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A medium–term model of Polish economy. An approach to transition problems

by

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There is a number of works devoted to modelling of Polish economy in the period of transition from the centrally planned to a market economy. Some 'new' examples K. Barteczko, A. Bocian, T. Kuszewski [7], A. Fusari [8], A. Karmann [9], M.Lipiec–Zajchowska, E. Sikora[10], J. Stefański [11]. W. Welfe [12]. This list of works is certainly incomplete. Every single day new papers appear, and – because of rapid changes in the economy – a part of them becomes out–of–date in a relatively short time. Most of papers published these days are of a rather general character. They contain mainly methodological considerations.

The present paper shows a general idea of a medium–term operational model of real and monetary flows in the economy for the time horizon of 5–10 years. The model incorporates several tens of basic economic variables that the Government as well as the Central Planning Bureau are interested in. A number of characteristic features make the model useful for controlling the development of the economy in the period of transition, e.g.

- financial interrelations between the subsytems of the economy,
- elements of tax and insurance system,
- elements of market mechanism generating consumers' demand and savings,
- an approach to modelling of the labour market,
- behavioral relations.

Limited volume of the paper does not allow us to present neither the computational version of the model adjusted to available statistical data nor the computational procedure for construction of scenarios of development of Polish economy in the nearest future.

When working on a model and a computational procedure every author has to tailor his or her intentions and ambitions to available statistical data. The most difficult stage of experiments with any model is its transformation to a

computable version. A number of scenarios of development of Polish economy up to 1999 were experimentally derived from the model. The following variables were incorporated in scenarios: outputs of subsystems (by 19 sectors; the list of subsystems is given below), productive investments, non–productive investments, utilization of production capacity, exports, imports, employment, unemployment, household incomes, savings, demand and supply on the consumer market, financial position of subsystems, taxes, foreign debt, etc.

A reader interested in the details should refer to [1]–[6].

Subsystems of economy

The following subsystems are distinguished in the model:

1. $SINE$ – state industrial firms (and co–operatives),
2. $MINE$ – municipal industrial firms,
3. $POIE$ – private industrial firms,
4. $STFA$ – state farms (and co–operatives),
5. $POFA$ – private farms,
6. $HOTR$ – domestic trade,
7. $HCOP$ – housing co-operatives,
8. $STBU$ – state budget,
9. $REBU$ – regional budget[1],
10. $NFHO$ – non–farm hoseholds,
11. $FAHO$ – farm households,
12. $FOTR$ – foreign trade,
13. $BANK$ – banking,
14. $DFOR$ – foreign (hard currency) market.

National economy is perceived as a system consisting of the above mentioned subsystems to which some (at least one to each) economic activities are assigned (see Table 1). The following activities are considered in the model:

(a) production,
(b) trade,
(c) investment,
(d) consumption.

Activities (a)–(d) are associated with both real and monetary flows. There are, however, activities associated only with monetary flows. They are called

(e) purely monetary activities.

Another activity,

(f) increase of reserves

is considered as a separate one.

[1] In the experimental computations both $STBU$ and $REBU$ were put together and called $BUDGET$.

Subsystems \ Activities	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Production	+	+	+	+	+									
Current consumption	+	+	+	+	+	+						+		
Productive investments	+	+	+	+	+									
Non–productive investments							+	+	+	+	+	+		
Industrial consumption										+	+			
Collective consumption							+	+	+					
Trade (domestic, foreign)						+						+		+
Financial activities	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Increase of reserves						+								

In the table (+) identifies activities assigned to a given subsystem.
Notice that foreign market is treated as a subsystem of the economy.

Prices

The following prices are accounted for:

- sales prices, c_j^z (for products bought by $HOTR$ and $FOTR$ in case of domestic products),
- wholesale prices, c_j^h ($HOTR$'s sales prices for all the buyers with except of $FOTR$ and households),
- retail prices, c_j^d (paid by households),
- "foreign" prices, ψ_j^D (for exported and imported products).

