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Modelling hierarchical dynamic systems for decentralized state estimation

by

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This paper presents two schemes for decentralized state estimation of hierarchical interconnected dynamical systems. Scheme-1 is the interaction free decoupled state estimation algorithm and it is inefficient compared to the centralized scheme. Scheme-2 is developed to improve the performance of the decoupled estimators by processing the estimated output error data (based on scheme-1) of each subsystem along with the state and parameter estimation technique of Tse and Wienert (1975). This, in turn, incorporates the effect of the state interaction terms into the decoupled filters. Both schemes are free from state information exchange between the sub-estimators and moreover, the telemetry and instrumentation costs reduces rapidly as the dimension of the composite system increases. A 10th order hierarchical interconnected dynamical model is used to illustrate the effectiveness of the proposed method based on scheme-2.
Keywords: Hierarchical systems; Estimation; Model

1. Introduction

For small to medium size linear dynamic systems, a centralised state estimation scheme has shown itself to be viable in terms of the processing burden placed on the computing systems. For large dimensional systems the conventional Kalman filter faces some computational difficulties, Kailath (1974), and moreover the processor loading grows steeply. However, as the size of the system increases the information exchanges between the subsystems increases rapidly, as does the cost of the telemetry system. Due to the physical configuration and high dimensionality of such large systems, a centralized state estimation scheme is neither economically feasible nor even necessary. To circumvent the above difficulty and in view of the advent of inexpensive computer hardware what is being done now is the introduction of minicomputer, microcomputer or microprocessor at each subsystem level of decentralized state estimation. A great deal of attention has been paid to the problem of decentralized state estimation of large scale systems and some interesting results have already been obtained by a number of research workers Venkateswarlu and Mahalanabis (1977), Sanders et al. (1978), Tacker and Sunders (1980), Davison and Chang (1986). Decentralized state estimation algorithms proposed by Tacker and Sanders (1980) for interconnected dynamic system still demand some information exchange amongst the subsystem estimators. Recently, Prasad et al. (1984) developed a state interaction model based decentralized estimation scheme which avoids all kinds of information exchanges. But, on the other hand, their method involves a considerable amount of computational burden while the state interaction model is augmented with the subsystem model, which in turn increases the order of subsystem estimators.

This paper proposes a complete decoupled state estimation scheme for the i th subsystem and considers the effect of state interconnection terms of other subsystems using combined state and parameter estimation techniques, Tse and Wienert (1975) in order to improve the performance of the decoupled filters. Our proposed scheme does not require any augmentation with state interaction model and moreover it avoids the information exchanges between the subsystems which in turn helps us to develop a decentralized state estimation of the composite hierarchical system.

The rest of the paper is organised as follows. In Section 2, we state the problem studied here for hierarchical interconnected discrete time systems. In Section 3, we then derive the decentralized state estimation algorithms for the composite system. Some simulation results of the proposed algorithms are then presented in Section 4. Discussion of the result and conclusions are presented in Section 5.

2. Problem statement

Let us consider a composite of "N" hierarchical interconnected subsystems $(S_i : i = 1, 2, 3, \dots)$ described by

$$X(k+1) = AX(k) + BU(k) + \Gamma\omega(k) \tag{1}$$

$$Y(k) = CX(k) + \eta(k) \tag{2}$$

where, $X(k) = (n \times 1)$; state vector

$U(k) = (p \times 1)$; input vector

$\omega(k) = (p \times 1)$; process noise vector; assumed to be white (uncorrelated) sequence with known covariance "Q".

$\eta(k) = (m \times 1)$; measurement noise vector; assumed to be white sequence with known convariance "R" structure and uncorrelated with $\omega(k)$ sequences.

The matrices A, B, Γ and C have their proper dimensions, and the ith subsystem which have all interconnections from the previous subsystems $\{S_j,\ j = 1, 2, \ldots, i-1\}$ can be described by the following pair of equations: S_i : $\{i$th subsystem $\}$, for $i = 1, 2, \ldots, N$.

$$X_i(k+1) = A_{ii}X_i(k) + \sum_{j=1}^{i-1} A_{ij}X_j(k) + B_iU(k) + \Gamma_i\omega(k) \tag{3}$$

$$y_i(k) = C_{ii}X_i(k) + \sum_{j=1}^{i-1} C_{ij}X_j(k) + \eta_i(k) \tag{4}$$

where $X_i(k) \in R^{n_i \times 1}$ is the state vector, $U(k)$ is the control vector, $\eta_i(k)$ is the scalar measurement noise and $y_i(k)$ is the scalar output of the ith subsystem. We have assumed that the pair (A_{ii}, C_{ii}) is completely observable. Thoughout our discussion, we shall assume that the input to the system is zero ($U(k) = 0$). The problem that we will study in the next section is given as follows:

For the given ith hierarchical interconnected subsystem model (3)–(4) (with $U(k) = 0$), we first consider the implementation of the complete decoupled (omission of state interactions) but asymptotic stable state estimation scheme. We then propose another scheme to improve the performance of this estimator by simply processing the estimated output error of the ith subsystem to obtain a state variable model of the interaction term by adopting the state and parameter estimation techniques of Tse and Wienert (1975). This in turn, improves the performance of the complete decoupled estimator of the ith subsystem.

