, a strategy s_i induces a strategy s_i' for every subgame T' of a given bargaining game. Similarly, a strategy vector s of T **induces** a strategy vector s' for T'. An equilibrium $s = (s_n, \ldots,, s_n)$ of the n–person game T is called **subgame perfect** if for all subgames T' of T the induced strategy combination s' is an equilibrium of T'. If a subgame T' would be reached, all players will want to rely on an equilibrium s' of T'. Subgame perfectness thus requires behavioral plans for subgames which are self–enforcing. Every non–subgame perfect equilibrium $s = (s_1, \ldots, s_n)$ is unreliable since there exists at least one subgame T' and at least one player who will want to deviate from the induced strategy combination s' when T' is really reached in the course of the game. If a bargaining game has only one subgame perfect equilibrium we will say that its **equilibrium solution** is unique.

In the game of Figure 2.1 there exists just one subgame perfect equilibrium, namely the strategy vector

$$(c-g,(a,\alpha,A)), \tag{2.2}$$

according to which player 2 accepts all ultimatum offers and player 1 asks for $d_1 = c - g$. A non-subgame perfect equilibrium would be, for instance, the strategy combination $(g,(a,\bar{\alpha},\bar{A}))$ according to which player 2 accepts only the actual demand $d_1 = g$ by player 1 and threatens not to accept any other proposal. Obviously, such a behavior can hardly be qualified as rational since non-acceptance of $d_1 = c/2$ means to choose a payoff of 0 instead of the positive amount $c/2$.

If also the demand $d_1 = c$ is possible there is a minor ambiguity since player 2 is indifferent between accepting and rejecting $d_1 = c$. If one assumes that all best replies are chosen with the same probability, the unique solution is the strategy vector

$$\begin{aligned}(c-g, \quad & 2 \text{ accepts all } d_1 < c \text{ and rejects } d_1 = c \\ & \qquad\qquad \text{with probability } \tfrac{1}{2}).\end{aligned} \tag{2.3}$$

Another way to justify $d_1 = c - g$ as the only solution demand is to rely on lexicographic preferences of the form that player 2 primarily cares for his monetary payoff and is only secondarily interested in punishing player 2 in case of a greedy demand d_1 .

Now one might object that ultimatum bargaining is a very extreme form of bargaining assuming either that only one party can commit itself to a certain demand or that one party can precede the other in making its commitment. In our view, this does not mean that ultimatum bargaining is practically irrelevant. One often faces situations where one can either accept a given proposal or have no agreement at all. Many shops treat their customers this way, usually we just do not consider such situations as bargaining games. One can easily generalize the ultimatum bargaining game to the well–known **alternating bid-bargaining games** studied by Ståhl (1972), Krelle (1975), and Rubinstein (1982). In alternating bid–bargaining first player 1 determines his demand d_1

with $0 \leq d_1 \leq c_1$ which then player 2 can accept or reject. If he accepts, the game ends with the payoff result $(d_1, c_1 - d_1)$. Here c_1 is the monetary reward which can be distributed if an agreement is reached immediately. If not, the second round of bargaining starts with player 2's demand d_2 with $0 \leq d_2 \leq c_2$ where c_2 is the monetary reward which can be distributed in round 2. Knowing d_2 player 1 can accept this proposal what yields the payoff vector $(c_2 - d_2, d_2)$ or reject it. In general, player 1 determines d_t if t is odd whereas player 2 is the demanding player in all even periods t.

If there exists a finite final round T for reaching an agreement, the rules for period T are exactly the ones of ultimatum bargaining since non–acceptance of d_T means that no agreement can be reached. Assume that the monetary amounts c_t are a decreasing function of t, i.e.

$$c_1 > c_2 > \ldots > c_{T-1} > c_T. \tag{2.4}$$

Since

$$(d_T = c_T - g, \quad 2 \text{ accepts } d_T < c_T \text{ and rejects } d_T = c_T \text{ with probability } \tfrac{1}{2}) \tag{2.5}$$

is the solution for every subgame starting in round T, the demanding player in period $T-1$ must leave at least $c_T - g$ for the other party, i.e. his optimal demand is

$$d_{T-1} = c_{T-1} - c_T \tag{2.6}$$

where we again implicitly assume that a player would use all optimal options with equal probabilities. Proceeding in this way yields

$$d_{T-\tau} = \begin{cases} \sum_{k=0}^{\tau}(-1)^{\tau-k} c_{T-k} - g & \text{if } \tau \text{ is even} \\ \sum_{k=0}^{\tau}(-1)^{\tau-k} c_{T-k} & \text{if } \tau \text{ is odd} \end{cases} \tag{2.7}$$

or

$$d_{T-\tau} = \sum_{k=0}^{\tau}(-1)^{\tau-k} c_{T-k} - (1 - (-1)^{\tau+1})\frac{g}{2}. \tag{2.7'}$$

Furthermore, any demand d_t satisfying

$$c_t - d_t > d_{t+1} \tag{2.8}$$

will be accepted whereas all lower demands will be rejected (in case of $c_t - d_t = d_{t+1}$ with probability 1/2). This shows that all alternating bid–bargaining games with $T < \infty$ and shrinking 'cakes' c_t according to condition (2.4) have an essentially unique equilibrium solution.

In the special case with

$$c_t = \delta^{t-1} c_1 \quad \text{with} \quad c_1 > 0 \quad \text{and} \quad 0 < \delta < 1, \tag{2.9}$$

which can be justified as discounting with the same constant discount factor δ for both players, the solution payoff vector $(d_1, c_1 - d_1)$ according to (2.7') is determined by

$$d_1 = c_1 \sum_{k=0}^{T-1} (-1)^{\tau-k} \delta^{T-1-k} - (1 - (-1)^{\tau+1}) \frac{g}{2}. \tag{2.10}$$

Neglecting the second term on the right hand–side of (2.10), which vanishes for $g \to 0$, one derives the solution share d_1/c_1 of player 1 as

$$d_1/c_1 = \sum_{k=0}^{T-1} (-1)^{T-1-k} \delta^{T-1-k}, \tag{2.11}$$

respectively as

$$d_1/c_1 = \frac{1 - (-1)^T \delta^T}{1 + \delta}. \tag{2.11'}$$

It is interesting to explore the limit of d_1/c_1 for $\delta \to 1$ and $\delta \to 0$. Because of

$$\lim_{\delta \to 1} d_1/c_1 = \begin{cases} 1 & \text{if T is odd} \\ 0 & \text{if T is even} \end{cases} \tag{2.12}$$

alternating bid–bargaining with a finite horizon implies the same extreme distribution as ultimatum bargaining if both players are very patient. The party, which is the last one to propose, can exploit its ultimatum bargaining power. If both parties are completely impatient in the sense of $\delta \to 0$, the first proposer has all the bargaining power as it is clearly demonstrated by

$$\lim_{\delta \to 0} d_1/c_1 = 1. \tag{2.13}$$

One can also approach the infinite time horizon. Since for $\delta < 1$

$$\lim_{T\to\infty} d_1/c_1 = \frac{1}{1+\delta}, \tag{2.14}$$

both players receive nearly the same share if δ is close to 1. For $\delta < 1$ the first proposer, player 1, gets more than player 2. Obviously, this analysis can be easily generalized to situations with different discount factors δ_1 and δ_2 satisfying $0 < \delta_1, \delta_2 < 1$. In case of $T = \infty$ one cannot solve alternating bid–bargaining models by backward induction as we have demonstrated it above for the case of $T < \infty$ and shrinking 'cakes' c_t. Furthermore, one has to define payoff vectors for all plays which do not lead to an agreement after a finite number of proposals. It seems natural (see Rubinstein, 1982) to assign the vector (0,0) of conflict payoffs to all such bargaining plays. In his well-known alternating bid–bargaining model with $T = \infty$ Rubinstein (1982) assumes that there is a constant amount to be divided but that both players have to discount their monetary incomes if an agreement is not reached immediately.

Let δ_1 and δ_2 with $0 < \delta_1$, $\delta_2 < 1$ denote player 1's, respectively 2's, discount factor. Let, furthermore, d_t denote the relative share of the constant amount which the demanding player wants to get in round t. With the help of this notation the payoff of player $i = 1, 2$ can be written as

$$u_i = \delta_i^{t-1} d_t \tag{2.15}$$

if i is the demanding player in period t and if this demand is the first accepted one and

$$u_i = \delta_i^{t-1}(c_t - d_t) \tag{2.16}$$

if player i is the accepting player in t and if d_t is the first accepted demand. Here we have normalized the payoffs or utilities by setting $u_i = 1$ if the whole amount would be given immediately to player $i = 1, 2$. Unlike in our previous version of ultimatum bargaining Rubinstein relies on continuous strategy sets, i.e. the possible demands are all real numbers $d_t \in [0, 1]$ and assumes that among two best replies a player will prefer the one yielding an earlier agreement.

Let player i be the demanding player in period $t, t+2, \ldots$ and assume that the play has reached period t. Obviously, player i will either want to make a proposal d_t which will be accepted by the other player j or plans to accept player $j's$ next proposal d_{t+1} since in period $t+2$ he faces the same situation as in period t except for the fact that the shares of the constant monetary amount have to be discounted. Observe that such a discounting is nothing else than a renormalisation of payoffs which should not affect the solution of a game. But if player i wants to accept d_{t+1} he might as well choose a proposal d_t which makes player j indifferent between receiving d_{t+1} in round $t+1$ and $c_t - d_t$ in round t, i.e.

$$1 - d_t = \delta_j d_{t+1}. \tag{2.17}$$

Similarly, player j can demand in period $t+1$ a share d_{t+1} which makes player i indifferent between his own demand d_{t+2} in $t+2$ and getting the residual share $1 - d_{t+1}$ in period $t+1$, i.e.

$$\delta_i d_{t+2} = 1 - d_{t+1}. \tag{2.18}$$

Since the game situation in $t+2$ is strategically equivalent to the one in period t (discounting is just a positive affine transformation of utilities), the solution behavior should be stationary in the sense of

$$d_{t+2} = d_t \quad \text{for all periods} \quad t = 1, 2, \ldots \tag{2.19}$$

Condition (2.19) can be formally derived by imposing subgame perfect and consistent equilibria in the way it has been done by Güth, Leininger, and Stephan (1991). Consistency requires the same solution for strategically equivalent games. For the case at hand, for instance, all subgames starting in odd and all subgames starting in even periods are strategically equivalent. The initial demands in those subgames therefore have to be identical as required by (2.19).