Model

Block I. Balances and technical constraints

Balance of products:

$$f + \sum_{p \in P} x^p = \sum_{p \in P} u^p + \sum_{p \in P} S^{Yp} Y^p + \sum_{p=6}^{11} S^{yp} y^p + \sum_{p=7}^{10} S^{zp} z^p + u^w + u^z + e + v. \tag{1.1}$$

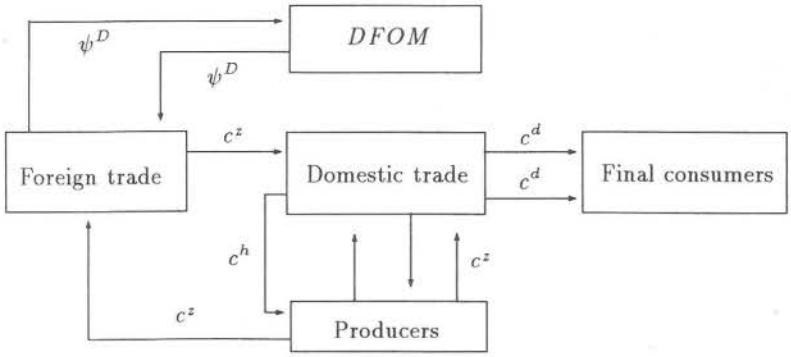
This is a rather general balance of products. There may be an inventory of products, not used by all the activities. The inventory is held by $HOTR$.

Equations relating intermediate consumption to intensity of production:

$$u^p = A^p x^p \quad (p \in P), \tag{1.2a}$$

$$u^w = a_{.w} h^w, \tag{1.2b}$$

$$u^z = a_{.z} h^z. \tag{1.2c}$$



c^d – retail prices
c^h – wholesale prices
c^z – selling prices
ψ^D – dollar prices

Figure 1

Intensities of $HOTR$ and $FOTR$ are defined in (1.5a), (1.5b) further on.

Imports:

$$f = f^z + f^i = f^k + f^e, \tag{1.3a}$$

$$f^z = \sum_{p\in P} \hat{\Gamma}^{up} u^p + \hat{\Gamma}^H (u^w + u^z), \tag{1.3b}$$

$$f^i = \hat{\Gamma}^Y \sum_{p\in P} S^{Yp} Y^p + \hat{\Gamma}^y \sum_{p=6}^{12} S^{yp} y^p, \tag{1.3c}$$

$$f^k = \sum_{p=7}^{10} \hat{\Gamma}^{zp} S^{zp} z^p, \tag{1.3d}$$

$$f^e = \hat{\Gamma}^e e. \tag{1.3e}$$

By assumption, imports are linearly related to current inputs (current import, f^Z), to investment (investment import, f^i), to final consumption (consumption, f^k), and to export (reexport, f^e).

Export:

$$e = \varepsilon S^e, \tag{1.4a}$$

$$\varepsilon \geq \varepsilon_{\min}. \tag{1.4b}$$

These relations determine the sectoral structure of exports as well as minimal exports.

Intensities of trade activities:

$$h^w = c^Z \left(\sum_{p \in P} x^p + f - e \right), \tag{1.5a}$$

$$h^z = \delta \varphi^D f^D + c^z e. \tag{1.5b}$$

Intensity of $HOTR$ is measured as total value (in sales prices) of products bought by the system. Assuming that all the (not exported) products and imports go throught $HOTR$ this intensity is the value of the domestic products traded.

Output (intensity of production activities):

$$x_j^p = \min \left\{ \gamma_j^{Mp} M_j^P, \beta_j^{xp} l_j^{xp} \right\}, \quad p \in P, \ j = 1, \ldots, n. \tag{1.6}$$

The relations between output on the one hand and production capacity and emplyment on the other hand are two–factor production functions of the Leontief–Koopmans type.

Dynamics of production capacities:

$$M^{pt} = (I - \hat{\mu}^{p,t}) M^{p,t-1} + \sum_{\tau=1}^{\theta} \hat{\delta}^{p,t,\tau} Y^{p,t-\tau}, \quad p \in P. \tag{1.7}$$

According to neoclassical concepts production capacity in a given period depends on the capacity in the previous period and on investment in the past (in last θ years; θ is the maximum investment lag). This is a push–type model.

Balance of labour force:

$$\sum_{p \in P} l^{xp} + l^{hw} + l^{hz} + l^{SM} + l^{BDP} + BEZ = ZSR. \tag{1.8}$$

Block II. Financial surpluses of subsystems

Financial surpluses of the subsystems to which production and trade activities are assigneed:

$$NFP^p = c^z x^p - c^h A^p x^p - p^{xp} l^{xp}, \quad p \in P, p \neq 5, \tag{2.1a}$$

$$NFP^5 = c^z x^5 - c^h A^5 x^5 - FKR, \tag{2.1b}$$

where

$$FKR = k^{x5} c^z x^5. \tag{2.1c}$$

In $POFA$ the are no wages. From farmers' incomes consumption and non–productive investment are financed. By assumption, they are in a relation to output

(in sales prices), $k^{x5}c^z x^5$ (k^{x5} – coefficient of proportionality).

$$NHW = c^h\left(\sum_{p\in P} x^p + f - e\right) - h^w - c^h u^w - p^{hw} l^{hw} + \\ +(c^d - c^h)(S^{z9} z^9 + S^{z10} z^{10}), \tag{2.2}$$

$$NHZ = c^z f + \delta\varphi^D e^D - h^z - c^h u^z - p^{hz} l^{hz}. \tag{2.3}$$

The last element of (2.2) is a retail margin.

