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Noncooperative Bargaining and the Core of an N – Person Characteristic Function Game [1]

by

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We investigate whether the core of an n - person characteristic function game can be supported by players' noncooperative behavior in some suitably defined bargaining model. In our bargaining model, players negotiate over (possibly infinitely) many periods and negotiations within one period consist of a sequence of finitely many proposals and responses to them. It is shown that for a totally balanced game the set of all payoff distributions attained by subgame perfect equilibrium points of the bargaining model with no discounting payoffs coincides with the core of the game if the equilibrium points satisfy the two conditions about low complexity of players' bargaining behavior: (i) stationarity and (ii) payoff – oriented response.

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1. Introduction

The purpose of this paper is to investigate a problem of coalition formation and payoff distributions in an n - person game in characteristic function form by presenting a noncooperative bargaining model. Our main concern is with how a cooperative outcome can be realized by players' noncooperative utility maximizing behavior under some specified rule about negotiation process.

Nash (1951) suggested that a cooperative game should be analyzed as a noncooperative game by constructing a bargaining model suitably defined in extensive form and by studying its noncooperative equilibrium points. In this noncooperative approach, we can clarify the assumptions that various cooperative solution concepts implicitly make about players' bargaining behavior if we reconstruct them as equilibrium points in a noncooperative bargaining game, and furthermore we can explain the multiplicity of cooperative solution concepts by various possible negotiation rules for the characteristic function game. This research program is now called the Nash program.

Up to now, several works towards carrying out the Nash program have been done for an n - person characteristic function game, stimulated by Harsanyi's (1974) earlier work on von Neumann-Morgenstern solution. Selten (1982) investigated the concept of a stable demand vector introduced by Albers (1975) in an n - person game with the one-stage property, and Binmore (1985) investigated the "asymmetric" Nash bargaining solution in 3-person games without the grand coalition (called the 3-person / 3-cake problem), and Gul (1989) investigated the Shapley value in a framework of an n - person market game. Kaneko (1986) also investigated von Neumann-Morgenstern solution from a different viewpoint of his convention-stability theory.

In this paper, we will consider the core among various cooperative solution concepts for a characteristic function game. Although the core is defined by a simple criterion of coalitional behavior of players, it is not a trivial question whether the core can be implemented by players' noncooperative behavior in some suitable bargaining model.

In our earlier work (Okada, 1991), we investigated the implementation problem of a Pareto efficient and individually rational outcome in the framework of a 2-person supergame. We incorporated a possibility of negotiations and of binding agreements for actions into the usual supergame model and considered a question whether a noncooperative equilibrium point in the bargaining game necessarily leads to a Pareto efficient and individually rational outcome. It turns out that the answer of this question depends on the complexity (or memory) of players' equilibrium strategies. The answer is affirmative if the two players employ stationary equilibrium strategies which are independent of the history of the game. We also pointed out that nonstationary (or history-dependent) equilibrium points may lead to Pareto inefficient outcomes even under such a strong institutional assumption on negotiations that players can reach binding agreements on their current and also future actions. In this paper, we will attempt to extend our investigation to an n-person game in characteristic function form.

The n - person bargaining model presented in this paper has the following features.

(1) Negotiations are done among relatively small number of players.

(2) The players can negotiate for (possibly infinitely) many periods. Negotiations within one period consist of a sequence of proposals and responses to them. There exists an upper limit of the number of proposals which can be made sequentially in one period.

(3) If one coalition is formed in some period, then the remaining players can continue their negotiations under the same rule at the next period.

(4) The possibility of renegotiations is allowed in a sense that, if the players fail in making any agreement in one period, then they can negotiate again under the same rule at the next period.

(5) Future payoffs are not discounted.

The aim of our analysis is to investigate a subgame perfect equilibrium point of our bargaining model which satisfies additional conditions about low-complexity of players' behavior.

When we attempt to explain a cooperative solution of a characteristic function game in the framework of noncooperative bargaining models, the two main questions should be considered:

(1) (the problem of bargaining rules) what kind of negotiation process should be designed?

(2) (the problem of strategic complexity) How complexity of players' bargaining behavior affects the equilibrium outcomes of the bargaining game?

Our main theorem shows that for an n - person totally balanced game in characteristic function form its core coincides with the set of all payoff distributions attained by subgame perfect equilibrium points of the bargaining game which satisfy the following two conditions on low-strategic complexity of players' bargaining behavior:

(i) stationarity: players' strategies are independent of the history of the game in past periods,

(ii) payoff–oriented response: every player responds to any proposal according only to the payoff offered to him.

It is also pointed out that players' complicated bargaining strategies which do not satisfy both conditions may lead to an outcome outside the core.

The paper is organized as follows. Section 2 defines an n - person game in characteristic function form and the core of it. Section 3 presents both informal and formal descriptions of our bargaining model. A subgame perfect equilibrium point of the bargaining game and the two conditions of stationarity and of payoff-oriented response are defined. Section 4 is devoted to the analysis of the equilibrium points of the bargaining game. Section 5 has concluding remarks.