3. Development of decentralized state estimator algorithm

We shall consider first the development of decoupled state estimation of the ith subsystem under consideration (described by equations (3) and (4)) and then discuss the improved version of this estimator by modelling the effect state interconnection terms in state variable form.

3.1. Decoupled state estimation

Scheme-1

The desired decoupled estimators structure can be obtained by considering the set of "$N = m$" state estimators for the decoupled system (for $i = 1, 2, \ldots, m$)

$$X_{id}(k+1) = A_{ii}X_{id}(k) + \Gamma_i\omega(k) \tag{5}$$

$$y_{id}(k) = C_{ii}X_{id}(k) + \eta_i \tag{6}$$

It is well known, Anderson and Moore (1979), the square–root Kalman filter for the above ith decoupled system can be written as:

$$\widehat{X}_{id}(k+1/k) = A_{ii}\widehat{X}_{id}(k/k-1) + K_{ip}(k)[y_i(k) - C_{ii}\widehat{X}_{id}(k/k-1)] \tag{7}$$

$$\widehat{y}_{id}(k+1/k) = C_{ii}\widehat{X}_{id}(k+1/k) \tag{8}$$

where, $K_{ip}(k) = A_{ii}K_i(k)$ = predicted Kalman gain for the i th subsystem. The Kalman gain $K_{ip}(k)$ can be obtained by employing the following equations:

$$\begin{aligned}&\begin{bmatrix} (C_{ii}P_i(k/k-1)C'_{ii}+R_i)^{1/2'} & K'_{is}(k) \\ 0 & S'_i(k/k) \end{bmatrix} \\ &= \mathbf{T}_1 \begin{bmatrix} R_i^{1/2'} & 0 \\ S'_i(k/k+1)C'_{ii} & S'_i(k/k-1) \end{bmatrix}\end{aligned} \tag{9}$$

$$\begin{bmatrix} S'_i(k+1/k) \\ 0 \end{bmatrix} = \mathbf{T}_2 \begin{bmatrix} S'_i(k/k)A'_{ii} \\ Q^{1/2'}\Gamma'_i \end{bmatrix} \tag{10}$$

$$K_{ip}(k) = A_{ii}K_{is}(k)[C_{ii}P_i(k/k-1)C'_{ii}+R_i]^{-1/2} \tag{11}$$

where: T_1, T_2 = Householder transformation matrices,
$P_i(k/k), P_i(k/k-1)$ = Error covariance matrices,
$S_i(k/k), S_i(k/k-1)$ = The square root matrices of $P_i(k/k)$ and $P_i(k/k-1)$,
Q, R_i = The input and output noise covariance matrices.

The i th subsystem estimator (7) is a stable filter, if A_{ii} is a stability matrix and the pair (A_{ii}, C_{ii}) is observable. This implies that all the eigenvalues of the $(A_{ii} - K_{ip}(k)C_{ii})$ are inside the unit circle. Under the same condition, it can be shown that the set of the "m" estimators (7) constitutes a sub-optimal decoupled estimator of the composite system (1)–(2). The decoupled estimators (7) while asymptotically stable (for $i = 1, 2, \ldots, m$) are only sub-optimal for the composite system and it may be desirable to improve the filter performance by introducing some corrective measure for the state interaction terms. It may be noted that the 1st subsystem estimator is optimal due to the inherent mode of interconnection of subsystems (3)–(4), for $i = 1$. Naturally, this is not equivalent to improvement of the performance of the 1 st subsystem estimator. The improved version of scheme-1 for the remaining estimators is discussed in the next subsection.

3.2. Improved version of decoupled state estimator

Let us define the estimated output error of i th subsystem as

$y_{ie}(k)$ = Actual output of i th subsystem − the estimated output of the same subsystem based on scheme-1.

$$= [y_i(k) - \widehat{y}_{id}(k/k-1)], \quad \text{for } = 1, 2, \ldots, m(N) \tag{12}$$

The estimated output error sequences for the i th subsystem are mainly due to the omission of the interaction terms which can be modelled by a linear dynamic discrete time system and it is described by the following pair of difference equations:

$$X_{ie}(k+1) = A_{ii,e}X_{ie}(k) + \sigma_i(k) \quad (13)$$

$$y_{ie}(k) = C_{ii,e}X_{ie}(k) + \xi_i(k) \quad (14)$$

where $X_{ie}(k) \in R^{n_i \times 1}$, $y_{ie}(k)$ is the scalar output from the system model (13)–(14) and $\sigma_i(k)$ and $\xi_i(k)$ are zero mean Gaussian noise with unknown covariances. Using the innovation theory, Kailath (1970), the state space model (13)–(14) with two white noise sources ($\sigma_i(k)$ and $\xi_i(k)$) can be replaced by an equivalent innovation representation with a single white noise source $\psi_i(k)$. The system (13)–(14) is equivalently represented by the following innovation model:

$$X_{ie}(k+1) = A_{ii,e}X_{ie}(k) + K_{ie}(k)\psi_i(k) \quad (15)$$

$$y_{ie}(k) = C_{ii,e}X_{ie}(k) + \psi_i(k) \quad (16)$$

where $\psi_i(k)$ is the innovation sequence with the zero mean and unknown covariances $R^*_{ie}(k)$ and $K_{ie}(k)$ is the Kalman gain of the equations (13)–(14). Both $R^*_{ie}(k)$ and K_{ie} can be calculated by adopting on–line procedure of Tse and Wienert (1975). The matrices $A_{ii,e}$ and $C_{ii,e}$ are assumed to have the following special structures:

$$A_{ii,e} = \begin{bmatrix} 0 & & \vdots & & I_{(n_i-1)\times(n_i-1)} \\ \cdots\cdots & \cdots\cdots & \vdots & \cdots\cdots & \cdots\cdots\cdots\cdots \\ \alpha_{i,1} & \alpha_{i,2}\cdots & \vdots & & \alpha_{i,n_i} \end{bmatrix}_{(n_i \times n_i)}$$

and

$$C_{ii,e} = [1.0 \;\; 0.0 \;\; \cdots\cdots\cdots \;\; 0.0]_{1\times n_i}$$

The order of the system matrix $A_{ii,e}$ and its parameters are to be estimated by off–line computations.

In order to estimate the parameters of the matrix $A_{ii,e}$, $R^*_{ie}(k)$ and $K_{ie}(k)$, we adopt the following procedure of Tse and Wienert (1975) for single output system. Let us denote P_{ie} the covariance matrix of the states in (15) and define,

$$r_i(l) = E\{y_{ie}(k+l)y'_{ie}(k)\} \quad (17)$$

and the equations (15)–(17) imply the following relations:

$$P_{ie} = A_{ii,e}P_{ie}A'_{ii,e} + K_{ie}R^*_{ie}K'_{ie} \quad (18)$$

$$r_i(0) = C_{ii,e}P_{ie}C'_{ii,e} + R^*_{ie} \quad (19)$$

$$r_i(l) = C_{ii,e}A^{l-1}_{ii,e}s_i, \quad l > 0 \quad (20)$$

$$s_i = A_{ii,e}P_{ie}C'_{ii,e} + K_{ie}R^*_{ie} \quad (21)$$

For $l = n_i + \tau$, the above equations (20)–(21), can be written together as compact form:

$$r_i(n_i+\tau) \;=\; \sum_{j=0}^{n_i-1} \alpha_{ij} r_i(j+\tau), \quad 1,2,\ldots,n_i \tag{22}$$

while the expression for linearly independent test of vectors utilized is as:

$$C_{ii,e}A_{ii,e}^{n_i} \;=\; \sum_{j=0}^{n_i-1} \alpha_{ij} C_{ii,e}A_{ii,e}^{j} \tag{23}$$

From equation (22) one can write the following expression:

$$R_i \;=\; \phi(n_i)\alpha_i \tag{24}$$

where $R_i \;=\; [r_i(n_i+1), r_i(n_i+2), \ldots, r_i(2n_i)]'$

$\alpha_i \;=\; [\alpha_{i,1}, \alpha_{i,2}, \ldots, \alpha_{i,n_i}]'$

and

$$\phi(n_i) \;=\; \begin{bmatrix} r_i(1) & r_i(2) & \ldots\ldots & r_i(n_i) \\ r_i(2) & r_i(3) & \ldots\ldots & r_i(n_i+1) \\ \vdots & \vdots & & \vdots \\ r_i(n_i) & r_i(n_i+1) & \ldots\ldots & r_i(2n_i-1) \end{bmatrix} \tag{25}$$

This matrix $\phi(n_i)$ is known as Hankel matrix and it is useful to determine the order of the unknown system. Let us define,

$$d_i(n_i) = |\det \phi(n_i)|, \quad \text{for } i = 2,3,\ldots,m(N)$$

and evaluate the value of $d_i(l)$, for $l = 1,2,\ldots,n_i,n_{i+1},\ldots$.
If,

$$\begin{aligned} d_i(l) &\neq 0 \quad \text{(greater than zero)}, && \text{for } l = 1,2,\ldots,n_i \\ &= 0, \quad \text{for } l = n_{i+1} && \text{or greater than } n_i \end{aligned}$$

then 'n_i' indicates the order of the unknown system. The same procedure can be adopted to determine the structures of the interaction model of other subsystems. After obtaining the structure of the unknown system, one can immediately calculate the elements of $\phi_i(n_i)$ in the following manner:

$$\widehat{r_i}(l) = \frac{1}{N_D} \sum_{k=1}^{N_D-1} y_{ie}(k+l)y'_{ie}(k) \tag{26}$$

where, N_D = Sufficient large number of data points.
Now, the estimate of α_i can be obtained from equation (24)

$$\widehat{R_i} = \widehat{\phi}_i(\widehat{n_i})\widehat{\alpha_i}, \quad \text{for } i = 1,2,\ldots,m(N) \tag{27}$$

Using equation (20), we can calculate s_i, as given below:

$$\begin{bmatrix} r_i(1) \\ r_i(2) \\ \vdots \\ r_i(n_i) \end{bmatrix} = \begin{bmatrix} C_{ii,e} \\ C_{ii,e}A_{ii,e} \\ \vdots \\ C_{ii,e}A_{ii,e}^{n_i-1} \end{bmatrix} s_i = \Delta_i s_i = s_i \tag{28}$$

We note that matrix Δ_i is the unit matrix of dimension $(n_i \times n_i)$ due to the special structure of $C_{ii,e}$ and $A_{ii,e}$.

