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# Bargaining in Changing Environments

by

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The paper extends the Rubinstein's strategic bargaining model to situations in which the players' interests concern the performance of a dynamic system. We develop the model in two stages: first, when the system evolves over time independently of the players, and second, when the players can influence the system's evolution. In both cases the evolution need not be stationary.

## 1. Introduction

Conflict of interests is a common phenomenon. In most conflicts there is a possibility of reaching a mutually beneficial agreement. Typically there is a set of various possible agreements, and there is, in turn, a conflict of interests about which agreement to conclude. Since no agreement can be unilaterally enforced, then the parties must negotiate.

The basic Nash's axiomatic approach (Nash,1950; Roth,1979) to bargaining is static in two respects. First, the outcome of negotiations is defined by a list of properties that it is required to satisfy. This means that the negotiation process itself is not modelled. Second, the subject of negotiations and the players' preferences do not change.

The strategic approach to bargaining problems (Rubinstein,1982; Sutton,1986; Osborne,Rubinstein,1990) removes the first above mentioned weakness concerning the statics of axiomatic models. But it only scratches the surface of the second problem.

This paper is an attempt to formulate a bargaining problem in such a way that it takes into account the dynamics of the negotiation process, and the dynamics of the environment in which the process takes place. And the model incorporates the possibility that negotiators influence the environment.

The paper is organized as follows. In section 2 we introduce the basic notation when outlining the idea of the Rubinstein's model of alternating offer bargaining. Section 3 is concerned with the situation in which the environment evolves over time. These changes influence the sets of feasible agreements as well as the partners' preferences about them. The evolution, however, is independent on the players.

In section 4, in turn, we allow the players to negotiate and, at the same time, to influence the changes of the environment. In fact, we consider a system which is controlled by two players who are interested in the system's performance. And that is why they negotiate about how to coordinate the control of the system. Thus, the two types of dynamics (of bargaining, and of controlling the system) overlap in the model. In all models we assume that players have complete knowledge about the game. We conclude in section 5.

## 2. Negotiation over the Partition of a Shrinking Pie

The simplest situation we shall discuss is the following. Two players bargain over how to divide a "pie" between them. The size of the pie is 1 and an agreement is a pair $(p_1,p_2)$, $p_1 + p_2 = 1$, where $p_i$ is Player $i$'s share of the pie. Each player wants to negotiate a possibly large share.

The bargaining process consists in making (alternating) proposals and reacting to the partner's proposals. In each time period $t \in \Theta$, $\Theta = \{0, 1, 2, ...\}$, one of the players, say $i$, proposes an agreement, i.e. $(p_1, p_2) \in P$, where

$$P = \{(p_1, p_2) : p_1 + p_2 = 1; p_1, p_2 \geq 0\}, \tag{1}$$

and the other player ($j$) reacts, i.e. accepts ($Y$) or rejects ($N$) the offer (in what follows we assume that Player 1 starts the process, i.e. makes an offer at $t = 0$).

In case the offer is accepted the game ends. When the offer is rejected we move to the next period, $t+1$, and the roles of the parties reverse - now Player $j$ makes an offer, and Player $i$ reacts. In case of rejection the game continues with alternating roles of the parties up to the moment when an offer is accepted. If, however, an agreement is not reached both parties receive $p_1 = p_2 = 0$.

The value of the pie to both parties diminishes over time in such a way that Player $i$'s utility $\overline{G}_i : [0,1] \times \Theta \to \Re$ is

$$\overline{G}_i(p_i, t) = \delta_i^t p_i, \tag{2}$$

where $\delta_i \in (0,1)$ is a discount factor of Player $i$. Then, making his or her bargaining decisions, a negotiator must weigh the possible advantages following from longer bargaining (a possibly larger negotiated share) against the losses caused by the flying time (worse "taste" of the pie).

At the time when the outlined model has been suggested by Rubinstein (1982), its novelty consisted in the fact that the bargaining process was explicitly modelled as a sequence of moves and counter-moves which follow from the non-cooperative behavior of negotiators.