2. An n-Person Game in Characteristic Function Form

An n-person game in characteristic function form is a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is the set of players, and v, the characteristic function, is a real -valued function on the family of all subsets of N with $v(\emptyset) = 0$. A nonempty subset S of N is called a coalition. The value $v(S)$ assigned to a coalition S is interpreted as a sum of money which the players in S can distribute among themselves in any way if they reach an agreement on a payoff distribution.

The characteristic function v is *0-normalized* if

$$v(i) = 0 \quad \text{for} \quad i = 1, \ldots, n,$$

and is *superadditive*

$$V(S \cup T) \geq v(S) + v(T)$$

for any two disjoint coalitions S and T. A zero-normalized superadditive characteristic function v is called *essential* if

$$v(N) > 0.$$

In this paper, we assume that the characteristic function v is zero-normalized, superadditive and essential whenever no specifications are given.

A payoff vector of a coalition S is a real-valued function on S , denoted by $x^S = (x_i^S)_{i \in S}$. A payoff vector x^S for S is called *feasible* if

$$\sum_{i \in S} x_i^S \leq v(S).$$

The set of all feasible payoff vectors for S is denoted by X^S. X_+^S denotes the set of all payoff vectors $x^S = (x_i^S)_{i \in S}$ in X^S satisfying $x_i^S \geq 0$ for all $i \in S$. A payoff vector $x^N = (x_i^N)_{i \in N}$ for N is simply denoted by $x = (x_i)_{i \in N}$ whenever no confusion arises.

DEFINITION 2.1 *A payoff vector $x = (x_1, \ldots, x_n)$ for N is said to be an imputation of (N, v) iff*

1. *individual rationality: $x_i \geq 0$ for all $i \in N$, and*

2. *Pareto efficiency: $\sum_{i \in N} x_i = v(N)$.*

The set of all imputations of (N, v) is denoted by X^*.

DEFINITION 2.2 *The* core *of a game (N, v) is defined by*

$$C(v) = \{x \in X^* | \sum_{i \in S} x_i \geq v(S) \quad \text{for all} \quad S \subseteq N\}.$$

DEFINITION 2.3 *(1) A* restricted game *of a game (N, v) is a pair (S, v_S) where S is a coalition of N and v_S is the characteristic function on S induced naturally by v, i.e.,*

$$v_S(T) = v(T) \quad \text{for all} \quad T \subseteq S.$$

(2) A game (N, v) is called totally balanced *iff every restricted game of it (including itself) has the nonempty core.*

We will provide a characterization of the core which is useful to our investigation. Let $x = (x_1, \ldots, x_n)$ be a payoff vector for N. We define *the maximum payoff for player i under x* by

$$m_i(x) = \max_{S: i \in S \subseteq N} \{v(S) - \sum_{j \in S, j \neq i} x_j\}.$$

The value $m_i(x)$ is the maximum payoff which player i can obtain by forming all possible coalitions including himself under the assumption that all other members in the coalitions receive the payoffs specified in $x = (x_1, \ldots, x_n)$.

PROPOSITION 2.1 *An imputation* $x = (x_1, \ldots, x_n)$ *of* (N, v) *is in the core* $C(v)$ *iff*

$$x_i = m_i(x) \quad \text{for all} \quad i = 1, \ldots, n.$$

The proof of Proposition 2.1 is left to the readers. Proposition 2.1 shows that for every imputation x in the core every player enjoys his maximum payoff under x.

3. A Noncooperative Bargaining Model for the Game (N,v)

3.1. Informal Description

We first provide an informal description of our bargaining model for an n-person characteristic function game (N, v). A precise definition of the rule of the bargaining game will be given in extensive form in the next subsection.

All players in N negotiate for coalition formations and payoff destributions over (possibly infinitely) many periods. Negotiations in every period start with a proposal of a (predetermined) player and responses by other players to it. If some player rejects the proposal, he can make a counterproposal. This process is repeated finitely many times within one period. A proposal is a pair of a coalition and a feasible payoff vector for the coalition. If a proposal is accepted and thus some coalition is formed, then negotiations will continue among the remaining players in the next period. Otherwise, negotiations will be repeated among the same set of players. Negotiations do not end until each of n players belongs to some coalition, More specifically, our bargaining game proceeds as follows.

In period 0, an ordering $a = (i_1, i_2, \ldots, i_n)$ over N is randomly selected. This ordering is called the proposership ordering, which is used to determine the first proposer at every period. Given $S \subset N$, let a_S denote the ordering over S which is naturally induced by α.

In every period $t = 1, 2, \ldots$, negotiations take place within the set N^t of players. This player set N^t in period t will be defined inductively by the rule of the game explained below with the initial condition of $N^1 = N$.

Rules:

(1) The first proposal: The first proposer is the player who is in the first position in N^t with respect to the ordering α_{N^t}. Let $i \in N^t$ be the first proposer. Player i proposes a pair (x^S, S) satisfying (i) $i \in S \subseteq N^t$ and (ii) $x^S = (x_j^S)_{j \in S} \in X_+^S$. The proposal (x^S, S) is called essential if $v(S) > 0$, and inessential otherwise. In what follows, we assume that every player makes an essential proposal when he becomes a proposer.

(2) Responses: When the first proposer i makes a proposal (x^S, S), all players in $S - \{i\}$ can either accept or reject it sequentially according to α_{N^t}. The two cases are possible.

