Control and Cybernetics

VOL. 21 (1992) No. 2

Production in a non–market economy

by

Jan Gadomski

Systems Research Institute
Polish Academy of Sciences
ul. Newelska 6
01–447 Warszawa
Poland

In the paper the axioms of a production function adequate for an analysis of a nonmarket or imperfect market economy are discussed. The new set of axioms is proposed and used for the specification of the production function with required properties. The production function proposed is applied in the analysis of the efficiency of the production factors in Polish industry.

1. Introduction

This paper deals with the production function of firms maximizing certain objective function (not neccesarily profit), operating in a non-competitive environment.

The neoclassical production theory (in its narrow sense) is irrelevant for this task because the properties of the neoclassical production function make it impossible for the firm to operate in regions where there is no factor substitution and/or the marginal cost of production is close to the minimum point.

The neoclassical production theory (in its wider Frischean sense) is also irrelevant for this purpose because:

- it does not provide a tool for empirical research;
- it is too technically oriented to suit economic investigation (makes no distinction between capital and labour as qualitatively different categories).

There are many reasons of ineffectiveness of firms. Let us name some of them. The best known case is the monopolistic firm producing at marginal cost close to its minimum or even at decreasing marginal cost. Another reason of ineffectiveness can be the true objective function of an enterprise — the statutory objective function can be biased by interests of various groups (those of workers, management etc.). In case of the command economy the outcome of bargaining between the firm and the central authority results in a specific equilibrium point of the firm. This point shows which of these agents outweighs the other in the bargaining process. Another reason of ineffectiveness in the command economy is a disequilibrium which is characteristic for this kind of economic system. In such economy the central economic authorities redistribute disequilibria according to their goals and preferences.

A transition from the command to the market economy requires, as the first step, deregulation of prices. Such measure contributes to the establishment of the economic equilibrium but is not a sufficient condition for the effectiveness of production. The aim of this paper is to indicate the links between effectiveness, factor productivities and effects of disequilibria. A production function is proposed which makes it possible to monitor the economic performance of an industry. In the analysis of effectiveness of the command economy there always emerges the problem of prices which do not reflect actual values of products, inputs and labour costs. It will be shown that in the economy where the technical progress is negligible, the problem of the significance of prices can be avoided — there exists a strict correspondence between the laws of production and production costs.

2. Production capacity, actual production and shortages

The aim of the model to be presented is the establishment of the relationship among production capacity, actual production, utilization rate and the shortages of supplies.

2.1. Production and shortages

All considerations are based on the assumption that the technical progress is very slow or negligible. Production capacity Q is defined as a two-factor function. The factors concerned are capital K and labour L. The production capacity function is formulated in the following way:

$$Q = F(K, L). \tag{1}$$

The properties of the production capacity function will be discussed later.

Production factors create a potential possibility of producing product Q provided that the supplies of energy, raw materials and other inputs of that kind are sufficient. Thus, the relation between actual production Y and the production capacity Q is as follows:

$$Y = \rho\, Q, \tag{2}$$

where parameter ρ, $0 < \rho < 1$, denotes the utilization rate of the production capacity.

Generally, it can be assumed that there is only one input X interpreted as an aggregate of various inputs of raw materials, energy etc. The relation between the input X and product Y is described by the following equation:

$$X = \gamma\, Y \tag{3}$$

where γ denotes fabrication coefficient, $\gamma > 0$.

Usually it is very hard to determine rate γ in absolute terms. Instead, the model of the process of production can be formulated in the following way:

$$Y = min[Q, X/\gamma] = min[F(K, L), X/\gamma]. \tag{4}$$

It follows from (4) that on the basis of (3) the demand for input is equal:

$$X = Q\, \gamma. \tag{5}$$

Note that if there is full utilization of production capacity, then

$$X = Q\,\gamma = Y\gamma \tag{6}$$

On the basis of the above equation it can be stated that if shortages occur, the loss of production can be defined as the difference between the full and the actual utilization of the production capacity:

$$Q - Y = F(K, L) - Y \tag{7}$$

caused by the shortage of supply of input X:

$$(Q - Y)\,\gamma. \tag{8}$$

2.2. The short–term production capacity

In the short–term analysis it has been assumed that the capital stock (machinery, buildings, etc.) is given and constant while the labour is the only variable factor. Hence the short–term production capacity is a function of one variable L :

$$Q = f(L) \tag{9}$$

The short–term production capacity function (9) should satisfy the following conditions:

C1. There is no product without employment, that is $f(0) = 0$.

C2. Potential productivity of labour $P\mathrm{L}$, $P\mathrm{L} = Q/L$ is null, if the employment is null:

$$P\mathrm{L} = \lim_{L \to 0} f(L)/L = 0$$

C3. Potential productivity of labour $P\mathrm{L}$ as a function of labour has a single maximum point for $L = L^*$, so that

$$\begin{aligned} \frac{d^2 P\mathrm{L}}{dL^2} &> 0, \quad L < L^*; \\ \frac{d^2 P\mathrm{L}}{dL^2} &= 0, \quad L = L^*; \\ \frac{d^2 P\mathrm{L}}{dL^2} &< 0, \quad L > L^*. \end{aligned}$$

C4. Function (9) has inflection point for $L = L_p$, $L_p < L^*$, so that

$$\begin{aligned}
\frac{d^2 f}{dL^2} &> 0, \quad 0 < L < L_p; \\
\frac{d^2 f}{dL^2} &= 0, \quad L = L_p; \\
\frac{d^2 f}{dL^2} &< 0, \quad L > L_p.
\end{aligned}$$

C5. Diminishing returns are expressed in relative terms, as the marginal elasticity of production capacity $\varepsilon(L)$ is defined to be $\varepsilon(L) = \frac{df}{dL}\frac{L}{f}$, ie. a decreasing function:

$$\frac{d\varepsilon}{dL} < 0, \quad \text{for all } L > 0.$$

Condition C1 states that there is no "free lunch", i.e. a single worker cannot produce anything in a real factory designed for hundreds of workers. Condition C2 states the same about productivity of that single worker. Condition C3 requires the productivity to increase with the growth of employment until a certain point; further increment of employment causes diminishing productivity. There is only one maximum of the productivity of labour. Condition C4 states that for the employment not exceeding certain level L_p, the increments of employment cause more than proportional increases of the production capacity. If the level of employment exceeds L_p then the increments of employment cause diminishing increases of the production capacity. Condition C5 generally implies that the ratio of the marginal potential productivity of labour to the potential productivity of labour is a decreasing function. In particular, the conditions C1,...,C4 imply two cases:

A. Production capacity is an increasing function of labour for all values of L ($L > 0$), and

$$\frac{df}{dL} > 0, \quad \frac{d^2 f}{dL^2} < 0\,, \quad L > L_p. \tag{10}$$

B. There exists a certain point $L_{p'}$, $L_{p'} > L_p$, such that:

$$\begin{aligned}
\frac{d^2 f}{dL^2} &< 0, \quad L < L_p < L_{p'}; \\
\frac{d^2 f}{dL^2} &= 0, \quad L = L_{p'}; \\
\frac{d^2 f}{dL^2} &> 0, \quad L > L_{p'};
\end{aligned} \tag{11}$$

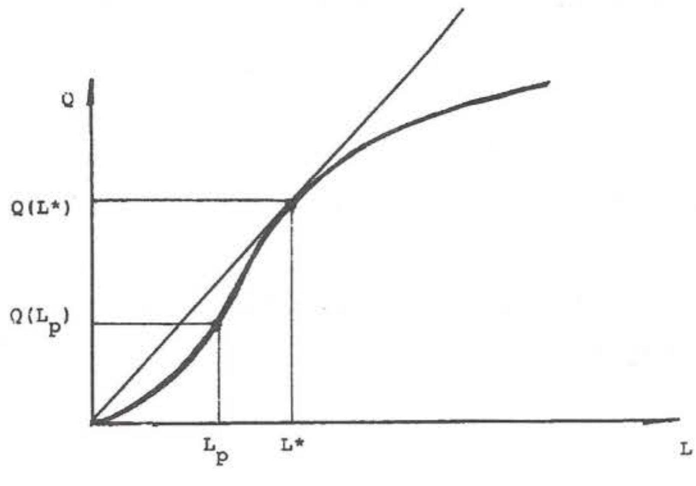


Figure 1A.

It can be noted that in Case A, whenever the employment exceeds the level L_p the neoclassical assumptions of the production function hold true.

Case B determines the existence of the maximum point of the production capacity at a certain level of employment L^{**}. Increment of the employment over that level results in the absolute decrease of the production capacity. The existence of point L^{**} is broadly justified by R. Frisch (Frisch, 1965).

The conditions C1,...,C5 and Cases A and B strictly determine the shapes of the short-term production capacity functions shown in Fig.1A and Fig.1B.

2.3. Short–term costs

As it was mentioned above in point 2.1 there are three elements involved in the process of production: two production factors (capital K and labour L) and material input X (energy, raw materials etc.). All these elements are included in the total cost of production C:

$$C = kK + wL + xX, \tag{12}$$

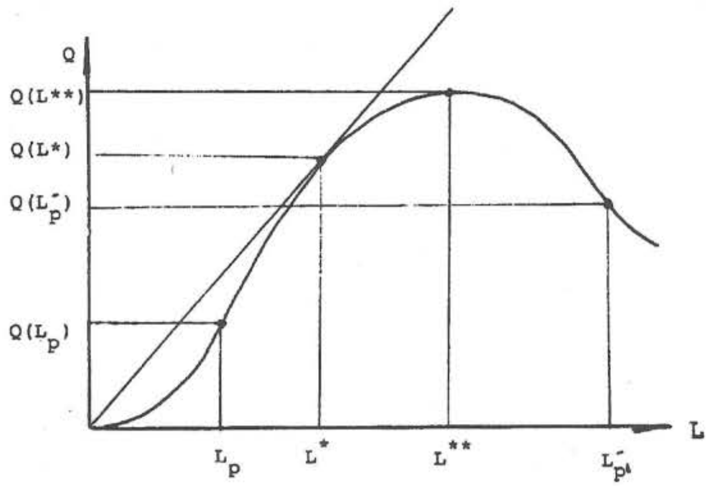


Figure 1B.

where: w – the wage rate,
k – interest or amortization rate,
X – amount of material input used in production,
x – unit price of the material input.

In the short–term analysis the capital K is assumed to be constant while the variable part of the total cost C (12) is made up of the components consisting of employment L and material input X.

Let us assume that the demand for input X is satisfied. On the basis of (3), (4), (5) and (6) the cost of producing product Q can be expressed in the following form:

$$C(Q) = kK + wL(Q) + xQ\,\gamma.$$

Assuming that the inverse f^{-1} of the function (9) exists, the above cost equation can be rewritten in the following form:

$$C(Q) = kK + wf^{-1}(Q) + xQ\,\gamma$$

or (13)

$$C(Q) = kK + wf^{-1}(Q) + x'Q\,\gamma,$$

where $x' = x\gamma$.

It should be noted that the last component in (12) and (13) is linearly dependent on the level of production so that the sum of the first two elements make up the internal (from the firm's point of view) cost.

The marginal cost is obtained by deriving equation (13):

$$MC = \frac{dC}{dL} = w\,(df/fL)^{-1} + x' \tag{14}$$

One can notice that the marginal cost (14) is always positive and because

$$\frac{dMC}{dL} = -w\left(d^2f/dL^2\right)/\left(df/dL\right)^3,$$

there exists a point L_p satisfying condition C4.

Since the variable cost VC can be defined as

$$VC = wL + x'Q,$$

then the average variable cost AVC can be expressed in the following form:

$$AVC = \frac{VC}{Q} = w\frac{L}{Q} + x'. \tag{15}$$

The average cost AC is defined in the following way:

$$AC = \frac{C}{Q} = \frac{kK}{Q} + \frac{wL}{Q} + x'. \tag{16}$$

It can be easily shown that both the average variable cost AVC and the average cost AC have minima in points satisfying conditions C1, C2, C3 and C4.

The meaning of the condition C5 is that it imposes in case B the maximum level of production from given capital K regardless of the level of employment L.

The cost functions derived from the production capacity function are shown in Fig.2A (Case A) and Fig.2B (Case B). It should be noted that in the above considerations we assumed that there are no shortages of supply so that production capacity is equal to actual production. In such a case the production capacity function is identical to the production function.

Let us now investigate the impact of shortages of supply on the costs C, AC, MC and AVC. Whenever the demand for input is not satisfied the available product will decrease according to (4). The production loss is expressed by (7). In this case the total cost of producing Y ($Y < Q$) has the following form obtained from (13):

$$C(Y) = kK + wf^{-1}(Q) + xY\gamma. \tag{17}$$

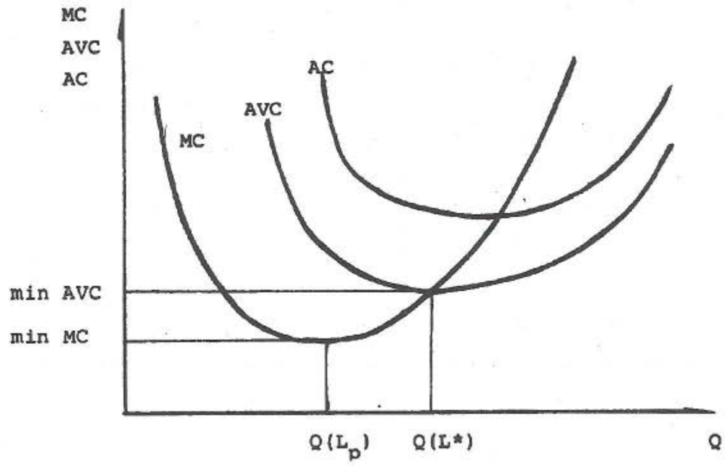


Figure 2A.