Since it is assumed that the parties have complete knowledge of the game, and they remember all the past moves, then a strategy for Player $i$ is a collection of mappings $\sigma_i = \{\sigma_i^t\}_{t=0}^{\infty}$, where $\sigma_i^t$ is a function of the history of the game, i.e.

$$\sigma_i^t : \prod_{s=0}^{t-1} P \to P$$

or

$$\sigma_i^t : \prod_{s=0}^{t-1} P \to \{Y, N\},$$

depending on whether $t$ is even or odd. It can be proven that each partition $(p_1, p_2) \in P$ can be supported by a Nash equilibrium of the game. This means that Nash equilibrium is not an appropriate concept for the bargaining game under consideration. In each such equilibrium Player $i$ always offers $p_i$, and accepts offers that are not worse than $p_i$. Because of the diminishing, over time, value of the pie such a strategy may be seen as involving threats (of rejecting offers) which are not credible in latter periods.

That is the reason why the notion of subgame perfect equilibrium is more appropriate for the bargaining game. It can be proven (Rubinstein,1982; Sutton,1986) that there exists a unique equilibrium in such a game. If we denote equilibrium strategies by $(\sigma_1^*, \sigma_2^*)$ then

$$\sigma_1^{*t}(p^0, p^1, \ldots, p^{t-1}) = p_1^* \quad \text{for all} \quad (p^0, p^1, \ldots, p^{t-1}) \in \prod_{s=0}^{t-1} P$$
$$\text{if } t \text{ is even,} \qquad (3)$$

$$\sigma_1^{*t}(p^0, p^1, \ldots, p^t) = \begin{cases} Y & \text{if } p_1^t \geq q_1^* \\ N & \text{if } p_1^t < q_1^* \end{cases} \quad \text{for all } (p^0, p^1, \ldots, p^t) \in \prod_{s=0}^{t} P,$$

$$\text{if } t \text{ is odd.}$$

This means that Player 1 *always* proposes an agreement $p^* = (p_1^*, p_2^*) \in P$, and *always* accepts a partner's offer if and only if she is offered not less than $q_1^*$. Equilibrium strategy for Player 2 can be written in an analogous way, i.e. he always offers $q^*$ , and accepts $p^*$. $p^*$ and $q^*$ are the solutions of the following equations

$$\begin{aligned} \overline{G}_1(q_1^*, 0) &= \overline{G}_1(p_1^*, 1) \\ \overline{G}_2(p_2^*, 0) &= \overline{G}_2(q_2^*, 1) \end{aligned} \qquad (4)$$

An agreement generated by the above equilibrium strategies is $p^*$ and it is reached at $t = 0$, i.e. Player 1's offer is *immediately* accepted by Player 2. For the utilities of the form (2) we obtain $p_i^* = \frac{1-\delta_2}{1-\delta_1\delta_2}$.

The above outlined model has been extended in a number of ways. Van Damme, Selten, and Winter (1990) considered a situation when the pie can be divided only in finitely many different ways, Hoel (1987) analyses a case when the proposer is chosen randomly at each period, durable offers were introduced by Stahl (1990). An important extension of the model allows a player to opt out of the negotiations (Wolinsky,1987; Binmore,Shaked,and Sutton,1989; Osborne, Rubinstein,1990). The next two sections of this paper develop the model in another direction, namely we consider the negotiations as a process which takes place in a dynamic environment.

## 3. Bargaining in an Evolving System

One of the essential assumptions of Rubinstein's model concerns the stationarity of preferences, what means that for $t \in \Theta$, $p \in P$, $q \in P$ we have $\overline{G}_i(p_i, t) >$

$\overline{G}_i(q_i, t+1)$ if and only if $\overline{G}_i(p_i, 0) > \overline{G}_i(q_i, 1)$. In this section we resign from this assumption. The underlying idea is that the players negotiate within a system that evolves over time. This changes need not be stationary, and they may influence the players' utilities. It will be convenient to describe the situation in a somewhat different way than previously.