(2a) If all players in $S - \{i\}$ accept (x^S, S), then it is agreed upon. In this case, the coalition S is formed and every player j in S receives the payoff x_i^S. And then, in period $t+1$ all players in $N^{t+1} = N^t - S$ have negotiations under the same rule as in period t with the proposership ordering $\alpha_{N^{t+1}}$.

(2b) If some player j in $S - \{i\}$ rejects (x^S, S), then he can make a counterproposal.

(3) Counterproposal: Player j counterproposes a pair (y^T, T) satisfying (i) $j \in T \subseteq N^t$ and (ii) $y^T = (y_k^T)_{k \in T} \in X_+^T$.

(4) Responses: All players in $T - \{j\}$ can either accept or reject player j's counterproposal (y^T, T). Then, the same rule as in (2) is applied.

(5) Finitely many proposals: The process from (2) to (4) is repeated until an agreement is reached. However, there exists an upper bound K of the number of successive proposals in every .period t where K is any fixed positive integer with $K \geq 2$. When no agreement is reached after K proposals, negotiations break down in period t.

(6) Renegotiations: When negotiations break down in period t, the players in N^t can negotiate again in the next period $t+1$ under the same rule as in period t. In this case, we have $N^{t+1} = N^t$.

(7) End: The game ends if and only if (i) every player in N belongs to some coalition and obtains an agreed upon payoff, or (ii) players in a coalition S with $v(S) = 0$ remain. In case (ii), we assume without loss of generality that the coalition S is formed and every member in S obtains zero-payoff. In the case of an infinite play, all players who fail to belong to any coalitions obtain zero-payoffs.

In what follows, the proposership ordering $\alpha = (i_1, i_2, \ldots, i_n)$ is fixed, and we put $\alpha = (1, 2, \ldots, n)$ for simplicity.

3.2. The Extensive Form

The rule of our bargaining model explained in the last subsection is formally described in the extensive form. The bargaining model in each period is called a component game. We will first define a component game.

(1) The Component Game

The component game with the player set S is defined by a seven-tuple

$$G(S) = (S, X, Z, P, A, h, c)$$

of which each element is explained below.

(i) S, a subset of N, is the set of players. Without any loss of generality, we write

$$S = \{1, ..., s\}.$$

(ii) X is the set of all personal positions where players make choices. X consists of the positions of the following types: for all $i \in S$,

$$p_i(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m), \quad m = 0, 1, \ldots, K-1$$

$$r_i(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m), \quad m = 0, 1, \ldots, K$$

where (x^{S_k}, S_k) is a proposal by player i_k for $k = 1, \ldots, m$. The notation $p_i(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m)$ means the position of player i where he makes a proposal under the history that the proposals (x^{S_k}, S_k) by players i_k $(k = 1, \ldots, m)$ have been rejected. When $m = 0$, the first proposal is made by player i at this position. The notation $r_i(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m)$ means the position of player i where he responds to player i_m's proposal (x^{S_m}, S_m) under the history that all previous proposals (x^{S_k}, S_k) by players i_k $(k = 1, \ldots, m-1)$ have been rejected and also that all responders in S_m preceding him have accepted the proposal (x^{S_m}, S_m).

(iii) Z is the set of all endpoints, which consists of those of the following types:

$$a(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m), \quad m = 1, \ldots, K$$

$$\tilde{a}(\{i_k, (x^{S_k}, S_k)\}_{k=1}^K)$$

where (x^{S_k}, S_k) is the proposal by player i_k.
The notation $a(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m)$ means the endpoint where player i_m's proposal (x^{S_m}, S_m) is accepted after all previous proposals (x^{S_k}, S_k) $(k = 1, \ldots, m-1)$ have been rejected. The notation $\tilde{a}(\{i_k, (x^{S_k}, S_k)\}_{k=1}^K)$ means the endpoint where negotiations break down in period t after all K proposals (x^{S_k}, S_k), $k = 1, \ldots, K$, have been rejected.

(iv) $P = [P_1, \ldots, P_S]$, the player partition, is a partition on X where P_i $(i = 1, \ldots, s)$ is the set of all personal positions of player i.

(v) A, the choice function, is a function which assigns a nonempty choice set to every personal position in X such that

$$A(p_i(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m)) = \{(x^T, T) | i \in T \subseteq S, |T| \geq 2, x^T \in X_+^T\}$$

$$A(r_i(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m)) = \{\text{accept, reject}\}.$$

(vi) h, the payoff function, is a function which assigns a payoff vector to every endpoint in Z such that

$$h(a(\{i_k, (x^{S_k}, S_k)\}_{k=1}^m)) = x^{S_m}$$

$$h(\tilde{a}(\{i_k, (x^{S_k}, S_k)\}_{k=1}^K)) = (0, \ldots, 0).$$

(vii) c, the continuation function, is a function which assigns a component game to every endpoint in Z such that

$$c(a(\{i_k, (x^{S_k}, S_k)\}_{k=1}^{m})) = G(S - S_m)$$

$$c(\tilde{a}(\{i_k, (x^{S_k}, S_k)\}_{k=1}^{K})) = G(S).$$

The game tree of the component game is constructed by the set of personal positions, X, the set of endpoints, Z, and the choice function, A, according to the rule explained in subsection 3.1. The component game $G(S)$ has no play when $S = \emptyset$, and has the unique payoff vector $(0, \ldots, 0)$ when $v(S) = 0$.

