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Voting procedure characteristics

by

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The present paper describes dynamic characteristics of voting procedures, which occur when the positions along a certain dimension (sometimes called ideological) of voters and candidates are taken into account. Using simulation technique the centrality or extremity of chosen voting procedures were shown.

1. Assumptions of the experiment

The essential part of the simulation method is based upon the concept of elementary support (Mercik, 1986).

Definition. Let $\{E_1, E_2, \ldots, E_m\}$ be a partition of n voters in which E_k is a subset of all voters having the same order over the set of candidates, $E_i \cap E_k = \emptyset$ for $i \neq k$. We call E_k the k-th elementary support.

Let us divide interval [0,1] into disconnected intervals $I_1, I_2, \ldots, I_m$ such that $\bigcup_k I_k = [0,1]$. Those intervals arise between all subsequent midpoints found between every pair of candidates' positions ($\equiv$ points along ideological dimension, which is represented by the interval [0,1] in this experiment). It is easy to see that positions of all voters from one elementary support belong to one and only one such constructed interval and there is no voter from any other elementary support whose position may be found inside the interval. If n is number of candidates we have $m = \binom{n}{2} + 1$ such intervals.

On the basis of this assumptions one may construct the following algorithm (Mercik, 1988):

STEP 1. For a given number of candidates n generate randomly their positions along the [0,1] interval.

STEP 2. Number the set of candidates giving the first number for the leftmost candidate and the n-th number for the rightmost candidate.

STEP 3. Establish all $I_1, I_2, \ldots, I_m$ intervals using generated positions.

STEP 4. Generate all the (m) orders over the set of candidates. They reflect the point of view of every elementary support. On the interval [0,1] such orders are generated by Euclidan distance between a candidate's position and the end of a given interval I_k $(k = 1, 2, \ldots, m)$ that is the closest to him or her.

STEP 5. According to the distribution of electorate find the number of voters whose positions belong to each of the elementary supports ($\equiv$ belong to all I_k).

STEP 6. For all the voters from each elementary support generate $j \in \{1, 2, \ldots, R-1\}$ number of candidates s/he is going to vote for (according to the selected voting procedure). This number may differ for different voters even if they belong to the same elementary support.

STEP 7. Votes are cast for candidates in the following way: if simulated j (chosen by a voter from I_k $(k = 1, 2, \ldots, m)$ number of candidates s/he is going to vote for) is greater than or equal to the sequential number of given candidate in the order generated by I_k then one vote is cast for him or her by this voter.

STEP 8. Iterate until a given number of runs is obtained and calculate the frequency of wins for all candidates.

In the sense of the above algorithm, the frequency of wins for any candidate depends evidently upon his or her position among candidates and the results of the simulation give an answer to the following question: what is the probability for any candidate to be a winner depending of his or her position in the ordered set of candidates. If such probability is greater (for candidates centrally positioned) than what one can expect from electorate distribution we observe the central tendency of a given voting procedure. Greater probability for extreme candidates allow us to observe the extremity of a given voting procedure.

The following voting procedures were investigated: Condorcet (simple majority), Schwartz, Dodgson, Black, Copeland, maximin, Borda, ϵ–Borda, Nanson, average rank, jury, plurality, plurality with run–off, Hare, Coombs, approval, negative, cumulative, disapproval.

Voting procedure	Cardinality of electorate	Positions			No solutions
		1	2	3	
Condorcet	460	0.055	0.288	0.652	0.005
Schwartz	50	0.314	0.293	0.391	.....
Dodgson	50	0.540	0.013	0.447	.....
Black	460	0.088	0.025	0.435	0.452
Copeland	100	0.060	0.256	0.445	0.239
Maximin	50	0.050	0.007	0.447	0.496
Borda	25	0.081	0.463	0.082	0.374
ϵ–Borda	50	0.108	0.252	0.112	0.528
Nanson	460	0.328	0.649	0.022	.....
Average rank	50	0.048	0.469	0.033	0.450
Jury	50	0.333	0.332	0.335	.....
Plurality	25	0.642	0.006	0.352	.....
Plurality with run–off	50	0.082	0	0.075	0.842
	100	0.100	0	0.089	0.811
Hare	25	0.050	0	0.054	0.896
Coombs	50	0.388	0.035	0.425	0.152
Approval	50	0.238	0.653	0.109	.....
Negative	25	0.282	0.104	0.300	0.314
Cumulative	460	0.386	0.188	0.425	.....
Disapproval	100	0.223	0.652	0.125	.....