Namely, there is a certain set $U$ of possible agreements. The parties must jointly decide about the chŏice of an element of $U$. Negotiations can take place over the periods $t \in \Theta = \{0, 1, \ldots, T-1\}$, where $T$ is finite. The bargaining process proceeds as previously, i.e. as a sequence of alternating offers and reactions.

There is also a certain dynamic system, the state of which $x(t) \in X$ influences the parties' objectives. The evolution of the system's state is described by a mapping $f : \Theta \times X \to X$, i.e.

$$x(t+1) = f(t, x(t)) \ , \quad t = 0, 1, \ldots, T-1, \tag{5}$$

where $x(0)$ is given.

Then, the utility of Player $i$ can be expressed with $\hat{G}_i : \Theta \times X \times U \to \Re$. In other words, if the players achieve an agreement about $p \in U$, at time $t$, then they attain utility levels $\hat{G}_i(t, x(t), p)$, $i = 1, 2$, and the game ends. In case they do not reach an agreement up to $T$, the attained utility levels will be equal to zero.

If the parties do not reach an agreement until the time $t-1$, then at $t$ we have the following set of attainable payoffs:

$$\hat{S}^t = \{(y_1, y_2) : y_1 = \hat{G}_i(t, x(t), p), \ \ i = 1, 2, \quad \text{for all } p \in U. \tag{6}$$

We make the following assumptions about $\hat{S}^t$, $t = 0, 1, \ldots, T-1$ :
(A1) $\hat{S}^t$ is compact,
(A2) Pareto–frontier $P(\hat{S}^t)$ is continuous,
(A3) if $t > s$, and $y \in P(\hat{S}^t)$, then there exists $z \in P(\hat{Z}^s)$ such that $z \geq y$.[1]
In addition we normalize the payoffs so that $\hat{S}^T = \{(0, 0)\}$.

Let us denote an offer made at time $t$ by $p^B(t)$. Then, at the same period the reaction to this offer takes place, and we denote it by $r(t)$. At time $t$ players remember the whole history of negotiations, i.e. all offers and reactions up to that time. Bargaining at $t$ means, however, that all the past reactions were

[1]By $z \geq y$ we mean $z_i \geq y_i$, $i = 1, 2$.

negative ($= N$).Therefore, when considering information sets at $t$ we can confine our interest to past offers.

It may happen that there exist two different offers $p^B(t)$ and $q^B(t)$ such that they yield the same payoffs, i.e. a point $y^B(t) \in \hat{S}^t$, where $y_i^B(t) = \hat{G}_i(t, x(t), p^B(t)) = \hat{G}_i(t, x(t), q^B(t))$. However, since the evolution of the system, i.e. the state $x(t+1)$, and thus $\hat{S}^{t+1}$, are independent on the past offers, then instead of $p^B(t)$ or $q^B(t)$ we can use, in strategic considerations, merely $y^B(t)$.This will simplify the notation and analysis. Denote then the information set at $t$ by $\hat{H}^t = (y^B(0), y^B(1), \ldots, y^B(t-1))$.

The form of a bargaining strategy of Player $i$ is analogous to that in partition of a pie problem, e.g. for Player 1 we have a sequence $\tilde{\sigma_1} = \{\sigma_1^t\}_{t=0}^{T-1}$, where

$$\sigma_1^t : \hat{H}^t \rightarrow S^t \quad \text{if } t \text{ is even,}$$

$$\sigma_1^t : \hat{H}^t \rightarrow \{Y, N\} \quad \text{if } t \text{ is odd.}$$

*A subgame perfect equilibrium* in our game is a pair of strategies $(\tilde{\sigma}_1^*, \tilde{\sigma}_2^*)$ that induces in every subgame a Nash equilibrium of that subgame. Then, we can determine $(\tilde{\sigma}_1^*, \tilde{\sigma}_2^*)$ by a backward analysis.

To simplify the reasoning we assume that if Player $i$ has a choice between two agreements $(p, t)$ (i.e. $p \in U$ achieved at $t$) and $(q, s)$ such that they yield the same payoff to him, then he will choose the earlier agreement.