We thus obtain two equations, namely (2.17) and

$$\delta_i d_t = 1 - d_{t+1} \tag{2.18'}$$

whose unique solution is given by

$$d_t^* = \frac{1 - \delta_j}{1 - \delta_i \delta_j} \quad \text{for} \quad t = 1, 2, \ldots \tag{2.20}$$

where i, respectively j, is the in period t demanding, respectively accepting player.

As shown by Rubinstein (1982) the alternating bid–bargaining game with $T = \infty$ and constant discount factors has a unique subgame perfect equilibrium whose demand behavior is described by equation (2.20) and which requires to accept only demands $d_t \leq d_t^*$ in round $t = 1, \ldots, T$. Here it is, of course, essential that Rubinstein assumes that all demanded shares d_t with $0 \leq d_t \leq 1$ are possible, i.e. that the monetary amount is completely divisible. As shown by van Damme, Selten, and Winter (1990) the alternating bid–bargaining model with realistic discount factors has quite a large set of subgame perfect equilibria even when the smallest positive money unit g is small compared to the amount which can be distributed.

Also for ultimatum bargaining games, i.e. in case of $T = 1$, Rubinstein's assumptions imply a unique solution, namely $d_1^* = c$ and acceptance of all demands d_1.

An advantage of the Rubinstein–model of alternating bids is that it can be approximated by solutions of games with $T < \infty$ which can be computed by backward induction. The infinite horizon game is thus a reasonable approximation for large, but finite horizon games. In our view, this is the most important reason for studying infinite horizon games. Furthermore, Rubinstein's analysis is certainly one of the most elegant and inspiring studies in bargaining theory.

If, however, the wide acceptance of the Rubinstein–model is mainly due to its unique subgame perfect equilibrium solution, this is a rather weak support since uniqueness of its equilibrium solution is purely a pathology of the highly unrealistic assumption that money is completely divisible. Furthermore, we have shown that there are lots of alternating bid–bargaining games with unique equilibrium solutions if money is completely divisible, e.g. all alternating bid bargaining games with shrinking cakes and $T < \infty$. It therefore seems justified to say that uniqueness of the equilibrium solution cannot be the decisive reason why the Rubinstein–model has received such a lot of attention in the bargaining literature.

Another reason for the prominence of the Rubinstein–model could be that alternating bids are seen as a typical aspect of real bargaining situations. Unfortunately, the solution play of the Rubinstein–model does not correspond to such an empirical support since the first proposal, the demand d_1^* according to equation (2.20), is already accepted. In our view, alternating moves are much more typical for bargaining situations with incomplete information where parties try to demonstrate their strength by risking conflict (see Güth and Selten, 1991, who show that one can signal bargaining strength by risking conflict in a bargaining game which allows for alternating concessions).

Another weakness of the Rubinstein–model is that its solution relies on a delicate choice between two indifferent options, a property which is typical for alternating bid–bargaining games with continuous action spaces. According to equations (2.17) and (2.18) both players are indifferent between accepting the other player's offer and waiting for the next round for which one expects acceptance of the own offer. Nevertheless, the Rubinstein–model assumes that both players choose the earlier agreement ,with probability 1 . Observe that this cannot be justified by a lexicographic preference with a secondary interest for short plays since time preferences are explicitly taken into account by the two discount factors. In other words, all the gains by reaching an agreement in period t instead of one in period $t+1$ go to the player who demands in period t. This shows that alternating bids are ultimatum games where ultimatum power is restricted to the gains of an earlier agreement.

To demonstrate that this restricted form of ultimatum power can be dramatic assume $\delta_1 = \delta_2 = \delta$ so that (2.20) implies

$$d_t^* = \frac{1}{1+\delta} \quad \text{for } t = 1, 2, \ldots \tag{2.21}$$

Clearly, $d_t^* \to 1$ for $\delta \to 0$, i.e. the demanding player receives nearly all the 'cake' if both players are very impatient. Actually, the situation of ultimatum bargaining, respectively $T = 1$, can be viewed as the limit case of the Rubinstein–model for $\delta_1 = \delta_2 = \delta \to 0$. Due to

$$\lim_{\delta_i \to 1} d_t^* = \lim_{\delta_i \to 1} \frac{1-\delta_j}{1-\delta_i\delta_j} = 1 \tag{2.22}$$

the extreme result $d_t^* = 1$ can be also approximated by making the demanding player very patient in the sense of $\delta_i \to 1$. It therefore seems justifed to say that the Rubinstein–model generalizes ultimatum bargaining games by allowing to vary ultimatum power continously.

Although we concentrate on individually rational bargaining behavior, we would like to mention the many experiments which have been performed with ultimatum bargaining games and alternating bid–bargaining models with various horizon parameters T and discount factors (see Güth and Tietz, 1990, for a rather recent survey). The experimental observations clearly indicate systematic deviations from the equilibrium solution according to monetary incentives. Accepting players, faced with greedy demands, are willing to sacrifice considerable amounts of money to punish the demanding player. There can be no doubt that bargainers are not only guided by monetary incentives, but also by **norms of distributive justice** which determine their aspirations and thereby their frustrations when these aspirations cannot be fulfilled due to a greedy demand.

We also should indicate that the bargaining games, discussed above, are probably the most well-known bargaining games with unique equilibrium solutions but by no means the only ones. Actually every non-degenerate bargaining game with perfect information (i.e. all information sets contain just one decision node) has a unique equilibrium solution since in a non-degenerate game no player will be indifferent between two different plays. To judge which bargaining model is most suitable to represent real bargaining situations would require a lot of field research to explore the rules of bargaining. In our view, different bargaining situations will often rely on different rules and we do not see any empirical support for the hypothesis that these rules determine bargaining games with unique equilibrium solutions.

3. Models with a multiplicity of equilibrium solution

The major weakness of most bargaining games with unique equilibrium solutions is the asymmetry of bargaining positions which is true for all bargaining games which have been explicitly taken into account in Section 2. We do not deny that bargaining situations in real life are rarely symmetric. But typically such asymmetries result from different evaluations of results, private information etc. and are not due to a bargaining procedure which, without any further justifcation, assigns more powerful bargaining positions to some parties and less powerful ones to others.

In our view, this weakness cannot be avoided by an initial chance move which implies the same chances for all parties to assume a certain position in a bargaining game. If this would be true, all injustice of the world could be justified by referring to an initial chance move implying equal a priori–expectations for all individuals in the society, i.e. a so-called veil of ignorance (see Harsanyi, 1955). As random dictatorship in social choice we do not view randomly determined asymmetric bargaining rules as an acceptable allocation procedure.

Furthermore, the equilibrium results of all bargaining games, explicitly considered in Section 2, will be usually **inefficient** if we apply them to situations where a proposal does not completely determine all the results but leaves some freedom of choice for the accepting player. If, for instance, ultimatum bargaining concerns bilateral trade, a proposal d_1 by player 1 may not be a vector of net trades, but simply a price vector p leaving player 2 the choice of the amounts he is willing to trade at this price vector p. In such a case player 1 will choose the price vector p^* which determines his optimal vector of net trades given that player 2 adjusts optimally to a given price vector.

In Figure 3.1 we have graphically illustrated this situation by the well-known trade box-diagramm for the special case of two commodities and two consumers, namely player 1 and 2. For players $j = 1,2$ and both commodities $i = 1,2$ the amount e_i^j is player j's endowment with commodity i. Thus $e^1 = (e_1^1, e_2^1)$, respectively $e^2 = (e_1^2, e_2^2)$, would be the allocation result with no trade. $x^{*2}(p)$ is player 2's offer curve assigning his optimal net trade vector $\overset{*2}{x}(p) - e^2$ to every price vector $p = (p_1, p_2)$. Anticipating that 2 will choose a net trade vector on his offer curve, player 1 will set the price vector p^* which determines the point $x^{*2}(p^*)$ on player 2's offer curve where his indifference curve J^1 is tangent to $x^{*2}(p)$. For all points $x^{*2}(p) \neq e^2$ this means that in $x^{*2}(p^*)$ the indifference curve J^1 of player 1 and the indifference curve of player 2 through $\overset{*2}{x}(p^*)$ intersect, i.e. the allocation result is inefficient.

In the following we will briefly sketch some bargaining models where at least the bargaining procedure as such assigns no special roles to certain bargaining parties. Typically such bargaining games will have many subgame perfect equilibria. It is therefore important to discuss game theoretic concepts which allow to select one of these equilibria as the only bargaining solution.

The most simple symmetric bargaining procedure is the one of **unanimity bargaining**. Assume again that a constant monetary reward c has to be allocated among n (≥ 2) bargaining parties or players $1, \ldots, n$. In unanimity bargaining

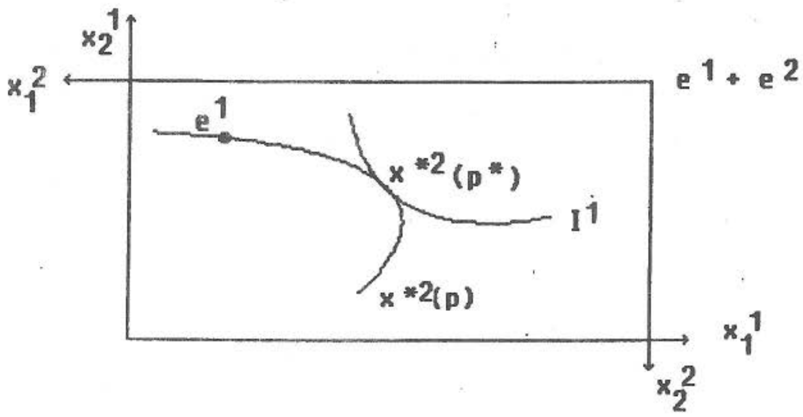

Figure 3.1. Bilateral trade when ultimatum bargaining power is restricted to choosing the price vector

every party $i = 1, \ldots, n$ independently commits itself to a demand d_i (≥ 0). If for the demand vector $d = (d_1, \ldots, d_n)$ with $d_i \geq 0$ for $i = 1, \ldots, n$ condition

$$\sum_{i=1}^{n} d_i > c \tag{3.1}$$

holds, we say that the demands are **inconsistent**. In case of inconsistent demands conflict results with 0–payoffs for all bargaining parties. If, however, demands are consistent, we have to distinguish two cases:

$$\sum_{i=1}^{n} d_i = c \tag{3.2}$$

and

$$\sum_{i=1}^{n} d_i < c \tag{3.3}$$

In case of (3.2) the whole 'cake' c is distributed and every player i receives his demand d_i. In case of (3.3) we speak of an **anti–conflict** since one can distribute more than demanded. One may simply assign the conflict payoffs to the case of anti–conflict or allow for another round of unanimity bargaining now for the residual share

$$c - \sum_{i=1}^{n} d_i. \tag{3.4}$$

Since the latter rule may lead to the absurd phenomenon of an infinite sequence of anti–conflicts, we assume that in case of (3.3) bargaining ends in conflict, i.e. there is only one round of unanimity bargaining. In equilibrium one will not observe an anti–conflict since every player j could increase his demand by (3.4).