Table 1. Probability of win for different positions, for different voting procedures, different cardinality of electorate

2. The results of simulations

1000 runs were done for different voting procedures and for $i = 3$ to $i = 9$ candidates one after another for an electorate consisting of changing numbers of voters and for described forms of electorate distribution (presented in Fig. 1) functions. The results of the simulation can be found in Table 1 and Figures 1 to 9. All voting procedures and results obtained are described in (Mercik, 1990). This is also the source for all figures.

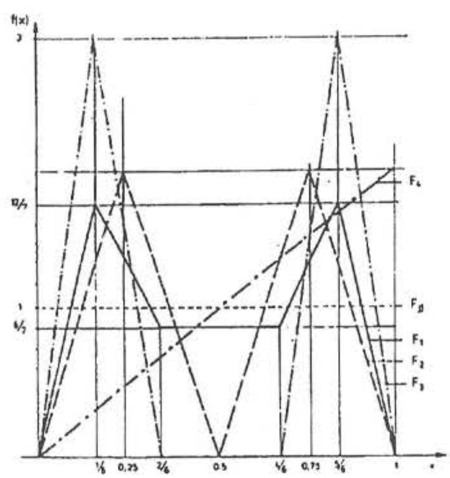

Figure 1. Five types of probability densities describing an electorate distribution the along interval [0,1] used in the computer experiments. Source: Mercik, 1988

3. Conclusions

One may draw the following conclusions:

1. In all the cases of electorate distribution — even the extreme ones — some voting procedures have tendency to give more chances for centrist candidates. For example, this characteristic is almost negligible for approval voting procedure.
2. There are also some procedures giving more chances for extreme candidates (alternatives). Among them the most known and used is plurality voting procedure.

3. The Jury voting procedure does not depend on candidates' positions.
4. The information about tendency of given voting procedure may be used when one analyses the so called entry problem, i.e. when a new enterer has to choose his or her position relatively to the other candidates.
5. The results show also that voting procedures have different efficiency in choosing one candidate from a set of more than three candidates.

The above characteristics depends also on cardinality of electorate. Such information may be useful when a new group decision making body is planned.

References

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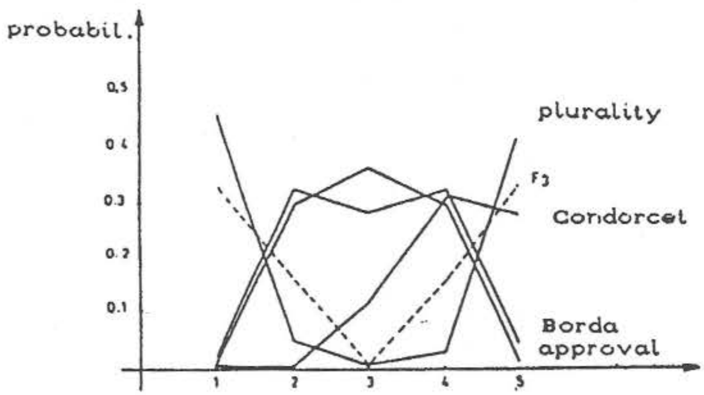

Figure 2. Positional probabilities of alternatives following from basic voting procedures (spatial model, 5 alternatives, electorate's distribution — **F3**)