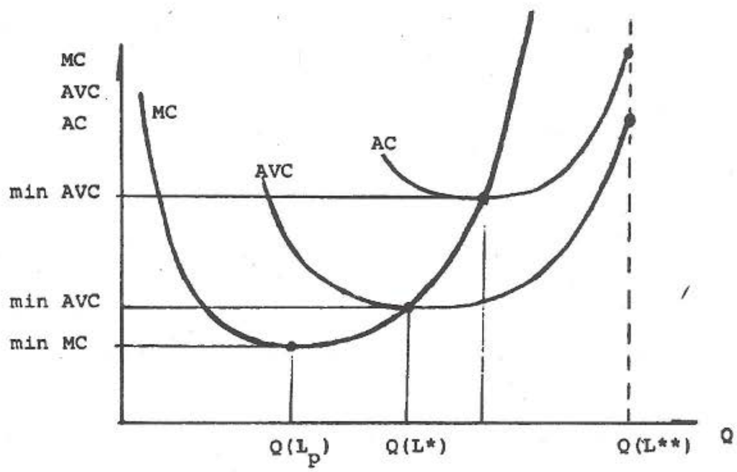


Figure 2B.

Having in mind the utilization rate ρ defined in (2), equation (17) can be rewritten into the following form:

$$C(Y) = kK + wf^{-1}(Y\gamma) + x'Y \tag{18}$$

The equation (18) expresses the actual cost as a function of the actual production Y. By deriving (18) with regard to Y one can obtain the actual marginal cost:

$$\begin{aligned} MC = \frac{dC}{dY} = w\frac{df^{-1}(Y\rho)}{dY} + x' = w(df/dL)^{-1}\rho + x' = \\ = w(df/dL)^{-1} + x' + w(df/dL)^{-1}(1-\rho)/\rho \end{aligned} \tag{19}$$

The above equation shows that the decrease of input unmatched by changes in the amount of factors engaged causes increase of the marginal cost by the term:

$$w(df/dL)^{-1}(1-\rho)/\rho.$$

The actual average cost AC in the case of shortage has the following form:

$$\begin{aligned} AC &= \frac{kK + wf^{-1}(Y\rho)}{Q\rho} + x' = \\ &= \frac{kK + wf^{-1}(Q)}{Q} + \frac{(kK + wf^{-1}(Y\rho))(1-\rho)}{Y\rho} \end{aligned} \tag{20}$$

Equation (20) shows that the shortage of supply results in the increment of the average cost by the term:

$$\frac{1-\rho}{\rho}\frac{kK + wL}{Q}.$$

Equations (19) and (20) show that the shortages (disequilibrium) do not affect the location of the characteristic points L_p, L^* , L^{**} determining minima of the cost curves. This property implies that there is strict correspondence between the short–term production function and the characteristic points of the short–term production costs invariant to the biases caused by disequilibrium.

2.4. The intermediate production capacity function

The intermediate production capacity function is a function describing production possibilities when both production factors (capital and labour) change while the technical progress is either slow or negligible. Hence it can be assumed that

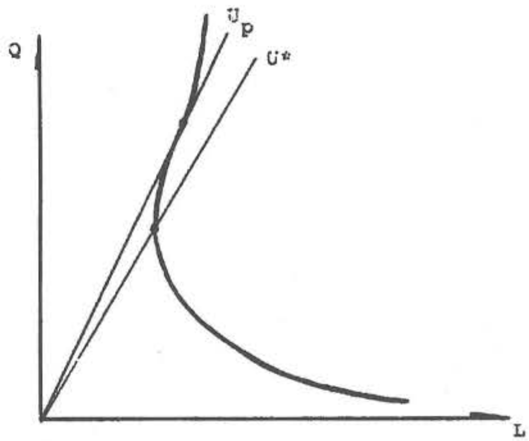


Figure 3A.

the productivity characteristics of such production processes are constant in time.

This attitude to the technical changes is important in the production modelling in the economy haunted by the technical and logistical problems of production (for example that of continuity of supply). These problems cause some sort of short–termism in regulation processes. The most suitable model for such system is the command economy or the economy in the process of transition from the command to the market economy.

The obvious way of modelling such type of economy is to assume the homogeneity of the production capacity function. One of the advantages of such an assumption is that one avoids the discussion about the distinction between the technical progress and the economy of scale.

Homogeneity of the two–factor production capacity function (1) satisfying the short–run conditions C1,..,C5 (A and B) implies the transformation of the characteristic points from one–dimensional points to the two–dimensional (on the $K \times L$ plane) ones:

$$\begin{array}{ccc} L_p & \longrightarrow & U_p\ , \\ L^\star & \longrightarrow & U^\star\ , \\ L^{\star\star} & \longrightarrow & U^{\star\star}\ , \\ L_{p'} & \longrightarrow & U_{p'}\ . \end{array}$$

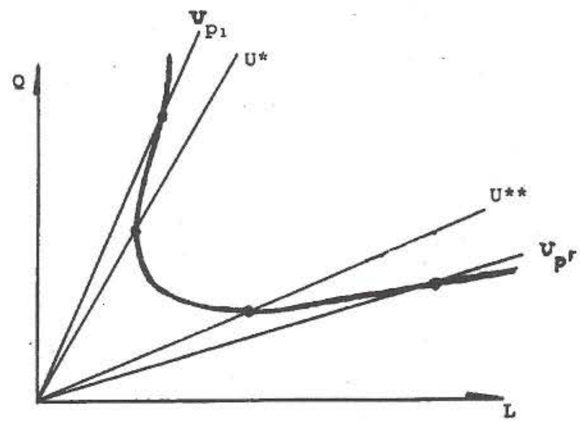


Figure 3B.

The meaning of the above transformation is that for a given technology there exists certain constant points U_p, $U^\star$, and in case B points $U^{\star\star}$ and $U_{p'}$. These points correspond to distinct values of the capital intensity of labour. In terms of the Frischean theory the relation of these values of the capital intensity of labour correspond to the rays intersecting isoquants (Fig.3A for case A and Fig.3B for case B respectively). It follows from the properties of the intermediary term production capacity function that the cost characteristics are not affected by the scale of production. In conclusion it can be stated that for the determination of the characteristic points (minima of cost curves) one has to know the relative (period to period) value of the utilization rate and the value of the production capacity function.

For the purpose of the empirical research the following formula is proposed:

$$Y_t = \rho_t \, Q(K_t, L_t) \tag{21}$$

where subscript t denotes the period of time, $t = 1, \dots, T$.