Aggregate financial surpluses and wages in subsystems to which production and trade activities are assigned:

$$NFI = \sum_{p\in P} NFP^p + NHW + NHZ, \tag{2.4a}$$

$$PŁP = \sum_{p=1}^{4} p^{xp} l^{xp}, \tag{2.4b}$$

$$PŁHW = p^{hw} l^{hw}, \tag{2.4c}$$

$$PŁHZ = p^{hz} l^{hz}, \tag{2.4d}$$

$$PPH = PŁP + PŁHW + PŁHZ. \tag{2.4e}$$

Wages in the economy:

$$PŁSM = p^{SM} + l^{SM}, \tag{2.5a}$$

$$PŁBD = p^{BDP} + l^{BDP}, \tag{2.5b}$$

$$PGN = PPH + PŁSM + PŁBD. \tag{2.5c}$$

Wages in the economy are equal to the sum of wages in all the subsystems to which production and trade activities are assigned plus wages in housing co–operatives and budget institutions.

Financial surplus:

$$DTD = \varphi^D(f^D - e^D). \tag{2.6}$$

Foreign trade balances:

$$SHD = -DTD, \tag{2.7a}$$

$$SHZ = \delta SHD. \tag{2.7b}$$

Notice that the surpluses (after conversion to zloty) are not *ex definitione* equal the financial surplus of $FOTR$ (taken with opposite sign). The surplus of the last subsystem depends on the domestic sales prices, which do not matter in determination of "foreign" surpluses.

Foreign debt:

$$ZAD^{D,t} = (1 + \delta^{D,t})ZAD^{d,t-1} - SHD^t. \tag{2.8}$$

Foreign debt in a given year depends on debt in the previous year, rate of interest, and balance of foreign trade.

Notice that by assumption all of the balance of foreign trade is used to pay back the foreign debt. Negative balance of trade increases foreign debt.

Block III. Monetary flows

All the equations of this block may be written in the following general form:

$$R_0^p + N^p + \sum_{q=1}^{14} F^{pq} = K^p + \sum_{q=1}^{14} F^{qp} + R_1^p \quad (p = 1, \ldots, 14), \tag{3.1}$$

where R_0^p, R_1^p – monetary reserves at the beginning and at the end of a given year, and

$$N^p = NFP^p, \quad p \in P, \tag{3.2a}$$
$$N^6 = NHW, \tag{3.2b}$$
$$N^9 = PPH, \tag{3.2c}$$
$$N^{11} = NHZ, \tag{3.2d}$$
$$N^{13} = DTR, \tag{3.2e}$$
$$N^{14} = DTD, \tag{3.2f}$$
$$N^p = 0, \quad p = 7, 8, 10. \tag{3.2g}$$

$$F^{1,8} = DBP + ISP, \tag{3.3}$$
$$F^{2,8} = DBK + ISK, \tag{3.4}$$
$$F^{4,8} = DBR + ISR, \tag{3.5}$$
$$F^{6,8} = DBH + ISH, \tag{3.6}$$

$$F^{7,p} = DSMP^p, \quad p = 1, 2, 4, 6, 8. \tag{3.7a}$$
$$F^{7,9} = WSM + SKM + CZY, \tag{3.7b}$$
$$F^{8,p} = POO^p + PON^p + POP^p + POD^p + ZUS^p + WAM^p, \quad p = 1, 2, \tag{3.8a}$$
$$F^{8,3} = POO^3 + PON^3 + POP^3 + POD^3 + ZUS^3, \tag{3.8b}$$
$$F^{8,4} = PRU + PON^4 + POP^4 + ZUS^4, \tag{3.8c}$$
$$F^{8,5} = PRI, \tag{3.8d}$$
$$F^{8,6} = PON^6 + POP^6 + POD^6 + ZUS^6 + WAM^6, \tag{3.8e}$$
$$F^{8,7} = POP^7 + ZUS^7, \tag{3.8f}$$
$$F^{8,9} = POL, \tag{3.8g}$$
$$F^{8,11} = POO^{11} + POP^{11} + POD^{11} + ZUS^{11}, \tag{3.8h}$$