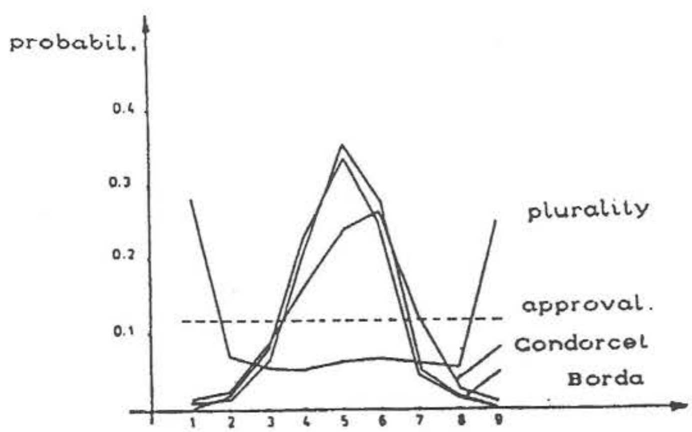

Figure 3. Positional probabilities of alternatives following from basic voting procedures (spatial model, 9 alternatives, electorate distribution — **F0** type)

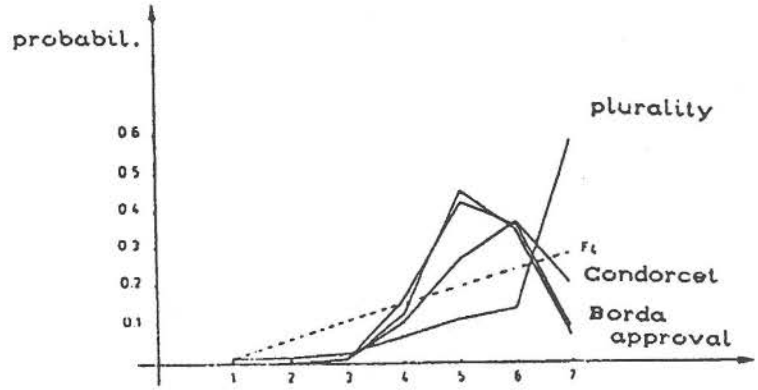

Figure 4. Positional probabilities of alternatives following from basic voting procedures (spatial model, 7 alternatives, electorate distribution — **F4** type)

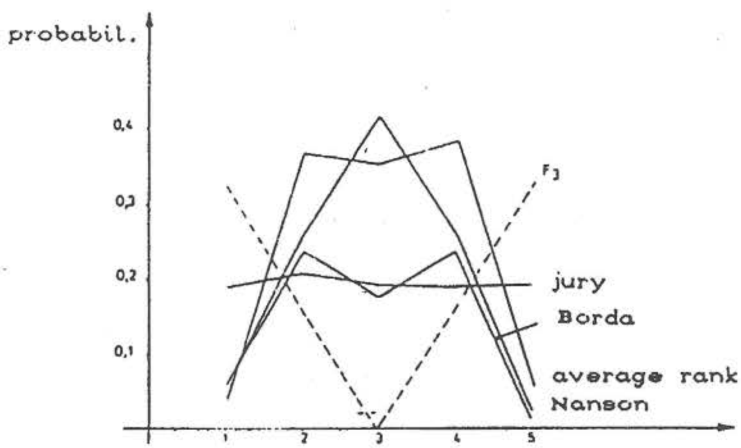

Figure 5. Positional probabilities of alternatives following from Borda–based voting procedures (5 alternatives, electorate distribution — **F3** type)