Finally, the matrices $K_{ie}(k)$ and $R^*_{ie}(k)$ are calculated recursively by the following procedure:

$$R^*_{ie}(k) = r_i(0) - C_{ii,e}P_{ie}(k)C'_{ii,e}, \quad P_{ie} = 0 \tag{29}$$

$$K_{ie}(k) = [s_i - \widehat{A}_{ii,e}P_{ie}C'_{ii,e}]R^{*\,-1}_{ie}(k) \tag{30}$$

$$P_{ie}(k+1) = \widehat{A}_{ii,e}P_{ie}\widehat{A}'_{ii,e} + K_{ie}(k)R^*_{ie}(k)K'_{ie}(k) \tag{31}$$

The predicted estimated states for the i th subsystem interaction model (13)–(14) can be written as

$$\widehat{X}_{ie}(k+1/k) = \widehat{A}_{ii,e}\widehat{X}_{ie}(k/k) \tag{32}$$

$$\widehat{X}_{ie}(k/k) = \widehat{X}_i(k/k-1) + K_{ie}(k)[y_{ie}(k) - C_{ii,e}\widehat{X}_{ie}(k/k-1)] \tag{33}$$

The improved version of decentralized state estimation of the i it subsystem (3)–(4) can thus be obtained by algebraic addition of the two equations (7) and (32). The schematic diagram of the proposed estimation algorithm is shown in Fig. 1. The proposed algorithm is described by the following sequence of steps:

ALGORITHM: (For scheme-2): For $i = 1, 2, 3, \dots, m(N)$

STEP-1: Estimate the state $\widehat{X}_{id}(k/k-1)$ of the i th decoupled subsystem (5)–(6), using the square–root Kalman filter equations (7)–(11).

STEP-2: Compute the i th subsystem estimated output error data using the expression (12) (i.e.: $y_i(k) - \widehat{y}_{id}(k/k-1)$)

STEP-3: The interaction terms involved in the i th subsystem (3)–(4) are modelled in the state variable form (13)–(14), which are finally expressed by the innovation model (15)–(16).

STEP-4: Order of the system (15)–(16) and the parameters of the system matrices $A_{ii,e}$ and $C_{ii,e}$ are estimated by (off-line computation) utilizing the equations (24)–(28).

STEP-5: Kalman gain $K_{ie}(k)$, noise covariance $R^*_{ie}(k)$ and state estimates $\widehat{X}_{ie}(k+1/k)$ of the i th innovation model (15)–(16) are then obtained recursively by employing the equations (29)–(33) (on–line computation).