3. The effectiveness analysis of Polish industry

In the analysis of the effectivness of Polish industry the following production function satisfying properties specified above was used (Gadomski, 1988):

$$Y_t = L_t\ \rho_t\ a\ U_t\ b\ exp(-cU_t)\ \varepsilon_t\ , \tag{22}$$

where Y_t – product of the industry in the year t;
U_t – capital intensity of labour in the year t;
ε_t – stochastic term;
a, b, c – constant parameters.

It can be shown that the form (22) of the production capacity function can be expressed in the following form with the transformed parameters:

$$Y_t = L_t\ \rho_t\ PL^\star\ \{(U_t/U^\star)\ exp[1-(U_t/U^\star)]\}^\beta\ \varepsilon_t, \tag{23}$$

where: $PL^\star$ – average productivity of labour function in the point $U^\star$, $PL^\star = PL(U^\star)$ – constant parameter,
β – substitution parameter, $\beta > 0$.

The relation between parameters of the form (22) and the parameters of the form (23) is given by the following set of expressions:

$$\begin{aligned} \beta &= b, \\ U^\star &= \beta/c, \\ PL^\star &= a\ (U^\star/e)^\beta. \end{aligned}$$

Having the values of the parameters $U^\star$ and β it is easy to determine the value of the parameter U_p:

$$U_p = U^\star[1 + (\sqrt{\beta}/\beta)],$$

while parameters $U^{\star\star}$ and $U_{p'}$ (if exist) are given, respectively, by the following formulae:

$$\begin{aligned} U^{\star\star} &= U^\star(\beta - 1)/\beta, \\ U_{p'} &= U^\star[1 - (\sqrt{\beta}/\beta)]. \end{aligned}$$

The existence of parameters $U^{\star\star}$ and $U_{p'}$ depends on the value of the substitution parameter β. The production capacity function (23) describes Case B and parameters $U^{\star\star}$ and $U_{p'}$ exist if $\beta > 1$. Otherwise $(0 < \beta < 1)$ the production capacity function corresponds to Case A.

The production capacity function (22) belongs to the class of VES production functions. Its main advantage is that it can be used to separate the proper effectiveness of production connected with the utilization of the production factors from the impact of shortages (disequilibrium). The main problem with function (22) is that one cannot determine separate values of the capacity utilization rate ρ_t and of parameter PL^*.

In the development of Polish industry in the period of 1970–1984 two subperiods can be distinguished. The first embracing years 1970-1980 can be described as a period of huge investment and fast growth, while the second which began in 1981 was the period of political crisis, decreased availability of western credits and changes in the central economic policy.

The differences between these subperiods consist not only in the investment rate and credit drawn but also in the changes of preferences of the central economic authorities. These preferences were expressed in the supply of industries with such inputs as imports, energy, etc.

Both subperiods had two similar features. The first one was that the industries did not reveal a tendency for technical progress. The second one was that the ways and means of the central economic authorities remained unchanged.

The reaction of the central economic authorities to the crisis consisted in the changes of priorities. In turn, this resulted in altering the disequilibrium structure. One of the aims of the analysis was the assessment of the effects of the change of priorities on the production efficiency.

In the research it was assumed that the values of the utilization rate ρ_t were constant within each period and different between them. For the estimation the following form of the equation (22) was used:

$$\ln (\mathrm{PL}_t) = a + b \ln (U_t) + c\, U_t + d\, D_t + \varepsilon_t, \tag{24}$$

where: a, b, c – parameters as in (22),

D_t – dummy variable connected with the periodization of the time series:

$$D_t = \begin{cases} 0, & 0 \le t \le 1980, \\ 1, & 1981 \le t \le 1984; \end{cases}$$

d – parameter showing relative change in the central priorities between subperiods.

The results obtained are presented in Table 1 and Table 2. All parameters have signs as expected and are statistically significant. Among the eight industrial branches analysed only one (textiles) belongs to Case A.

The trajectories of the capital intensity of labour of each of the eight branches analysed are shown in Figures 4 to 11. These charts prove the inertial behaviour of Polish industry in the period of 1970–1984. This inertia could be attributed, inter alia, to the economic crisis: despite the fact that the supply of labour (either in terms of number of employees or manhours) was non-increasing. The continued growth of capital adversely influenced the efficiency of production in most industries.

Moreover, it can be stated that at the end of the period of 1970–1984 most industries operated in unfavourable regions of costs. Six out of eight industries analysed operated in the region of decreasing variable cost. Among these six industries there was one operating in the region of decreasing marginal cost. Only two industries operated in the region of increasing variable cost: the paper and pulp industry and the textile industry.

The values of parameter *d* reveal the relative changes in supplies to industries resulting in vast changes in the utilization of the production capacity.

The relative changes of the utilization of the production capacity between the periods of 1970–1980 and 1981–1984 in industries are as follows:

1.	Fuel and energy	-9.6%
2.	Metallurgy	+60.0%
3.	Machine	+15.5%
4.	Chemical	-60.8%
5.	Mineral mining	-19.6%
6.	Paper and pulp	-11.8%
7.	Textile	-25.0%
8.	Food processing	-10.0%

It can be concluded from the above data that the improvement of the utilization of the production capacity in the metallurgy and machine industries not only did not compensate for the losses caused by supply shortages in the remaining industries but was accompanied by deterioration of factor productivities.

However, revaluation of the capital in the state-owned industries made it impossible to extend the research for the later years. One can state on the basis of rough data that the development tendencies of the capital and employment continued up to 1989. Such circumstances created specific conditions for the transition process from the command to the market economy.

On the basis of the above presented research an attempt to forecast some aspects of the transition process can be made. Three main stages of that process can be distiguished: (i) establishment of equilibria of both the product and supply markets due to radical decrease of demand caused by imposed financial discipline, positive interest rate and reduction of subventions, decrease of production caused by decrease of demand; (ii) adaptation of firms to the diminished demand and production targets: cost optimization of inputs and labour redundancy; (iii) adjustment of capital to the optimum level (profit maximization).

The research presented above indicates the main obstacle to the success of the stabilization programme — the monopolistic structure of most of Polish industries. Introduction of free prices plus the monopolistic structure and decrease of demand cause the increase of the average production costs and may provide positive profits to producers. Such outcome — a typical monopolistic trap — could seriously endanger the feasibility of the stabilization programme and of the whole transition process.

Reference

FRISCH R. (1965), *Theory of production*, D. Reidel, Dordrecht.

GADOMSKI J. (1988), Production and the non-market behaviour of firms (in Polish), internal report, Systems Research Institute, Polish Academy of Sciences, Warsaw.