Clearly, every demand vector d satisfying (3.2) is an equilibrium of the unanimity bargaining game. Since this game has no proper subgames, every such demand vector is, furthermore, subgame perfect, i.e. unanimity bargaining has a vast multiplicity of bargaining solutions if the smallest positive money unit g is small compared to c and even more if money is completely divisible.

In his famous article Nash (1950 and 1953) proposed an ingenious solution to resolve this troublesome non–uniqueness of the bargaining solution. The so–called **cooperative Nash–solution** maximizes the product of dividends, i.e. the agreement payoffs minus the conflict payoffs. Since in our example conflict payoffs are assumed to be 0, what can always be achieved by an appropriate renormalization of utilities, the dividend is just player i's demand d_i. Maximization of

$$L(d, \lambda) = \prod_{i=1}^{n} d_i - \lambda \left[\sum_{i=1}^{n} d_i - c \right] \tag{3.5}$$

with respect to d_j yields

$$\prod_{i=1, i \neq j}^{n} d_i = \lambda \ \text{ for } \ j = 1, \ldots, n. \tag{3.6}$$

Thus the cooperative Nash–solution is given by

$$d_j^* = c/n \ \text{ for } \ j = 1, \ldots, n. \tag{3.7}$$

Nash (1953) justifies this solution by a convincing set of axioms as well as by a constructive ad hoc–selection procedure for unanimity bargaining. Here we do not describe Nash's familiar arguments. There are also new axiomatic characterizations of the cooperative Nash–solution which support and clarify his bargaining solution (Lensberg, 1982).

Let us consider the same bargaining situation, i.e. there is a positive amount c of money to be distributed and all parties receive 0 in case of conflict, but with parties making concessions c_i instead of demands. More specifically assume that there are T (≥ 1) rounds of concession making according to the following rules: In every round $t = 1, \ldots, T$, if it is reached, all parties $i = 1, \ldots, n$ must independently choose to make a concession c_i^t meaning that it is willing to give up c_i^t of the cake. Since we assume that a concession is binding, it is reasonable to require

$$c_i^t \geq c_i^\tau \quad \text{for all} \quad t > \tau \quad \text{and} \quad i = 1, \ldots, n, \tag{3.8}$$

where we assume that all previous concessions are always known. Bargaining stops with an agreement in period t if

$$\sum_{i=1}^{n} (c - c_i^t) = nc - \sum_{i=1}^{n} c_i^t \leq c, \tag{3.9}$$

i.e. if the total concession $\sum_{i=1}^{n} c_i^t$ is large enough to satisfy all demands. In such a case party i receives $c - c_i^t$, i.e. what it has not conceded to the others. Observe that this implicitly assumes that the positive residual amount

$$\sum_{i=1}^{n} c_i^t - (n-1)c \tag{3.10}$$

in case of an anticonflict in period t is lost for the players.

If no agreement is reached in round t, i.e. if (3.9) is not satisfied, bargaining stops in round t with conflict if in round t no player i has made a further concession in the sense of $c_i^t > c_i^{t-1}$. For $t = 1$ the corresponding condition is $c_i^1 > 0$ for at least one party i. Otherwise bargaining continues with $t + 1$ as long as $t + 1 \leq T$. In round T conflict results if (3.9) is not satisfied.

It is assumed that there exists a positive smallest money unit g and that all concessions as well as c are integer multiples of g. We refer to this model as a **concession bargaining game** with T possible rounds of concession making.

Except for the different rules for the case of anticonflicts concession bargaining for $T = 1$ is identical to unanimity bargaining. Thus in the last round T the cooperative Nash–solution implies

$$c_j^T = \frac{n-1}{n} c, \tag{3.11}$$

if for all players i the following condition is fulfilled:

$$c_j^{T-1} \leq \frac{n-1}{n}c, \tag{3.12}$$

Condition (3.12) essentially states that player i has not yet precluded the cooperative Nash–solution demand c/n for unanimity bargaining by his previous concessions. Anticipating this result for all subgames starting in period T one can look at a subgame starting in period $T-1$ with concessions $c_1^{T-2}, \ldots, c_n^{T-2}$. According to (3.12) a player j receives a lower payoff in period T if his concession c_j^{T-1} is larger than $(n-1)c/n$. Consequently, player j will either want to make the minimal positive concession $c_j^{T-1} = c_j^{T-2} + g$ in order to prevent conflict or no concession, i.e. $c_j^{T-1} = c_j^{T-2}$, if he expects others to prevent conflict by positive concessions. Similar results are true for all earlier periods $t < T-1$. Thus concession bargaining leads to minimal concessions till the very last period T where the situation is similar to that one of unanimity bargaining.

For the limit $g \to 0$, i.e. when the positive smallest money unit becomes infinitely small, the final round T is reached with no concession so that the bargaining parties are dividing c in round T as in unanimity bargaining. This shoes that unanimity bargaining can be viewed as the limiting case of concession bargaining for all finite time horizons $T < \infty$ if money becomes more and more divisible. Thus unanimity bargaining is far more realistic than it might look at first sight.

Similar to the Rubinstein-model with alternating bids one can, of course, introduce time costs of bargaining, e.g. a certain positive utility K_i (> 0) which respresent player i's cost of delaying the agreement by one period. These bargaining costs are sunk if a player incurs the cost $(t-1)K_i$ of bargaining for t periods even when bargaining ends in period $t = 2, \ldots, T$ with no agreement. A less realistic assumption would be that the cost $(t-1)K_i$ of bargaining for t periods are only due if in period t an agreement is reached. In the latter case it is possible that in later rounds the bargaining parties have lost all their interest in reaching an agreement. If, for instance,

$$(t-1)\sum_{i=1}^{n} K_i > c, \tag{3.12}$$

bargaining costs would exceed what can be distributed.

If bargaining costs are sunk and the final round T is reached, the solution $c^T = (c_1^T, \ldots, c_n^T)$ of all subgames in round T is determined by (3.11) as for the case of no bargaining costs. But unlike in the situation of no such costs, a player j in period $T-1$ can be interested in achieving an early agreement. If, for instance,

$$0 < \sum_{i=1}^{n} (c - c_i^{T-1}) - c < K_j, \tag{3.13}$$

it would have been better for player j to concede the amount between the inequality signs of (3.13), i.e. to choose

$$c_j^{T-1} = \sum_{i=1, i \neq j}^{n} \left(c - c_i^{T-1}\right), \tag{3.14}$$

than to delay the agreement what causes the additional bargaining cost K_j.

4. Incomplete information

In bargaining situations we can distinguish three kinds of information deficits: **Stochastic uncertainty** results from stochastic events with given objective or subjective probabilities. Game theory takes care of stochastic uncertainty by relying on cardinal utility functions which allow to evaluate lotteries. **Strategic uncertainty** expresses that bargainers are not sure about the future behavior of their opponents. We resolve strategic uncertainty by applying game theoretic solution concepts as, for instance, the subgame perfect equilibrium in Section 1. **Incomplete information**, in general, means that at least one bargaining party is not sure about the bargaining rules as expressed by the game tree, respectively the extensive form. In his ingenious article Harsanyi (1967/8, Part I, II, III) showed that all information deficits about the rules of a game can be reinterpreted as information deficits concerning other players' utility functions. We refer to the expected utility or payoff functions of player i as to player i's possible **types**.

The essential trick to resolve incomplete information is to transform it into strategically equivalent stochastic uncertainty by assuming a fictitious initial chance move determining the types of all players (Harsanyi, 1967/8, Parts I, II, Ill). We want to illustrate this with the help of the simple concession bargaining game with $c = 1$, two players, i.e. $n = 2$, $T = 2$, and two possible concessions, namely 0 and $\frac{1}{2}$, for both parties. Incomplete information concerns the (sunk) cost of bargaining which both players expect to be either 0 or $\frac{1}{4}$. We denote by p the probability by which 2 expects $K_1 = 0$ and by q player 1's probability for $K_2 = 0$ where $0 < p, q < 1$. With the complementary probability a player expects bargaining cost of 1/4 for his opponent.

The game is graphically illustrated in Figure 4.1. A play starts at the origine o with the fictitious initial chance move determining the possible vectors (K_1, K_2) with $K_i \in \{0, \frac{1}{4}\}$ for $i \in \{1, 2\}$. The probabilities for the four chance moves are attached in brackets to the respective branches. According to the information sets, as graphically illustrated by encircling the decision nodes among which the deciding player cannot distinguish, player 1 knows only his own bargaining cost K_1 when choosing between his two possible concessions $c_1^1 = 0$ and $c_1^1 = 1/2$. Player 2 is neither aware of K_1, nor of c_1^1 when he decides between $c_2^1 = 0$ and $c_2^1 = 1/2$. This shows that both information deficits concerning the other's bargaining cost are well represented. Since, furthermore, player 1's conditional probability for $K_2 = 0$ is q regardless whether his own type is $K_1 = 0$ or $K_1 = 1/4$, player 1's beliefs concerning K_2 correspond to the probabilities of the fictitious chance move. A similar statement holds for player 2's beliefs about K_1.

Since the rules of the game in Figure 4.1 are assumed to be commonly known, i.e. the game is one with complete information, one can say that by introducing the fictitious initial chance move we have transformend incomplete intormation, i.e. informations deficits concerning the rules of bargaining, into strategically equivalent stochastic uncertainty resulting from partially unobservable chance moves.

We call a strategy s_i of player i **dominated** if player i has another strategy $\hat{s}_i$ which for all possible behavioral constellations of i's co–players is never worse and at least for one such behavioral constellation better; i.e. whatever his coplayers do, player i will never prefer s_i to $\hat{s}_i$,whereas the opposite statement is wrong. **Repeated elimination of dominated strategies** requires to eliminate simultaneously for all players all dominated strategies. Since in the reduced game new strategies can be dominated, this step may have to repeated. At least in bargaining games where all parties have only finitely many strategies, this procedure must stop after finitely many steps with a reduced game in which no player has a dominated strategy. Observe that an equilibrium point of the reduced game is also one in the original game since in the original game there can be no better reply to the strategy combinations of the reduced game than in the reduced game itself.