$$F^{8,12} = WEM, \tag{3.8i}$$
$$F^{9,p} = WZY^p, \quad p = 1,2,4,6,11, \tag{3.9a}$$
$$F^{9,3} = FKN, \tag{3.9b}$$
$$F^{9,7} = KMZ, \tag{3.9c}$$
$$F^{9,8} = SSN, \tag{3.9d}$$
$$F^{9,12} = KBZ^9, \tag{3.9e}$$
$$F^{10,5} = FKR, \tag{3.10a}$$
$$F^{10,8} = SRI, \tag{3.10b}$$
$$F^{10,12} = KBZ^{10}, \tag{3.10c}$$
$$F^{11,8} = DHZ, \tag{3.11a}$$
$$F^{11,12} = KBZ^{11}, \tag{3.11b}$$
$$F^{12,8} = SEM, \tag{3.12a}$$
$$F^{12,p} = SKB^p, \quad p = 9,10, \tag{3.12b}$$
$$F^{12,11} = SKB^{11} + SHZ, \tag{3.12c}$$
$$F^{12,13} = KZR, \tag{3.12d}$$
$$F^{12,14} = KZD, \tag{3.12e}$$

$$F^{13,12} = \rho SKR, \tag{3.13}$$
$$F^{14,12} = \delta SKD. \tag{3.14}$$

Quantities K^p are defined as follows:

Investment outlays in the subsystems to which production activities are assigned:

$$K^p = c^h S^{Y^p} Y^p, \quad p \in P. \tag{3.15}$$

Investment outlays in $HOTR$ and $FOTR$:

$$K^6 = c^h S^{y6} y^6, \tag{3.16a}$$
$$K^{11} = c^h S^{y11} y^{11}. \tag{3.16b}$$

Investment and consumption in $HCOP$ (purchases and maintenance):

$$K^7 = c^h S^{y7} y^7 + c^h S^{z7} z^7 + PŁSM. \tag{3.17}$$

Investment and consumption in budget–financed institutions:

$$K^8 = c^h S^{y8} y^8 + c^h S^{z8} z^8 + PŁBD. \tag{3.18}$$

Costs in household:

$$K^9 = c^d S^{z9} z^9, \tag{3.16a}$$
$$K^{10} = c^d S^{z10} z^{10} \tag{3.16b}$$

Block IV. Institutional constraints

Constraints of this block reflect law in force, tax system and the welfare benefit system. Variables from (3.3) and (3.4) are interrelated with previously introduced ones, and are also related to the following additional variables:

AMO^p – depreciation (exogenous, by assumption),

$$ZYB^p = \begin{cases} NFP^p - POO^p - POP^p - PON^p + \\ -WAM^p - ZUS^p, & p = 1, \\ NFP^p - POP^p - WAM^p - ZUS^p, & p = 2, \\ NFP^p - POO^p - PON^p - ZUS^p, & p = 3, \\ NFP^p - PRU - POP^p - ZUS^p, & p = 4, \\ NEP^p - PRI, & p = 5, \\ NHW - POO^p - POP^p - PON^p + \\ -WAM^p - ZUS^p, & p = 6, \\ NHZ - POO^p - POP^p - WAM^p + \\ -ZUS^p, & p = 11, \end{cases} \tag{4.1}$$

$$DON = PGN + FKN + \sum_{p=1,2,4,6,11} WZY^p. \tag{4.2}$$

$$ZYN^p = \begin{cases} ZYB^p - POP^p, & p = 1,2,3,6,11 \\ ZYB^p, & p = 4,5. \end{cases} \tag{4.3}$$

WBB^p – value of buildings (exogenuously given)

List of institutional equations

Sales tax:

$$POO^p = \beta_1^p c^z x^p, \quad p = 1,2,3, \tag{4.4a}$$
$$POO^6 = \beta_1^6 h^w, \tag{4.4b}$$
$$POO^{11} = \beta_1^{11} h^z, \tag{4.4c}$$

Wage tax:

$$POP^p = \beta^p p^{xp} l^{xp}, \quad p = 1,\ldots,4. \tag{4.5a}$$
$$POP^6 = \beta^6 PŁHW, \tag{4.5b}$$
$$POP^7 = \beta_2^7 PŁSM, \tag{4.5c}$$
$$POP^{11} = \beta_2^{11} PŁHZ. \tag{4.5d}$$
$$\tag{4.5e}$$

