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On Construction of a Cooperative Fuzzy Game in International Fuzzy Decision Environments : a Possibilistic Approach

by

Fumiko Seo and I. Nishizaki

Kyoto Institute of Economic Research,
Kyoto University,
Yoshida-honmachi, Sakyoku, Kyoto, 606, JAPAN

A construction of an n-person cooperative fuzzy game in international fuzzy decision environments is considered. The international decision environments for a game is examined in two-fold : internal and external for a coalition, in which group decision situations are explicitly taken into account. The fuzzy game is constructed on diversified evaluations in group decision making for assessing the characteristic functions of the game. The diversification of evaluation is represented as possibility distribution in fuzzy set theory. The optimizing rule for fuzzy decisions is applied for the construction of a fuzzy game and the properties of its solutions, nucleolus and augmented nucleolus, are examined. In particular, importance for the coalitions to consider external information from outside coalitions is suggested.

1. Introduction

This paper intends to construct an n-person cooperative fuzzy game for international conflict solving in fuzzy decision environments, in which multiple decision makers exist.

The existence of group decision making in international decision environments is a source of fuzziness in assessing the value of a cooperative game among countries. This paper treats the fuzziness in international decision making for conflict solving in terms of the possibility distribution functions in fuzzy set theory.

The fuzziness in the international decision environments is twofold: internal and external. Let $N \triangleq \{1, 2, \ldots, n\}$ be the set of all countries under the considerations, which are defined as players in a game and T denotes a collection of non–empty subsets S of N, where S is called an international coalition. The fuzziness in the internal decision environments is formed by the existence of multiple decision makers in coalitions $S \subseteq T \subseteq N$. The fuzziness in the external decision environments is formed by the existence of the external coalitions, or "adverse coalitions" (Aubin 1982), $\bar{S} \triangleq T - S$, $\bar{S} \subset T \subseteq N$. $\bar{S}$ also includes multiple decision makers.

The fuzziness in these international decision environments occurs from diversifications of multiple evaluation for a game in coalitions S and external coalitions $\bar{S}$. An explicit recognition of the fuzziness due to the varied evaluation leads to construct an optimizing decision. The fuzzy set theory, developed by Zadeh, etc., has defined "optimizing" decisions in a fuzzy environment on the membership function $\mu_{\tilde{D}}(x_{opt})$ using the maximin rule (Bellman & Zadeh 1970):

$$\mu_{\tilde{D}}(x_{opt}) = \max_x \min \{\mu_o(x), \mu_c(x)\} \tag{1}$$

or

$$x_{opt} = arg \left(\max_x \min \{\mu_o(x), \mu_c(x)\} \right), \tag{2}$$

where x is a decision variable and $\mu_o(x)$ and $\mu_c(x)$ are the membership functions of an objective and a constraint, respectively. This paper concerns constructing an n–person cooperative fuzzy game using the fuzzy decision rule in the multiple decision environments.

Consider an n–person cooperative game in characteristic function form. Denote $S \subseteq N \triangleq \{1, 2, \ldots, n\}$ as a coalition of players and $\mathcal{S}$ as a family of coalitions, which is a collection of all coalitions including the S. Then an n–person cooperative game in characteristic function form is described with a correspondence $(S, J(S))$, $S \subseteq N$, mapping coalitions $S \in \mathcal{S}$ into feasible subsets $J(S) \subset R^S$ of multiutilities (multigains) of the coalitions S. We denote the game as $\Gamma \triangleq (\mathcal{S}, J(S))$.

A cooperative fuzzy game has been constructed by Aubin (1981, 1982), Butnariu (1978), etc. on the concept of a fuzzy coalition which is defined with the rate of participation τ of a player i in a coalition S, $S \subset N$, and denoted as $\tau_i^S \in [0,1]$, instead of $\tau_i^S \in \{0,1\}$ in the case of crisp coalitions. The fuzzy game is described as $F\Gamma = (\mathcal{T}, J(\tau))$, $\tau \in \mathcal{T}$, where $\mathcal{T} \subset [0,1]^n$ denotes a family of fuzzy coalitions τ. In this model, values of the characteristic function $J(\tau) \subset R^\tau$ have still been assessed as the crisp numbers, where $R^\tau \triangleq \tau \cdot R^n = R^{S_\tau}$ and S_τ denotes the set of active players, the support $S_\tau = \{i \in N,\ \tau_i > 0\}$, in the fuzzy coalition.