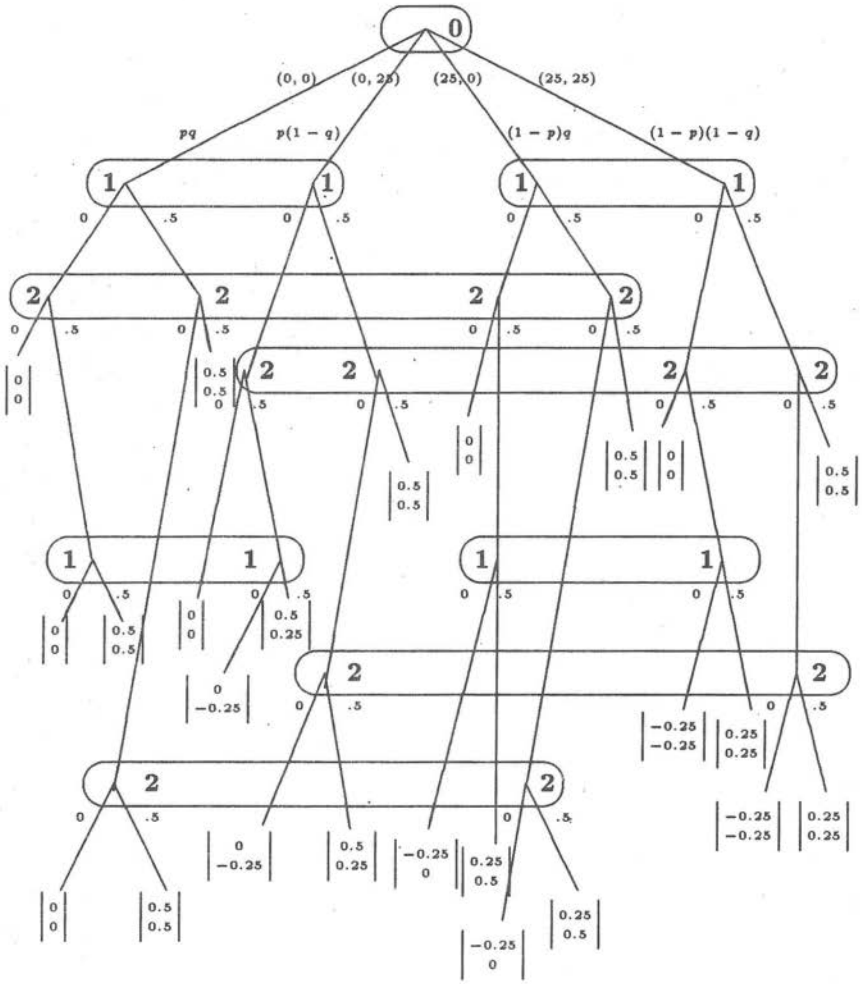

Figure 4.1. A simple concession bargaining game with incomplete information about bargaining costs

It is intuitively clear why players want to avoid dominated strategies. If there is the slighest doubt whether a given strategy of another player will be chosen or not, one can simply avoid the risk of making a false decision by relying on undominated strategies. Repeated elimination of dominated strategies can be applied by repeating this argument where it is, of course, essential that all players rely on the same reasoning process. One might object that doubts about the decisions of others should be reflected formally, e.g. by a bargaining model where one explicitly takes into account that a strategy may be chosen by small, but positive mistake probabilities (Selten, 1975). For the game of Figure 4.1 the result is the same regardless whether one assumes that every move must be chosen with a small positive mistake probability or not. We therefore rely on repeated elimination of dominated strategies, as described above, when solving the bargaining game of Figure 4.1.

To simplify our terminology we say that a player has conceded if he has chosen the concession $1/2$ instead of 0. The lower information sets of a player are reached if his opponent conceded in round 1 but if he himself did not. Obviously, both players will want to concede in round 2 after such a unilateral concession by their opponent in round 1. Eliminating all strategies which prescribe not to concede in round 2 yields a reduced game in which not conceding (in round 1) is again dominated. Thus repeated elimination of dominated strategies implies that both players always concede, i.e. they reach an agreement to split the 'cake' $c = 1$ evenly already in round 1 regardless of the type constellation (K_1, K_2) chosen by the initial fictitious chance move.

The bargaining game of Figure 4.1 is very simple and should only demonstrate how one can graphically visualize bargaining games with incomplete information. Observe that player i with given bargaining cost $K_i = 0$ has to determine also how he would decide as the type $K_i = 1/4$ which has coplayer expects. This is due to the fictitious initial chance move which can select all expected type constellations so that, when solving the game, it does not matter which type constellation is the true one. Only when deriving the actual solution play one will rely on the type constellation which is actually present.

More complex bargaining games with incomplete information usually cannot be solved by repeated elimination of dominated strategies. Typically the reduced game after applying this procedure still contains a multiplicity of equilibrium solutions. Games in which other players observe decisions of opponents, whose type they do not know, before deciding themselves are called **signaling games**. The equilibria of such games can be **signaling equilibria**, i.e. they reveal the types of players, or **pooling equilibria**, i.e. one cannot infer the true type of a player by observing his behavior. It is, of course, also possible to have partially revealing equilibria which signal some type characteristics and conceal others. Here we do not discuss such equilibria since they will be explicitly considered in the second part of our paper where we study in great detail a bargaining game with incomplete information. General eharacteristics of bargaining with incomplete information are also discussed by Güth and Selten (1991).

Part 2. How to sell a lemmon

Instead of continuing our rather abstract discussion of bargaining models we want to consider now a specific problem, namely how to sell a lemon. **A lemon** is a bad used car whose quality is known to the seller but not necessarily to the potential buyer (see Akerlof, 1970). As suggested by Akerlof (1970, Section III) many other markets are similar to markets of used cars whose true quality is known to the sellers, but not to the buyers. According to AKERLOF (1970) good and bad used cars have to be sold at the same price, i.e. he precludes signaling a car's quality. From this he concludes that owners of good used cars must keep their cars so that only lemons will be traded.

Akerlof (1970) does not specify the market decision process. He implicitly seems to rely on simultaneous bids of many buyers and many sellers like in one shot–double oral auctions. As for Akerlof (1970) asymmetry of information is the crucial aspect of our game model: Whereas a seller knows the true quality of his car, a buyer has only probabilistic beliefs concerning this car's quality. Compared to Akerlof (1970) we assume a sequential decision process allowing to signal a car's quality and bilateral bargaining between just one seller and one potential buyer of a used car.

The bargaining model is a signaling game since the potential buyer might infer from a previous decision of the car owner whether the car is good or bad. We therefore discuss the conditions for signaling, i.e. type revealing, and pooling, i.e. non–revealing, equilibria. To limit the set of equilibria we rely on the concept of **uniformly perfect equilibria** (see Harsanyi and Selten,1988, who implicitly define this concept, and Güth and van Damme, 1991a and b, for previous applications).

A game without inferior or duplicate strategies (see Harsanyi and Selten,

1988) is ϵ –uniformly perturbed if every choice has to be used with the same small positive minimum probability ϵ. An equilibrum q of the unperturbed game is called uniformly perfect if there exists a sequence $\epsilon^1, \epsilon^2, \ldots$ of mistake probabilities ϵ^k with $\epsilon^k \to 0$ for $k \to \infty$ and equilibria q^k of its ϵ^k–uniformly perturbed games with $q^k \to q$ for $k \to \infty$. In other words: A uniformly perfect equilibrium is stable against small uniform perturbations of the game.

In the following we will first describe the game model and then discuss its uniformly perfect signaling and pooling equilibria. In our Conclusions we will summarize our results and indicate the use of equilibrium selection by which one can derive an unique prediction if signaling and pooling equilibria coexist. We also will try to relate our analysis to the rather general discussion in the first part of our paper.

5. The game model

Let V, respectively V^* , be the owner of a bad, respectively good, used car which he wants to sell. His potential customer is denoted by K. Customer K expects the car to be a lemon, i.e. a bad quality car, with probability w and a good car with the complimentary probability $1 - w$ where $0 < w < 1$. These expectations are assumed to be known to the seller. Thus according to our discussion in Section 4 of the first part K's incomplete information about the car's quality is adequately taken into account by an fictitious initial chance move as graphically represented in Figure 5.1. Since only the owner learns about the car's quality, K's uncertainty is preserved, i.e. K's incomplete information is transformed into stochastic uncertainty resulting from a partially unobservable chance move.

After learning whether the car is a lemon, what happens with probability w, or a good car, an event with probability $1 - w$, the owner V, respectively V^*, of the car must decide between P and $\bar{P}$, if the car is a lemon, and between P^* and $\bar{P}^*$ if not. The decision P, respectively P^* , can be interpreted as a warranty certificate (which the owner must buy from a garage) or as a repair which is cheaper for V^* than for V. In case of $\bar{P}$, respectively $\bar{P}^*$, the car is offered for sale without such a repair. For the sake of simplicity and to allow an easy graphical illustration as by Figure 5.1 we assume that there are only two prices for which the car can be offered, namely H and L where $C > H > L > 1$.

Here C is the value of the good car for K and 1 the one for V^*. A lemon is assumed to have 0–value for V and K. The decision for the high price H is denoted by H for V after P, by h for V after $\bar{P}$ and by H^*, respectively h^*, for V^* after P^*, respectively $\bar{P}^*$. The low price moves are indicated by the letter 'n' in a similar way.