Sectoral security rate:

$$ZUS^p = \beta_3^p p^{xp} l^{xp}, \quad p = 1, \ldots, 4. \tag{4.6a}$$
$$ZUS^6 = \beta_3^6 PŁHW, \tag{4.6b}$$
$$ZUS^7 = \beta_3^7 PŁSM, \tag{4.6c}$$
$$ZUS^{11} = \beta_2^{11} PŁHZ. \tag{4.6d}$$
$$\tag{4.6e}$$

Depreciation:

$$WAM^p = \beta_4^p AMO^p, \quad p = 1, 2, 3, 6, 11. \tag{4.7}$$

Income tax:

$$POD^p = \beta_5^p ZYB^p, \quad p = 1, 2, 3, 6, 11. \tag{4.8}$$

Property tax:

$$PON^p = \beta_6^p WBB^p, \quad p = 1, 3, 6. \tag{4.9}$$

Profit distributed to employees:

$$WZY^p = \beta_7^p ZYN^p, \quad p = 1, 2, 4, 6, 11. \tag{4.10}$$

Taxes paid by socialized sector of agriculture:

$$PRU = \beta_8^p NFP^4. \tag{4.11}$$

Taxes paid by non–socialized sector of agriculture:

$$PRI = \beta_9^5 NFP^5. \tag{4.12}$$

Taxes paid by non–farm hauseholds:

$$POL = \beta_{10}^9 DON. \tag{4.13}$$

Net income of $HFHO$:

$$DNN = DON + SSN - POL - CZY + KMZ^9 + KBZ^9 - SKM^9 - SKB^9. \tag{4.14}$$

Net income of $GDRI$:

$$DNR = FKR + SRI + KBZ^{10} - SKB^{10}. \tag{4.15}$$

Unemployment benefits:

$$PBEZ = k^{BEZ} p^{BDP} BEZ. \tag{4.16}$$

Total net income:

$$DNL = DNN + DNR + PBEZ \tag{4.17}$$

Block V. Behavioral equations

Equations of this block are divided into 9 groups. All of them seem clear and are supplemented only with brief comments.

Labour efficiency in production subsystems influenced by consumer price index:

$$\beta^{xp,t} = \beta^{xp,t-1}(I + \hat{\gamma}^{xp,t} SWZT^{GODO,t}), \quad p \in P, \tag{5.1}$$

where

$$SWZT^{GODO,t} = c^{d,t-1} Z^{i,t} : c^{d,t-1} Z^{i,t-1} - 1, \tag{5.2a}$$

(rate of increase of household purchase)

$$Z = S^{z9} z^9 + S^{z10} z^{10} \tag{5.2b}$$

(market supply of products)

Consumer price index and efficiency "pressure" on wages in production subsystems:

$$p^{xp,t} = p^{xp,t-1}\left\{I + \hat{\mu}^{xp,t} SWKU^t + \hat{\theta}^{xp,t}\left[\hat{\beta}^{xp,t} : \hat{\beta}^{xp,t-1} - I\right]\right\}, \; p \in P, \tag{5.3}$$

where

$$SWKU^t = c^{d,t-1} Z^{i,t} : c^{d,t-1} Z^{i,t-1} - 1, \tag{5.4}$$

(consumer price index)

Capacity utilization related to labour efficiency in production subsystems:

$$\gamma_j^{MP,t} = \min\left\{1, \gamma_j^{MP,t-1}\left[1 + \theta_j^{\gamma p,t}\left(\frac{\beta_j^{xp,t}}{\beta_j^{xp,t-1}} - 1\right)\right]\right\},$$

$$p \in P, \; j + 1, \ldots, n \tag{5.5}$$

(capacity utilization increases along with the increase in labour efficiency).

Wages in trade influenced by consumer price index and labour efficiency measured as turnover per employee (in comparable prices):

$$p^{hw,t} = p^{hw,t-1}\left[1 + \mu^{hw,t} SWKU^t + \theta^{hw,t}\left(\frac{h_{t-1}^{w,t}}{h^{w,t-1}} \cdot \frac{l^{hw,t-1}}{l^{hw,t}} - 1\right)\right] \tag{5.6a}$$

where

$$h_{t-1}^{w,t} = c^{z,t-1}\left(f^t + \sum_{p \in P} x^{p,t} - e^t\right), \tag{5.6b}$$

$$p^{hz,t} = p^{hz,t-1}\left[1+\mu^{hz,t}SWKU^t+\theta^{hz,t}\left(\frac{h^{z,t}_{t-1}}{h^{z,t-1}}\cdot\frac{l^{hz,t-1}}{l^{hz,t}}-1\right)\right] \tag{5.7a}$$

and

$$h^{z,t}_{t-1} = \delta^{t-1}\varphi^{D,t-1}f^{D,t}+c^{z,t-1}e^t \tag{5.7b}$$

(expressions in paretheses in (5.6a), (5.7a) stand for annual rate of sales value per employee in $HOTR$, $FOTR$, in sales prices of the previous year).