This paper presents a new concept of an n–person cooperative fuzzy game which introduces the fuzziness in the values of the characteristic function while the rates of participation of players i, $i \in S \subset N$, are treated as bivalent, $\tau_i \in \{0,1\}$. Thus the cooperative fuzzy game is constructed as $F\Gamma \triangleq (\mathcal{S}, \tilde{J}(S))$ where $\tilde{J}$ denotes the fuzzy characteristic functions as multivalent real–valued functions mapping the coalitions $S \in \mathcal{S}$ into feasible fuzzy subsets $\tilde{J}(S) \subset R^S$ of multiutilities (multigains) of the coalitions S. $\tilde{J}(S)$ is the fuzzy coalition value. The fuzziness of $\tilde{J}(S)$ is assumed to come from the diversification of multiple evaluation in the decision environments and evaluated as a possibility distribution.

In the next section, formation of the cooperative fuzzy game in the fuzzy decision environments is discussed. In Section 3, a method of "optimizing" the evaluation of fuzzy coalition values is examined. In Section 4, properties of the solution of the fuzzy games are discussed. In Section 5, a numerical example of the optimizing decisions in the fuzzy environments is presented. Finally concluding remarks are provided with a group decision interpretation embodying external information.

2. Formation of a cooperative fuzzy game in the international fuzzy environments

Now we shall consider the fuzzification rule of the characteristic function of a game in the international fuzzy decision environments. The fuzziness in the assessment of characteristic functions is represented with the possibility distributions. Possibility theory, developed by Zadeh (1978), Dubois and Prade (1985), etc., concerns the quantification of judgement and treats imprecise information for an event as a matter of degree under the possibility measure. A diversification of evaluation by multiple decision makers can properly be represented by a possibility distribution. Let x^S be multiutility of a coalition $S \subseteq N$ and x_i be a value of utility of a player $i \in S$ for the coalition $S \subseteq N$. We treat x_i as a fuzzy variable assessed in the internal and external fuzzy environments. The fuzziness is assessed with the possibility distributions of the multiutility x^S. The possibility distribution is constructed as

$$\Pi_{X^S} \triangleq \{(x_i, \pi_{X^S}(x_i))\}, \quad i \in S, \tag{3}$$

or, in the vector representation,

$$\Pi_{X^S} \triangleq (x, \pi_{X^S}(x)), \tag{4}$$

where $x_i \in x \subseteq R^n$ and π_{X^S} is the possibility distribution function defined with the membership function $\mu_{\tilde{J}(S)}(x)$ of the fuzzy coalition value $\tilde{J}(S)$,

$$\pi_{X^S}(x) = \mu_{\tilde{J}(S)}(x). \tag{5}$$

The fuzziness in the characteristic functions, which is introduced from diversified evaluation for the coalition values, is formed in the internal and external fuzzy decision environments. The membership function represents the degree of evaluation for x^S and $x^{\bar{S}}$ by multiple decision makers.

The value of the fuzzy characteristic function $\tilde{J}(S)$ (the fuzzy coalition value) is treated in terms of transferable utility. In other words, the rule of the game presumes that utility of a game is transferable and shared by players i, $i \in S$. The rates of transfer for the multiutility play the same role as the price for the commodities (Aubin 1982). A cooperative game whose utility is measured in the side–payments is described as $\Gamma \triangleq (\mathcal{S}, v(S))$. As a natural extensions of the game, a cooperative fuzzy game with the side–payments is described as $F\Gamma \triangleq (\mathcal{S},\tilde{v}(S))$ where the fuzzy coalition value $\tilde{v}(S)$ is assessed in terms of its diversification with the possibility distribution function

$$\pi_{X^S}(x) = \mu_{\tilde{v}(S)}(x). \tag{6}$$

3. "Optimizing" evaluation of the value of coalitions

Before derivation of solution concepts of the fuzzy cooperative game $F\Gamma \triangleq (\mathcal{S},\tilde{v}(S))$, the fuzzy coalition value $\tilde{v}(S)$ for S should have been "optimized" in advance with the maximin decision rule in the fuzzy decision environments. Let $\mu_S(x) \triangleq \mu_{\tilde{v}(S)}(x)$. The optimal decision $\tilde{D}^S$ for multiutility x^S in a coalition S is defined by

[Optimizing rule]

$$\mu_{\tilde{D}^S}(x_{opt}) = \max_x \min\{\mu_S(x), \mu_{\hat{S}}(x)\} \tag{7}$$

where $\mu_{\hat{S}}(x) \triangleq \mu_{\hat{v}(S)}(x)$ and $\hat{v}(S)$ is the characteristic function of S assessed by the external coalition. $\hat{v}(S)$ shows external information concerning the assessment of $v(S)$ for a coalition S. We will define the external information for the characteristic function $v(S)$ as

$$\hat{v}(S) \triangleq \bigcup_{S \subset T \subseteq N} (\tilde{v}(T) \ominus \tilde{v}(T-S)). \tag{8}$$