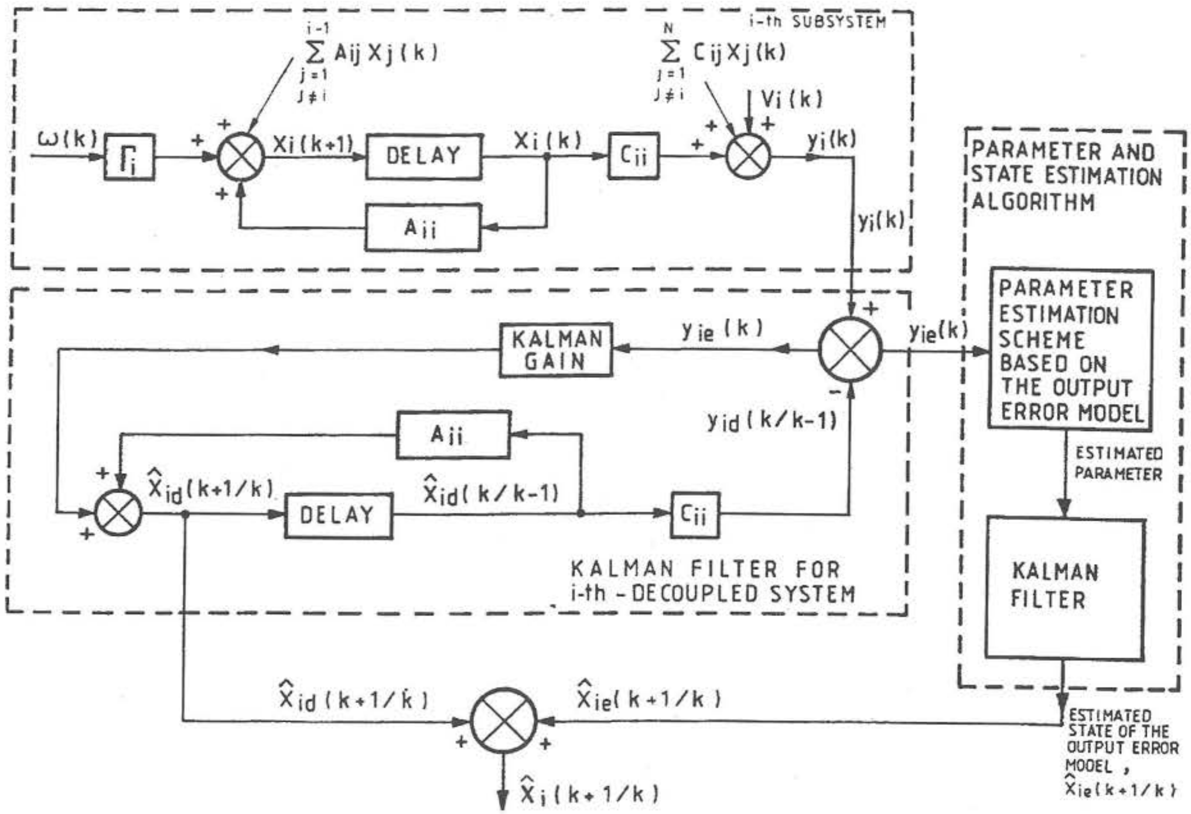

FIG. 1 SCHEMATIC DIAGRAM FOR DECENTRALIZED STATE ESTIMATION FOR THE i-th SYSTEM

<u>STEP-6:</u> Compute,

$$\widehat{x}_i(k+1/k) = \widehat{X}_{id}(k+1/k) + \widehat{X}_{ie}(k+1/k)$$

using the equations (7) and (32).

<u>STEP-7:</u> Continue the steps 1, 2, 5 and 6 for on–line decentralized state estimation for interconnected hierarchical systems (3)–(4).

4. Results of simulation study

In order to illustrate the effectiveness of the proposed algorithm of the preceding Section 3.2, let us consider a 10th order system model which arises out of hierarchical interconnections of three subsystems. The explicit model of the composite system (with $U(k) = 0$) is given as

$$X(k+1) = \tag{34}$$

$$= \left[\begin{array}{cccc|ccc|ccc}
0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.205 & 1.222 & -2.727 & 2.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
-0.101 & 0.101 & 0.0 & -0.949 & 0.336 & -1.46 & 2.1 & 0.0 & 0.0 & 0.0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
0.1 & -0.001 & 0.02 & 0.01 & -0.5 & 0.006 & -0.01 & 0.34 & -1.48 & 2.1
\end{array}\right] X(k) +$$

$$+ \begin{bmatrix}
-0.059 & 0.0002 & 0.0015 & 0.0003 & -0.0002 & 0.0002 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0002 & -0.0002 & 0.0003 & -0.059 & 0.0002 & 0.0015
\end{bmatrix}' \omega(k)$$

$$Y(k) = \begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0
\end{bmatrix} X(k) + \eta(k) \tag{35}$$

The above system has been simulated using the following initial data; while the Subsystem-1, 2 and 3 have the dimensions 4, 3 and 3 respectively.

$$\begin{aligned} E\{X(0)\} &= [-0.6\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0]' \\ E\{\omega(k)\omega(k)'\} &= Q = \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}; \\ E\{\eta(k)\eta(k)'\} &= R = \begin{bmatrix} 0.01 & 0.0 & 0.0 \\ 0.0 & 0.01 & 0.0 \\ 0.0 & 0.0 & 0.01 \end{bmatrix}; \end{aligned}$$

We have considered the following initial data in order to obtain centralized and decentralized state estimate of the composite system.

Initial data for centralized estimate:

$$\begin{aligned} \widehat{X}(0/-1) &= [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]' \\ P(0/-1) &= 0.1 \times I_{10\times 10} \end{aligned}$$

With this data, we obtain the Kalman gain at 400 th iteration as given below:

$$K_p = \begin{bmatrix} 0.016 & -0.04 & -0.02 \\ 0.015 & -0.05 & -0.03 \\ 0.016 & -0.05 & -0.03 \\ 0.017 & -0.04 & -0.03 \\ -0.06 & 0.5 & -0.029 \\ -0.1 & 0.65 & -0.11 \\ -0.1 & 0.79 & -0.21 \\ 0.022 & -0.5 & -0.57 \\ 0.033 & -0.7 & -0.73 \\ 0.061 & -1.0 & -0.83 \end{bmatrix}$$

Initial data for complete decoupled state estimation, i.e. scheme-1:

$$\widehat{X}_{id}(0/-1) = [0\ 0\ 0\ 0]' \tag{36}$$
$$P_{id}(0/-1) = 0.01 \times I_{n_i \times n_i}; \quad \text{for } i = 1, 2, 3 \tag{37}$$

With the above initial data, we obtain the following 'Kalman gain' at 400 th iteration for subsystem-1 to subsystem-3.