Wages in housing co-operatives influenced by consumer price index and labour efficiency measured as consumption per employee (in comparable prices):

$$\begin{aligned} p^{SM,t} &= p^{SM,t-1}\left[1+\mu^{SM,t}SWKU^t+\right. \\ &\quad \left.+\theta^{SM,t}\left(\frac{c^{h,t-1}Z^{SPMI,t}}{c^{h,t-1}Z^{SPMI,t-1}}\cdot\frac{l^{SM,t-1}}{l^{SM,t}}-1\right)\right] \end{aligned} \tag{5.8a}$$

where

$$Z^{SPMI} = S^{z7}z^7 \tag{5.8b}$$

(final consumption in $HCOP$).

Wages in budget institutions are influenced by consumer price index and labour efficiency measured as consumption per employee (in comparable prices):

$$\begin{aligned} p^{BDP,t} &= p^{BDP,t-1}\left[1+\mu^{BDP,t}SWKU^t+\right. \\ &\quad \left.+\theta^{BDP,t}\left(\frac{c^{h,t-1}Z^{BDP,t}}{c^{h,t-1}Z^{BDP,t-1}}\cdot\frac{l^{BDP,t-1}}{l^{BDP,t}}-1\right)\right] \end{aligned} \tag{5.9a}$$

where

$$Z^{BDP} = S^{z8}z^8 \tag{5.9b}$$

(final consumption in budget entitites).

Final resources of households:

$$ZPL^t = DNL^t+\left[OSZ^{t-1}+OSW^{t-1}\right](1+o^t). \tag{5.10}$$

Purchases of households as influenced by price and income:

$$\tilde{Z}^i = \Omega[c^d]^{-1}ZPL \tag{5.11}$$

(substitution demand function).

Voluntary savings of households:

$$OSZ = DNL - c^d\tilde{Z}^i. \tag{5.12}$$

Excess demand:

$$RPP = \tilde{Z}^i - Z^i. \tag{5.13}$$

Forced savings:

$$OSW = c^d NPP, \tag{5.14a}$$

where

$$NPP_i = \begin{cases} RPP_i, & \text{gdy } RPP_i > 0, \\ 0, & \text{gdy } RPP_i \leq 0. \end{cases} \tag{5.14a}$$

Final remarks

Econometricians dealing with mathematical models are aware of deficiencies and controversies of assumptions of various models, models of national economies included. Assumptions of our models are certainly deficient and controversial too. The authors themselves would be able to criticize them very sharply. We think, however, that the model has to be developed in the following directions, allowing

– to describe more precisely the tax system and its relations to processes in the real sphere of economy, and especially to production and investment processes, e.g. in the model presented herein taxes depend on outputs, costs, wages; but the real processes of production and investment do not depend on taxes; taxes do not form any basis for allocation of factors of production,

– to develop the block of institutional constraints related to settlements between the subsystems of the economy and the banking sector; in the model presented here only some of the constraints are taken into account,

– to capture interrelationships between prices and rates of exchange (total and sectoral); in the model only some of the interrelations are incorporated,

– to take into account the potential consequences of privatization.

The postulates forwarded above implicitly indicate weak points of the model. Some of them reflect our will to incorporate into the model the phenomena and processes which have been either impossible or very difficult to model not so long ago, because of difficulties in collecting statistical data, or simply because of nonexistence of some of the phenomena.

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[2] Budowa systemu modeli dla wyznaczania ścieżek wzrostu gospodarki narodowej. Etap II. Weryfikacja modelu ze szczególnym uwzględnieniem współpracy z zagranicą, rolnictwa i polityki finansowej.
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[3] Budowa systemu modeli dla wyznaczania ścieżek wzrostu gospodarki narodowej. Etap III. Opracowanie scanariusza wzrostu gospodarki polskiej do 1995 r.
(Construction of a system of models for determination of growth paths of national economy. Stage III: Preparation of the scenario of growth for Polish economy until 1995; in Polish). Elaborated under the leadership of Professor Z. Czerwiński. Research program: CBP 02.15/1.1.4, Poznań 1988.