Without loss of generality we can assume for a while that $T$ is odd. This means that at $T-1$ it is Player 1's turn to make an offer. Since $S^T = \{(0,0)\}$, then Player 2 is ready to accept any outcome from the set $A(T-1) = S^{T-1}$. Then, Player 1 chooses an offer $y^B(T-1)$ from the set $\boldsymbol{y}(T-1)$ of her best offers,

$$\boldsymbol{y}(T-1) = \{y \in S^{T-1} : y = arg \max_{z \in A(T-1)} z_1\}.$$

Since $y^B(T-1) \in A(T-1)$ then $r(T-1) = Y$. In period $T-2$ Player 1's acceptance set is $A^{T-2} = \{y \in S^{T-2} : y \geq y^B(T-1)\}$, and Player 2 chooses his offer from $\boldsymbol{y}(T-2) = \{y \in S^{T-2} : y = arg \max_{z \in A(T-2)} z_2\}$.

In general, at $t$ we have: if there is Player $i$'s turn to make an offer then Player $j$'s *acceptance set* is:

$$A(t) = \{y \in S^t : y_j \geq y_j^B(t+1)\} \tag{7}$$

(by definition $y_j^B(T) = 0$, and Player $i$ chooses $y^B(t) \in \boldsymbol{y}(t)$, where the *set of best offers* of Player $i$ is

$$\boldsymbol{y}(t) = \{y \in S^t : arg \max_{z \in A(t)} z_i\}. \tag{8}$$

Finally, we obtain $\boldsymbol{y}(0)$, and each $y^B(0) \in \boldsymbol{y}(0)$ is immediately accepted. The above outlined process of obtaining $(\tilde{\sigma}_1^*, \tilde{\sigma}_2^*)$ is illustrated in Figure 1.

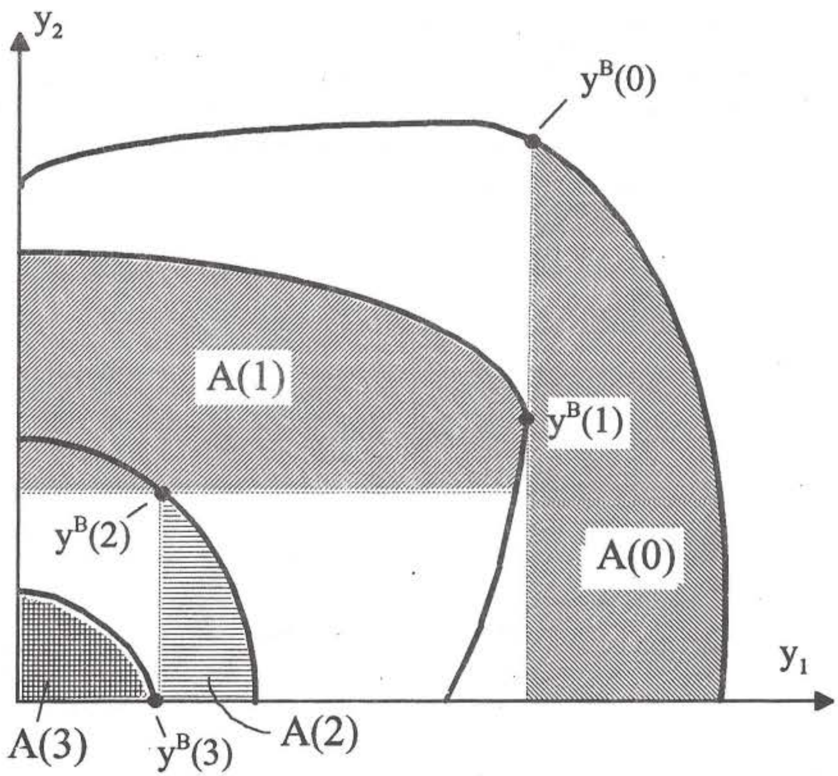


Figure 1. Illustration of the process of the determination of subgame perfect equilibrium strategies

It is worth emphasizing that if $\boldsymbol{y}(0)$ is not a singleton then we have many subgame perfect equilibrium outcomes in the game . This possibility could not take place in the partition of a pie game.