By the bounded difference rule (Zadeh 1975),

$$\mu_{\hat{S}}(x) = \max_{S \subset T \subseteq N} \{\max(0, \mu_T(x) - \mu_{T-S}(x))\}, \tag{9}$$

where $\mu_T(x) \triangleq \mu_{\tilde{v}(T)}(x)$ and $\mu_{T-S}(x) \triangleq \mu_{\tilde{v}(T-S)}(x)$. The implication of $\hat{v}(S)$ is an "aggregate" of the external evaluation for the coalition value of S by members of outside coalitions $\bar{S} \subset T \subseteq N$. The evaluation of the characteristic functions $v(S)$ and $\hat{v}(S)$ is "fuzzified" due to the existence of multiple evaluation in the internal and external decision environments and the diversification of evaluation by the multiple decision makers in every coalition, assessed in the form of the possibilistic distributions, represented by membership functions $\mu_S(x)$ and $\mu_{\hat{S}}(x)$, respectively, using (3) and (6).

Let X be a feasible set of multiutilities. The fuzzy "optimal" value $x_{opt} \in \tilde{D}^S \subset X$ for a coalition S is characterized by its membership function $\mu_{\tilde{D}^S}(x_{opt})$ with the optimizing rule (7).

Notice that $\mu_S(x)$ and $\mu_{\hat{S}}(x)$ can be treated as the fuzzy numbers. Then the equation (9) can be rewritten as

$$\mu_{\hat{S}}(z) = \max_{S \subset T \subseteq N} \left\{ \sup_{z=x-y} \min\{\mu_T(x), \mu_{T-S}(y)\} \right\}. \tag{10}$$

In particular, the L-R type representation of fuzzy numbers (Dubois and Prade 1978, 1980),

$$M \triangleq (m, \gamma, \delta)_{LR}, \tag{11}$$

can be constructed through a $[0,1]$-normalization of the $\mu_S(x)$-values and with regression analysis to yield a least square estimation, if desired, and used for fast computations. In the particular form,

$$\mu_M(x) = \begin{cases} L((m-x)/\gamma) & \text{for } x \le m, \gamma > 0, \\ R((x-m)/\delta) & \text{for } x \ge m, \delta > 0, \end{cases} \tag{12}$$

where m is the mean value of M and γ and δ denote left and right spreads, respectively. The formula for subtraction of two fuzzy numbers , M and N, is

$$(m, \gamma, \delta)_{LR} - (n, \zeta, \eta)_{LR} = (m - n, \gamma + \eta, \delta + \zeta)_{LR}. \tag{13}$$

The optimizing evaluation $\mathring{v}\ (S)$ of multiutility for a coalition $S \subseteq N$ in the fuzzy decision environments proceeds with the following behavioral rules: Let α be an acceptable threshold level of the $\mu_S(x)$-value for the coalition S on consideration of the external information.

[Behavioral rules]

(I) Conjoint case of $\tilde{v}(S)$ and $\hat{v}(S)$.

(1) When $\max_x \min\{\mu_S(x), \mu_{\hat{S}}(x)\} > \alpha$,

$$\mathring{v}\ (S) = x_{opt} = \max\left\{x \mid \max_x \min\left(\mu_S(x), \mu_{\hat{S}}(x)\right)\right\} \tag{14}$$

(see Figure 1(A)). Equation (14) assures the uniqueness of x_{opt}-value. Due to the supperadditivity of the characteristic functions, usually $x_{opt} \le \hat{x}^{hgt}$, where $\hat{x}^{hgt} = \mu_{\hat{S}}^{-1}(\text{hgt}(\hat{S}))$ and $\text{hgt}(\hat{S}) \triangleq \text{hgt}(\hat{v}(S)) = \sup_{x \in X} \mu_{\hat{S}}(x)$.

(2) When $\max_x \min\left\{\mu_S(x), \mu_{\hat{S}}(x)\right\} \le \alpha$, with an assigned $\bar{\beta}$-value, $\bar{\beta} \le \alpha$,

$$\text{(i)} \quad \mathring{v}\ (S) = x^R_{\beta-opt} = \max\left\{x \mid \mu_S(x) \ge \bar{\beta}\right\} \quad \text{for} \quad x^R_{\beta\text{-}opt} \le \hat{x}^{hgt} \tag{15}$$

(see Figure 1(B)). Exceptionally in an inefficient case,

$$\text{(ii)} \quad \mathring{v}\ (S) = x^L_{\beta\text{-}opt} = \min\left\{x \mid \mu_S(x) \ge \bar{\beta}\right\} \quad \text{for} \quad x^L_{\beta\text{-}opt} \ge \hat{x}^{hgt}. \tag{16}$$

The β-value is a revised acceptance level of $\mu_S(x)$ for $\tilde{v}(S)$ on consideration of the external information $\hat{v}(S)$ in the assessment of $v(S)$ and is obtained as a result of a compromise in the coalition S.