Industry No	Parameters*				
	a	b	c	d	
1	0.2223 (6.6894)	2.0503 (7.9247)	-0.5801 (6.7100)	-0.1015 (9.9856)	R2 = .98 DW = 2.78 F = 246.8
2	1.0168 (13.2366)	2.0744 (8.7567)	-0.7622 (6.79411)	0.4702 (3.9107)	R2 = .94 DW = 2.33 F = 52.8
3	1.8202 (3.9913)	1.8144 (6.0075)	-1.7694 (3.6869)	0.1442 (1.2489)	R2 = .96 DW = 2.53 F = 79.8
4	0.8783 (13.7792)	2.4526 (16.2359)	-0.9375 (12.0648)	-0.9375 (9.9748)	R2 = .99 DW = 1.98 F = 327.4
5	0.7475 (4.0287)	1.9068 (9.4548)	-0.9871 (5.4506)	-0.2182 (6.4531)	R2 = .98 DW = 2.15 F = 103.1
6	0.7591 (3.0405)	1.1009 (5.4437)	-0.1261 (2.2385)	-0.1261 (2.1731)	R2 = .96 DW = 1.76 F = 93.6
7	0.6019 (4.9743)	0.7795 (12.8971)	-0.3071 (1.9086)	-0.2871 (17.0116)	R2 = .98 DW = 2.11 F = 148.4
8	1.9563 (6.5996)	1.4374 (4.6921)	-0.8599 (2.9964)	-0.1060 (3.2631)	R2 = .96 DW = 1.51 F = 100.1

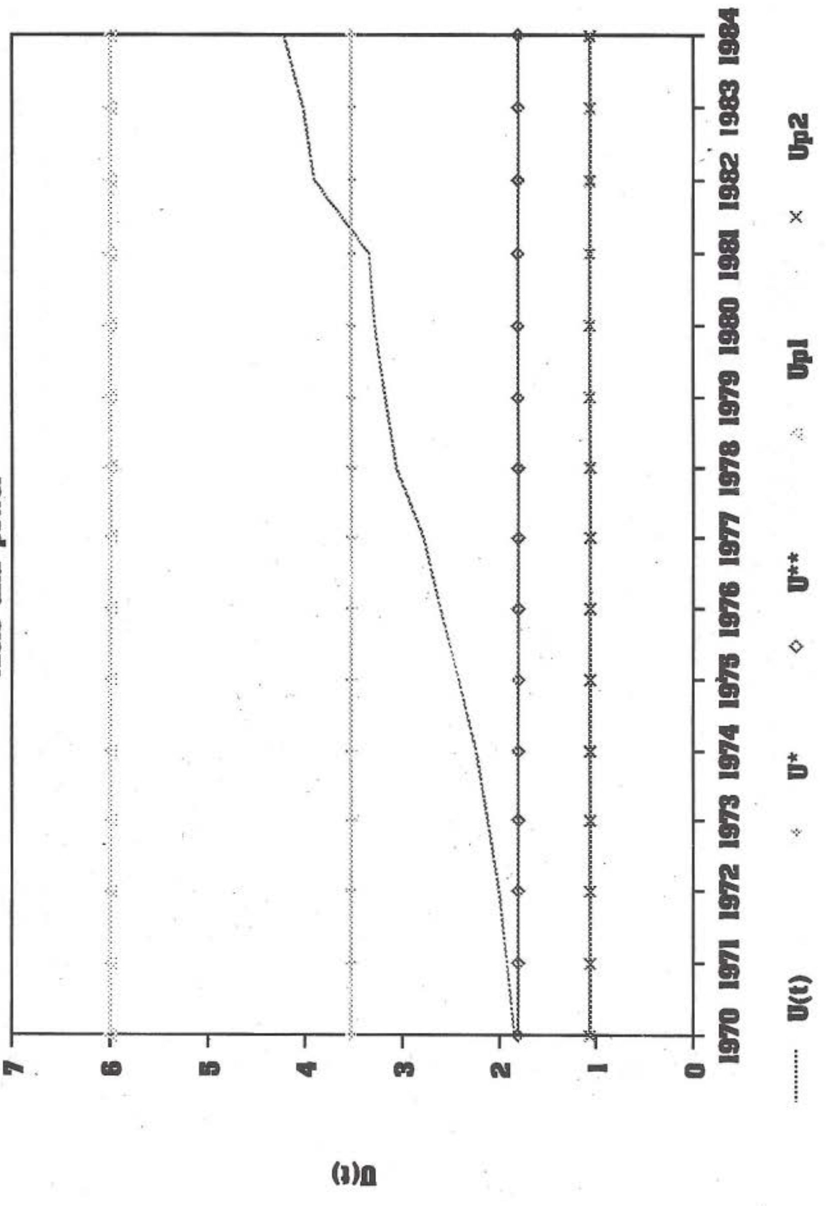
* Numbers in parentheses are values of t–statistics

Table 1. Parameters of equation (24)

Industry No	β	$U^{\star}$	$U^{\star\star}$	U_{p_1}	U_{p_2}	$PL^{\star}$
1	2.0503	3.5343	1.8105	6.0027	1.0060	3.330
2	2.0744	2.7216	1.4096	4.6112	0.8319	2.770
3	1.8144	1.0254	0.4602	1.7866	0.2641	1.050
4	2.6161	1.5494	1.5494	4.2866	0.9456	2.190
5	1.9068	1.9316	0.9186	3.3306	0.5328	1.100
6	1.1410	1.9633	0.2430	3.8012	0.1255	1.450
7	0.7795	2.5380	—	5.4125	—	1.730
8	1.4374	1.6714	0.5087	3.0656	0.2773	3.510

Table 2. Parameters of the production capacity function (23)