Note, that it appears that the sequences of subgame perfect equilibrium strategies have a relatively simple form. This, in particular, means that the equilibrium strategies at $t$ does not depend on the past history of bargaining. This does not mean, however, that these strategies are stationary. Recall that in the game over partition of a pie each player always proposed the same partition, and that his acceptance levels were stable. This is not the case in the game of this section.

We shall mention one more aspect. Up to now we have assumed that Player 1 makes an offer first (i.e. at $t = 0$). Suppose, however, that now she has a freedom to decide whether to move first or not. This yields the problem of whether it is advantageous to be the first proposer.

It appears that the answer to that question depends on the relative positions, and on shapes of the sets $S^0, S^1, \ldots, S^T$ of feasible outcomes. Figure 2 illustrates the two possible cases, i.e. when Player 1 prefers to begin negotiations, and when she would choose to be the second to move.

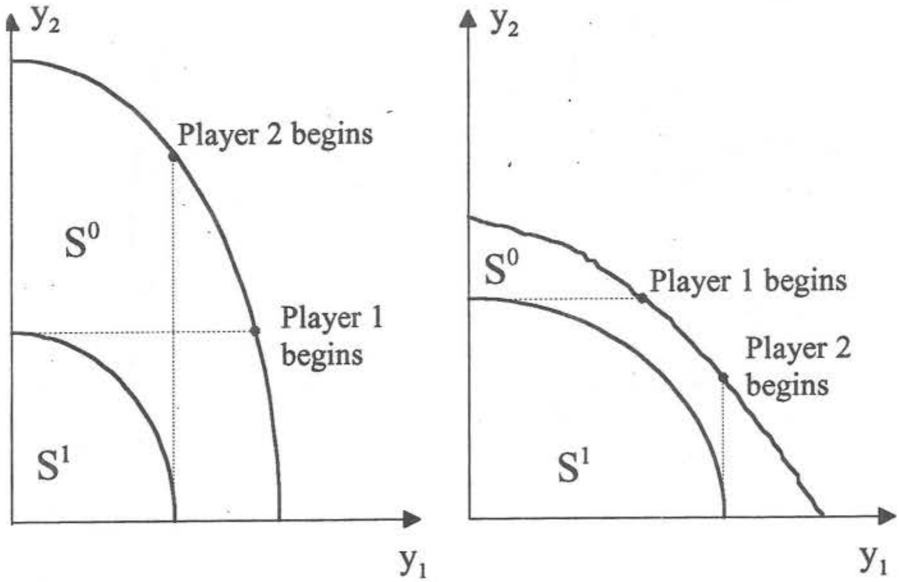


Figure 2. Illustration of the two possible types of solutions to the problem of whether to be the first proposer

# 4. Negotiations in a Controlled System

Now we shall analyze a situation which is an extension of the model from last section. Namely, now the dynamic system does not evolve independently of the players, it is controlled by the negotiating parties. And they negotiate over how to control the system. An important feature of the situation is that the bargainers can influence the system's evolution during the negotiation process. Such an approach has been suggested by Stefanski and Cichocki (1987), and then addressed in Houba (1989) and Cichocki and Stefanski (1990). It seems that to a similar family of models belongs the game introduced by Okada (1991), in which the parties are involved in playing a repeated game.

Player $i$'s decision vector applied at time $t$ is denoted by $u(t) \in U_i^t$, $i = 1, 2$. The dynamics of the system is described by a state equation

$$x(t+1) = F(t, x(t), u_1(t), u_2(t)), \tag{9}$$

$t = 0, 1, \ldots, T-1$. When the system is in a state $x(t)$, then its future evolution can be determined by decision sequences

$$\tilde{u}_i^t = (u_i(t), u_i(t+1), \ldots, u_i(T-1)), \quad i = 1, 2.$$

We assume the stage–additive form of the players' objectives:

$$G_i(t, x(t), \tilde{u}_1^t, \tilde{u}_2^t) = \sum_{s=t}^{T-1} g_i(x(s), u_1(s), u_2(s)). \tag{10}$$

We also assume the feedback information structure in the system, i.e. the players know the current state $x(t)$. *Control strategies*[2] are denoted by $\tilde{\gamma}_i^t = \{\gamma_i^s\}_{s=t}^{T-1}$, where $\gamma_i^s : X^t \to U_i^t$, $i = 1, 2$.