[4] Budowa systemu modeli dla wyznaczania ścieżek wzrostu gospodarki narodowej. Etap IV. Weryfikacja modeli pod kątem ich lepszego dostosowania do dostępnego materiału statystycznego, wprowadzenie zestawu założeń odzwierciedlających reakcję podmiotów gospodarczych na zmianę cen, przeprowadzenie obliczeń testujących.
(Construction of a system of models for determination of growth paths of national economy. Stage IV: Verification of models from the point of view of their better adaptation to available statistical material; introduction of a system of assumptions reflecting reactions of economic agents to price changes, conduct of test calculations; in Polish). Elaborated under the leadership of Professor E. Panek. Research program: CBP 02.15/1.1.4, Poznań 1989.

[5] Budowa systemu modeli dla wyznaczania ścieżek wzrostu gospodarki narodowej. Etap V. Opracowanie procedur obliczeniowych prowadzących do rozwiązania kolejnych bloków modeli przy zmieniających się układach cen. Opracowanie wariantów scenariuszy rozwoju gospodarki Polski do 1994 i 2000r.
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Appendix

List of symbols

To simplify notation, time index is dropped in equations without lagged variables. Time index is explicitly written only in equations with lagged variables.

n – number of products ($j = 1, \ldots, n$),

P – set of subsystems with productive activity assigned ($p \in P$),

T – planning horizon,

θ – maximal investment lag,

f – imports

f^z – imports consumed by productive and trade activities,

f^i – imports consumed by investment activities,

f^k – imports consumed by consumption activities,

f^e – imports consumed by reexport,

$\hat{\Gamma}^Y$ – diagonal matrix of shares of imports in investment,

$\hat{\Gamma}^y$ – diagonal matrix of shares of imports in investment (productive sphere),

$\hat{\Gamma}^{up}$ – diagonal matrices of shares of imports in investment (non–productive sphere),

$\hat{\Gamma}^{zp}$ – diagonal matrices of shares of imports in final consumption,

$\hat{\Gamma}^e$ – diagonal matrix of shares of imports in reexport,

x^p – outputs,

u^p – intermediate consumption,

u^w – intermediate consumption in domestic trade,

u^z – intermediate consumption in foreign trade,

Y^p – productive investment,

S^{Yp} – matrix of sectoral structure of productive investment (by origin of inputs),

y^p – non–productive investments,

S^{yp} – matrix of sectoral structure of non–productive investments,

z^p – consumtion,

S^{zp} – matrix of sectoral structure of inputs in consumption activities,

v – increases of reserves,

e – exports,

ε – total exports,

$\varepsilon_{\min}$ – minimal exports,

S^e – sectoral structure of exports,

h^w – activity levels of domestic trade,

h^z – activity levels of foreign trade,

A^p – matrix of intermediate input coefficients,

$a_{.w}$ – intermediate input coefficients (related to unit of intensity of domestic trade,

$a_{.z}$ – intermediate input coefficients (related to unit of intensity of foreign trade,

c^z – sales prices,

c^h – wholesale prices,
c^d – retail prices,
δ – rate of exchange of Polish zloty vs. US dollar,
φ^D – (import and export) prices on hard currency markets,
γ_j^{Mp} – maximal capacity utilization,
M_j^p – production capacity,
β_j^{xp} – labour efficiency,
l_j^{xp} – coefficients of current labour inputs,
μ_j^p – capacity depreciation,
$\delta_j^{p,t,\tau}$ – ration of investment output in year t to investment outlays in year $t-\tau$,
l^{hw} – employment in domestic trade,
l^{hz} – employment in foreign trade,
l^{SM} – employment in housing co–operatives,
l^{BDP} – employment in budget institutions,
BEZ – unemployment,
ZSR – labour force,
p^{xp} – unit wages,
NFP^p – gross financial surplus,
NHW – gross financial surplus in $HOTR$,
NHZ – gross financial surplus in $FOTR$,
p^{hw} – wage stake in $HOTR$,
p^{hz} – wage stake in $FOTR$,
NFI – total gross financial surplus in subsystems to which productive or trade activities are assigned,
$PŁP$ – wages in subsystems to which productive activities are assigned,
$PŁHW$ – wages in $HOTR$,
$PŁHZ$ – wages in $FOTR$,
$PBEZ$ – unemployment benefits,
k^{BEZ} – the ratio of unemployment benefits to the average wage in the budgetary sphere,
PPH – wages in subsystems to which productive or trade activities are assigned,
ZYB^p – gross profits from productive and trade activities,
ZYN^p – net profits from productive and trade activities,
$PŁSM$ – wages in housing co–operatives,
$PŁBD$ – wages in budgetary institutions,
PGN – wages in the economy as a whole,