(II) Disjoint cases of $\tilde{v}(S)$ and $\hat{v}(S)$.

$$\overset{\circ}{v}(S) = x^R_{\beta\text{-}opt} = \max\{x \mid \mu_S(x) = \beta \le \bar{\beta}\} \tag{17}$$

with an assigned $\bar{\beta}$-value, $\bar{\beta} \le \alpha$, and

$$\bar{\beta} \ge \beta(m \parallel \tilde{v}(S), \hat{v}(S) \parallel) \ge 0, \tag{18}$$

where $m \parallel \tilde{v}(S), \hat{v}(S) \parallel$ denotes a "distance" of $\tilde{v}(S)$ from $\hat{v}(S)$ (Figure 2). A β-value is assessed depending on the distance and is set as $\beta = f(m)$ where f is a decreasing function of m.

We can use the fuzzy number calculation (subtraction) for evaluation of the distance. Let $\tilde{v}(S) = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$ and $\hat{v}(S) = (\underline{n}, \overline{n}, \zeta, \eta)_{RL}$, where $\underline{m}, \underline{n}$ and $\overline{m}, \overline{n}$, are the lower and upper modal values of m and n as the mean values, respectively. The $m \parallel \tilde{v}(S), \hat{v}(S) \parallel$ is represented with the fuzzy distance function as a subtraction of two fuzzy numbers,

$$\tilde{d}^S(x) \triangleq \mid \tilde{v}(S) - \hat{v}(S) \mid = (\mid \underline{m} - \overline{n} \mid, \mid \overline{m} - \underline{n} \mid, \gamma + \eta, \delta + \zeta)_{LR}. \tag{19}$$

As a scalarized, "parametric" representation for the fuzzy distance evaluation, we can get, for example,

$$m \parallel \tilde{v}(S), \hat{v}(S) \parallel \triangleq \arg\left[\text{hgt}\left(\tilde{d}^S(x)\right)\right] = \arg\left(\sup_{x \in X} \mu_{\tilde{d}^S}(x)\right) \tag{20}$$

(see Figure 3).

With the device of "parameterization" of the optimizing decisions for fuzzy evaluation of the coalition values, derivation of solutions of the fuzzy cooperative game is straightforward.

4. Derivation of solutions for the fuzzy cooperative game

Solution concepts of the fuzzy cooperative game are examined in terms of the core and the nucleolus with the "parameterized" (crispened) values of the fuzzy characteristic functions.

Define the fuzzy excess of a coalition S in the fuzzy game $F\Gamma \triangleq (\mathcal{T}, \overset{\circ}{v}(S))$, $S \subseteq N$, as

$$\tilde{e}(S,x) \triangleq \overset{\circ}{v}(S) - \sum_{i \in S} x_i, \quad S \subset N, \tag{21}$$

where $\overset{\circ}{v}(S)$ denotes the optimal value of a coalition S under the fuzzy decision environments.

The core of the fuzzy game is defined with the coalitional rationality:

$$\tilde{e}(S,x) \leq 0, \tag{22}$$

and the collective rationality:

$$\sum_{i \in N} x_i = \overset{\circ}{v}(N). \tag{23}$$

where $\overset{\circ}{v}(N)$ is assessed as the mean value of the fuzzy number $\tilde{v}(N)$ which represents a possibility distribution of evaluation for the grand coalition N. In the equations (22) and (23), decisions in the fuzzy environments are "defuzzified." The core does not always exist. Then, by relaxing (22) in the definition of the core, the concept of a quasi-core (ε–core) is introduced. The (strong) ε–core is defined with

$$\tilde{e}(S,x) \leq \varepsilon \qquad \text{for all } S \neq \phi, N, \tag{24}$$

and (23).