(2) The Bargaining Game Γ

The extensive form of our bargaining game Γ is constructed from the family $\{G(S)\}_{S \subseteq N}$ of component games defined above. Γ starts with the component game $G(N^1)$ with $N^1 = N$. All endpoints of $G(N^1)$ are connected to appropriate component games $G(N^2)$ by the continuation function in $G(N^1)$, and all endpoints of $G(N^2)$ are also connected to appropriate component games $G(N^3)$ by the continuation function in $G(N^2)$, etc. Γ is constructed in this way as far as the continuations of component games are possible. When we want to emphasize the upper bound K of the number of successive proposals in every period in Γ, we write Γ^K instead of Γ.

Let x be any personal position of player i in the extensive form of Γ. Then, there exist uniquely (1) a sequence $\{G(N^k)\}_{k=1}^{t}$ of component games, and (2) endpoints z^k in all $G(N^k)$, $k = 1, \ldots, t-1$, and (3) a personal position x^t of player i in $G(N^t)$ such that x is corresponding to the sequence $(z^1, \ldots, z^{t-1}, x^t)$ under the construction rule of Γ. The sequence $(z^1, \ldots, z^{t-1}, x^t)$ is called the *history* of x. In what follows, the position x is identified with the history $(z^1, \ldots, z^{t-1}, x^t)$ of itself when no confusion arises. When $x = (z^1, \ldots, z^{t-1}, x^t)$, we say that x is a position in period t in Γ, We assume that at his every position of Γ every player can know the history of the position. The bargaining game Γ is formally described as an extensive game with perfect information, and of infinite length.

Let z be a play of Γ. Similarly to the case of a personal position, there exist uniquely (1) a sequence of component games, $\{G(N^t)\}_{t=1}^{T}, T < \infty$ or $T = \infty$, and (2) endpoints z^t in all $G(N^t)$, $t = 1, \ldots, T$ such that z is corresponding to the sequence $(z^1, \ldots, z^T)$ under the construction rule of Γ. When $T < \infty$, z is said to have length T, and to have infinite length, otherwise. The payoff function h of Γ assigns a payoff vector $h(z) = (h_1(z), \ldots, h_n(z))$ to every play z of Γ satisfying $h_i(z) = x_i^S$ if player i belongs to some coalition S and obtains a payoff x_i^S on the play z, and 0 otherwise.

(3) Strategy and Payoff

A (pure) strategy σ_i for player i in Γ is defined by a function that assigns to every position x of player i in Γ a choice $\sigma_i(x)$ at x. In this paper, we will not consider any randomization of choices by players. Let Π_i denote the set of player i's strategies in Γ. Given a strategy combination $\sigma = (\sigma_1, \ldots, \sigma_n)$ of Γ, a play z of Γ is uniquely determined. Then, the payoff $H_i(\sigma)$ of player i for σ is defined by

$$H_i(\sigma) = h_i(z).$$

Let $H(\sigma)$ denote the payoff vector $(H_1(\sigma), \ldots, H_n(\sigma))$ for σ. Given a strategy combination $\sigma = (\sigma_1, \ldots, \sigma_n)$ of Γ and a subgame $\tilde{\Gamma}$ of Γ, let $\sigma_{|\tilde{\Gamma}}$ denote the strategy combination in $\tilde{\Gamma}$ induced by σ. Similarly to Γ, we can define the payoff for player i in $\tilde{\Gamma}$ when $\sigma_{|\tilde{\Gamma}}$ is played, which is denoted by $H_i(\sigma_{|\tilde{\Gamma}})$.

3.3. Equilibrium Points

We define the concept of a subgame perfect equilibrium point which we will employ as the noncooperative solution concept for Γ. We also introduce two conditions about low-complexity of an equilibrium point: (i) stationarity and (ii) payoff-oriented response.

DEFINITION 3.1 *A strategy combination $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ of Γ is said to be a* Nash equilibrium point *of Γ iff for all $i = 1, \ldots, n$, $H_i(\sigma^*) \geq H_i(\sigma^*/\sigma_i)$ for all $\sigma_i \in \Pi_i$ where σ^*/σ_i is the strategy combination obtained from σ^* by replacing σ_i^* with σ_i.*

DEFINITION 3.2 *A Nash equilibrium point $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ of Γ is called a* subgame perfect equilibrium point *iff it induces a Nash equilibrium point on every subgame of Γ.*

DEFINITION 3.3 *A Nash equilibrium point $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ of Γ is called* stationary *iff the following holds for every $i = 1, \ldots, n$: for every position $x = (z^1, \ldots, z^{t-1}, x^t)$ of player i in Γ in every period t, $\sigma^*(x)$ depends only on x^t.*

Definition 3.3 says that in a stationary equilibrium point players' behavior at every position in every period t does not depend on the history of the game before period t, but may depend on the intra-period history x^t of the game within period t. In this paper, we call such an equilibrium point stationary because it induces the same strategies of players on every subgame of Γ that starts with the first proposal in every period.