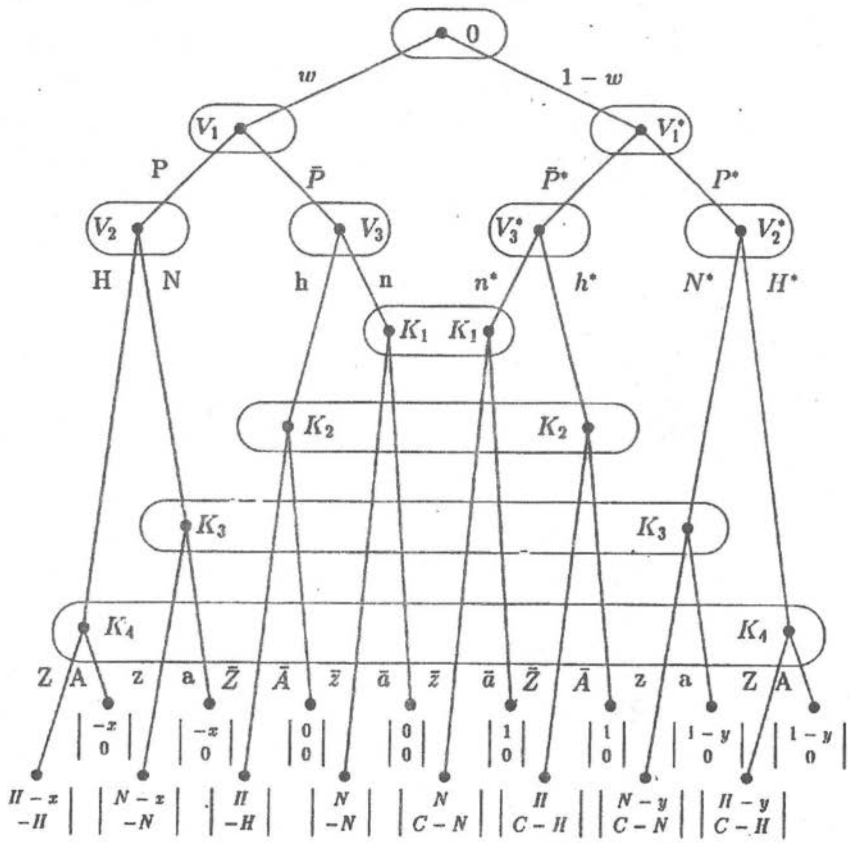

Figure 5.1. The bargaining model for selling a lemon

Customer K learns about the price and whether the car has been repaired or not, but not about its quality. Knowing this he must choose between buying (the decisions with the letter 'z') and not buying (denoted by 'a').

The payoff vectors are attached to the endpoints of the game tree. The upper component is the owner's payoff, the lower the one of K. Since we only give two payoffs, we rely on the usual convention that the payoff of a non–existing type is the same as the one of its existing type. For the case at hand this means that both, V and V^*, evaluate all endpoints in the same way, namely according to the upper component of the payoff vector.

The positive parameter x, respectively y ($< x$), is the cost which the repair implies for V, respectively V^*. All other parameters have been described above. If the car is sold, the owner receives the price, H or N, from which he must deduct his repair cost x or y in case of a repair. For K the payoff is C minus the price if the car is the good one and 0 minus the price if not. If the car is not sold, K's payoff is zero whereas the owner's payoff is determined by the value of the car, 0 or 1, minus the repair cost x or y in case of a repair.

We consider all games satisfying the following parameter restrictions:

$$\begin{aligned} &0 < w < 1,\ 0 < y < x < H,\ y < H - 1, \\ &1 < N < H < C,\ H \neq (1-w)C \neq N \end{aligned} \tag{5.1}$$

The assumptions $x < H$ and $y < H - 1$ are imposed to avoid dominated and thereby inferior strategies. By the conditions $H \neq (1-w)C \neq N$ we exclude border cases where in the unperturbed game the buyer could be indifferent between buying or not. Since $(1-w)C$ is the expected value of the car according to K's a priori expectations, we exclude prices which are as high as the expected value of the car for buyer K.

When deriving the solution it will be convenient to have an easy notation for the various agents of customer K, i.e. for customer K in his four different information sets of Figure 5.1. The potential buyer who has to decide between $\bar{z}$ and $\bar{a}$ is denoted by K_1, whereas K_2 decides between $\bar{Z}$ and $\bar{A}$, K_3 between z and k, and K_4 between Z and A. This shorthand is indicated in Figure 5.1 by giving the subscripts of K in brackets. In the same way we also distinguish the three agents of V and V^* in Figure 5.1.

6. Uniformly perfect equilibria in pure strategies

In the following ϵ is always a small positive number and a perturbed game always an ϵ–uniformly perturbed game of the game in Figure 5.1 which is the unperturbed game. According to our shorthand a mixed strategy vector q can be written as

$$q = (q_A, q_K)$$

with

$$q_A = (q_V, q_{V^*})$$
$$q_V = (q_{V_1}, q_{V_2}, q_{V_3})$$
$$q_{V^*} = (q_{V_1^*}, q_{V_2^*}, q_{V_3^*})$$

and

$$q_k = (q_{K_1}, q_{K_2}, q_{K_3}, q_{K_4}).$$

Since all choices are binary choices, mixed strategies are completely described by specifying one probability. So $q_{K_4}(Z) = 1 - \epsilon$ would mean, for instance, that K_4 accepts with maximal probability in the ϵ–uniformly perturbed game. We do not distinguish between the pure strategy Z and the mixed strategy $q_{K_4}(Z) = 1$. So q can also be a pure strategy vector.

We first concentrate on uniformly perfect equilibria of the unperturbed game in pure strategies. Although our game model is rather simple, it has $2^{10} = 1024$ possible pure strategy combinations. Fortunately, many of them can be exeluded with the help of the following propositions.

Proposition 6.1 *In a uniformly perfect equilibrium (in pure strategies) the choice of the low price by V_j and V_j^* and its acceptance implies non–acceptance of the high price, i.e. if V_3 and V_3^* ask for the low price and K_1 chooses $\bar{z}$, then K_2 has to choose $\bar{A}$; if V_2 and V_2^* ask for the low price and K_3 chooses z, then K_4 has to use A.*

Proof. Assume the contrary. Then the respective seller could increase his price from N to H and still sell his car. He thus has a profitable deviation. ■

Proposition 6.2 *In a uniformly perfect equilibrium (in pure strategies) the two seller agents V_j and V_j^* with $j = 2, 3$ choose the high price if the two buyer agents responding to them choose the same move, i.e. they both accept or reject.*

Proof. If both buyer agents accept with maximal probability $1-\epsilon$ in an ϵ–uniformly perturbed game the seller earns a revenue of $(1-\epsilon)H$ by choosing $H(H^*)$, whereas $N(N^*)$ yields only $(1-\epsilon)N$. Similarly, $H(H^*)$ yields ϵH instead of ϵN, implied by $N(N^*)$, if both buyer agents reject with maximal probability. ∎

If the two buyer agents responding to the high or low price of the same seller agents, decide differently in an uniformly perfect equilibrium in pure strategies, Proposition 6.1 implies that the low (high) price must be the accepted (rejected) one. The only possibilities for different decisions of two buyer agents responding to the same seller agent are therefore $(\bar{z}, \bar{A})$ and (z, A). This implies

Proposition 6.3 *In a uniformly perfect equilibrium (in pure strategies) V_j and V_j^* choose the low price if both buyer agents responding to them use different moves.*

Proof. Since only the low price will be accepted according to $(\bar{z},\bar{A})$ or (z,A), it is better to choose the low price. ∎

Proposition 6.4 *In a uniformly perfect pooling equilibrium (in pure strategies), i.e. V_j and V_j^* for $j = 1, 2, 3$ take the same choice, the buyer agents responding to them accept both prices if $(1-w)C > H$, reject both prices if $N > (1-w)C$, and accept (reject) the low (high) price if $H > (1-w)C > N$.*

Proof. Consider an ϵ–uniformly perturbed game and a buyer agent who has to decide between accepting or rejecting an offer. If both sellers V_j and V^* choose the same strategy his conditional probability that the car is a lemon is the a priori probability w regardless whether he faces the actually chosen price or not. This is a simple consequence of Bayes–rule and the fact that the actually chosen price is realised with maximal probability $1-\epsilon$ in every perturbed game. ∎

Proposition 6.5 *If V_2 and V_2^* as well as V_3 and V_3^* choose the same price in a uniformly perfect equilibrium in pure strategies, then V_1 must choose $\bar{P}$ and V_1^* his move $\bar{P}^*$.*

Proof. Because of Proposition 6.4 the buyer decisions depend on whether $(1-w)C > H$, or $H > (1-w)C > N$, or $N > (1-w)C$. In the first case both prices are accepted, i.e. all seller agents deciding on the price will choose the high price. Similarly, the low price is chosen in case of $H > (1-w)C > N$, whereas in case of $N > (1-w)C$ the high price is chosen since both prices are rejected. As neither the price proposals, nor buyer's behavior are influenced by the repair decision, it is obviously better not to invest in a costly repair. ∎

With the help of Proposition 6.1 to 6.5 we are now able to determine all uniformly perfect equilibria in pure strategies. In Table 6.1 we consider all 16 pure strategies s_K of the buyer and determine with the help of Proposition 6.1 to 6.5 together with some minor additional calculations the best reply q_A to s_K. There are only three strategies s_K where the strategy combination (q_A, s_K) with q_A being the best reply to s_K is consistent with Proposition 6.4, namely s_K^1, s_K^6, and s_K^{16}. In all other cases the buyer's behavior is not as predicted by Proposition 6.4 when we assume that the seller's behavior is the best reply to s_K: If the best reply q_A to s_K satisfies the assumptions of Proposition 6.4, the buyer's behavior should only depend on which of the three conditions in Proposition 6.4 is true. Nevertheless the strategies s_K^j with $j \neq 1, 6$, and 16 introduce additional dependencies although the best reply q_A to s_K^j satisfies the conditions of Proposition 6.4. This proves

Proposition 6.6 *If (q_A, s_K) is a uniformly perfect equilibrium (in pure strategies), then s_K is either the pure strategy s_K^1, s_K^2, or s_K^{16} described in Table 6.1.*

For $s_K^j = s_K^1$, s_K^6, and s_K^{16} not only q_A listed in Table 6.1 is a best reply to s_K^j, but it is also true that s_K^j is a best response to q_K. Since according to all three best responses q_A the seller always chooses the same price, the buyer's behavior is predicted by Proposition 6.4 which implies the behavior described by s_K^j. This shows that in every ϵ–uniformly perturbed game it is an equilibrium point to choose the moves in line s_K^1, s_K^6, or s_K^{16} of Table 6.1 with maximal probability. Consequently, the pure strategy vectors in line s_K^1, s_K^6, and s_K^{16} are the only uniformly perfect equilibria in pure strategies.