The nucleolus of the fuzzy cooperative game whose coalition values are represented by the possibility distributions is defined as

$$\begin{aligned} \mathcal{N}(F\Gamma) \;\triangleq\; & \{x \mid H(\tilde{e}(S_1,x),\ldots,\tilde{e}(S_{2^n-2},x)) \\ & \leq_L H(\tilde{e}(S_1,y),\ldots,\tilde{e}(S_{2^n-2},y)), \forall y \in X(F\Gamma)\}, \end{aligned} \tag{25}$$

where $H : R^{2^n-2} \rightarrow R^{2^n-2}$ and $X(F\Gamma)$ is a set of imputations $x \in R^n$ of the fuzzy game, i.e.,

$$X(F\Gamma) = \left\{ x \mid x_i \geq \overset{\circ}{v}(\{i\}),\ \forall i \in N,\ \sum_{i \in N} x_i = \overset{\circ}{v}(N) \right\}. \tag{26}$$

In $\overset{\circ}{v}(\{i\}) \triangleq \tilde{v}(\{i\})$, the fuzziness comes from imprecise recognition about the $v(\{i\})$-value, which occurs from a shortage of proper information due to the isolation of a player i. The $\leq_L$ denotes a lexicographic ordering. The existence of the nucleolus and its uniqueness can be proved straightforwardly after the defuzzification (Schmeidler 1969).

5. Numerical Example

A numerical example of the decision environments for five countries is presented, which is formulated as a five person cooperative fuzzy game. The characteristic function $\tilde{v}(S)$ is assessed with a possibility distribution of multigains x^S for international coalitions $S \subseteq N = \{1,2,3,4,5\}$. Formation of the coalitions increases total gains (or decreases total losses) of players in S. The fuzzy coalition value $\tilde{v}(S)$ is assessed with a possibility distribution which is represented as an L-R type fuzzy number and depicted in Table 1. The values for $v(T), v(T-S), \hat{v}(S)$ and $\overset{\circ}{v}(S)$ are assessed in the processes of deriving the optimizing decisions. Figure 4 explains the assessment of a $\overset{\circ}{v}(S)$-value. Figure 5 depicts the evaluation of the disjoint case.

In the conjoint case (I–(1) in Section 3), the threshold level of acceptance for the values of the membership function of the derived optimal coalition values $\overset{\circ}{v}(S) = x_{opt}$ is set as $\alpha = 0.4$. In the other conjoint case (I–(2)), a revised acceptance level by a compromise in S is set as $\bar{\beta} = 0.3$ and the optimal evaluation for $\mu_S(x) \geq \bar{\beta}$ derives the corresponding $\overset{\circ}{v}(S)$-value in (15).

In the disjoint case II, a new acceptance level based on the external information $\beta \leq \bar{\beta} = 0.3$ is assessed as a linear function of the fuzzy distance evaluation from (18) using the L-R type fuzzy number calculation (19). In particular, this example uses the following definition,

$$m \parallel \tilde{v}(S), \hat{v}(S) \parallel = \min_{x} \left[\arg\left\{\mathrm{hgt}(\tilde{d}^S(x))\right\}\right], \quad S \in \bar{\bar{S}}, \tag{27}$$

where $\bar{\bar{S}}$ denotes a family of disjoint coalitions, S and $\bar{S}$. Let

$$\ell_0 = \min_{S \in \bar{\bar{S}}} \left(m \parallel \tilde{v}(S), \hat{v}(S) \parallel\right), \tag{28}$$

$$\ell_1 = \max_{S \in \bar{\bar{S}}} \left(m \parallel \tilde{v}(S), \hat{v}(S) \parallel\right). \tag{29}$$

The β-value for a coalition S is calculated with

$$\beta_S = -\frac{3m \parallel \tilde{v}(S), \hat{v}(S) \parallel}{10(\ell_1 - \ell_0)} + \frac{3\ell_1}{10(\ell_1 - \ell_0)}. \tag{30}$$

The β-value for a characteristic function $\tilde{v}(S)$ of a coalition S varies between the maximum value 0.3 and the minimum value 0.0 proportionally to the distance m. The result is summarized in Table 2 and Table 3. Finally, the nucleolus as a solution concept of this game is obtained by solving the linear programming problem repetitively. The value of the nucleolus for each player is shown and compared with a non-fuzzy case in Table 4.

6. Concluding remarks

An n-person cooperative game theory has been applied for an international conflict resolution problem and solutions for the nucleolus have been presented (Seo and Sakawa 1990). This paper extends this research for the international fuzzy decision environments in which the existence of multiple decision makers are explicitly considered. The existence of external evaluation by the outside coalitions $\bar{S}$ are often crucial for the assessment of the value of coalitions in the complex and value–conflicting real world. When only internal evaluation for the coalition values of a game is taken into account, the simple majority rule or its variations may be applied. This situation may incline to use restricted information for the coalition. This paper presents an alternative approach which intends to modify the internal evaluation for a coalition S with the intentional utilization of external information formed by the outside coalitions $\bar{S}$. An investigation in this paper suggests that the values of coalitions will be usually underestimated when the external evaluation is not taken into account, if the superadditivity is satisfied (compare Table 1 with Table 2). Thus the introduction of the external considerations has an effect on the nucleolus values and brings a different allocation among countries.