Fig. 4 Capital intensity of labour
fuels and power



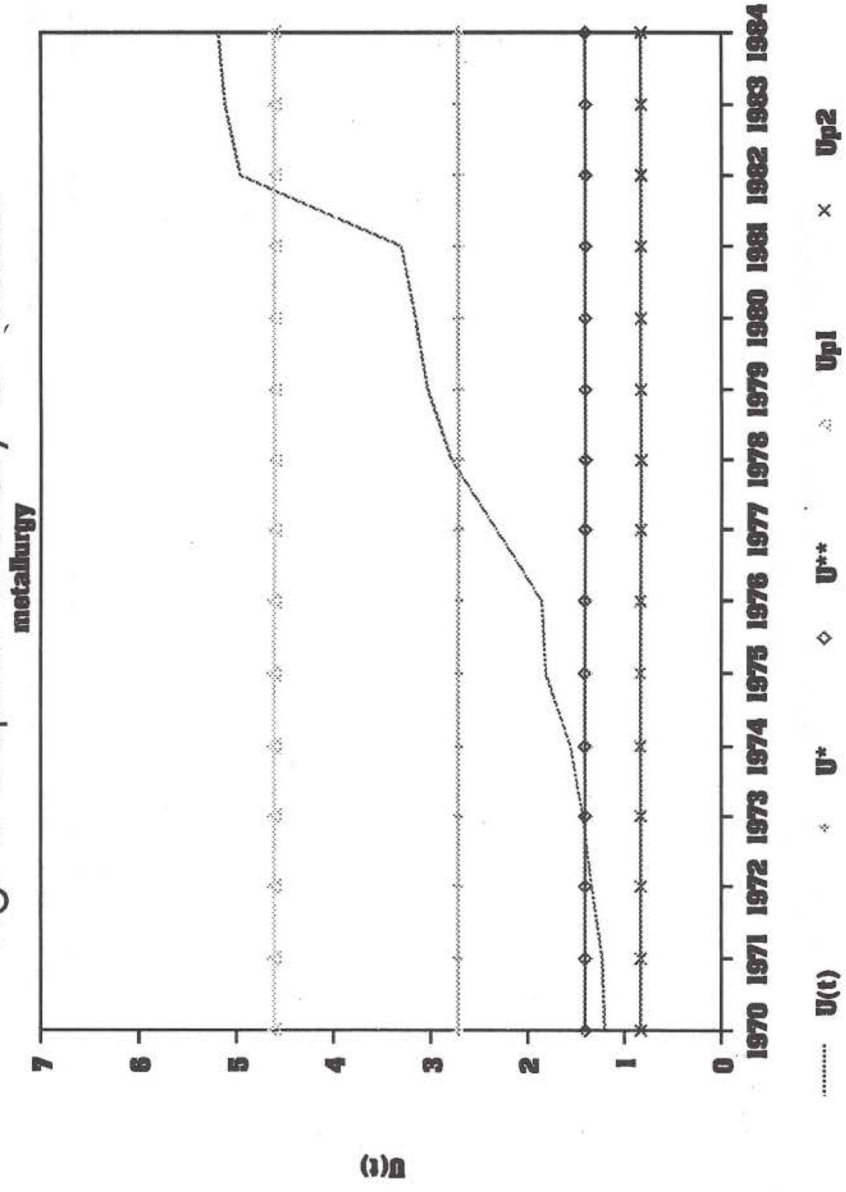


Fig. 5 Capital intensity of labour

Fig. 6 Capital intensity of labour

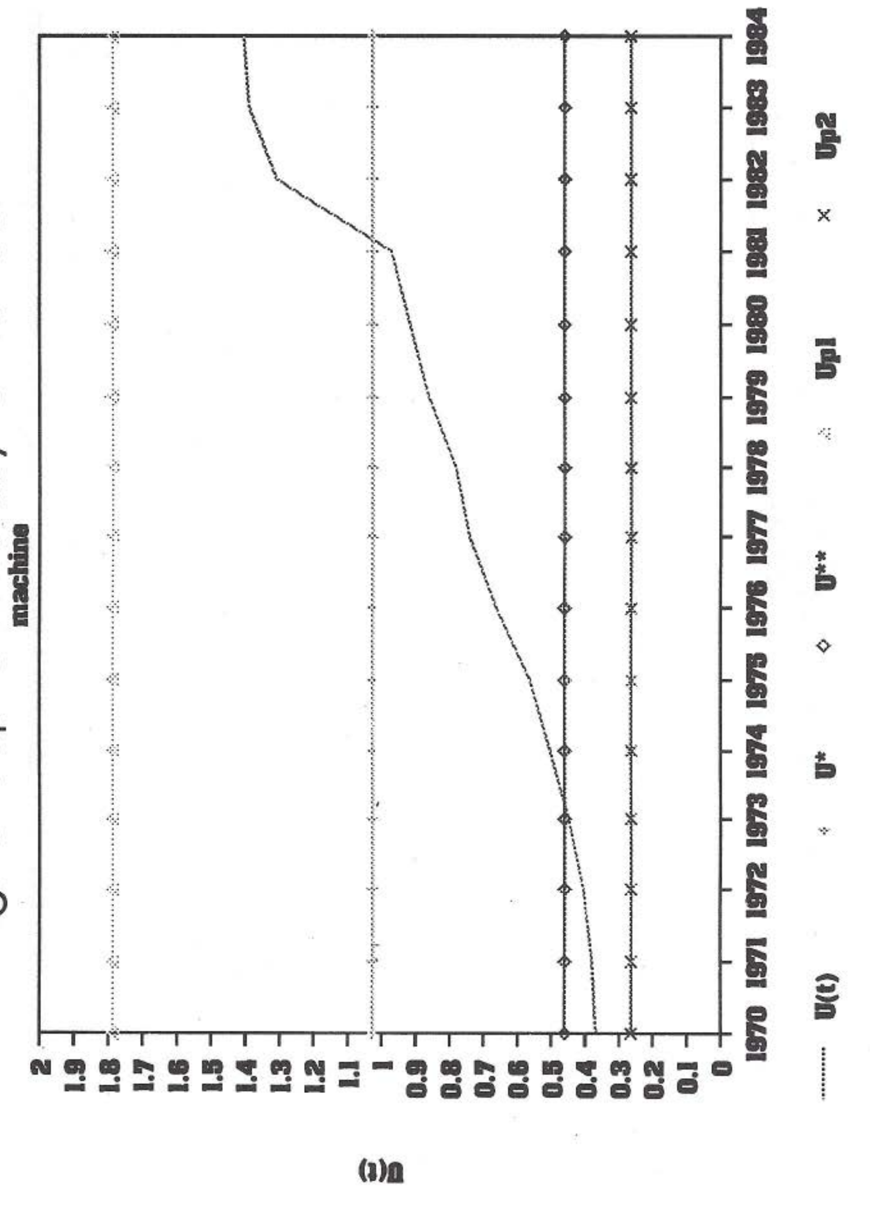
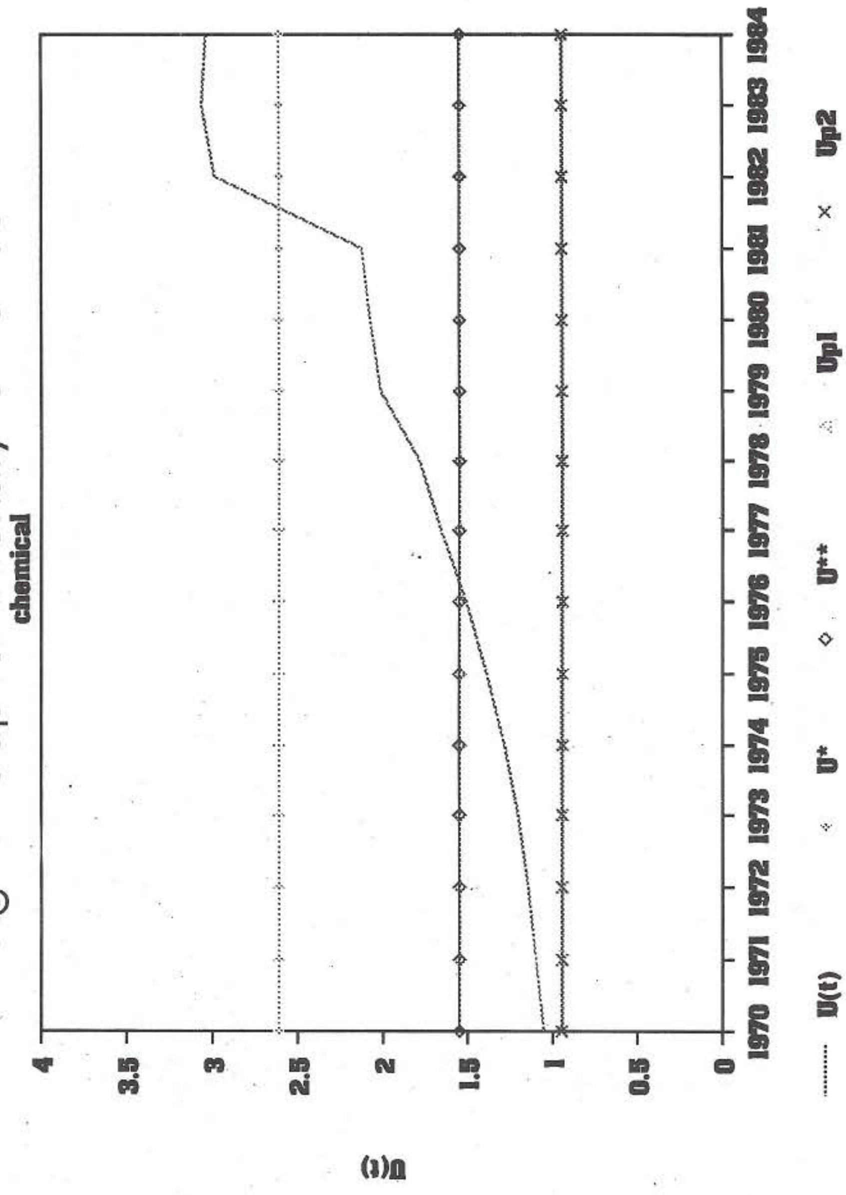