Theorem 6.6 *The game described by Figure 5.1 has only three uniformly perfect equilibria in pure strategies, namely the strategy vectors s^1, s^6, and s^{10}, listed in Table 6.2, which also describes the parameter region for which the corresponding equilibrium exists.*

It is an interesting result of Theorem 6.6 that there is a unique uniformly perfect equilibrium in pure strategies for all parameter constellations satisfying our

Name of pure strategy s_K	s_K				the best reply q_A to s_K						excluded by Proposition 6.4 or equilibrium in the range
					q_V			q_{V^*}			
	q_{K_1}	q_{K_2}	q_{K_3}	q_{K_4}	q_{V_1}	q_{V_2}	q_{V_3}	$q_{V_1^*}$	$q_{V_2^*}$	$q_{V_3^*}$	
s_K^1	$\bar{z}$	$\bar{Z}$	z	Z	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$(1-w)C > H$
s_K^2	$\bar{z}$	$\bar{Z}$	z	A	$\bar{P}$	N	h	$\bar{P}^*$	N^*	h^*	$\emptyset$
s_K^3	$\bar{z}$	$\bar{Z}$	a	Z	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\emptyset$
s_K^4	$\bar{z}$	$\bar{Z}$	a	A	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\emptyset$
s_K^5	$\bar{z}$	$\bar{A}$	z	Z	—	H	n	—	H^*	n^*	$\emptyset$
s_K^6	$\bar{z}$	$\bar{A}$	Z	A	$\bar{P}$	N	n	$\bar{P}^*$	N^*	n^*	$H > (1-w)C > N$
s_K^7	$\bar{z}$	$\bar{A}$	a	Z	—	H	n	—	H^*	n^*	$\emptyset$
s_K^8	$\bar{z}$	$\bar{A}$	a	A	$\bar{P}$	H	n	$\bar{P}^*$	H^*	n^*	$\emptyset$
s_K^9	$\bar{a}$	$\bar{Z}$	z	Z	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\emptyset$
s_K^{10}	$\bar{a}$	$\bar{Z}$	z	A	$\bar{P}$	N	h	$\bar{P}^*$	N^*	h^*	$\emptyset$
s_K^{11}	$\bar{a}$	$\bar{Z}$	a	Z	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\emptyset$
s_K^{12}	$\bar{a}$	$\bar{Z}$	a	A	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\emptyset$
s_K^{13}	$\bar{a}$	$\bar{A}$	z	Z	P	H	h	P^*	H^*	h^*	$\emptyset$
s_K^{14}	$\bar{a}$	$\bar{A}$	z	A	—	N	h	—	N^*	h^*	$\emptyset$
s_K^{15}	$\bar{a}$	$\bar{A}$	a	Z	P	H	h	P^*	H^*	h^*	$\emptyset$
s_K^{16}	$\bar{a}$	$\bar{A}$	a	A	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$N > (1-w)C$

Table 6.1. The 16 possible pure strategies s_K and the best replies q_A to s_K (a parameter condition gives the range where the strategy combination (q_A, s_K) is consistent with Proposition 6.4, the symbol '$\emptyset$' indicates that this range is empty; '—' indicates that the best reply is not uniquely determined by our parameter restrictions)

parameter restrictions (5.1). Although the model of Figure 5.1 is a signaling game, none of the three equilibria s^1, s^6 and s^{16} of Table 6.2 is a signaling equilibrium, i.e. the buyer never learns about the true type of the car before buying or not buying. This also explains why no seller type V_1 or V_1^* invests in a costly repair. A repair would only pay if it would signal a better quality of the car. The pooling equilibria s^1, s^6 and s'^{16} therefore exclude that a seller type invests into a costly repair.

Whether the car is sold or not and at which price is purely determined by its a priori–expected value $(1-w)C$ for the potential buyer. If this value exceeds

Name of uniformly perfect equilibrium in pure strategies	q_A						q_K				Parameter restrictions
	q_V			q_{V^*}							
	q_{V_1}	q_{V_2}	q_{V_3}	$q_{V_1^*}$	$q_{V_2^*}$	$q_{V_3^*}$	q_{K_1}	q_{K_2}	q_{K_3}	q_{K_4}	
s_1	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\bar{z}$	$\bar{Z}$	z	Z	$(1-w)C > H$
s_6	$\bar{P}$	N	n	$\bar{P}^*$	N^*	n^*	$\bar{z}$	$\bar{A}$	z	A	$H > (1-w)C > N$
s_{16}	$\bar{P}$	H	h	$\bar{P}^*$	H^*	h^*	$\bar{a}$	$\bar{A}$	a	A	$N > (1-w)C$

Table 6.2. The uniformly perfect equilibria in pure strategies of the game in Figure 5.1

the high price, this is the price at which it is bought. If this value is between both prices, it is sold at the low price. The car cannot be sold if its expected value $(1-w)C$ is even smaller than the low price N.

Although the car is never sold for $N > (1-w)C$, we nevertheless can predict the prices chosen by the various seller agents. In the same way we also know the prices and acceptance decisions of all seller, respectively buyer agents, even of those who never have to decide according to the actual play. Since all strategy vectors of Table 6.2 exclude a repair, the seller agents V_2 and V_2^* as well as the buyer agents K_3 and K_4 are never asked to move. Nevertheless we know how they would decide since we have solved the unperturbed game via its uniformly perturbed games in which all agents are asked to move with positive probability. This clearly indicates how the concept of uniformly perfect equilibria limits speculation on beliefs in unreached information sets and induces local rationality.

Although our model assumes only two possible sales prices, the high price H and the low price N, our results allow us to speculate what would happen if prices could be chosen more freely. Obviously in a uniformly perfect equilibrium in pure strategies the seller will not ask for prices exceeding the expected value $(1-w)C$ of the car. If there are many possible prices smaller than $(1-w)C$, he will ask for the highest one. Observe, furthermore, that a uniformly perfect equilibrium excludes positive sales prices smaller than 1. Since V^* would never sell at a price smaller than 1, the buyer would infer that the car is a lemon and therefore prefer not to buy. If there exists a positive smallest money unit g

and if all integer multiples of g can be chosen as prices, the highest price in the interval from 1 to $(1-w)C$ will be chosen. If this interval is empty, no trade will take place since the expected value of the car for the potential buyer K is smaller than 1, i.e. the value of the good car for the seller. This is the situation envisaged by Akerlof (1970) who predicted such a low posterior probability for a good quality car that no trade will take place.

7. On the possibility of signaling

According to Theorem 6.1 there can be no signaling of the car's true quality via the repair or price decision of the seller if one relies on the concept of uniformly perfect equilibria in pure strategies. In the following it will be shown that signaling becomes possible if we allow for mixed behavioral strategies, i.e. if every seller or buyer agent is allowed to randomize between his two possible moves.

Although we think that we have explored all possibilities for uniformly perfect equilibria, we will not try to prove that the three equilibria described in Theorem 6.1 and the three uniformly perfect equilibria, which will be discussed below and which all prescribe randomization for one seller and one buyer agent, are the only uniformly perfect equilibria of the game described in Figure 5.1. What we want to show is mainly that signaling the car's true quality is possible at all and that there exist generic regions of parameter constellations satisfying (5.1) for which uniformly perfect pooling and signaling equilibria coexist.

To have a simple notation π, respectively π^*, is the probability with which V_1, respectively V_1^*, uses his move P, respectively P^*. In an ϵ-uniformly perturbed game the behavioral strategy of V_1 and V_1^* are both indicated by π^ϵ. Since at most one of these seller's agents voluntarily randomizes, this notation cannot cause any confusion. Furthermore, μ_j denotes the probability by which the buyer agent K_j accepts the offer of the seller.

Proposition 7.1 *The strategy vector $\hat{q} = (\hat{q}_V, \hat{q}_{V^*}, \hat{q}_K)$ with*

$$\begin{aligned}
\hat{q}_V &= (\hat{\pi}, H, h),\\
\hat{q}_{V^*} &= (P^*, H^*, h^*),\\
\hat{q}_K &= (\bar{a}, \bar{A}, z, \mu_4)
\end{aligned}$$

where

$$\hat{\pi} = \frac{1-w}{w} \cdot \frac{C-H}{H}$$
$$\mu_4 = \frac{x}{H}$$

is a uniformly perfect equilibrium of the game in Figure 5.1 if the following three conditions are satisfied:

$$x > N$$
$$x > \frac{H}{H-1} y,$$

and

$$H > (1-w)C.$$

Proof. Whether for an agent one move is better than the other depends only on the payoff implications of his move given that his information set is actually reached. We will therefore analyse how his conditional payoff expectation, given that the information set is actually reached, is influenced by his own move. For the buyer agent K_4 we must show that his moves yield the same payoff. Let π^ϵ denote the probability for the move P by the seller agent V_1 for which K_4's conditional payoff expectation

$$\frac{-w\pi(1-\epsilon)H + (1-w)(1-\epsilon)^2(C-H)}{w\pi(1-\epsilon) + (1-w)(1-\epsilon)^2}$$

for acceptance is equal to 0 in an ϵ–uniformly perturbed game. For π^ϵ one obtains

$$\pi^\epsilon = \frac{1-w}{w} \cdot \frac{C-H}{H}(1-\epsilon).$$

Clearly, $\pi^\epsilon \rightarrow \hat{\pi} = \frac{1-w}{w} \cdot \frac{C-H}{H}$ for $\epsilon \rightarrow 0$. If V_1 chooses P with probability π^ϵ in the ϵ–uniformly perturbed game, K_4 can freely randomize. Similarly, let μ_4^ϵ be the probability for the move Z by the buyer agent K_4 which makes V_1 indifferent between his two possible moves. Since V_1's conditional payoff for P is

$$(1-\epsilon)\mu_4 H + \epsilon\,(1-\epsilon)N - x$$

whereas it is

$$(1-\epsilon)\,\epsilon\, H + \epsilon^2 N$$

for $\bar{P}$, one obtains

$$\mu_4^\epsilon = \frac{x + \epsilon\,[(1-\epsilon)H - (1-2\,\epsilon)N]}{(1-\epsilon)H}$$

and $\mu_4^\epsilon \to \mu_4 = \frac{x}{H}$ for $\epsilon \to 0$. For V_1^* the move P^* is better than $\bar{P}^*$ if

$$\epsilon\,[(1-\epsilon)N + \epsilon] + (1-\epsilon)\,[\mu_4^\epsilon H + 1 - \mu_4^\epsilon] - y >$$
$$\epsilon\,[\epsilon N + 1 - \epsilon] + (1-\epsilon)\,[\epsilon H + 1 - \epsilon]\,.$$

Since $\mu_4^\epsilon \to \mu_4$ for $\epsilon \to 0$, this condition is satisfied for ϵ (> 0) suffciently small if

$$\mu_4 H + 1 - \mu_4 > 1 + y$$

or

$$x\frac{H-1}{H} > y.$$

For V_2, respectively V_2^*, the high price is better than the low one if

$$\mu_4^\epsilon H > (1-\epsilon)N.$$

Since $\mu_4^\epsilon \to \mu_4$ for $\epsilon \to 0$ and $\mu_4 H > N$ is equivalent to $x > N$, the choice of H by V_2 is optimal if ϵ (> 0) is sufficiently small and if $x > N$. For V_3 the high price is better since both price proposals are accepted only with minimal probability ϵ. For the same reason also V_3^* prefers the high price.