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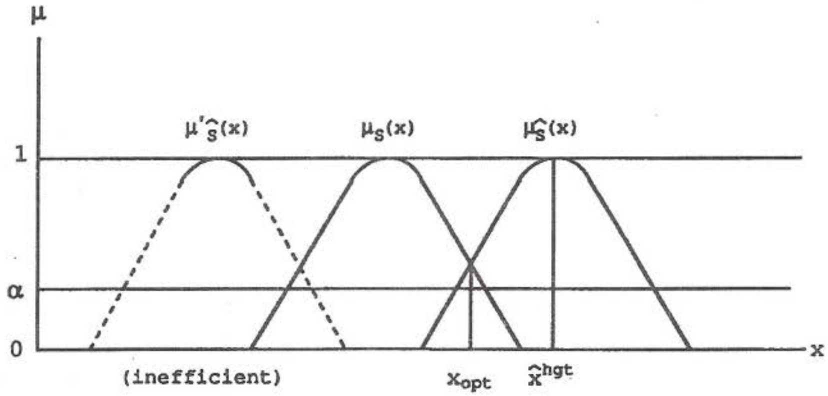
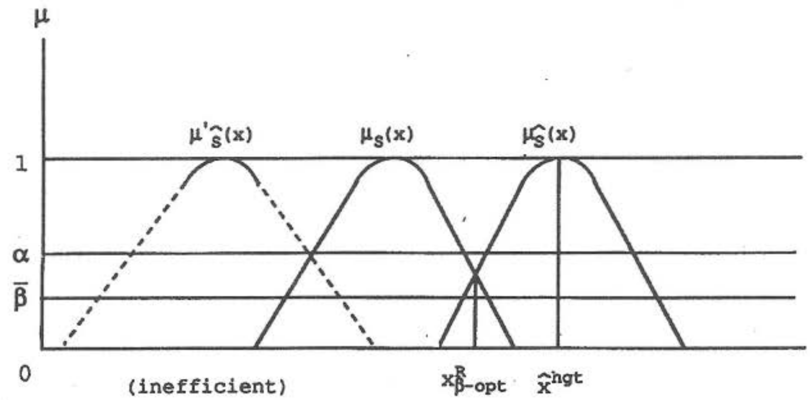


(A)



(B)

Figure 1. The optimizing evaluation $\overset{\circ}{v}\,(S)$ of multiutility (Conjoint case).

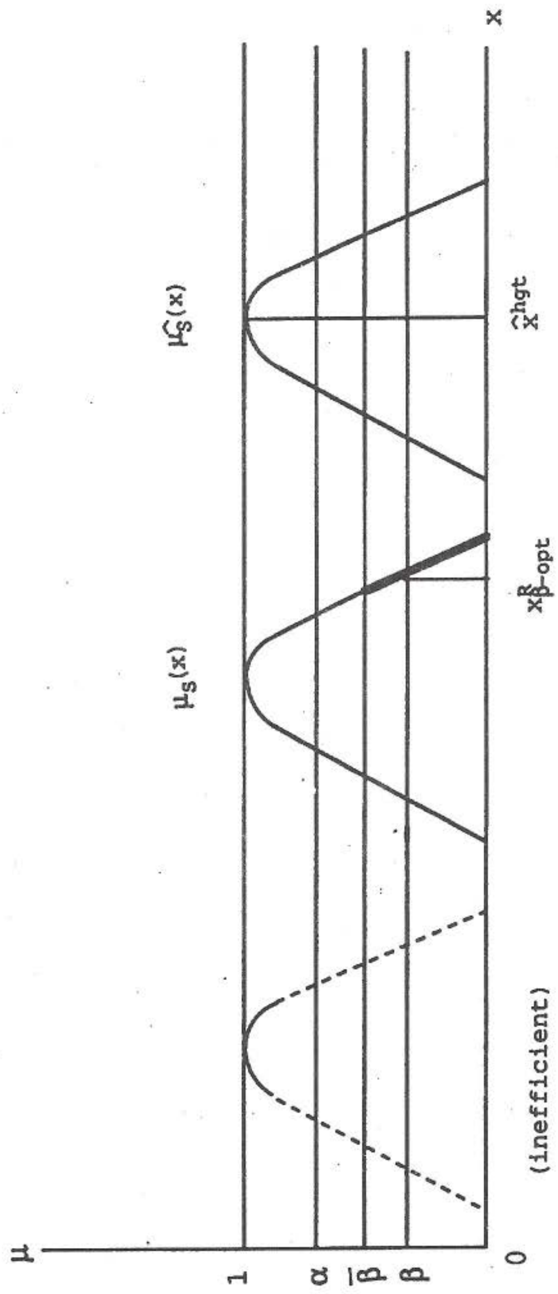
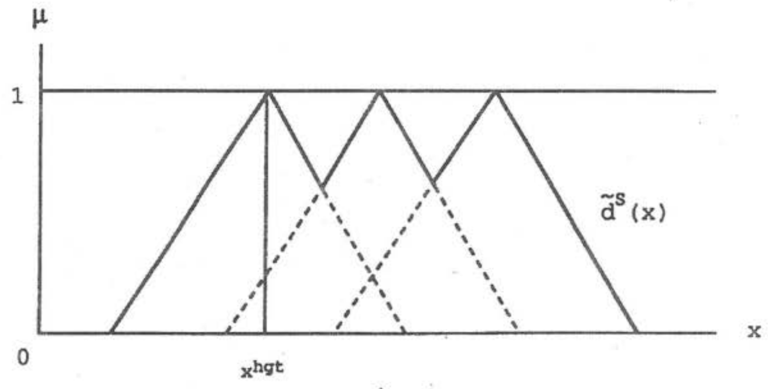
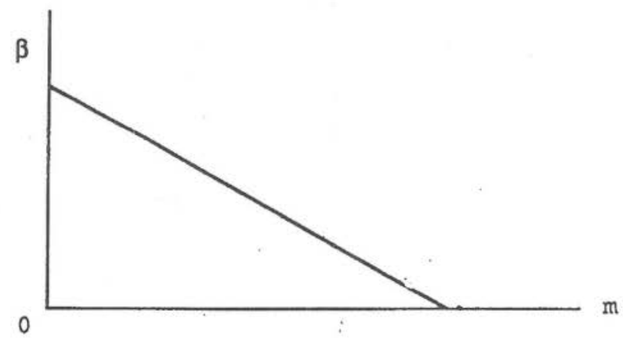