Fig. 7 Capital intensity of labour



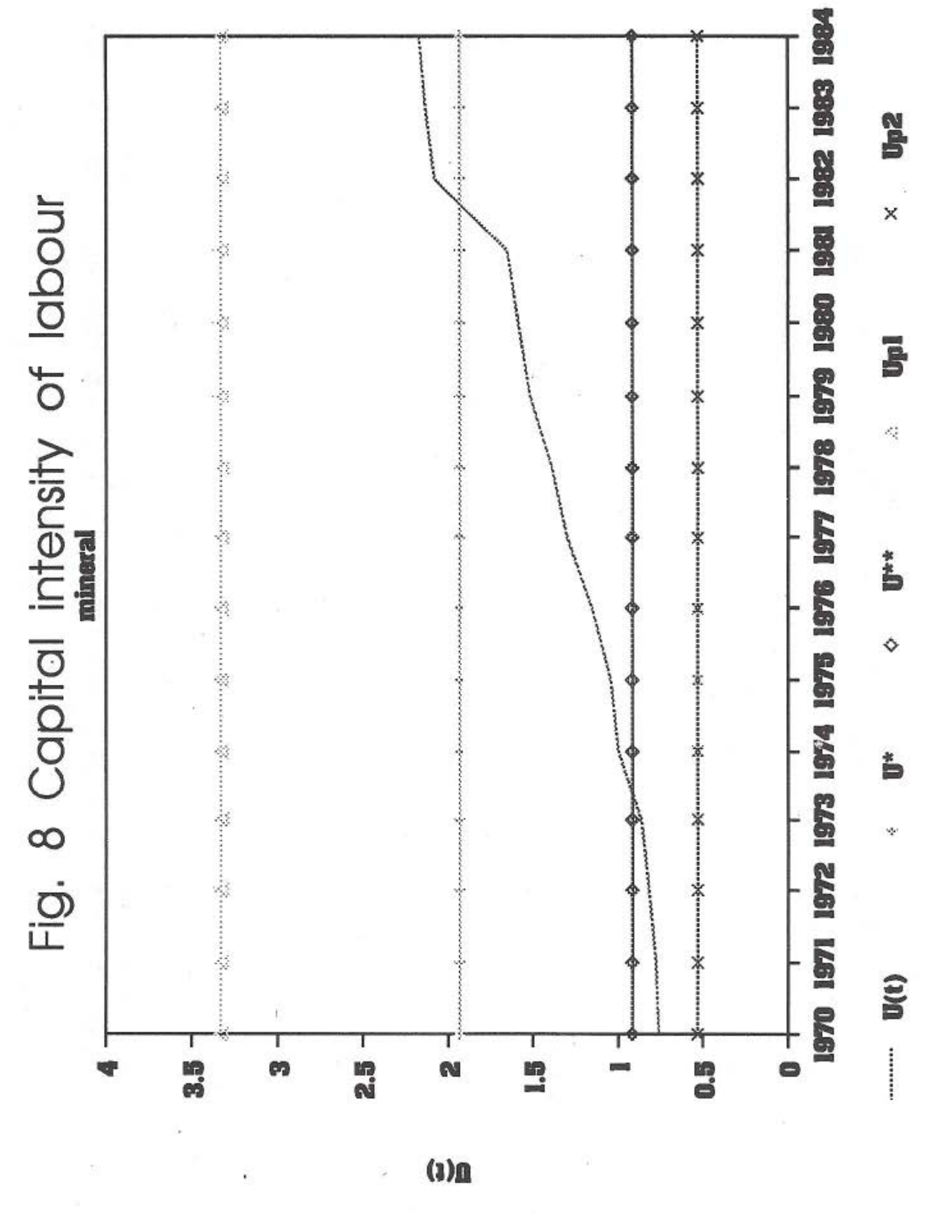


Fig. 8 Capital intensity of labour

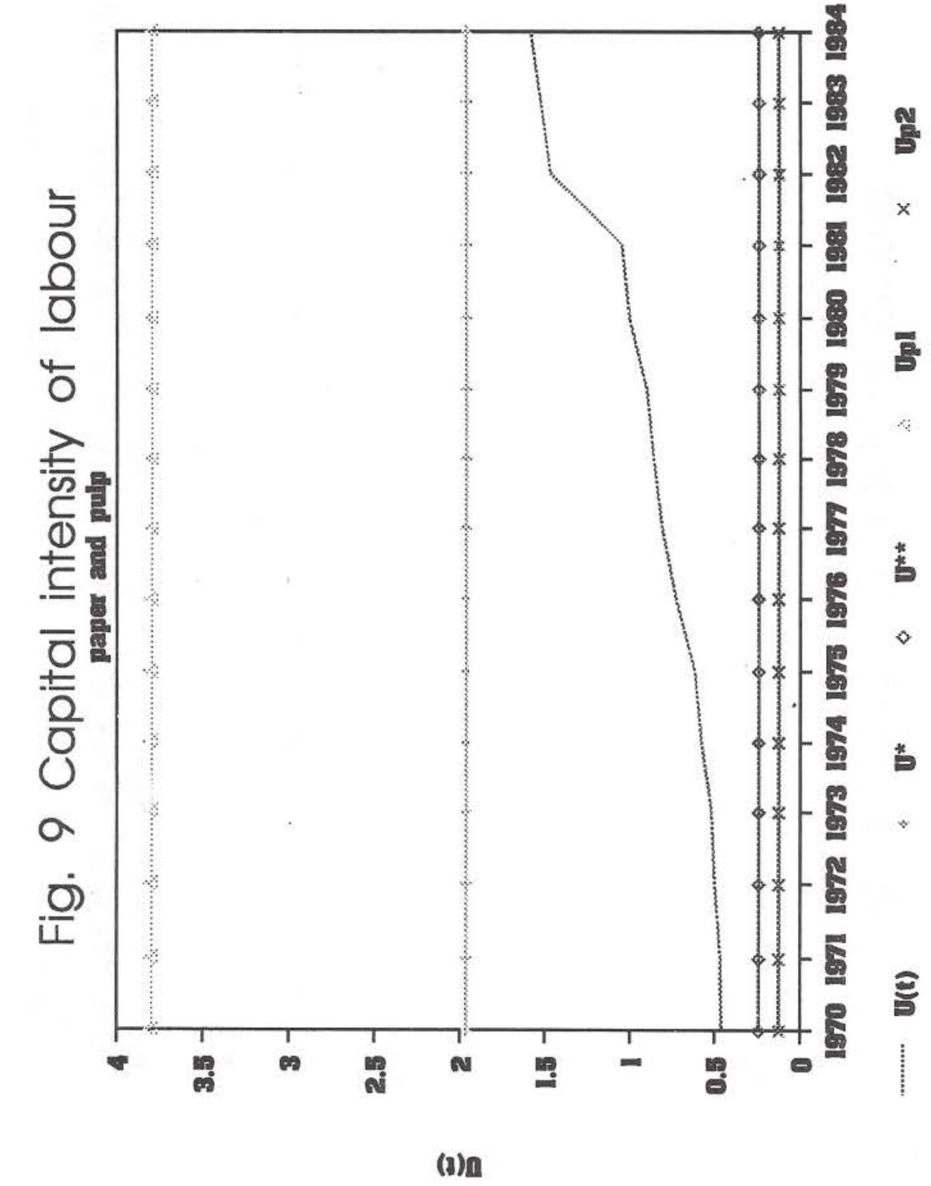


Fig. 9 Capital intensity of labour
paper and pulp
U(t)
0
0.5
1
1.5
2
2.5
3
3.5
4
1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984