Remark. A Nash equilibrium point $\sigma^\star = (\sigma_1^\star, \ldots, \sigma_n^\star)$ of Γ is stationary iff there exists a strategy combination $\sigma^S = (\sigma_i^S)_{i\in S}$ for every component game $G(S)$, $S \subseteq N$, such that the following holds for every $i \in N$: for any position $x = (z^1, \ldots, z^{t-1}, x^t)$ of player i in Γ, we have

$$\sigma^\star(x) = \sigma_i^S(x^t).$$

where x^t is a position of player i in the component game $G(S)$. Then, $\sigma^\star$ is called the strategy combination of Γ defined by the family $\{(\sigma_i^S)_{i\in S}\}_{S\subseteq N}$ of strategy combinations for all component games $G(S)$.

Finally, we define a low-complexity condition about players' response rule in the bargaining model.

DEFINITION 3.4 *A stationary equilibrium point $\sigma^\star = (\sigma_1^\star, \ldots, \sigma_n^\star)$ of Γ is said to have* payoff oriented *response iff, for every component game $G(S)$, there exists a payoff vector $a^S = (a_i^S)_{i\in S}$ such that every $\sigma_i^\star$ prescribes the following response rule of player i in $G(S)$: for any proposal (y^T, T), $i \in T \subseteq S$, accept it if $y_i^T \geq a_i^S$, and reject it otherwise. The payoff a_i^S is called* player i's acceptable payoff *in the component game $G(S)$.*

The condition of payoff-oriented response means that every player employs a simple response rule which depends only on the payoff offered to him, independent of the history of the game. Every player accepts a proposal whenever he is offered at least as much as his acceptable payoff. Remark that every player's acceptable payoff depends on the component game. We can interpret in the two ways why a player may employ such a simple response rule in negotiations. One way is to assume that the capacity of the information processing system embodied in a player is bounded either physically or with high costs and that he has to employ a low-complexity response rule by this reason. Another interpretation is that a player employs such a simple response rule with strategic consideration. For example, low-complexity strategies may have a focal point effect such that all players are more likely to select them with mutual expectations that the other players also select them.

4. A Characterization of Equilibrium Points of the Bargaining Game Γ

In this section, we will characterize the subgame perfect equilibrium point of the bargaining game Γ which satisfies both conditions of stationarity and of payoff oriented response. In what follows, we assume that an equilibrium point is subgame perfect and stationary.

LEMMA 4.1 *Let $\sigma^\star = (\sigma_i^\star)_{i\in N}$ be an equilibrium point of Γ such that $x^\star = H(\sigma^\star)$. For every $i \in N$ let Γ_i be the subgame of Γ starting with player i's move to make the last proposal in the component game $G(N)$. Then*

$$H_i(\sigma^\star|_{\Gamma_i}) = m_i(x^\star)$$

where $m_i(x^\star)$ is player i's maximum payoff under $x^\star$.

PROOF: By definition, we have

$$m_i(x^\star) = \max_{S:i\in S\subseteq N}[v(S) - \sum_{j\in S, j\neq i} x_j^\star]$$

Let S^i be the coalition which attains $m_i(x^\star)$. For any x_i with $m_i(x^\star) > x_i$, define $w = (w_j)_{j\in S}$ in X_+^S by

$$w_i = x_i \quad \text{and}$$

$$w_j = x_j^\star + (m_i(x^\star) - x_i)/(|S^i| - 1), \quad j \in S^i, \quad j \neq i.$$

Then, we have $\sum_{j\in S^i} w_j = v(S^i)$ and $w_j > x_j^\star$ for all $j \in S^i, j \neq i$. Suppose that player i makes the last proposal (w, S^i) in the component game $G(N)$. Also suppose that all responders except the last one, say player k; accept this proposal. If player x accepts it, then the proposal (w, S^i) is agreed upon and thus he obtains w_k. On the other hand, if he rejects it, negotiations break down in period t and thereafter renegotiations will take place in the next period $t+1$. Since $\sigma^\star$ is stationary, player k will obtain $x_k^\star$ in such a case. Apparently it is optimal for him to accept (w, S_i). By the "backward induction" argument, we can see that all other players in S_i also accept (w, S_i) in $\sigma^\star$. Therefore player i can obtain at least x_i in $\sigma^\star$. On the other hand, suppose that player i proposes (y^T, T), $i \in T$, with $y_i^T > m_i(x^\star)$. Then,

$$\sum_{j\in T, j\neq i} y_j^T + y_i^T \leq v(T) \leq \sum_{j\in T, j\neq i} x_j^\star + m_i(x^\star).$$

Since $y_i^T > m_i(x^\star)$, there exists some player $j \in T, j \neq i$, such that $y_j^T < x_j^\star$. Assume that (y^T, T) is accepted in $\sigma^\star$. Then player j can increase his payoff from y_j^T to $x_j^\star$ by rejecting (y^T, T). This contradicts that $\sigma^\star$ is a subgame perfect equilibrium point of Γ. Hence, player i's last proposal (y^T, T) must be rejected when $\sigma^\star$ is employed. Afterward, renegotiations will take place in the next period, and player i will obtain the equilibrium payoff $x_i^\star$ since $\sigma^\star$ is stationary. Since $m_i(x^\star) \geq x_i^\star$, we must have $H_i(\sigma^\star_{|\Gamma_i}) = m_i(x^\star)$ in order for $\sigma^\star$ to be a subgame perfect equilibrium point of Γ. ∎

Lemma 4.1 shows that, once every player becomes the last and K-th proposer in the component game $G(N)$, he can enjoy the maximum payoff under the equilibrium payoff vector. This implies that, when he responds to the last second proposal, he will reject it whenever he is offered less than this maximum payoff.