In order to facilitate further considerations we introduce a notation

$$J_i(t, x(t), \tilde{\gamma}_1^t, \tilde{\gamma}_2^t) = G_i(t, x(t), \tilde{u}_1^t, \tilde{u}_2^t), \tag{11}$$

where the control sequences $(\tilde{u}_1^t, \tilde{u}_2^t)$ are generated by strategies $(\tilde{\gamma}_1^t, \tilde{\gamma}_2^t)$, i.e. $u_i(s) = \gamma_i^s(x(s))$, $s = t, t+1, \ldots, T-1$, $i = 1, 2$.

At the beginning of each time period $t$ before reaching an agreement the parties bargain, i.e. Player $i$ makes an offer, and Player $j$ reacts (or vice versa).

[2]Please note the difference between control strategies and bargaining strategies. Implementation of the former control the the system's evolution, while implementation of bargaining strategies are the sequences of offers and reactions in negotiations.

Then, if an agreement is reached it is immediately applied up to $T$. If, however, players do not decide to cooperate, the decisions $u_k^P(t)$ generated by *preagreement strategies* $\tilde{\gamma}_k^P = \{\gamma_k^{Pk}\}_{t=0}^{T-1}$, $k = 1,2$, are implemented. The choice of preagreement strategies will be addressed later. In case the parties do not reach an agreement at all, the outcome of the game will be

$$y_i^P = J_i(0, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P), \quad i = 1,2. \tag{12}$$

Note, that now this outcome is not given exogenously, but it depends on the players' strategies.

Let us denote $Y$

$$Y_i(t, x(0), \tilde{\gamma}_1, \tilde{\gamma}_2) = J_i(0, x(0), \tilde{\gamma}_1, \tilde{\gamma}_2) - J_1(t, x(t), \tilde{\gamma}_1^t, \tilde{\gamma}_2^t), \tag{13}$$

where $\tilde{\gamma}_i^t$ is a subsequence of $\tilde{\gamma}_i$, and $x(t)$ is a point on the system's trajectory generated by $(\tilde{\gamma}_1, \tilde{\gamma}_2)$.

If an agreement is made at time $t \in \Theta$, and it concerns the application of strategies $(\tilde{\gamma}_1^{A\tau}, \tilde{\gamma}_2^{A\tau})$, then the final ayoffs are:

$$y_i = Y_i(\tau, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P) + J_1(\tau, x^P(\tau), \tilde{\gamma}_1^{A\tau}, \tilde{\gamma}_2^{A\tau}), \quad i = 1,2, \tag{14}$$

where the state $x^P(\tau)$ results from applying $(\tilde{\gamma}_1^P, \tilde{\gamma}_2^P)$ up to $\tau - 1$. It is clear that in this case the players' control strategies will consist partly of $\tilde{\gamma}^P$ and partly of $\tilde{\gamma}^A$.

Suppose now, that we are at time $t$, before an agreement was made. When negotiating, the players must know the set of payoffs attainable at $t$:

$$\begin{aligned} S^t &= S(t, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P) = \\ &= \{(y_1, y_2) : y_i = Y_i(t, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P) + J_i(t, x^P(t), \tilde{\gamma}_1^t, \tilde{\gamma}_2^t), \\ &\qquad \text{for all} \quad \tilde{\gamma}_1^t \in \tilde{\Gamma}_1^t, \tilde{\gamma}_2^t \in \tilde{\Gamma}_2^t, \ i = 1,2\} \end{aligned} \tag{15}$$

where $\tilde{\Gamma}_i^t$ is a set of admissible $\tilde{\gamma}_i^t$.