$$\begin{aligned} K_{1p} &= [0.047\ \ 0.044\ \ 0.042\ \ 0.04]'; && \text{Kalman gain for subsystem-1} \\ K_{2p} &= 10^{-4} \times [0.46\ \ 0.45\ \ 0.42]'; && \text{Kalman gain for subsystem-2} \\ K_{3p} &= [0.027\ \ 0.023\ \ 0.021]'; && \text{Kalman gain for subsystem-3} \end{aligned}$$

Initial data for scheme-2: We have used here the same initial data as in scheme-1. It is logical to expect that the filter performance based on scheme-1 is suboptimal due to the omission of interaction terms in subsystems-2 and 3 (see equations (5)–(6)), while the subsystem-1 filter performance is optimal. The estimated

output error data (for subsystems-2 and 3) based on scheme-1 are processed to compensate the modelling error by employing state and parameter estimation algorithm of Tse and Wienert (1975). The corresponding model parameters and Kalman gain for subsystems-2 and 3 are given below:
Estimated parameters and Kalman gains:

(i) Subsystem-2:

$$\widehat{A}_{22,e} = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 0.2009 & -1.299 & 2.088 \end{bmatrix}, \quad C_{22,e} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix},$$

$$K_{2e} = \begin{bmatrix} 1.349 & 2.114 & 2.924 \end{bmatrix}',$$

(ii) Subsystem-3:

$$\widehat{A}_{33,e} = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 0.8301 & -2.619 & 2.787 \end{bmatrix}, \quad C_{33,e} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix},$$

$$K_{3e} = \begin{bmatrix} 2.569 & 4.877 & 8.354 \end{bmatrix}',$$

Figs. 2–6 shows the actual states and centralized estimates of the states $X_1(k)$, $X_5(k)$, $X_6(k)$, $X_8(k)$ and $X_{10}(k)$ of the composite system. Decentralized state estimates based on scheme-1 and scheme-2 are also presented in the same figures to compare their results with the actual value and centralized estimates.

5. Concluding remarks

The decoupled state estimation algorithm based on scheme-1 has been tested by considering a hierarchical interconnected system of order 10. It has been noticed that the subsystem-1 estimated states based on scheme-1 are same as for the centralized scheme. But remaining subsystems state estimates (based on scheme-1) are inefficient compared to centralized scheme. It has been observed that the performance of decoupled filters (based on scheme-1) significantly improved while each subsystem estimated output error is processed individually along with the combined state and parameter estimation technique of Tse and Wienert (1975). The improved version of proposed decoupled state estimation algorithm (scheme-2) does not require any information exchange between the sub–estimators which in turn reduces the instrumentation and telemetry costs compared to centralized scheme. Moreover, the algorithms proposed in Sections 3.1 and 3.2 show that as the number of subsystems increases, the computational burden reduces very rapidly compared to the centralized scheme. The simulated results of all different estimation algorithms are compared and presented in Figs. 2–6.

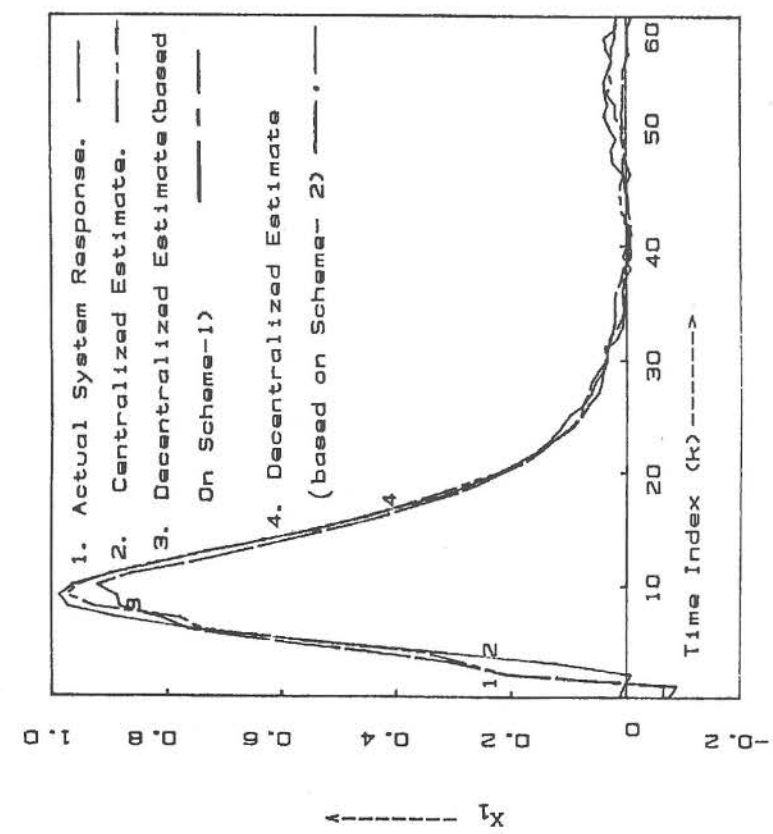

Fig. 2. Actual and Estimated State of the subsystem-1.