LEMMA 4.2 *Let (N, v) be a totally balanced game. Then, for any imputation $x^\star$ in the core $C(v)$, there exists an equilibrium point $\sigma^\star = (\sigma_i^\star)_{i\in N}$ of G such that $H(\sigma^\star) = x^\star$.*

PROOF: Since (N, v) is totally balanced, for every restricted game (S, v_S) of (N, v) we can select a payoff vector $x^{\star S}$ in the core of (S, v_S). First define a strategy $\pi_i^{\star N}$ for every player i in the component game $G(N)$ as follows:

(1) propose (x*, N) , and

(2) for any proposal (x^S, S), $i \in S$,

$$\text{accept it if } x_i^S \geq x_i^\star, \text{ and reject it otherwise.} \tag{4.1}$$

Similarly, define the strategy $\pi_i^{\star S}$ for player i in the component game $G(S)$, $S \subset N$, with $x^\star$ replaced by $x^{\star S}$. Let $\sigma^\star$ be the stationary strategy combination in Γ defined by $\{(\pi_i^{\star S})_{i \in S}\}_{S \subseteq N}$. Clearly $H(\sigma^\star) = x^\star$. We will show that $\sigma^\star$ is a subgame perfect equilibrium point of Γ by the " backward induction " argument.
Claim 1: $\sigma^\star$ is a Nash equilibrium point of Γ.
(PROOF of Claim 1) First assume that the first proposer, player 1, deviates from $\sigma^\star$ in period t $(= 1, 2, \ldots)$. Suppose that he proposes $(x^T, T), 1 \in T, x_1^T > x_1^\star$ in order to increase his payoff. Since $x^T \in X_+^T$ and $x^\star \in C(v)$, we have

$$\sum_{i \in T} x_i^T \leq v(T) \leq \sum_{i \in T} x_i^\star.$$

Since $x_1^T > x_1^\star$, there exists some player $j \in T$, $j \neq 1$, such that $x_j^T < x_j^\star$. Let j be the first responder in T among such players. Player j rejects (x^T, T) and counterproposes $(x^\star, N)$ from (4.1). All other responders in N except player 1 accept this counterproposal if they conform to $\sigma^\star$. Therefore, if player 1 accepts it, $(x^\star, N)$ is agreed upon and he will obtain the equilibrium payoff $x_1^\star$. On the other hand, if he rejects it, he is in the same position to make a proposal. The same argument as above is applied, and the possible outcome in period t is either that he obtains the equilibrium payoff $x_1^\star$ or that negotiations break down. When negotiations break do'vn, renegotiations will take place in the next period $t+1$. Since $\sigma^\star$ is stationary, the same arguments as in period t hold in the next period. Therefore, player 1 can not increase his payoff by deviating from $\sigma^\star$. Next, assume that any other player $i \in N$, $i \neq 1$, deviates from $\sigma^\star$. Suppose that he rejects player 1's proposal $(x^\star, N)$ and counterproposes (x^T, T), $i \in T$, $x_i^T > x_i^\star$. By the same reason as in the case of player 1, (x^T, T) is rejected by some player in $\sigma^\star$ and player i can not increase his payoff by deviating from $\sigma^\star$. This completes the proof of claim 1.

In the next claims 2 and 3, we assume that the component game is $G(N)$.

Claim 2. $\sigma^\star$ prescribes to every player $i \in N$ the optimal response to the last proposal (y^T, T) in every period.

(PROOF of claim 2) First consider the case $y_i^T \geq x_i^\star$. If player i accepts (y^T, T), he will obtain the payoff y_i^T or negotiations will break down , depending on whether all succeeding responders accept it or not. In case that negotiations break down, he will obtain the equilibrium payoff $x_i^\star$ in the next period since $\sigma^\star$ is stationary. Therefore, he can obtain either y_i^T or $x_i^\star$ by accepting (y^T, T). On the other hand, if he rejects (y^T, T), negotiations break down and he will obtain $x_i^\star$ in the next period. Since $y_i^T \geq x_i^\star$, it is optimal for him to accept (y^T, T). In case $y_i^T < x_i^\star$, it is clearly optimal for him to reject (y^T, T). This completes the proof of claim 2.

Claim 3. $\sigma^\star$ prescribes to every player $i \in N$ the optimal proposal when he makes the last proposal in every period.