Figure 2. The optimizing evaluation $\mathring{v}\ (S)$ of multiutility (Disjoint case).



(A) A fuzzy distance function (L-R type)



(B) Assignment function

Figure 3. Assignment of the β-values.

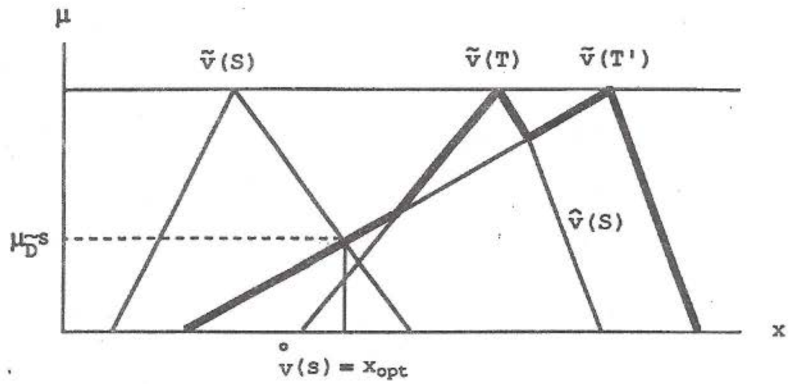
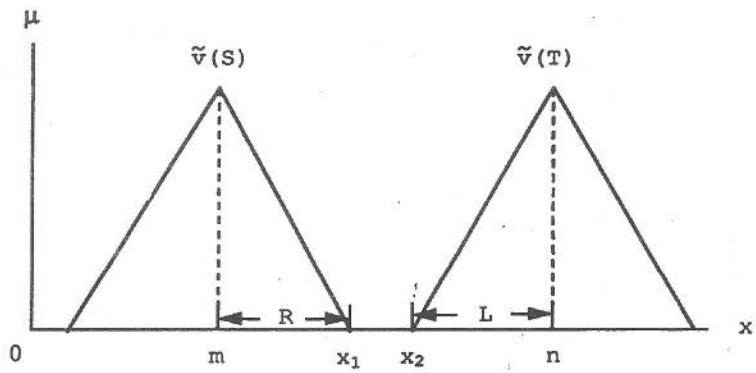


Figure 4. Assessment of the $\overset{\circ}{v}$ (S)-value.



$x_2 - x_1 > 0 \cdots$ Distinct case
$x_2 - x_1 \leq 0 \cdots$ Conjuct case

Figure 5. Evaluation of the disjunction.