It remains to prove that also the buyer agents want to use their moves prescribed by $\hat{q}_K$ with maximal probability. K_1's conditional payoff expectation for acceptance is negative if

$$(1-w)\,\epsilon\,(C-N) < w(1-\pi^\epsilon)N.$$

Since $\pi^\epsilon \to \hat{\pi}$ for $\epsilon \to 0$, this condition is fulfilled for ϵ (> 0) sufficiently small if $\hat{\pi} < 1$, i.e. $(1-w)C < H$. For K_2 acceptance is worse if

$$(1-w)\,\epsilon\,(C-H) < w(1-\pi^\epsilon)N,$$

i.e. also if $(1-w)C < H$. Acceptance by K_3 is optimal for $(1-w)(1-\epsilon)(C-N) > w\pi^{\epsilon}N$. Since $\pi^{\epsilon} \to \hat{\pi}$ for $\epsilon \to 0$ and

$$(1-w)(C-N) > w\hat{\pi}N$$

is equivalent to $H > N$, this condition is always fulfilled for ϵ (> 0) sufficiently small.

This shows that every ϵ–uniformly perturbed game with ϵ sufficiently small has an equilibrium point q^{ϵ} which requires all pure choices of the strategy vector $\hat{q}$ in Proposition 3.1 to be realised with maximal probability $1-\epsilon$ and whose components π^{ϵ} and μ_4^{ϵ} converge to $\hat{\pi}$, respectively μ_4, for $\epsilon \to 0$. Thus the strategy vector $\hat{q}$ is the limit of equilibria q^{ϵ} in ϵ–uniformly perturbed games for $\epsilon \to 0$ as required by the concept of uniformly perfect equilibria. ∎

Proposition 7.2 *The strategy vector $\tilde{q} = (\tilde{q}_V, \tilde{q}_{V^*}, \tilde{q}_K)$ with*

$$\begin{aligned}
\tilde{q}_V &= (\tilde{\pi}, N, h), \\
\tilde{q}_{V^*} &= (P^*, N^*, h^*), \\
\tilde{q}_K &= (\bar{a}, \bar{A}, \mu_3, A)
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\pi} &= \frac{1-w}{w} \cdot \frac{C-N}{N} \\
\mu_3 &= \frac{x}{N}
\end{aligned}$$

is a uniformly perfect equilibrium of the game in figure 5.1 if the following three conditions are satisfied:

$$\begin{aligned}
x &> \frac{N}{N-1}\,y \\
x &< N \\
(1-w)C &< N
\end{aligned}$$

Proof. For V_1 to be indifferent between P and $\bar{P}$ in an ϵ–uniformly perturbed game K_3 must use μ_3^{ϵ} with

$$\mu_3^{\epsilon} = \frac{x + \epsilon\,(H(1-2\epsilon) + \epsilon N)}{(1-\epsilon)N}$$

where $\mu_3^\epsilon \rightarrow \mu_3$ for $\epsilon \rightarrow 0$. For K_3 to be indifferent between his two moves z and a the seller agent V_1, in turn, must behave according to π^ϵ with

$$\pi^\epsilon = \frac{(1-\epsilon)(1-w)(C-N)}{wN}$$

where $\pi^\epsilon \rightarrow \hat{\pi}$ for $\epsilon \rightarrow 0$. The seller agent V_1^* prefers P^* if

$$(1-\epsilon)(\mu_3^\epsilon N + 1 - \mu_3^\epsilon) + \epsilon\,(\epsilon H + 1 - \epsilon) - y >$$
$$\epsilon\,(\epsilon N + 1 - \epsilon) + (1-\epsilon)(\epsilon H + 1 - \epsilon).$$

Since $\mu_3^\epsilon \rightarrow \mu_3$ for $\epsilon \rightarrow 0$, this condition is satisfied for ϵ (> 0) sufficiently small if

$$\mu_3 N + 1 - \mu_3 - y > 1$$

or $\mu_3(N-1) > y$ which is eqivalent to $(N-1)x > Ny$. Seller agent V_2, respectively V_2^*, prefers the lower price since it is accepted with probability μ_3^ϵ whereas the high price is accepted only with the minimum probability ϵ. Seller agent V_3, respectively V_3^*, prefers the higher price since both prices are rejected with maximal probability. For the buyer agent K_1 rejecting is better since

$$(1-w)\,\epsilon^2(C-N) < w(1-\pi^\epsilon)\epsilon N$$

holds for ϵ (> 0) sufficiently small. Similarly, $\bar{A}$ is better for K_2 since

$$(1-w)\,\epsilon\,(1-\epsilon)(C-H) < w(1-\pi^\epsilon)(1-\epsilon)H$$

holds for ϵ (> 0) sufficiently small due to $\pi^\epsilon \rightarrow \hat{\pi}$ for $\epsilon \rightarrow 0$ and $\tilde{\pi} < 1$ because of $(1-w)C < N$. Finally, seller agent K_4 prefers A if

$$(1-w)(1-\epsilon)\,\epsilon^2(C-H) < w\pi^\epsilon \epsilon H$$

which is also true for ϵ (> 0) sufficiently small. Thus $\tilde{q}$ is the limit of equilibrium q^ϵ of ϵ–uniformly perturbed games with π^ϵ and μ_3^ϵ and with maximal probability for the choices of $\tilde{q}$ otherwise. ■

Proposition 7.3 *The strategy vector* $\bar{q} = (\bar{q}_V, \bar{q}_{V^*}, \bar{q}_K)$ *with*

$$\begin{aligned}
\bar{q}_V &= (\bar{P}, H, h),\\
\bar{q}_{V^*} &= (\pi^*, H^*, h^*),\\
\bar{q}_K &= (\bar{z}, \mu_2, z, Z)
\end{aligned}$$

where

$$\begin{aligned}
\pi^* &= 1 - \frac{w}{1-w} \cdot \frac{H}{C-H}\\
\mu_2 &= 1 - \frac{y}{H-1}
\end{aligned}$$

is a uniformly perfect equilibrium of the game in Figure 5.1 if the following three conditions are satisfied:

$$\begin{aligned}
\frac{y}{x} &< \frac{H-1}{H}\\
y &< \frac{H-1}{H}(H-N)\\
(1-w)C &> H
\end{aligned}$$

Proof. For V_1^* to be indifferent between P^* and $\bar{P}^*$ in an ϵ–uniformly perturbed game K_2 must rely on μ_2^ϵ with

$$\mu_2^\epsilon = \frac{\epsilon\,[(1-\epsilon)N + \epsilon] + (1-\epsilon)\,[H(1-\epsilon)+\epsilon]}{(1-\epsilon)(H-1)} + \\ + \frac{-(1-\epsilon) - \epsilon\,[N(1-\epsilon) + (1-\epsilon)\,\epsilon] - y}{(1-\epsilon)(H-1)}$$

which converges to $\mu_2 = 1 - \frac{y}{H-1}$ for $\epsilon \to 0$. For K_2 tq be indifferent between his two moves $\bar{Z}$ and $\bar{A}$ the seller agent V_1^* must repair with probability

$$\pi^\epsilon = 1 - \frac{w(1-\epsilon)H}{(1-w)(C-H)}$$

which converges to π^* for $\epsilon \to 0$. For V_1 the move $\bar{P}$ is optimal if

$$(1-\epsilon)^2 H + \epsilon\,(1-\epsilon)N - x < (1-\epsilon)\,\mu_2^\epsilon\, H + \epsilon\,(1-\epsilon)N.$$

Since $\mu_2^\epsilon \rightarrow \mu_2$ for $\epsilon \rightarrow 0$, this condition is fulfilled for ϵ ($> O$) sufficiently small if $H(1-\mu_2) < x$ or $\frac{y}{x} < \frac{H-1}{H}$. For V_2, respectively V_2^*, the high price is better since both prices are accepted with maximal probability $1-\epsilon$. For V_3 the high price yields $\mu_2^\epsilon H$ whereas the low price implies a conditional payoff expectation $(1-\epsilon)N$. Since $\mu_2^\epsilon \rightarrow \mu_2$ for $\epsilon \rightarrow 0$ and $\mu_2 H > N$ is equivalent to $\frac{H-1}{H}(H-N) > y$, seller agent V_3 prefers the high price. This also proves that h^* is optimal for V_3^*. Acceptance is optimal for K_1 if

$$(1-w)(1-\pi^\epsilon)(C-N) > w(1-\epsilon)N.$$

Since $\pi^\epsilon \rightarrow \pi^*$ for $\epsilon \rightarrow 0$, this condition holds for ϵ (> 0) sufficiently small if

$$(1-w)(1-\pi^*)(C-N) > wN$$

or $H > N$. For K_3 acceptance is optimal if

$$(1-w)\pi^\epsilon(C-N) > w\,\epsilon\,N.$$

Due to $\pi^\epsilon \rightarrow \pi^*$ for $\epsilon \rightarrow 0$ this condition is fulfilled for ϵ (> 0) sufficiently small if π^* is positive, i.e. if $H < (1-w)C$. Finally, acceptance is K_4's better choice if

$$(1-w)\pi^\epsilon(C-H) > w\,\epsilon\,H.$$

Again this is true for ϵ (> 0) sufficiently small if π^* is positive, i.e. if $H < (1-w)C$. Thus $\bar{q}$ is the limit of equilibria q^ϵ of ϵ–uniformly perturbed games with π^ϵ and μ_2^ϵ and with maximal probability $1-\epsilon$ for the pure moves of $\bar{q}$. ■

According to all three equilibria $\hat{q}$, $\tilde{q}$ and $\bar{q}$ signaling is possible. If, for instance, V_1 happens to choose $\bar{P}$, what he does with probability $1-\hat{\pi}$ according to $\hat{q}$, and if $\hat{q}$ is expected to be the solution, than buyer K would conclude that the car is a lemon. Similarly, the car's quality is revealed with probability $w(1-\tilde{\pi})$ if $\tilde{q}$ is the solution. According to $\bar{q}$ the car's quality is revealed with probability $(1-w)\pi^*$, namely if V_1^* happens to decide and if he chooses his move P^*. This shows that the main screening device is the repair decision since the repair cost is lower for V^* than for V, the owner of a lemon.