Note that in this section we have not make assumptions about the sets from the sequence $\tilde{S} = \{S^t\}_{t=0}^T$. In particular it is important to check whether the sets from $\tilde{S}$ posses the property analogical to assumption A3 in the last section. The reason is that if $\tilde{S}$ did not have that property then the agreement resulting from subgame perfect equilibrium would not be achieved at $t = 0$. The following proposition answers the above question.

PROPOSITION 1 *For any time instants $t, s \in \Theta$, such that $t > s$ and for every $y^t \in S^t$ there exists a pair of outcomes $y^s \in S^s$ such that $y^s \geq y^t$.*

*Proof.* Denote by $y^{Pt}$ a point (on the plane of utilities) which belongs to a trajectory yielded by preagreement strategies, i.e. $y_i^{Pt} = Y_i(t, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P)$, $i = 1, 2$, and analogously denote $y^{Ps}$. Both $y^{Pt}$ and $y^{Ps}$ lie on the same trajectory generated by $(\tilde{\gamma}_1^P, \tilde{\gamma}_2^P)$. By definition the set $S^s$ includes terminal points of all trajectories for which their initial part, from $y^{P0}$ to $y^{Ps}$ is generated by $(\tilde{\gamma}_1^P, \tilde{\gamma}_2^P)$. Among these, there are also all such trajectories which over the time from $s$ to $t$ coincide with the trajectory yielded by $(\tilde{\gamma}_1^P, \tilde{\gamma}_2^P)$. Thus, the end points of the above trajectories, which constitute the set $S^t$, belong to the set $S^s$ :

$$S(t, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P) \subseteq S(s, x(0), \tilde{\gamma}_1^P, \tilde{\gamma}_2^P), \quad \text{for} \quad t > s \tag{16}$$

From (16) the proposition straightforward follows ■

The reasoning of the proof is illustrated in Figure 3. The process can be followed when we look at a point $y^{Pt}$ which moves along the trajectory generated by preagreement strategies.

As follows from (16) with the elapsing time the set of attainable payoffs is diminishing, and because of the feature described by Proposition 4.1 it is not beneficial to negotiate too long. This proposition emphasizes the advantage of reaching an early agreement over long–lasting bargaining, and reveals the fact that time is of value to both players.

For given preagreement control strategies the sequence $\tilde{S}$ of the sets of feasible outcomes is fixed. Then, because we proved proposition 4.1, further analysis of the alternating–offer bargaining process is analogous to that in the previous section.

There is, however, one difference in $\tilde{S}$ we want to mention. Namely, when parties negotiate in an evolving system then achieving an agreement even in very late periods, up to $T - 1$, makes sense. This is not always the case, however, when bargaining in a controlled system. In such a system there is no use to talk about an agreement at time periods $t > T_B$, where