Note: $\tilde{v}(T)$ denotes $\tilde{v}(T) \ominus \tilde{v}(T-S)$
$\tilde{v}(T')$ denotes $\tilde{v}(T') \ominus \tilde{v}(T'-S)$

S	$\tilde{v}(S)$		
	m	α	β
{1}	1	1	2
{2}	2	1	2
{3}	2	1	1
{4}	3	1	1
{5}	3	1	3
{1,2}	50	10	5
{1,3}	55	15	20
{1,4}	70	20	10
{1,5}	75	20	10
{2,3}	60	20	30
{2,4}	80	30	40
{2,5}	90	30	35
{3,4}	90	30	35
{3,5}	95	30	40
{4,5}	100	40	45
{1,2,3}	120	40	50
{1,2,4}	130	50	70
{1,2,5}	135	50	70
{1,3,4}	150	50	60
{1,3,5}	155	50	65
{1,4,5}	160	55	70
{2,3,4}	200	60	80
{2,3,5}	210	60	80
{2,4,5}	210	60	80
{3,4,5}	220	50	60
{1,2,3,4}	300	50	20
{1,2,3,5}	310	55	40
{1,2,4,5}	330	55	60
{1,3,4,5}	350	60	70
{2,3,4,5}	350	60	80
{1,2,3,4,5}	400	50	30

Table 1. Possibility distributions of fuzzy coalition values assessed as the fuzzy number.

S	$\overset{\circ}{v}(S)$	h	β	Case
{1}	1.875	0.5625	—	I–(1)
{2}	2.857	0.5714	—	I–(1)
{3}	2.642	0.3585	0.3585	I–(2)
{4}	3.720	0.1792	0.2798	II
{5}	5.198	0.0583	0.2672	II
{1,2}	53.626	0.0000	0.2748	II
{1,3}	69.252	0.0000	0.2874	II
{1,4}	77.000	0.0492	0.3000	II
{1,5}	82.000	0.1212	0.3000	II
{2,3}	81.075	0.1690	0.2975	II
{2,4}	101.538	0.4615	—	I–(1)
{2,5}	107.907	0.4884	—	I–(1)
{3,4}	114.012	0.3140	0.3140	I–(2)
{3,5}	120.934	0.3516	0.3516	I–(2)
{4,5}	126.287	0.4158	—	I–(1)
{1,2,3}	170.000	0.0000	0.0000	II
{1,2,4}	198.411	0.0000	0.0227	II
{1,2,5}	204.293	0.0000	0.0101	II
{1,3,4}	205.614	0.0000	0.0731	II
{1,3,5}	216.068	0.0000	0.0605	II
{1,4,5}	228.411	0.0000	0.0227	II
{2,3,4}	264.272	0.2443	0.1966	II
{2,3,5}	274.272	0.2721	0.1966	II
{2,4,5}	278.304	0.1250	0.1462	II
{3,4,5}	272.590	0.0000	0.1235	II
{1,2,3,4}	315.966	0.0000	0.2017	II
{1,2,3,5}	340.924	0.0440	0.2269	II
{1,2,4,5}	366.757	0.3874	0.3874	I–(2)
{1,3,4,5}	377.769	0.6033	—	I–(1)
{2,3,4,5}	379.924	0.6259	—	I–(1)
{1,2,3,4,5}	400.000	—	—	—

$$h \triangleq \max_x \min \{\mu_S(x), \mu_{\hat{S}}(x)\}$$

In Case I-(2), $\beta \triangleq h,\ \beta \geq \bar{\beta}$

Table 2. Fuzzy "optimal" values $(\alpha = 0.4, \bar{\beta} = 0.3)$.

S	$m \parallel \tilde{v}(s), \hat{v}(s) \parallel$	β
{4}	66	0.2798
{5}	71	0.2672
{1,2}	68	0.2748
{1,3}	63	0.2874
{1,4}	58	0.3000
{1,5}	58	0.3000
{2,3}	59	0.2975
{1,2,3}	177	0.0000
{1,2,4}	168	0.0227
{1,2,5}	173	0.0101
{1,3,4}	148	0.0731
{1,3,5}	153	0.0605
{1,4,5}	168	0.0227
{2,3,4}	99	0.1966
{2,3,5}	99	0.1966
{2,4,5}	119	0.1462
{3,4,5}	128	0.1235
{1,2,3,4}	97	0.2017
{1,2,3,5}	87	0.2269

Table 3. Fuzzy distance evaluation in the disjunct case.

player	Value	
	Non–fuzzy case	Fuzzy case
1	58.0000	54.0924
2	58.0000	60.2474
3	78.0000	71.2594
4	98.0000	91.0404
5	108.0000	119.3604

Table 4. Nucleous of the fuzzy game.