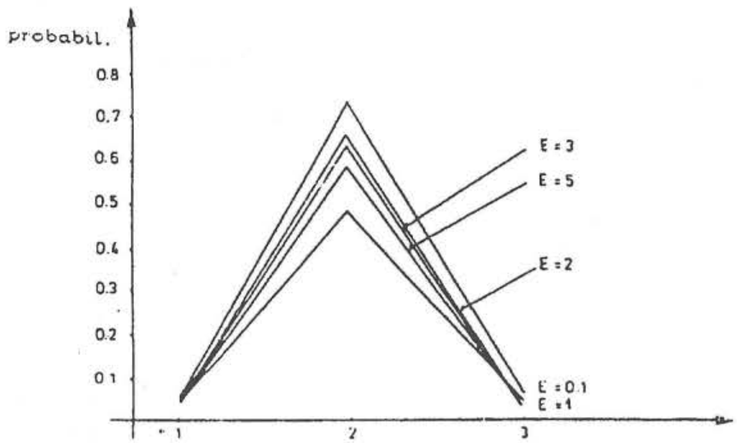

Figure 6. Positional probabilities of alternatives following from average rank voting procedure ($n = 3$,**F3**) with different values $\epsilon = 0.1, 1, 2, 3, 5$.

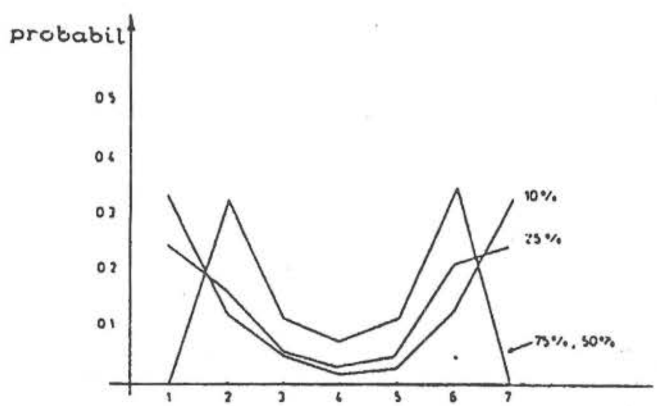

Figure 7. Positional probabilities of alternatives following from negative voting procedure ($n = 7$, **F3**) with indices of negation: 10%, 25%, 50% and 75%.

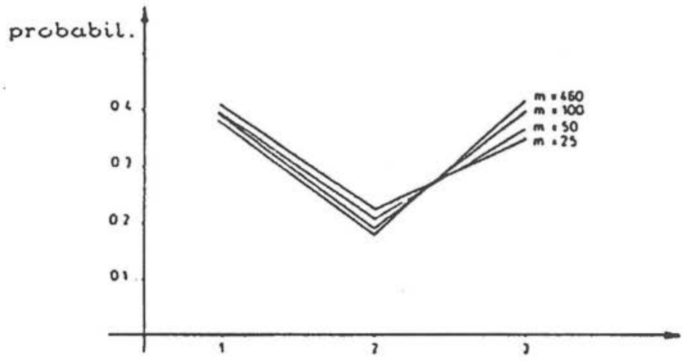

Figure 8. Positional probabilities of alternatives following from cumulative voting procedure ($n = 3$, **F3**) with number of voters: 25, 50, 100 and 460

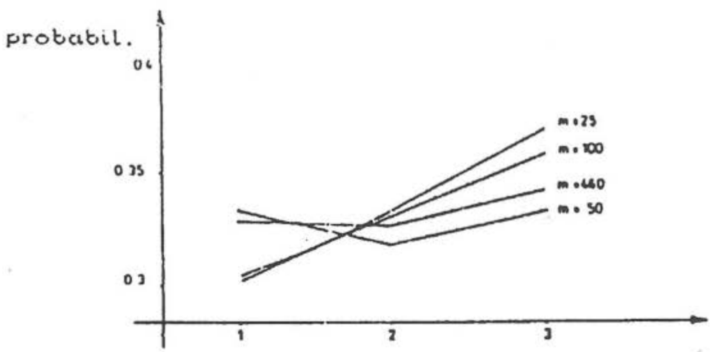

Figure 9. Positional probabilities of alternatives following from jury voting procedure ($n = 3$, **F3**) with number of voters: 25, 50, 100 and 460