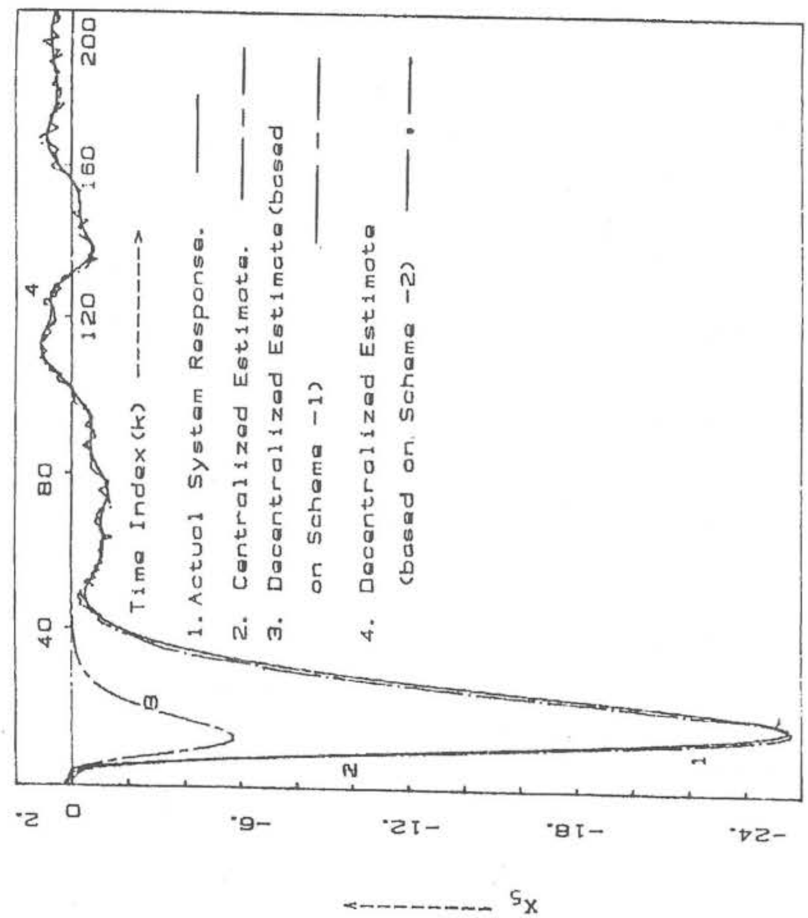

Fig. 3 Actual and Estimated State of the subsystem-2.

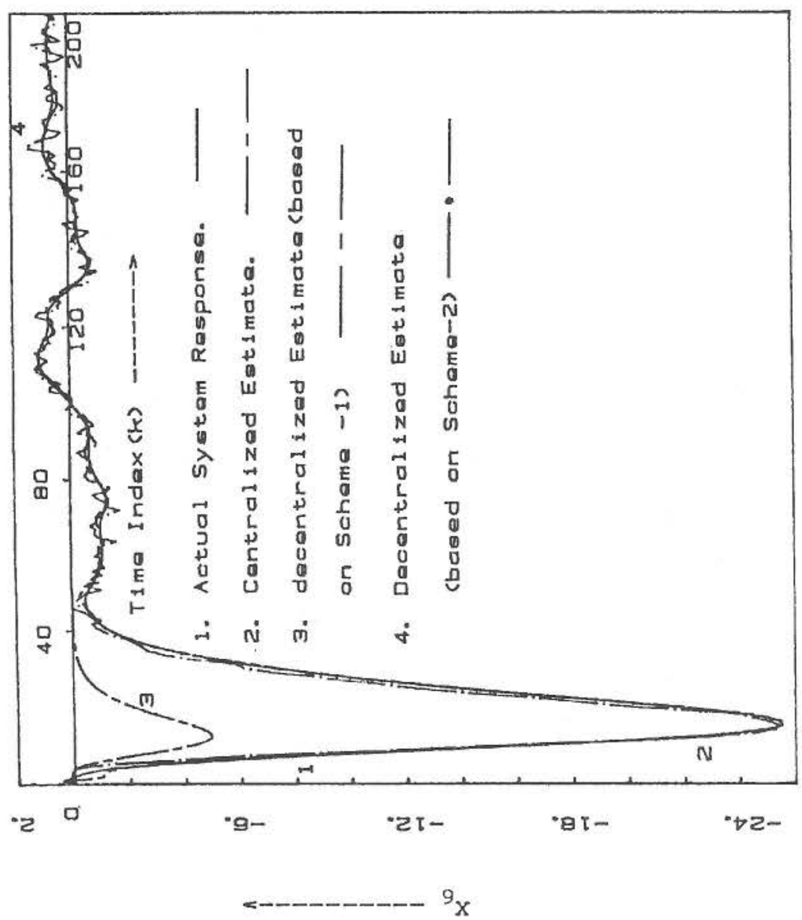

Fig. 4 Actual and Estimated State of the subsystem-2.