DON – personal income of non-farm households,
DNN – net income of non-farm households,
DTD – deficit in foreign trade on hard currency markets,
SHZ – balance of foreign trade,
δ^D – interest rate for foreign debts,
ZAD^D – foreign debt,
R_p^0, R_p^1 – beginning and end of financial reserves,
F^{pq} – financial flow from subsystem q to subsystem p,
N^p – financial surplus,
DBP – budget subsidies to current activities of $SINE$,
DBK – budget subsidies to current activities of $MINE$,
DBR – budget subsidies to current activities of $STFA$,
DBH – budget subsidies to current activities of $HOTR$,
DSM – budget subsidies to current activities of $HCOP$,
DHZ – budget subsidies to current activities of $FOTR$,
WSM – household deposits in housing co–operatives,
KMZ – housing loans,
SKM – amortization of housing loans,
CZY – rent paid $HCOP$ by $HFHO$,
POO^p – sales tax,
PON^p – property tax,
POP^p – wage tax,
POD^p – income tax,
ZUS^p – social insurence rate,
AMO^p – depreciation,
WAM^p – depreciation transferred to central budget,
PRU – tax paid by the socialized agricultural sector,
PRI – tax paid by the non–socialized agricultural sector,
POL – tax paid by non-farm households,
WEM – additional money supply,
WZY^p – profit distributed to employees,
FKN – consumption fund of non–socialized industry,
SSN – social benefits (pensions, disability pensions, stipends, other benefits) for non-farm households,
SRI – social benefits for farm households,
KBZ^p – loans,
SKB^p – amortization of loans,
KZD – foreign loans,

SKD – amortization of foreign loans,

K^p – non–productive and non-trade costs,

WBB^p – value of buildings,

β_1^p – sales tax rate,

β_2^p – wage tax rate,

β_3^p – social security rate,

β_4^p – budget share in depreciation fund,

β_5^p – income tax rate,

β_6^p – property rate,

β_7^p – employees' share in profits,

β_8^4, β_9^5 – tax rate in socialized and non–socialized sector of agriculture,

β_{10}^9 – tax rate for non-farm households,

DNR – net income of $GDRI$,

DNL – net income of households,

ν_j^{xp} – coefficient of reaction of labour efficiency to increase of consumption,

$SWZT^{GODO}$ – rate of increase in households' consumption,

Z^i – market supplies for households,

μ_j^{xp} – coefficient of reaction of wages to increase of consumer price index,

$SWKU$ – consumer price index,

θ_j^{xp} – coefficient of reaction of wages to increase in labour efficiency,

$\theta_j^{\gamma p}$ – coefficient of reaction of capacity utilization to labour efficiency,

μ^{hw} – coefficient of reaction of wages in $HOTR$ to increase of consumer price index,

μ^{hz} – coefficient of reaction of wages in $FOTR$ to increase of consumer price index,

μ^{SM} – coefficient of reaction of wages in $HCOP$ to increase of consumer price index,

μ^{BDP} – coefficient of reaction of wages in $BUDGET$ to increase of consumer price index,

θ^{hw} – coefficient of reaction of wages in $HOTR$ to labour efficiency,

θ^{hz} – coefficient of reaction of wages in $FOTR$ to labour efficiency,

θ^{SM} – coefficient of reaction of wages in $HCOP$ to labour efficiency,

θ^{BDP} – coefficient of reaction of wages in $BUDGET$ to labour efficiency,

ZPL – financial reserves of households,

OSZ, OSW – personal savings: voluntary and forced, respectively,

o – interest rate for personal deposits,

Z^t – consumer demand,

Ω – vector of parameters of demand function,

RPP – total excess demand,

NPP – vector of excess demand,

ω^4, ω^5 – ratios of average inputs in $STFA$ and $IGRO$ to the average in agriculture,

Z^{SPMI} – current consumption in $HCOP$,

k^{x5} – ratio of consumtion and investment outlays to income in $POFA$,

S^p – financial state of the subsystem p,

FKR – consumption of $GDRO$,

$\hat{\alpha}$, $\hat{\delta}$ – diagonal matrices composed of vectors α, δ,

I – unit matrix,

$\mathbf{1} = (1, \ldots, 1)$.