$$T_B = max\{t \in \Theta : \bar{S}^t \neq \emptyset\}, \tag{17}$$

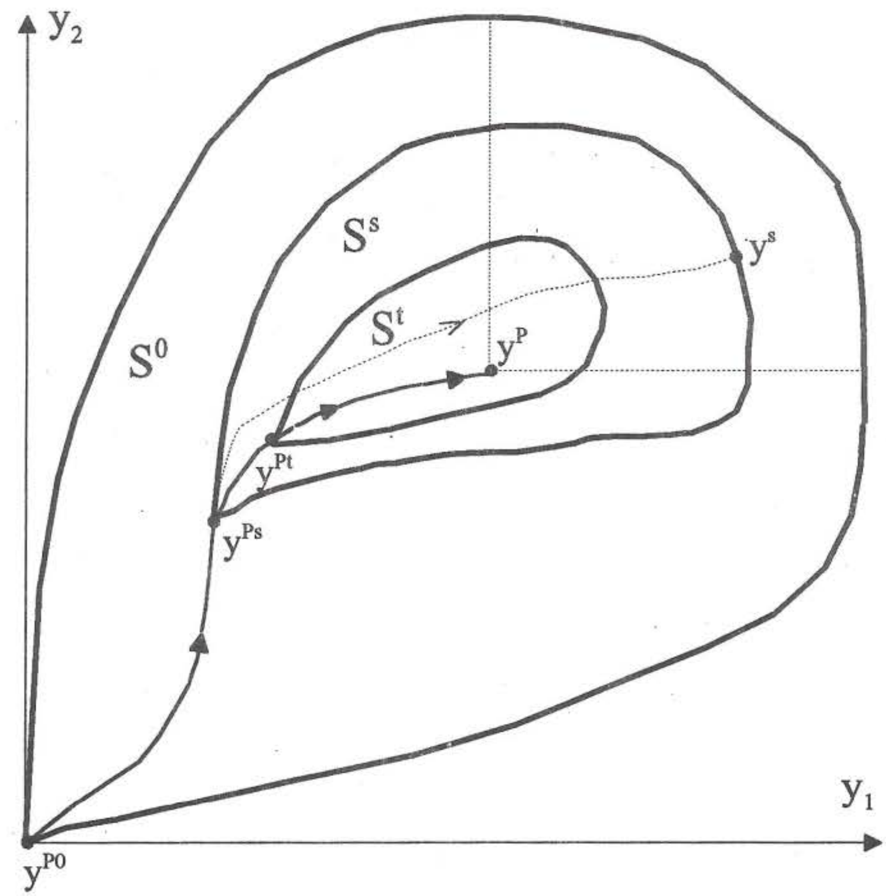


Figure 3. The sets of feasible agreement outcomes, changing during a negotiation process

where, in turn

$$\bar{S}^t = \{y \in S^t : y > y^P\}, \tag{18}$$

where $y^P$ is the end of the preagreement trajectory, determined by (12).

A natural choice of preagreement strategies is the feedback Nash equilibrium. Such an approach is very likely to be adopted by real life bargainers since it reflects what would happen if the parties did not *try* to negotiate. We can imagine, however, highly sophisticated players who take into account the fact that $(\tilde{\gamma}_1^P, \tilde{\gamma}_2^P)$ influence the shape and the positions of the sets $S^0, S^1, \ldots, S^{T-1}$ as well as the position of $y^P$. The corresponding problem they must solve in such a case will be addressed in another paper.

Here we illustrate only one aspect. Let us come back for a moment to the problem of deciding whether it is advantageous to begin negotiations. In the context of negotiations in a controlled system an answer to that question may depend on the choice of preagreement strategies. Such a situation is illustrated in Figure 4.

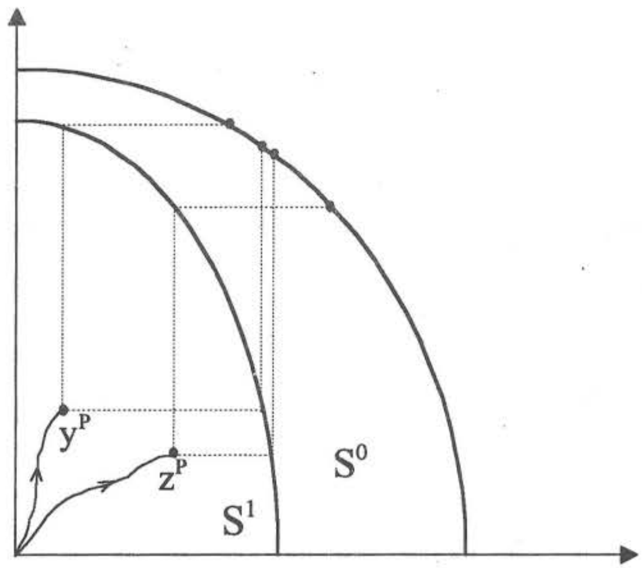


Figure 4. It may depend on preagreement strategies whether it is advantageous to begin negotiations with a first offer

## 5. Concluding Remarks

In the paper we introduce an extension of the strategic bargaining model to the case when the parties' goals are connected with performance of a dynamic system. We develop the model in two stages: first, the system evolves independently of the players, and second, when players control the evolution of the system. The differences in comparison with the partition of a shrinking pie problem are emphasized, as well as the strategic differences between the two analyzed situations. Subgame perfect equilibria for the extended case have been derived and the specific problems, which do not arise in simpler situations, are emphasized.

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