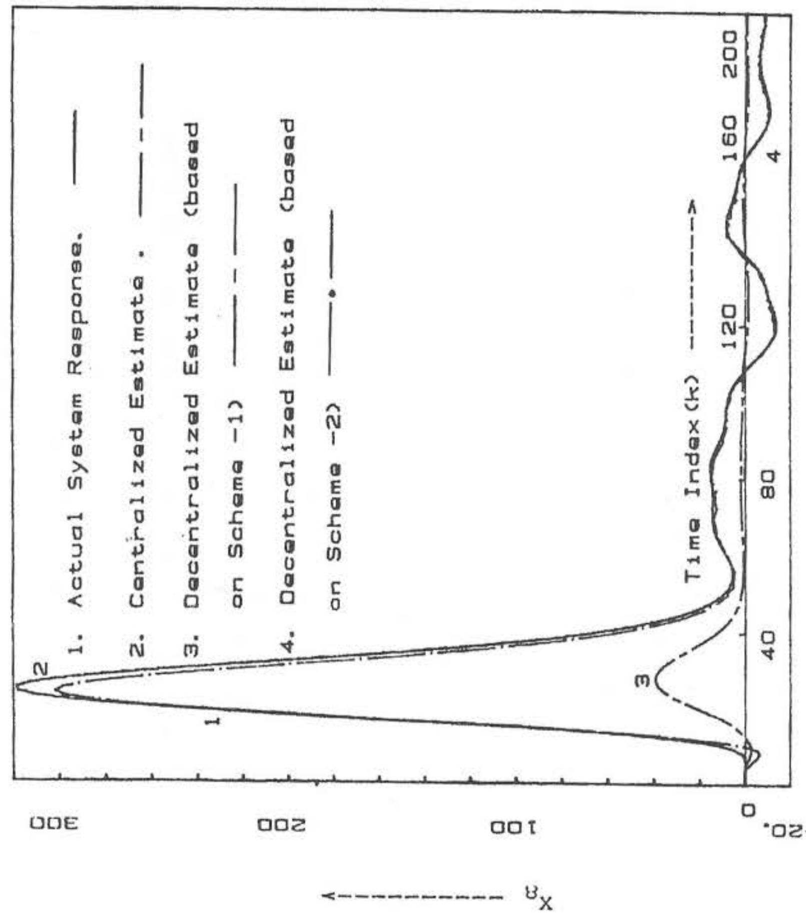

Fig. 5 Actual and Estimated State of the subsystem-3.

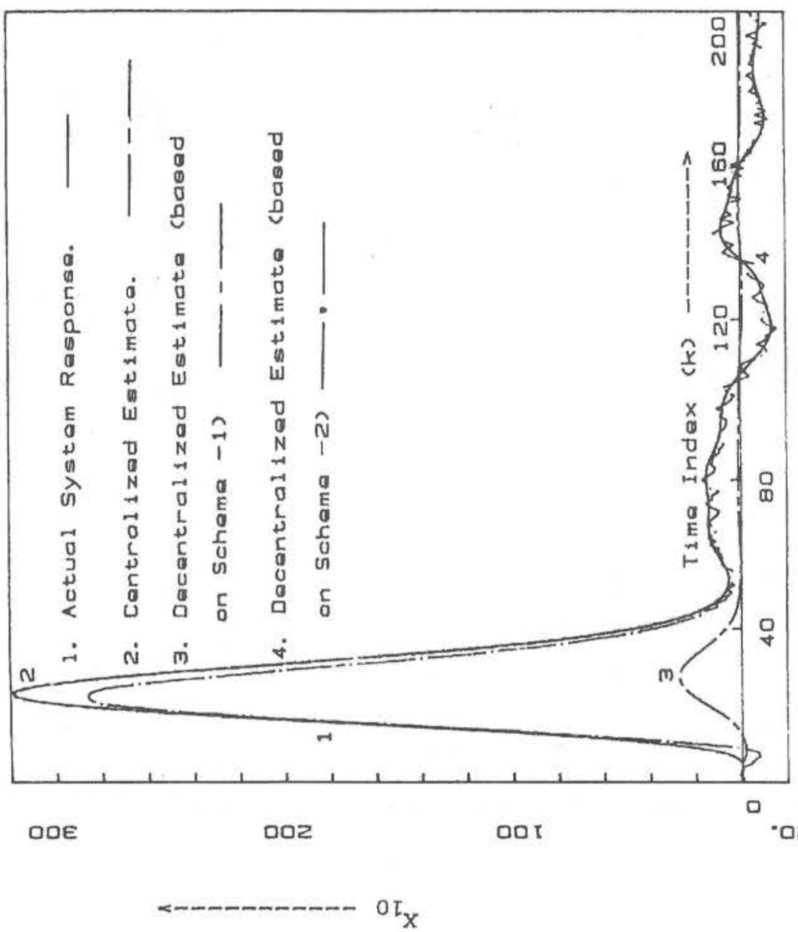

Fig. 6 Actual and Estimated State of the subsystem-3.

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