(PROOF of claim 3) When $\sigma^\star$ is employed, player i proposes $(x^\star, N)$ and this is accepted. If he proposes (y^T, T), $i \in T$, with $y_i^T > x_i^\star$, then it is rejected by some responder as we have seen in the proof of claim 1 . Therefore, negotiations break down and he will obtain $x_i^\star$ in the next period. Hence, it is optimal for him to propose $(x^\star, N)$. This completes the proof of claim 3.

By the induction argument starting from claims 2 and 3, we can show that $\sigma^\star$ prescribes to every player $i \in N$ the optimal choice when he responds to or makes the k-th ($k = 1, \ldots, K-1$) proposal. Since the same arguments as above hold for all subgames of Γ starting with the component games $G(S)$, $S \subset N$, we can prove that $\sigma^\star$ is a subgame perfect equilibrium point of Γ. ■

THEOREM 4.3 *Let (N, v) be a totally balanced game. Then, there exists a subgame perfect equilibrium point $\sigma^\star$ of Γ with $H(\sigma^\star) = x^\star$ satisfying (i) stationarity and (ii) payoff-oriented response if and only if $x^\star$ is in the core of (N, v).*

PROOF: The if-part is proved by Lemma 4.2. We will prove the only-if part . Let $\sigma^\star$ be any subgame perfect equilibrium point of Γ satisfying the two conditions in the theorem and let $x^\star = H(\sigma^\star)$. Let $a^N = (a_i^N)_{i\in N}$ be the acceptable payoff vector of $G(N)$ in $\sigma^\star$. Then, since $\sigma^\star$ prescribes players' optimal responses to the last players for proposal in $G(N)$, we can show that

$$a_i^N = H_i(\sigma^\star) \quad \text{for all} \quad i \in N. \tag{4.2}$$

From Lemma 4.1, we can show that every player $i \in N$ enjoys his maximum payoff $m_i(x^\star)$ under the equilibrium payoff vector $x^\star$ once he becomes the last proposer in period 1. Therefore, from the condition that $\sigma^\star$ prescribes players' optimal responses to the last second proposal, we must have

$$a_i^N = m_i(x^\star) \quad \text{for all} \quad i \in N. \tag{4.3}$$

From (4.2) and (4.3), we have

$$x_i^\star = m_i(x^\star) \quad \text{for all} \quad i \in N. \tag{4.4}$$

We can prove $x^\star \in C(v)$ from (4.4) and Proposition 2.3 if $x^\star$ is an imputation of (N, v). Finally, we will show that $x^\star$ is an imputation of (N, v). Suppose not. Then, $\sum_{i\in N} x_i^\star < v(N)$ and there exists an imputation $x = (x_i)_{i\in N}$ with $x_i > x_i^\star$ for all $i \in N$. Let $i \in N$ be the last proposer on the play of $\sigma^\star$ in period 1, and assume that player i counterproposes (x, N). Then, (x, N) must be accepted from (ii) payoff oriented response since $x_i > x_i^\star = a_i^N$ for all $i \in N$. Therefore, player i can increase his payoff from $x_i^\star$ to x_i. A contradiction. ■

Theorem 4.3 shows that for a totally balanced game the set of all equilibrium payoff vectors in our bargaining model Γ coincides with the core of the game if the equilibrium point satisfies both (i) stationarity and (ii) payoff oriented response. The essense of this result is explained as follows. Suppose that an imputation $x = (x_i)_{i \in N}$ not in the core is agreed upon in an equilibrium point of Γ. First the stationarity implies that the acceptable payoff of every player i in the component game $G(N)$ is equal to his equilibrium payoff x_i, because $\sigma^\star$ prescribes his optimal response to the last proposal in every period. Secondly, assume that there exists at least one coalition S such that $\sum_{i \in S} x_i < v(S)$. The players in this coalition are not satisfied with the proposal because they can find a feasible payoff vector y^S such that all of them are better off in y^S than in x. Suppose that one of them rejects the proposal x and counterproposes (y^S, S) (if there remains the opportunity to make a proposal). Then all other members in the coalition S accept this counterproposal because of the payoff oriented response and thus the proposer himself is better off. Therefore every member in S has an incentive to reject the imputation x not in the core and thus such an imputation can not be supported by an equilibrium point of Γ. This argument restates a usual view to players' bargaining behavior underlying the definition of the core in the framework of noncooperative game theory.

Finally, we investigate what outcomes will result in our bargaining model if players employ more complicated response rule than payoff-oriented one. For an illustration, we consider a simple case that only two successive proposals are possible within one period, i.e, $K = 2$, and also that the game (N, v) has the one-stage property which is introduced by Selten (1982). Here a characteristic function game is said to have *the one - stage property* if for every coalition S of N

$$v(S) > 0 \quad \text{implies} \quad v(N - S) = 0.$$

For the game (N, v) with the one-stage property, if one coalition reaches an agreement, then no further negotiations will take place among the remaining players.