It is interesting to note that in case of $\tilde{q}$ the price for the repaired car is lower than the one for the unrepaired car. The reason for this counterintuitive result

equilibrium	$\hat{q}$	$\tilde{q}$	$\bar{q}$
probability of signaling	$\frac{H-(1-w)C}{H}$	$\frac{N-(1-w)C}{N}$	$\frac{(1-w)C-H}{C-H}$

Table 7.1. The signaling probability implied by $\hat{q}$, $\tilde{q}$, and $\bar{q}$

is that, since every price proposal for the unrepaired car is rejected, the seller tries to exploit optimally the possibility of a mistake by the buyer.

Proposition 7.1, 7.2, and 7.3 together show that signaling the car's true quality is possible accorrding to the concept of uniformly perfect equilibria, but does not accur with probability 1.

The signaling probabilities for the three signaling equilibria are given by Table 7.1 in the original parameters of the game, described by Figure 5.1. $\hat{q}$ and $\tilde{q}$ are both signaling equilibria for $w > (C - N)/C$ and then the signaling probability for $\hat{q}$ is greater than the one for $\tilde{q}$ since $H > N$. The equilibria $\bar{q}$ and $\hat{q}$ (respectively $\tilde{q}$) are valid for different parameter constellations. The signaling probability implied by $\bar{q}$ can be larger than the one of $\hat{q}$ and also smaller than the one of $\tilde{q}$. Here one must, of course, remember that the initial chance move is purely fictitious so that these probabilities only express the buyer's expectations. For the seller signaling occurs with probability $1 - \hat{\pi}$ according to $\hat{q}$, with probability $1 - \tilde{\pi}$ according to $\tilde{q}$, and not at all according to $\bar{q}$ if the car is a lemon. ln case of a good car the seller expects no revelation of the car's quality according to $\hat{q}$ and $\tilde{q}$ and with probability π^* according to $\bar{q}$.

8. Discussion of results and concluding remarks

We first want to discuss the possibility that at least two uniformly perfect equilibria can coexist. Due to Theorem 6.1 there is no parameter constellation satisfying the parameter restriction (5.1) for which the corresponding game has two uniformly perfect equilibria in pure strategies. Clearly, also $\hat{q}$ and $\tilde{q}$ exclude each other since $\hat{q}$ requires $x > N$ whereas for $\tilde{q}$ one needs $x < N$.. Similarly, neither $\hat{q}$ and $\bar{q}$ nor $\tilde{q}$ and $\bar{q}$ can coexist since $\bar{q}$ relies on $(1 - w)C > H$ whereas

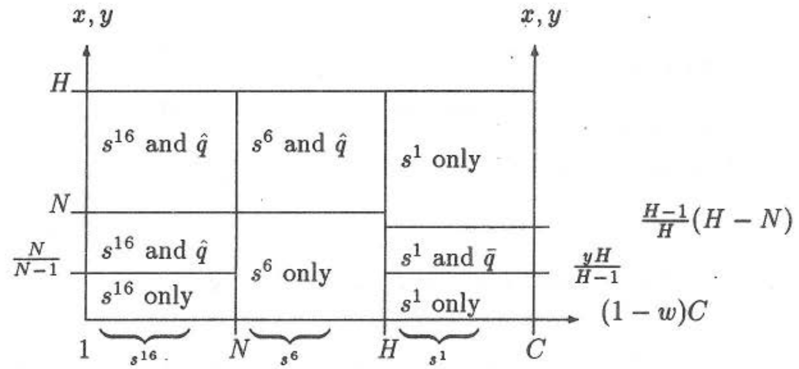

Figure 8.1. The regions of uniformly perfect equilibria for the game of Figure 5.1 with parameters 8.1

the expected value $(1-w)C$ of the car has to be smaller than H, respectively N, according to $\hat{q}$, respectively $\tilde{q}$. In order to demonstrate coexistence of uniformly perfect equilibria we therefore have to show that there are parameter constellations satisfying condition (5.1) such that the game of Figure 5.1 has exactly one equilibrium in pure strategies and one with two agents using mixed behavioral strategies.

In Figure 8.1 we graphically illustrate the situation for the parameter constellations with

$$C = 5 > H = 4 > N = 2, x > y = 1/2, \text{ and } 0 < w < 1 \tag{8.1}$$

where we exclude, of course, the border cases $w = \frac{C-H}{C}$, $w = \frac{C-N}{C}$, $x = N$, $x = \frac{N}{N-1}y$ and $x = \frac{H}{H-1}y$. Although there are regions with only one uniformly perfect equilibrium which is always in pure strategies, coexistence of one pure and one non–pure uniformly perfect equilibrium can also be observed (see Figure 8.1).

Whereas s^1, respectively s^6, can only coexist with $\bar{q}$, respectively $\hat{q}$, the pure strategy equilibrium s^{16} can go along with $\hat{q}$ and $\tilde{q}$. Since all boundaries in Figure 8.1 depend continuously on the game parameters, Figure 8.1, furthermore, illustrates a generic situation. Thus both phenomena, namely coexistence of one pure and one non–pure equilibrium or just one uniformly perfect equilibrium in pure strategies, are generic.

uniformly perfect equilibrum	s^1	s^6	s^{16}	$\hat{q}$	$\tilde{q}$	$\bar{q}$
probability of sale	1	1	0	$w\hat{\pi}\mu_4+$ $+(1-w)\mu_4$ $=(1-w)\frac{xC}{H^2}$	$w\tilde{\pi}\mu_3+$ $+(1-w)\mu_3$ $=(1-w)\frac{xC}{N^2}$	$w\mu_2+(1-w)\cdot$ $\cdot(\pi^*+(1-\pi^*)\mu_2$ $1-\frac{wCy}{(C-H)(H-1)}$
sales price in case of sale	H	N	—	H	N	H

Table 8.1. The probability of sale and the sales price for all 6 uniformly perfect equilibria

Although uniformly perfect equilibria do not allow to speculate freely about beliefs in unreached informed sets, they do not resolve strategic uncertainty completely. In the regions of Figure 8.1 with two equilibria the strategic advice remains ambiguous. If, for instance, s^{16} and $\hat{q}$ coexist, s^{16} presribes $\bar{P}^*$ for V_1^* whereas $\hat{q}$ tells V_1^* to use P^*. Similarly, K_3 is supposed to accept according to $\hat{q}$ and to reject according to s^{16}. Like other refinement concepts (see van Damme, 1991, and Güth, 1992, for a survey) uniform perfectness is only an attempt to define a necessary condition for individually rational decision making which, in general, will not be sufficient to resolve strategic uncertainty completely.

Attempts to resolve strategic uncertainty completely are theories of equilibrium selection (see Harsanyi and Selten, 1988, as well as Güth and Kalkofen, 1989). Güth and van Damme (1991a and 1991b) have applied equilibrium selection to signaling games and shown how to select one of possibly many uniformly perfect equilibria as the unique solution of the game. A similar study would be possible for the game of Figure 5.1.

The economic implications of the six uniformly perfect equilibria can be illustrated with the help of Table 8.1 which gives the probability of sale as well as the price in case of a sale for the three pure and the three non-pure uniformly perfect equilibria derived in sections 6 and 7 where we express the sales probabilities both by the parameters of the behavioral strategies and by the original parameters of the game.

Whereas the pure strategy equilibria imply either sale (s^1 and s^6) or non–sale (s^{16}), the non–pure equilibria imply both sale and non–sale with positive probability. Since $H > N$, the sales probability for $\tilde{q}$ is larger than the one for $\hat{q}$. The sales probability of $\bar{q}$ can be larger or smaller than the ones implied by $\hat{q}$ and $\tilde{q}$. Furthermore, every uniformly perfect equilibrium in Table 8.1 implies a unique sales price which can be the high one (in case of s^1, $\hat{q}$ and $\bar{q}$) or the low one (in case of s^6 and $\tilde{q}$).

The game of Figure 5.1 is highly restrictive since it allows only to choose between two prices, namely a high and a low one. Furthermore, one might want to allow the seller not to sell at all. Nevertheless some aspects will probably be true also for more general models, e.g. that signaling has to rely on non–pure choice behavior if there is only one screening choice like repairing the car or the generic coexistence of pooling and signaling equilibria. As already indicated when discussing the implications of Theorem 6.6, the possibility of more than two price proposals does not seem to cause problems. What matters more is the possibility of prices exceeding the car's expected value $(1 - w)C$.

The bargaining process in Figure 5.1 is of the most simple form. The seller first can repair the car and then chooses his sales price which the potential buyer can either accept or reject. One might argue that this is a much too simple procedure since we often observe long bargaining disputes before a settlement or conflict results. However, in most cases even a long dispute will end with a final offer which the other party then can only accept or reject. It is this final stage of the bargaining process what we try to capture by the game model of Figure 5.1.

Of course, in a bargaining game with incomplete information exchanges of arguments might affect the beliefs of the uninformed players. If, however, it is in the interest of V^* to give some information about the quality of the car, the owner V of the lemon can imitate V^* and also obtain the same advantages. In a uniform equilibrium screening has to be costly in the sense that V^*'s behavior cannot be imitated without a payoff loss by the seller type V who wants to sell a lemon.

Most bargaining models like, for instance, Rubinstein's celebrated approach consider highly stylized situations, e.g. the division of a unit cake. In economic life bargaining is just one aspect of the economic environment. So a bargaining model has to capture both, the economic environment with all its strategically relevant institutional details and the prevailing rules of bargaining. In our view, the game model of Figure 5.1 demonstrates the difficulty of this task. Although the economic structure is rather simple (just two types of a used car, which can be sold at two possible prices, and the simple procedure of ultimatum bargaining), it is by no means trivial to solve such a 'simple model'.

The great art in bargaining theory is to develop a model which captures the richness of strategically essential institutional details and also the typical rules of bargaining and which still be analysed analytically. In our view, one crucial institutional aspect is the existence of private information which is also the decisive feature of the game in Figure 5.1. With respect to bargaining rules one can rely on successive concession or final proposals as we did and as it is often assumed in the bargaining literature.

Game theory has provided thorough investigations of stylized bargaining rules. Master pieces which strongly influenced the game theoretic school of bargaining theory were Nash (1951 and 1953) and Rubinstein (1982) whose models were both generalized to environments with incomplete information (see Harsanyi and Selten, 1972, and Rubinstein, 1985).But we need many more such master pieces to complete the 'gallery' of game theoretic bargaining studies.

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