U(t)
U*
U**
Upl
Up2

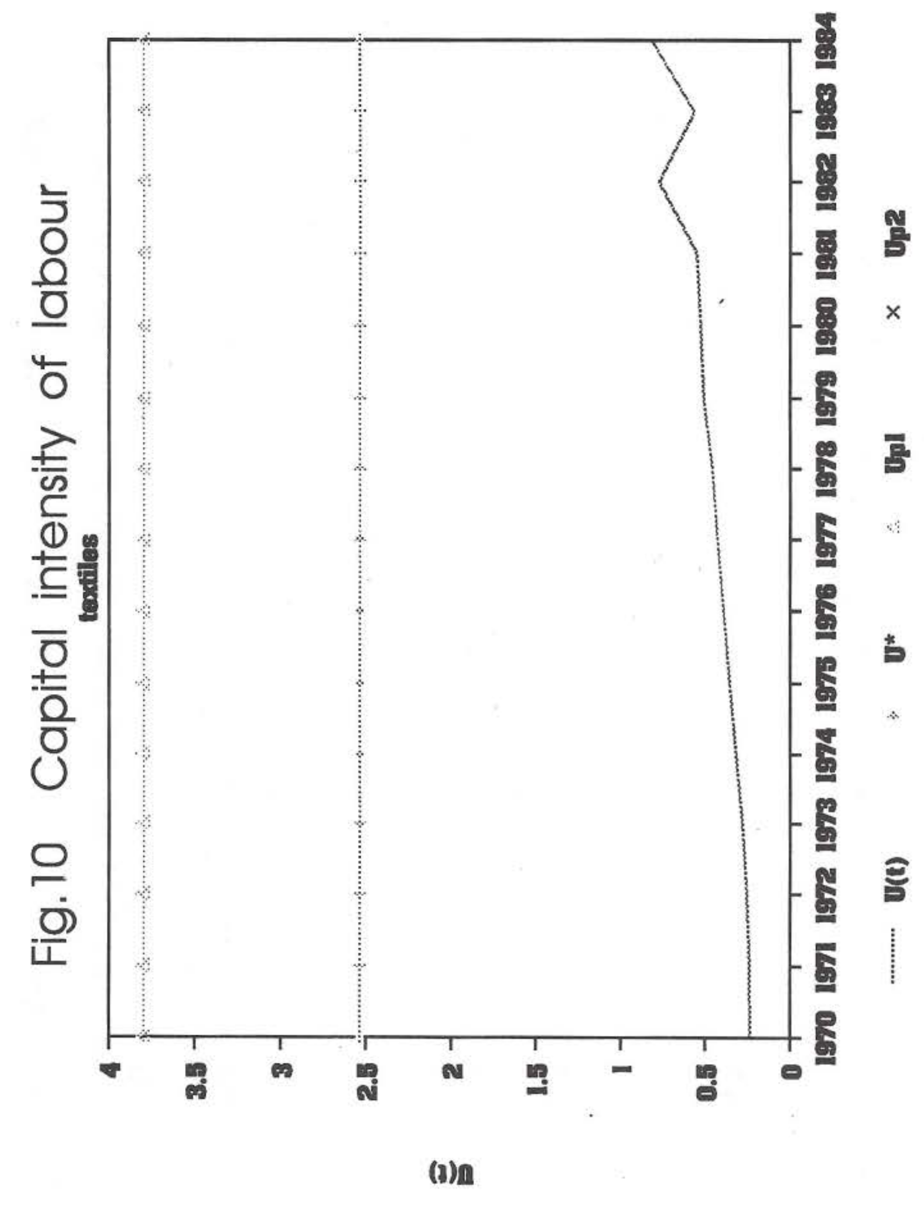


Fig.10 Capital intensity of labour
textiles
U(t)
4
3.5
3
2.5
2
1.5
1
0.5
0
1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984
U(t)
U*
Up1
Up2

Fig.11 Capital intensity of labour $_{\text{food}}$