PROPOSITION 4.4 *Let (N, v) be a game with the one-stage property. Then there exists an equilibrium point $\sigma^\star = (\sigma_i^\star)_{i \in N}$ of Γ^2 such that $H(\sigma^\star) = x^\star$ if and only if $x^\star$ is an imputation satisfying $m_k(x^\star) = x_k^\star$ for some $k \neq 1$ (the first proposer).*

PROOF: (if-part) If a coalition S with $v(S) > 0$ is formed in period 1, no further negotiations among players in $N - S$ will take place since (N, v) has the one-stage property. Therefore, it is sufficient for us to define a strategy $\pi_i^{\star N}$ of every player i for the component game $G(N)$ only. We first introduce some notations. For any $i \in N$, let S^i be a coalition which attains $m_i(x^\star)$, and let $x^{\star i}$ denote the feasible payoff vector for S^i such that $x_i^{\star i} = m_i(x^\star)$ and $x_j^{\star i} = x_j^\star$ for all $j \in S^i$, $j \neq i$. Define a strategy $\pi_i^{\star N}$ for player i in $G(N)$ as follows.

player 1: (1) propose $((0,0),\{1,k\})$, and
(2) for any last proposal (y^S, S), $1 \in S$,
accept it if $y_i^S \geq x_i^\star$, and reject it otherwise

player k: (1) propose $(x^\star, N)$, and
(2) reject the proposal $((0,0),\{1,k\})$ of player 1, and
for any other proposal (y^S, S) of player 1,
accept it if $y_i^S \geq m_i(x^\star)$ for all $i \in S, i \geq k$, and
reject it otherwise, and
for any last proposal (y^S, S),
accept it if $y_i^S \geq x_i^\star$, and reject it otherwise

player i ($\neq 1, k$): (1) propose $(x^{\star i}, S^i)$, and
(2) employ the same response rule as player k.

Let $\sigma^\star = (\sigma_i^\star)_{i \in N}$ be the stationary strategy combination of Γ^2 defined by $\{\pi_i^{\star N}\}_{i \in N}$. Clearly, on the play of $\sigma^\star$, player k becomes the last proposer in period 1, and his proposal is accepted. Therefore, $H(\sigma^\star) = x^\star$. We can show without much difficulty that $\sigma^\star$ is a subgame perfect equilibrium point of Γ^2 in the same way as in Lemma 4.2.

(only-if part) It follows from Lemma 4.1 that there exists some $k \neq 1$ such that $m_k(x^\star) = x_k^\star$. By the rule of Γ, it is clear that $x^\star$ is a feasible payoff vector for N. Suppose that $x^\star$ is not an imputation. Then $\sum_{i \in N} x_i^\star < v(N)$. This implies $x_k^\star < m_k(x^\star)$. A contradiction. ■

If an equilibrium point of Γ does not satisfy payoff oriented response, the players may employ complicated response rules. In fact, the equilibrium point constructed in the proof of Proposition 4.4 prescribes to a player the response rule which depends on many factors other than the payoff offered to him: (1) the payoffs offered to all other responders succeeding him and (2) how many proposals were made from the beginning of the period, or equivalently, how many proposals are possible by the end of the period.

In the equilibrium point, some player k rejects the first proposal and obtains the opportunity to make the last proposal. By exploiting this opportunity strategically, he enjoys the maximum payoff under the equilibrium point but the other player may not. Remark that this equilibrium point in favor of the last proposal is supported by other players' response rules stated above which are not payoff-oriented. Such complicated response behavior is not necessarily beneficial to them.

5. Concluding Remarks

We have presented a noncooperative bargaining model for an n-person characteristic function game and have investigated the problem of whether an equilibrium point of the bargaining game necessarily leads to a payoff distribution in the core of the game. When we explore cooperation among players in noncooperative bargaining model where they seek their own payoffs, it is obvious that the possibility of cooperation depends on the rule of negotiation process, i.e., the way in which proposals can be made and agreements can be reached. Our investigation shows that it also depends on the strategic complexity of players' bargaining behavior. Theorem 4.3 shows that an equilibrium point of the bargaining game necessarily leads to an outcome in the core if every player employs a "simple" bargaining strategy such that (i) his behavior is independent of the history of the game in past periods and (ii) he responds to proposals according only to the payoff offered to him. If players employ more complicated response rule in such a "strategic" way that their responses also depend on how many opportunities for proposals are left by the end of the period, then negotiations may result in an outcome outside the core. Such complicated bargaining strategies do not necessarily benefit players.

It seems to us that this result poses an interesting question of how institutional complexity (or rule complexity) of the game and strategic complexity of players' behavior are related when we consider noncooperative implementation of cooperation . For example, in an institutionally simple model of the supergame where there exists neither possibility of negotiations nor of binding agreements, the Folk theorem shows that players' history-dependent strategies incorporating a sophisticated punishment rule are necessary to the establishment of a cooperative outcome. The simple history – independent strategies never produce cooperation in the supergame model. On the other hand, once the game is institutionally organized in a way that negotiations become possible among players and a specified rule of negotiations is established, simple history

– independent strategies are sufficient and also necessary for the realization of cooperation. Complicated history-dependent strategies do not necessarily lead to cooperation.

This paper is our first attempt towards noncooperative game approach to an n - person characteristic function game. Many problems are left unsolved. We have assumed that players do not discount their future payoffs. It is an interesting question how the assumption of discounted payoffs affects the equilibrium outcomes of our bargaining game. It is also necessary for us to study strategic complexity of players' bargaining behavior more formally and more thoroughly. We will do these studies in future papers.

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